$$\mathrm{sign}(x) = \begin{cases} +1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

$$\theta(x) = \begin{cases} 1 & x \geqslant 0 \\ 0 & x < 0 \end{cases}$$

$$\delta(x) = \frac{d}{dx}\theta(x)$$

$$[x] = \text{Integer part of } x$$

1. $\int x|x-1|dx;$
2. $\int \sin|x|\cos x dx;$
3. $\int \cos|x|dx;$
4. $\int |x|\cos x dx;$
5. $\int x|\cos x|dx;$
6. $\int |x|\cos|x|dx;$
7. $\int \sin|x|\cos|x|dx;$
8. $\int e^{|x|}dx;$
9. $\int e^{|1-x|}dx;$
10. $\int \frac{|1-x|}{|x|}dx;$
11. $\int |1-x^2||x|dx;$
12. $\int |1-x^2|x dx;$
13. $\int \ln|x|dx;$
14. $\int_0^{\pi/2} (\cos^2(\cos x)+\sin^2(\sin x))dx;$
15. $\int |1-e^x|dx;$
16. $\int x\delta(x-x^2)dx;$
17. $\int x\theta(x)dx;$
18. $\int \frac{1}{|x|}dx;$
19. $\int \ln\left(\frac{1}{|x|}\right)dx;$
20. $\int [x]x dx;$
21. $\int (-1)^{|x|}dx;$
22. $\int (-1)^{[x]}dx;$
23. $\int (|1+x|-|1-x|)dx;$
24. $\int (x+|x|)^2dx;$
25. $\int [x]|\sin(\pi x)|dx;$
26. $\int_0^{2\pi} e^{inx}e^{-imx}dx;\ m,n\in Z$
27. $\int_0^{\pi} \frac{\sin(nx)}{\sin x}dx;$
28. $\int_0^{\infty} x^n e^{-x}dx;\ n\in N$
29. $\int_0^1 x^m(1-x)^n dx;\ m,n\in Z$
30. Legendre polynomial is defined as

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n\ ,\ n=0,1,2..$$

Proof that:

$$\int_{-1}^{1} P_n(x)P_m(x)dx = \begin{cases} 0 & n \neq m \\ \frac{2}{2n+1} & m = n \end{cases}$$

31. Using 30. proof that:

$$\int_{-1}^{1} x^m P_n(x) = 0\ ,\quad m < n\ .$$

32. $\int \min(\sqrt{x},2)dx;$
33. $\int \max(|x|,4)dx;$
34. $\int \max(4-x^2,2)dx;$
35. $\int \min(5-x^2,1,x)dx;$
36. $\int \max(\cos x,\frac{1}{2})dx;$
37. $\int \max(\sin x,0)+\min(\sin x,0)dx;$
38. $\int \min(\ln x,\frac{2\ln 2}{x})dx;$
39. $\int \min(|x|,|1-x|)dx;$
40. $\int \max(\cos x,\sin x)dx;$

41. $\int \varphi(x)dx$,

where $\varphi(x)$ is distance of x to the nearest integer number.

42. Find $f(x)$ if $f'(x^2) = \frac{1}{x}$.

43. Find $f(x)$ if $f'(\cos^2 x) = \sin^2 x$.

44. Find $f(x)$ if $f'(\ln x) = \begin{cases} 0, & x = 0 \\ 1, & 0 < x \leqslant 1 \\ x, & 1 < x \end{cases}$

45. $\int xf''(2x)dx$.

46. Proof that Bessel function of the integer index:

$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)d\theta \ ,$$

satisfies the equation:

$$x^2 J''(x) + xJ'(x) + (x^2 - n^2)J_n(x) = 0$$

47. The Psi-function is defined as:

$$\psi(x) = -\gamma + \int_0^1 \frac{1 - t^{x-1}}{1-t}dt$$

Proof that $\psi(x)$ satisfies the recursive equation:

$$\psi(x+1) \ = \ \psi(x) + \frac{1}{x}$$

48. Proof, using definition in 47., that:

$$\frac{d^n}{dx^n}\psi(x+1) \ = \ \frac{d^n}{dx^n}\psi(x) + \frac{(-1)^n n!}{x^{n+1}}$$

49. Proof, using definition in 30., that:

$$\int P_n(x)dx \ = \ \frac{1}{2n+1}\left[P_{n+1}(x) - P_{n-1}(x)\right]$$

50. Proof, using definition in 30., that:

$$\int (1-x^2)^{\frac{n}{2}-1}P_n(x)dx \ = \ \frac{1}{n}(1-x^2)^{n/2}P_{n-1}(x)$$