Evgeny Epelbaum, RUB

Facing up to contemporary challenges in light nuclei,
Argonne Theory Institute, 1.8.2012

A renormalizable approach to NN scattering with nonperturbative pions

in collaboration with Jambul Gegelia, based on arXiv:1207.2420

Outline

- Introduction
  - two nucleons at very low energies (pionless EFT)
  - inclusion of pions: KSW vs Weinberg
- Two nucleons without the NR expansion
  - perturbative pions: recovering KSW at NLO
  - nonperturbative pions at LO
- Summary & outlook

Pion-less EFT for two-nucleon scattering

Effective Lagrangian (Heavy Baryon): for $Q \ll M_\pi$ only zero-range interactions

$$\mathcal{L}_{\rm eff} = N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m} \right) N - \frac{1}{2} C_1^0 (N^\dagger N)^2 - \frac{1}{2} C_2^0 (N^\dagger \vec{\sigma} N)^2 - \frac{1}{4} C_1^2 (N^\dagger \vec{\nabla}^2 N)(N^\dagger N) + \text{h.c.} + \ldots$$

Goal: E(F)T for NN scattering at typical CMS momenta * $\sqrt{m_N E_B} \ll Q \ll M_\pi$

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* The answer is, in fact, known since > 6 decades: Effective Range Theory Blatt, Jackson '49; Bethe '49

Pionless EFT: natural scattering length

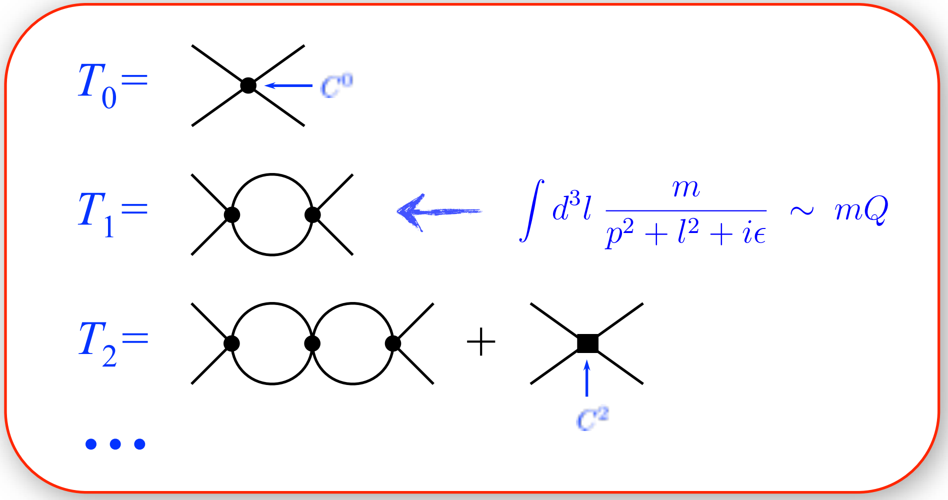
Scattering amplitude (S-waves):

$$S = e^{2i\delta} = 1 - i\left(\frac{km}{2\pi}\right) T,$$

$$T = -\frac{4\pi}{m}\frac{1}{k\cot\delta - ik} = -\frac{4\pi}{m}\frac{1}{\left(-\frac{1}{a} + \frac{1}{2}r_0k^2 + v_2k^4 + v_3k^6 + \ldots\right) - ik}$$

- **Natural case**

$$|a| \sim M_\pi^{-1},\ |r| \sim M_\pi^{-1},\ \ldots \longrightarrow T = T_0 + T_1 + T_2 + \ldots = \frac{4\pi a}{m}\Big[\underset{\substack{\uparrow\\ \sim Q^0}}{1} - \underset{\substack{\uparrow\\ \sim Q^1}}{iak} + \underbrace{\left(\frac{ar_0}{2} - a^2\right)k^2}_{\sim Q^2} + \ldots\Big]$$



EFT expansion based on NDA for C^i, i. e. $C^i \sim Q^0$, reproduces the ERE for T.

Pionless EFT: large scattering length

In reality: $a_{^1S_0} = -23.741 \text{ fm} = -16.6\, M_\pi^{-1} \qquad a_{^3S_1} = 5.42 \text{ fm} = 3.8\, M_\pi^{-1}$

Large scatt. length → shallow (virtual) bound state → need to resum certain graphs (fine tuning beyond NDA...)

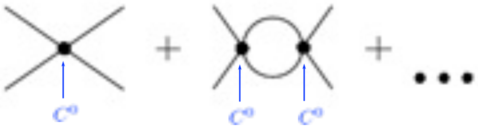
- **KSW approach for the case $|a| \gg M_\pi^{-1}$** Kaplan, Savage & Wise '97

Keep ak fixed, count $a \sim Q^{-1}$:

$$T = -\frac{4\pi}{m}\frac{1}{\left(-\frac{1}{a}+\frac{1}{2}r_0k^2+v_2k^4+v_3k^6+\ldots\right)-ik} = \frac{4\pi}{m}\frac{1}{(1+iak)}\Bigg[\underset{\sim Q^{-1}}{a} + \underbrace{\frac{a r_0}{2(a^{-1}+ik)}}_{\sim Q^{0}}k^2 + \underset{\sim Q^{1}}{\ldots}\Bigg].$$

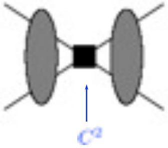
DR + Power Divergence Subtraction: $C^0 \sim 1/Q, \;\; C^2 \sim 1/Q^2, \;\; \ldots$

$T^{(-1)} =$



$$= \frac{-C^0(\mu)}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]},$$

$T^{(0)} =$



$$= \frac{-C^2(\mu)\,k^2}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]^2},$$

where:

Equivalent approaches
(modulo higher-order terms)

- **NDA for C^i but $m \sim 1/Q$**
  Weinberg
- **EFT with dibaryon fields: NDA for C^i and $m \sim Q^0$**
  Tarrus Castella, Soto

Chiral EFT
for two-nucleon scattering

Goal: EFT for NN scattering at typical CMS momenta $Q \sim M_\pi$

KSW: treat pion exchange in perturbation theory:

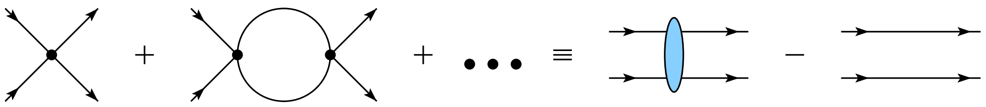
straightforward, consistent, but poor convergence...

