ADVANCED DIFFERENTIAL EQUATIONS.

1. Constructing DEs calculate the integrals:

$$\begin{aligned}
I_1 &= \int_0^\infty dx\; e^{-ax^2} \cos bx; \\
I_2 &= \int_0^\infty dx\; e^{-ax^2} \sin bx; \\
I_3 &= \int_0^\infty dx\; \frac{1-\cos ax}{x} e^{-kx}; \\
I_4 &= \int_0^\infty dx\; \frac{\sin \alpha x}{x} \frac{\sin \beta x}{x} e^{-kx}.
\end{aligned}$$

2. Reduce to the Riccati equation and solve it:

$$x^4(y' - y^2) = e^{\frac{a}{x}}.$$

3. Reduce to the Riccati equation and solve it:

$$xy' = 2y^2 + \ln^4(x).$$

4. Reduce to the Riccati equation and solve it:

$$y' = \alpha \cos(\alpha x) y^2 + \alpha \cos^3(\alpha x).$$

5. Solve equation, when $f(x)$ and $g(x)$ are arbitrary functions:

$$y' = \frac{f'}{g} y^2 - \frac{g'}{f}.$$

6. Solve equations when $f(x)$ is an arbitrary function:

$$y' = x^2(y-f)^2 + f'.$$

7. Solve equations when $f(x)$ is an arbitrary function:

$$y' = y^2 - \frac{f''}{f}.$$

8. Solve the following equation (hint: one can introduce the generation function $f(x) = \sum_{i=1}^{\infty} \omega_i x^{i-1}$):

$$\omega_n = \frac{\alpha}{n-1} \sum_{i=1}^{n-1} \omega_i \omega_{n-i}, \qquad \omega_1 = 1.$$

9. Solve the following equation (hint: one can introduce the generation function $f(x) = \sum_{i=0}^{\infty} \omega_i x^i$:

$$\omega_n = \frac{1}{n} \sum_{i=0}^{n-1} \omega_i \alpha^{n-i}, \qquad \omega_0 = 1.$$

10. Solve the equation of the Abel's type

$$yy' = \left(1 - \frac{1}{x}\right) y + 1.$$