Weinberg: both LO contact terms & OPEP must be resummed:

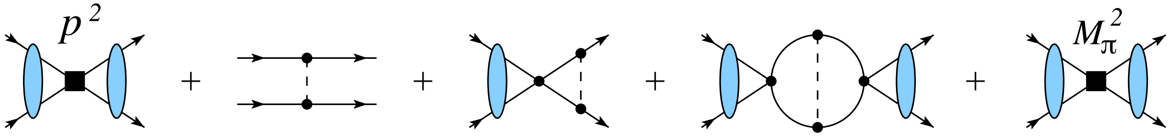
phenomenologically successful but renormalization rather intransparent...

KSW approach (perturbative pions)

$\mathcal{A}_{-1} =$



$\mathcal{A}_0 =$



Low Energy Theorems at NLO

Cohen, Hansen '99

$$k \cot\delta = -a^{-1} + \tfrac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \ldots$$

$$\begin{aligned} v_2 &= \frac{g_A^2 m}{16\pi F_\pi^2}\left(-\frac{16}{3a^2 M_\pi^4} + \frac{32}{5a M_\pi^3} - \frac{2}{M_\pi^2}\right) \\ v_3 &= \frac{g_A^2 m}{16\pi F_\pi^2}\left(-\frac{16}{3a^2 M_\pi^6} - \frac{128}{7a M_\pi^5} + \frac{16}{3 M_\pi^4}\right) \end{aligned}$$

	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
theory	-3.3	17.8	-108.	-0.95	4.6	-25.
NPWA	-0.5	3.8	-17.	0.04	0.7	-4.0

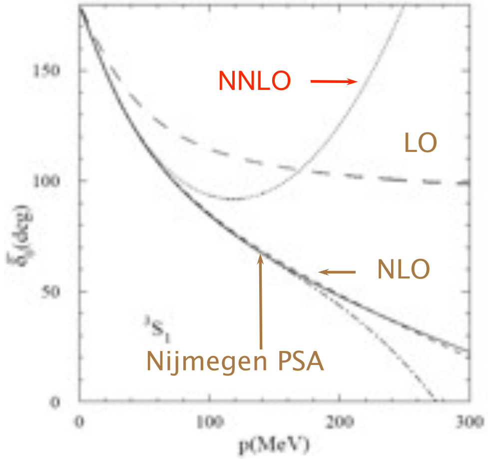
spin-singlet spin-triplet

Higher-order calculations also show problems in S=1 channels

Mehen, Stewart '00

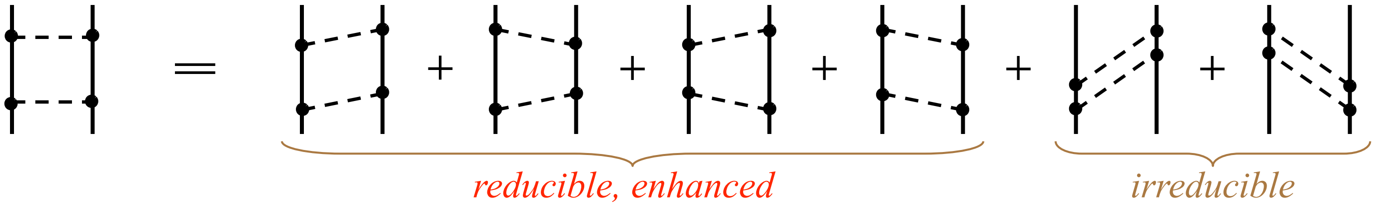
⇨ it seems necessary to treat pions non-perturbatively at $p \sim M_\pi$

see, however, Beane, Kaplan, Vuorinen, arXiv:0812.3938...



Non-perturbative pions: Weinberg‘s approach

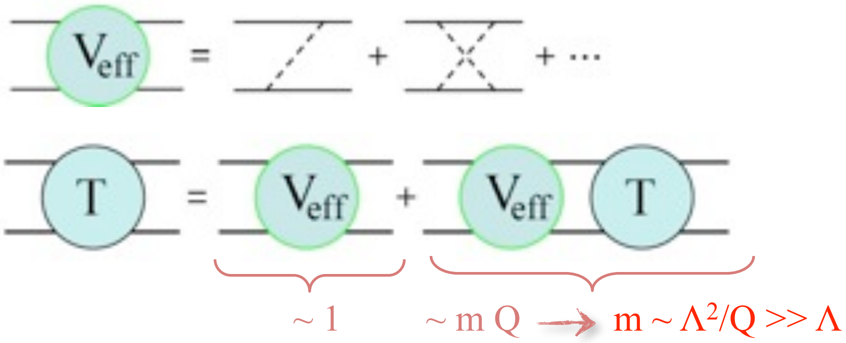
Perturbation theory fails due to infrared enhancement in reducible diagrams.



$$\frac{1}{E_{NN}-E_{\Psi}} = \frac{m_N}{\vec{p}^2-\vec{q}^2} \sim \frac{m_N}{Q^2} \gg \frac{1}{Q} \qquad \frac{1}{E_{NN}-E_{\Psi}} \sim \frac{1}{M_\pi} \sim \frac{1}{Q}$$

Weinberg‘s approach

- Irreducible contributions can be calculated using ChPT
- Reducible contributions enhanced and should be resummed



Two nucleons à la Weinberg

$V_{\rm cont}$,V_π grow with increasing momenta ⇒ LS equation must be regularized & renormalized

$$T(\vec{p},\vec{k}\,) = \Big[V_{\rm cont}(\vec{p},\vec{k}\,) + V_\pi(\vec{p},\vec{k}\,)\Big] + \int \frac{d^3q}{(2\pi)^3}\Big[V_{\rm cont}(\vec{p},\vec{q}\,) + V_\pi(\vec{p},\vec{q}\,)\Big]\frac{m}{k^2-q^2+i\epsilon}T(\vec{q},\vec{k}\,)$$

Complication: iterations of V generate UV divergences in T of a higher dimension which cannot be absorbed into $V_{\rm cont}$, need infinitely many counter terms even at LO (OPEP)

Static OPEP in coordinate space:

tensor operator: $S_{12} = 3\,\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2$

$$V_{1\pi}(\vec{r}\,) = \left(\frac{g_A}{2F_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[M_\pi^2\,\frac{e^{-M_\pi r}}{12\pi r}\left(S_{12}(\hat{r})\left(1+\frac{3}{M_\pi r}+\underbrace{\frac{3}{(M_\pi r)^2}}_{\text{singular potential in all S=1 channels}}\right)+\vec{\sigma}_1\cdot\vec{\sigma}_2\right)-\frac{1}{3}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta^3(r)\right]$$

(solutions of the Schröd Eq. still exist in repulsive cases)

- need counter terms in all spin-triplet partial waves
- infinite number of counter terms needed even in a given channel

Two nucleons à la Weinberg

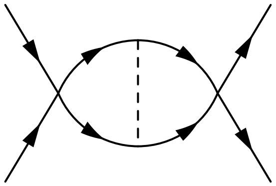
Inconsistency issue (?) of Weinberg‘s approach

Kaplan, Savage, Wise ’97

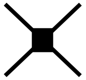
Consider iterations of the LO potential $V_{\rm LO} = V_{1\pi} + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$ in the LS equation

$$T = V + \int V G_0 V + \int \int V G_0 V G_0 V + \ldots \quad \text{where} \quad G_0 = \frac{m}{\vec{p}^{\,2} - \vec{l}^{\,2} + i\epsilon}$$

The 2n-th iteration will generally produce (among other) overall Log-divergences $\times (Q m_N)^{2n}$ where $Q \in \{ |\vec{p}|, \; M_\pi \}$ (in spin-singlet channels no powers of $|\vec{p}|$ can appear)

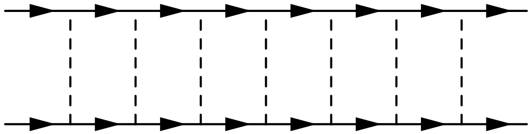


$$\propto \frac{1}{d-4} \frac{g_A^2 C^2}{256 \pi^2 F^2} m_N^2 \, M_\pi^2$$



⟶ must include: $D_0 M_\pi^2 = \Big[\delta D_0 + \underbrace{D(\mu_0) + \frac{g_A^2 C^2}{256 \pi^2 F^2} \boxed{m_N^2} \ln \left(\frac{\mu}{\mu_0} \right) \Big]}_{D_0^r(\mu)} M_\pi^2$

$m_N M_\pi \sim Q^0$ ⟶ must be resummed...



$\propto \dfrac{1}{d-4} \, \vec{p}^{\,6} \, m_N^6$ (spin-triplet) ⟶ even more serious...

However, numerical estimations show no enhancement of renormalized higher-order counter terms Gegelia, Scherer, Int. J. Mod. Phys. A21 (2006) 1079

Two nucleons à la Weinberg

How to renormalize the Schrödinger equation

Lepage, nucl-th/9697929

1. Introduce a ***finite*** cutoff $M_\pi \ll \Lambda \sim \Lambda_{\text{hard}}$
   All symmetries can be preserved   Slavnov '71; Djukanovic et al.'05, Hall, Pascalutsa '12
2. Tune $C_i(\Lambda)$ to low-energy observables ← (implicit) renormalization
3. Check self-consistency by means of error-plots (Lepage-plots)

   Predictive power easily understood in terms of Modified Effective Range Theory...

How **not** to renormalize the Schrödinger equation: an infinite cutoff limit

Removing Λ by taking the limit $\Lambda \to \infty$ may yield finite results for the amplitude but does not qualify for a consistent renormalization in the EFT sense. It is only justified if all necessary counterterms are included...   EE, Gegelia, EPJA 41 (2009) 341

$$T = \frac{\alpha_1 + \alpha_2\Lambda + \alpha_3\Lambda^2}{\beta_1 + \beta_2\Lambda + \beta_3\Lambda^2} \left\{ \begin{array}{ll} \xrightarrow{\Lambda \to \infty} & T = \dfrac{\alpha_3}{\beta_3} \\[2ex] \xrightarrow{\text{renormalization}} & T = \dfrac{\alpha_1 + \alpha_2\mu + \alpha_3\mu^2}{\beta_1 + \beta_2\mu + \beta_3\mu^2} \end{array} \right.$$

Two nucleons à la Weinberg

Nuclear EFT with nonperturbative pions: Current strategies

- Solve the A-body Schrödinger equation for chiral potentials regularized with a finite cutoff
- If the cutoff is to be removed, higher-order corrections to the potential must be treated in perturbation theory Pavon Valderrama '10,'11; Long, Yang '12

  This is, however, insufficient since already the LO LS equation is not renormalizable...

The quest

An approach as efficient as Weinberg's (i.e. nonperturbative pions) with renormalization as transparent as in KSW

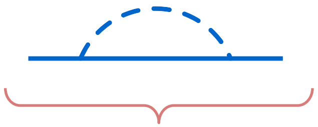
The idea

The linearly divergent UV behavior of the LO LS equation (and thus the inconsistency issue of Weinberg's approach) is not fundamental. Refraining from HB/NR expansion of the propagators (analogously to EOMS baryon ChPT in the 1N sector) naturally leads to a renormalizable LO equation.

Baryon ChPT

Relativistic Baryon ChPT:

The problem: nucleon mass (hard scale) in the propagators spoils the power counting...



scaling according to NDA: ~ Q^3

$$\xrightarrow{\text{chiral limit}} \quad \delta m_N = -\frac{3g_A^2 m^3}{(4\pi F_\pi)^2}\left(16\pi^2 L(\mu) + \frac{1}{2}\ln\frac{m^2}{\mu^2}\right) + \mathcal{O}(d-4)$$

Solutions:

- heavy-baryon expansion Jenkins, Manohar '91, Bernard et al. '92
- IR approach Ellis, Tang; Becher, Leutwyler '99
- EOMS: standard covariant + DR + finite subtractions Gegelia, Japaridze'99; Fuchs et al.'03

(based on the observation that terms violating PC are always analytic in soft scales)

$$m_N = m - 4c_1^r M^2 + \frac{3g_A^2 m}{32\pi^2 F^2} M^2 - \frac{3g_A^2}{128\pi^2 F^2} M^3 + \mathcal{O}(M^4)$$

← $\widetilde{\text{MS}}$, Gasser et al.'88

$$\rightarrow m - 4c_1^{r\prime} M^2 - \frac{3g_A^2}{128\pi^2 F^2} M^3 + \mathcal{O}(M^4)$$

← after additional subtraction (EOMS)

NN scattering revisited

- use manifestly Lorentz-invariant Lagrangian, decompose the fermion propagator as

$$\frac{\not p + m}{p^2 - m^2 + i\epsilon} = \frac{2\,m\,P_+ + (\not p - m\,\not v\,)}{p^2 - m^2 + i\epsilon} = \underbrace{\frac{2\,m\,P_+}{p^2 - m^2 + i\epsilon}}_{\text{include nonperturbatively}} + \underbrace{\ldots}_{\text{treat as correction}}$$

- resum the LO contact interactions + (static) OPEP

The LO equation (for details on the derivation see Djukanovic et al., Few Body Syst. 41 (2007) 141)

$$T_0\left(\vec p\,', \vec p\right) = V_0\left(\vec p\,', \vec p\right) - \int \frac{d^3\vec k}{(2\,\pi)^3}\, V_0\left(\vec p\,', \vec k\right) \underbrace{\frac{m^2}{2\left(\vec k^2 + m^2\right)\left(p_0 - \sqrt{\vec k^2 + m^2} + i\epsilon\right)}} T_0\left(\vec k, \vec p\right)$$

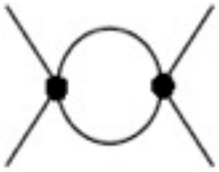
reduces to the usual $\dfrac{m}{(\vec p^{\,2} - \vec k^{\,2} + i\epsilon)}$ in the nonrelativistic limit

- well-known equation Kadyshevsky ’68
- by no means unique: many similar EQs emerge from 3-dim reduction of the Bethe-Salpeter EQ maintaining the same unitarity cut Blankenbecler-Sugar, Gross, ...

NN scattering revisited

1/m-expansion

Consider the loop integral which enters the bubble diagram



$$I = \frac{1}{(2\,\pi)^3}\int d^3\vec{k}\;\theta(\Lambda - |\vec{k}|)\,\frac{1}{\left[\vec{k}^2 + m^2\right]\left[p_0 - \sqrt{\vec{k}^2 + m^2} + i\,0^+\right]}$$

$$= \frac{1}{4\pi^2\sqrt{m^2+p^2}}\left[p\ln\frac{\Lambda\sqrt{m^2+p^2} + p\sqrt{\Lambda^2+m^2}}{\Lambda\sqrt{\Lambda^2+m^2} - p\sqrt{m^2+p^2}} - 2\sqrt{m^2+p^2}\ln\frac{\Lambda+\sqrt{\Lambda^2+m^2}}{m} + 2p\tanh^{-1}\frac{p}{\Lambda} - m\tan^{-1}\frac{\Lambda}{m} - 2\pi i p\right]$$

- **Expand in Λ (first) and then in 1/m**

$$I = \frac{1}{4\pi^2}\left[-\frac{2i\pi p}{m} - 2\ln\frac{\Lambda}{m} - m^2(\pi + \ln 4) + \mathcal{O}\left(\frac{1}{m^2}, \frac{1}{\Lambda}\right)\right]$$

- **NR (HB) approach: first expand in 1/m and then in Λ**

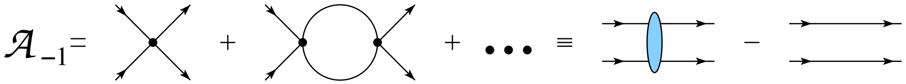
$$I = \frac{1}{4\pi^2}\left[-\frac{2i\pi p}{m} - \frac{4\Lambda}{m} + \mathcal{O}\left(\frac{1}{m^2}, \frac{1}{\Lambda}\right)\right]$$

  - same low-energy physics; different UV behavior compensated by the counter terms
  - perfectly fine in perturbative setting (where NDA is applicable as in ChPT)
  - an infinite number of counter terms will have to be included when resumming OPEP

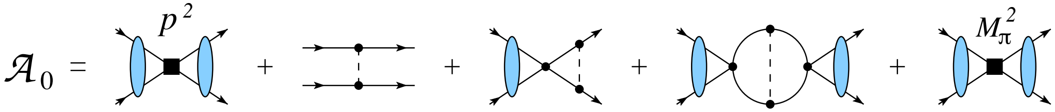
NN scattering revisited:
perturbative pions (KSW) at NLO

KSW approach revisited

Expansion of the amplitude:



$$\mathcal{A} = \mathcal{A}_{-1} + \mathcal{A}_0 + \mathcal{A}_1 + \cdots$$



LO amplitude: $\mathcal{A}_{-1} = \frac{-C}{1 - C\, I(p)} = \frac{-C_R(\nu)}{1 - C_R(\nu)\, I_{\rm R}(p,\nu)}$

The loop integral I(p) can equally well be computed in DR:

$$I(p) \overset{\text{DR}}{=} -\frac{\bar\lambda m_N^2}{8\pi^2} + \frac{m_N^2 \ln\frac{m_N}{\mu}}{4\pi^2} - \frac{m_N^3 + 2i\, p\, m_N^2}{8\pi\sqrt{m_N^2+p^2}} + \frac{p\, m_N^2 \sinh^{-1}\left(\frac{p}{m_N}\right)}{4\pi^2\sqrt{m_N^2+p^2}} - \frac{m_N^2}{4\pi^2}$$

with $\bar\lambda = -\frac{1}{n-3} - \gamma - \ln(4\pi)$

same as HB KSW
modulo
higher-order terms

NDA scaling: $I(p) \sim m_N Q$ → just $\overline{\text{MS}}$ would be insufficient...

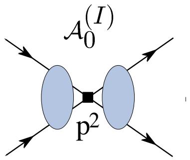
Renormalize by subtraction: $I_{\rm R}(p,\nu) = I(p) - I(i\,\nu) = \boxed{-\frac{m(\nu + i\, p)}{4\pi} + \mathcal{O}(p^2,\nu^2)}$

→ proper scaling after renormalization!

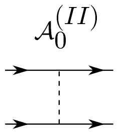
KSW approach revisited

Subleading contributions:

absent in NR KSW but scales as Q^3 (higher order)

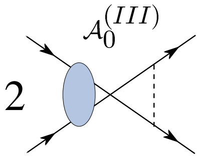


$$\mathcal{A}_0^{(I)} = -\mathcal{A}_{-1}^2 \left[\boxed{\frac{C_{2R}\, m_N^2 \left(2m_N^2 + p^2 - 2m_N\sqrt{m_N^2+p^2}\right)}{8\pi\, C_R}} - \frac{2\, C_{2R} p^2}{C_R^2} \right]$$



$$\mathcal{A}_0^{(II)} = \frac{g_A^2}{4F^2} \left(-1 + \frac{M^2}{4\,p^2} \ln \frac{M^2+4p^2}{M^2} \right)$$

← same as in NR KSW



$$\mathcal{A}_0^{(III)} = \frac{g_A^2}{2F^2}\, \mathcal{A}_{-1} \left[I_R(p,\nu) - M^2 I_{1\mathrm{loop}} \right]$$

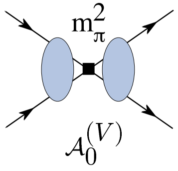
← same as in NR KSW up to higher-order ($1/m^2$) terms in A_{-1}, I_R and $I_{1\mathrm{loop}}$

where the 1-loop integral is given by:

$$I_{1\mathrm{loop}} = \frac{m_N^2}{2} \int \frac{d^n k}{(2\,\pi)^n} \frac{1}{[k^2+m_N^2]\left[p_0 - \sqrt{k^2+m_N^2} + i\epsilon\right][(k-p)^2+M^2]}$$

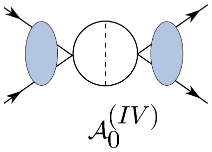
$$= -\frac{m_N}{8\pi p}\left[\tan^{-1}\left(\frac{2p}{M}\right) + \frac{i}{2}\ln\frac{M^2+4p^2}{M^2}\right] + \mathcal{O}\left(p, M\right),$$

KSW approach revisited



$$\mathcal{A}_0^{(V)} = -\frac{D_{2R} M^2}{C_R(\nu)^2} \mathcal{A}_{-1}^2$$

← same as in NR KSW modulo higher-order ($1/m^2$) terms in A_{-1}



$$\mathcal{A}_0^{(IV)} = \frac{g_A^2}{4F^2} \mathcal{A}_{-1}^2 \left[M^2 I_{2\text{loop}} - I_R(p,\nu)^2 \right]$$

where:

$$I_{2\text{loop}} = \frac{m_N^4}{4} \int \frac{d^n k_1 d^n k_2}{(2\,\pi)^{2\,n}} \frac{1}{[k_1^2 + m_N^2] \left[p_0 - \sqrt{k_1^2 + m_N^2} + i\epsilon \right]} \frac{1}{[k_2^2 + m_N^2] \left[p_0 - \sqrt{k_2^2 + m_N^2} + i\epsilon \right] [(k_1 - k_2)^2 + M^2]}$$

$$= \frac{m_N^2}{16\pi^2} \left[\frac{\ln 8}{4} - \frac{2C}{\pi} - \frac{7\zeta(3)}{2\pi^2} - \frac{1}{2} \ln \frac{M^2 + 4p^2}{\boxed{m_N^2}} + i \tan^{-1} \left(\frac{2p}{M} \right) \right] + \mathcal{O}\,(p, M)$$

no need to promote $D_{2R}M^2$ to LO when treating pions nonperturbatively!
(no inconsistency issue)

This has to be compared with the NR KSW result:

$$I_{2\text{loop}}^{\text{HB,PDS}} = \frac{m_N^2}{16\pi^2} \left[1 - \frac{1}{2} \ln \frac{M^2 + 4p^2}{\mu^2} + i \tan^{-1} \left(\frac{2p}{M} \right) \right]$$

← the difference is accounted for by a finite shift in D_{2R}

To summarize, we recover exactly the results of NR KSW (modulo higher-order terms)

NN scattering revisited:
nonperturbative pions (W) at LO

Weinberg's approach revisited

$$T_0\left(\vec{p}\,',\vec{p}\right) = V_0\left(\vec{p}\,',\vec{p}\right) - \int \frac{d^3\vec{k}}{(2\,\pi)^3}\, V_0\left(\vec{p}\,',\vec{k}\right) \frac{m^2}{2\left(\vec{k}^2+m^2\right)\left(p_0-\sqrt{\vec{k}^2+m^2}+i\epsilon\right)}\, T_0\left(\vec{k},\vec{p}\right)$$

with: $V_0 = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M_\pi^2} + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$

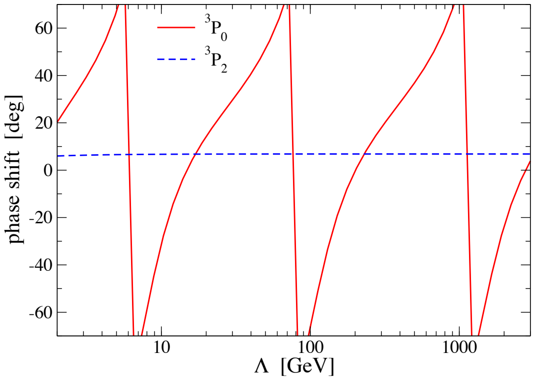
- only Log-divergences → LO Eq. perturbatively renormalizable (in the QFT sense):

  $T_0 = V_0 - V_0 G_0 V_0 + V_0 G_0 V_0 G_0 V_0 - \ldots$ ← all UV divergences absorbable in $C_S,\ C_T$

  → it is safe to remove the cutoff: $\Lambda \to \infty$

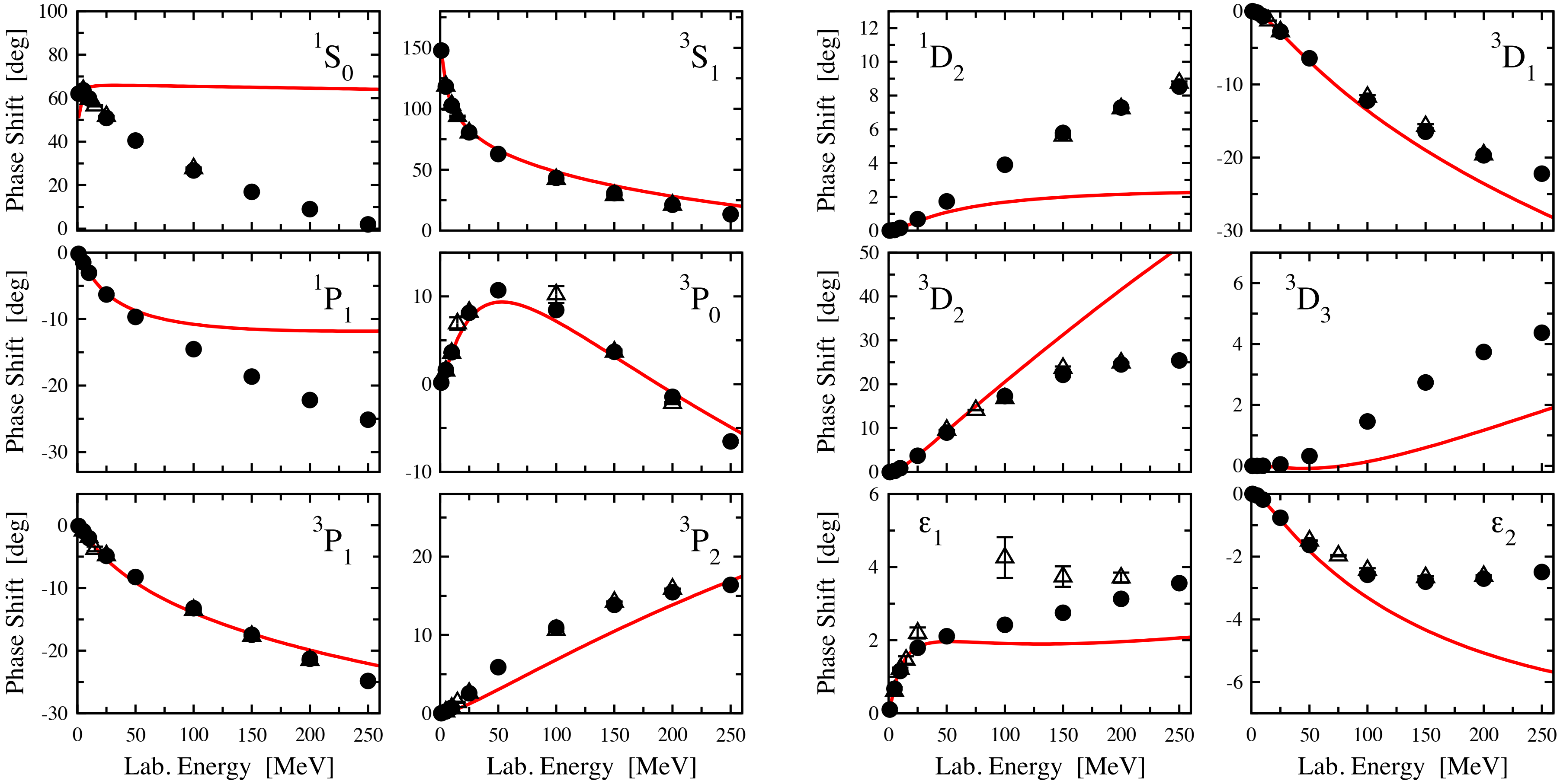
- the only finite subtractions needed to maintain the power counting affect the values of $C_S,\ C_T$ → LO Eq. also renormalizable in the EFT sense

- nonperturbatively, the UV behavior is the same as for the Skorniakov-Ter-Martirosyan Eq. (Schröd. Eq. with $1/r^2$ potential in 2 spatial dimensions) → nonunique solutions may exist for strong-enough attractive cases (only $^3P_0$ in this particular scheme)



Weinberg‘s approach revisited

Cutoff-independent results for neutron-proton phase shifts at LO



The deuteron BE at LO is 2.15 MeV.

Low-energy theorems: KSW vs Weinberg

Predictions for coefficients in the ERE in the $^1S_0$ channel

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW from Ref. [23]	fit	fit	-3.3	18	-108
LO Weinberg	fit	1.50	-1.9	8.6(8)	$-37(10)$
Nijmegen PWA	-23.7	2.67	-0.5	4.0	-20

Predictions for coefficients in the ERE in the $^3S_1$ channel

$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW from Ref. [23]	fit	fit	-0.95	4.6	-25
LO Weinberg	fit	1.60	-0.05	0.8(1)	$-4(1)$
Nijmegen PWA	5.42	1.75	0.04	0.67	-4.0

Summary & outlook

New formulation of Weinberg's approach without employing the NR expansion

- LO equation renormalizable → cutoff can be safely removed
- no inconsistency issues
- (fairly) good agreement with the data at LO

Some benefits of the new formulation

- transparent renormalization, no 1/Λ artifacts, partial resummation of 1/m terms

To avoid any misunderstanding:

- Nothing conceptually wrong with the original W. approach (if Λ is kept finite)
- The obtained LO equation/amplitude is by no means unique
- I made no fundamental statements about PC (finite parts of contact terms)
- No implications for PC in the original W. approach

Work in progress & outlook

- higher-order corrections, chiral extrapolations, external probes, ...

Spares…

Effective Range Expansion

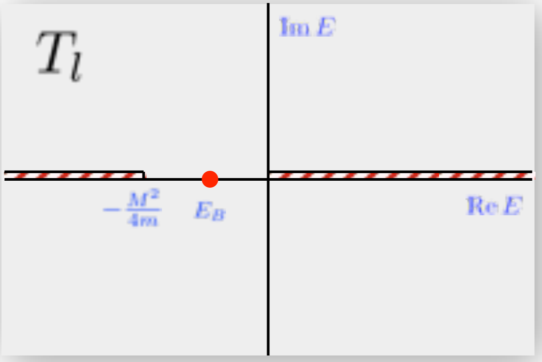
Blatt, Jackson ’49; Bethe ‘49

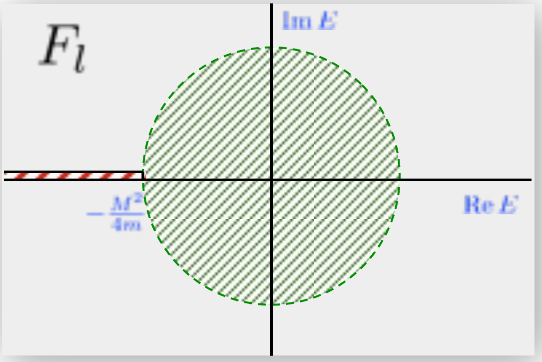
Nonrelativistic nucleon-nucleon scattering (uncoupled case):

effective-range function

$S_l(k) = e^{2i\delta_l(k)} = 1 + i\frac{mk}{2\pi}T_l(k)$ where $T_l(k) = \frac{4\pi}{m}\frac{k^{2l}}{F_l(k) - ik^{2l+1}}$ and $F_l(k) \equiv k^{2l+1}\cot\delta_l(k)$

If $V(r)$ satisfies certain conditions, F_l is a meromorphic function of k^2 near the origin





→ effective range expansion (ERE):

$$\boxed{F_l(k^2) = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + \ldots}$$

The analyticity domain depends on the range M^{-1} of $V(r)$ defined as

$M = \min(\mu)$ such that $\int_{R>0}^{\infty} |V(r)|\, e^{\mu r} dr = \infty$ (for strongly interacting nucleons $M = M_\pi$)

2N beyond ERE: Low-Energy Theorems

Both ERE & $\not\pi$-EFT provide an expansion of NN observables in powers of k/M_π , have the same validity range and incorporate the same physics

$\Longrightarrow$ **ERE ~ $\not\pi$-EFT**

Beyond π-less EFT: higher energies, LETs...

Two-range potential $V(r) = V_L(r) + V_S(r)$, $M_L^{-1} \gg M_H^{-1}$

- $F_l(k^2)$ is meromorphic in $|k| < M_L/2$
- $$\boxed{F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]}$$

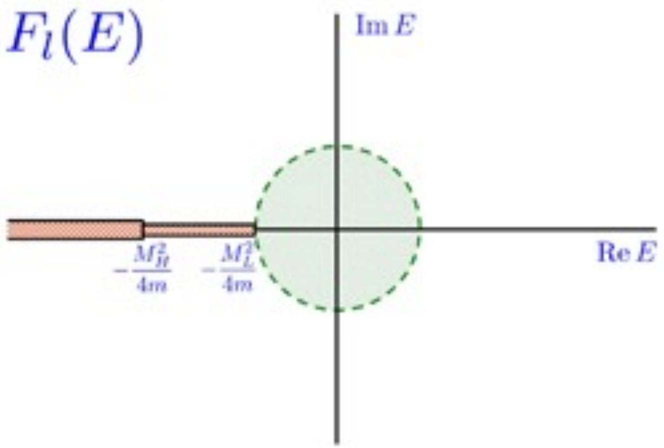
modified effective range function

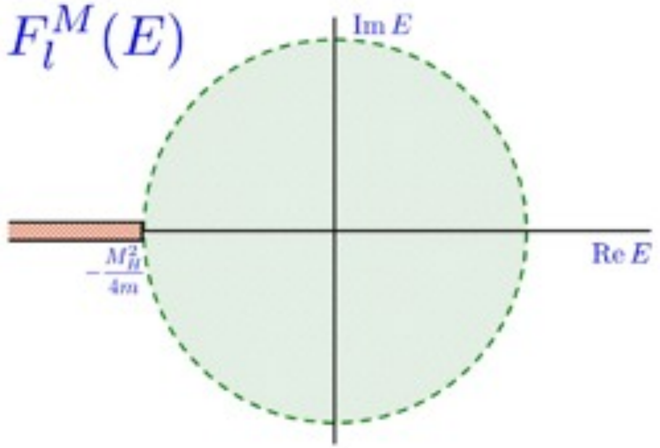
Haeringen, Kok '82

$$\underbrace{f_l^L(k)}_{\text{Jost function for } V_L(r)} = \lim_{r\to 0}\left(\frac{l!}{(2l)!}(-2ikr)^l \underbrace{f_l^L(k,r)}_{\text{Jost solution for } V_L(r)}\right)$$

$$M_l^L(k) = Re\left[\frac{(-ik/2)^l}{l!} \lim_{r\to 0}\left(\frac{d^{2l+1}}{dr^{2l+1}} \frac{r^l f_l^L(k,r)}{f_l^L(k)}\right)\right]$$

Per construction, F_l^M reduces to F_l for $V_L = 0$ and is meromorphic in $|k| < M_H/2$





2N beyond ERE: Low-Energy Theorems

Example: proton-proton scattering

$$F_C(k^2) = C_0^2(\eta)\, k\, \cot[\delta(k) - \delta^C(k)] + 2k\,\eta\, h(\eta) = -\frac{1}{a^M} + \frac{1}{2} r^M k^2 + v_2^M k^4 + \ldots$$

where $\underbrace{\delta^C \equiv \arg\Gamma(1+i\eta)\,,}_{\text{Coulomb phase shift}}$ $\eta = \frac{m}{2k}\alpha\,,$ $\underbrace{C_0^2(\eta) = \frac{2\pi\eta}{e^{2\pi\eta}-1},}_{\text{Sommerfeld factor}}$ $h(\eta) = \mathrm{Re}\Big[\underbrace{\Psi(i\eta)}_{\text{Digamma function } \Psi(z)\equiv\Gamma'(z)/\Gamma(z)}\Big] - \ln(\eta)$

MERE and low-energy theorems

Long-range forces impose correlations between the ER coefficients (low-energy theorems)

Cohen, Hansen '99; Steele, Furnstahl '00

$$\boxed{F_l \;\equiv\; k^{2l+1}\cot\delta_l \;=\; -\frac{1}{a} \;+\; \frac{1}{2}rk^2 \;+\; v_2k^4 \;+\; \ldots \;=\; \frac{A_l\, F_l^L \;-\; k^{4l+2}}{A_l \;+\; F_l^L}}$$

$$\underbrace{\hphantom{\frac{A_l\, F_l^L - k^{4l+2}}{A_l + F_l^L}}}_{\text{depend on } F_l^M \text{ and quantities calculable from } V_L}$$

where $F_l^L = k^{2l+1}\cot\delta_l^L,\;\; A_l = (F_l^M + M_l^L)|f_l^L(k)|^2$

Compute $\delta_l^L(k),\; f_l^L(k),\; M_l^L(k)$ from V_L and use first n coefficients in the MERE as input

$$F_l^M(k^2) = -\frac{1}{a^M} \;+\; \frac{1}{2}r^M\, k^2 \;+\; v_2^M\, k^4 \;+\; v_3^M\, k^6 \;+\; v_4^M\, k^8 \;+\; \ldots$$

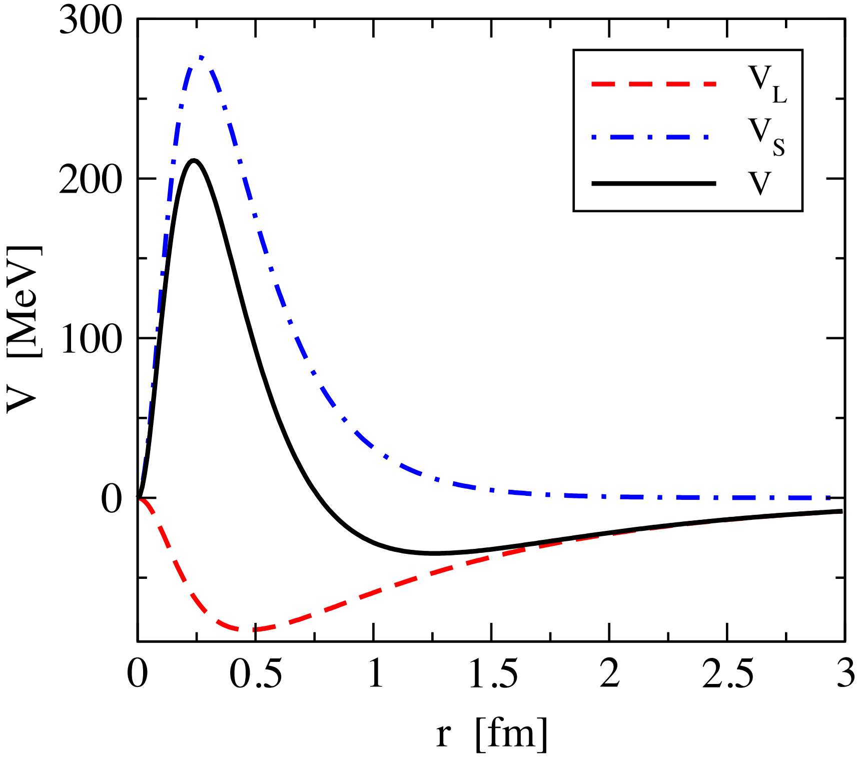
⟹ reproduce first n ERE coefficients and make predictions for all the higher ones (LETs)

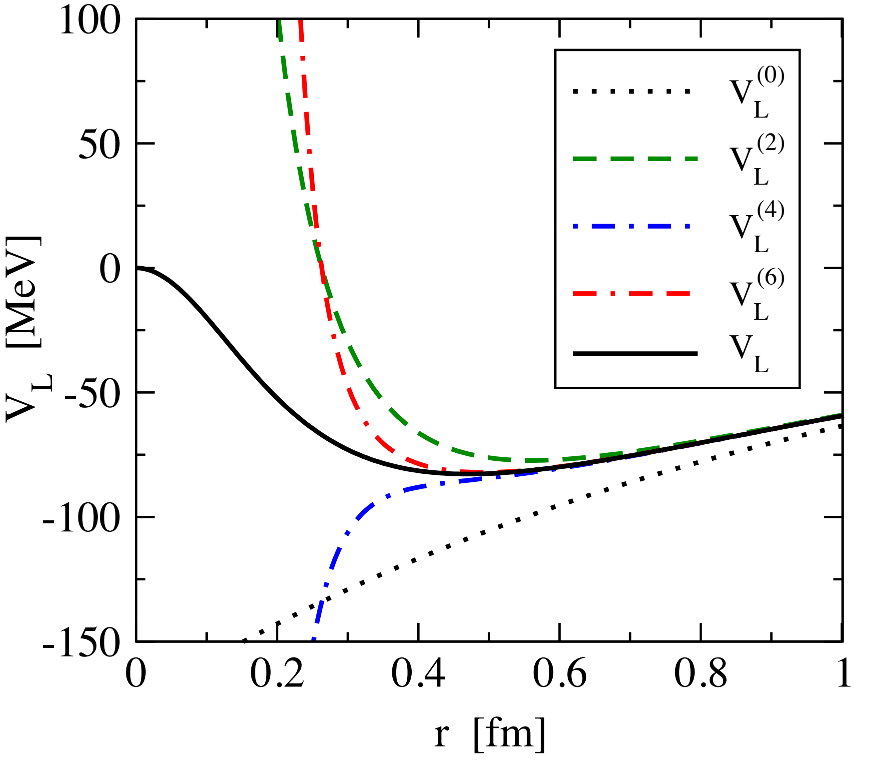
„Chiral“ toy model

The model: $V(r) = V_S(r) + V_L(r)\,, \qquad V_{S,L}(r) = A_{S,L}\,\frac{(m_S r)^2}{1+(m_S r)^2}\,e^{-m_{S,L} r}$

„Chiral“ expansion:

$$V_L(r) = V_L^{(0)}(r) + V_L^{(2)}(r) + V_L^{(4)}(r) + \mathcal{O}((m_S r)^6) \qquad V_L^{(2\nu)}(r) = \frac{(-1)^\nu}{(m_S r)^{2\nu}}\,A_L\,e^{-m_L r}$$





„Chiral“ toy model