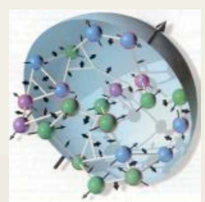
Generalized Parton Distributions in χPT

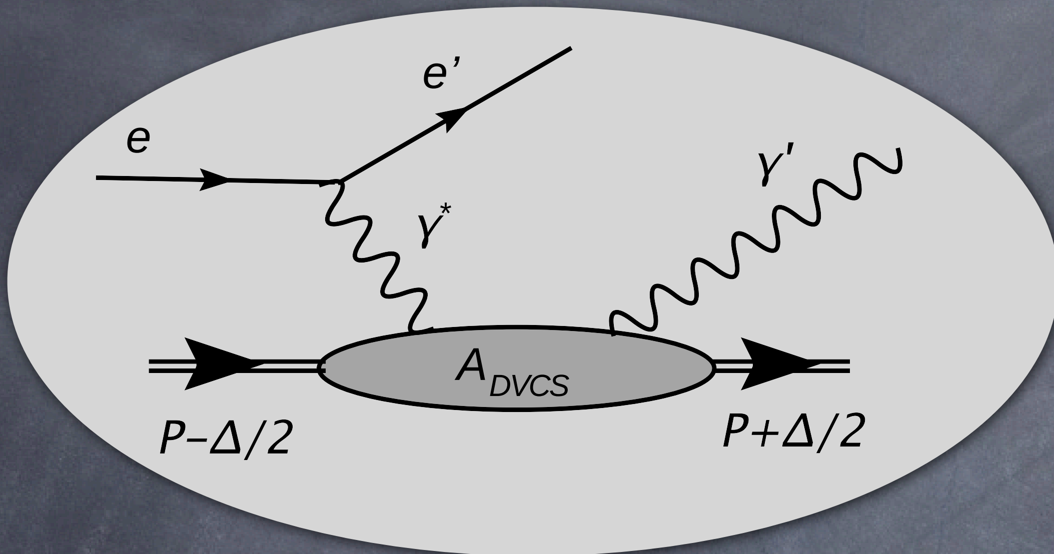
Nikolai Kivel

in collaboration with

M. Polyakov & A. Vladimirov



Deeply Virtual Compton Scattering



Q^2 large $\gg 1/R_N$

$Q^2/s = x_{Bj}$ fixed

$\Delta^2 \sim 1/R_N \ll Q^2$

Highly virtual photon interacts with the quarks and gluons inside nucleon

DVCS Amplitude

Breit Frame

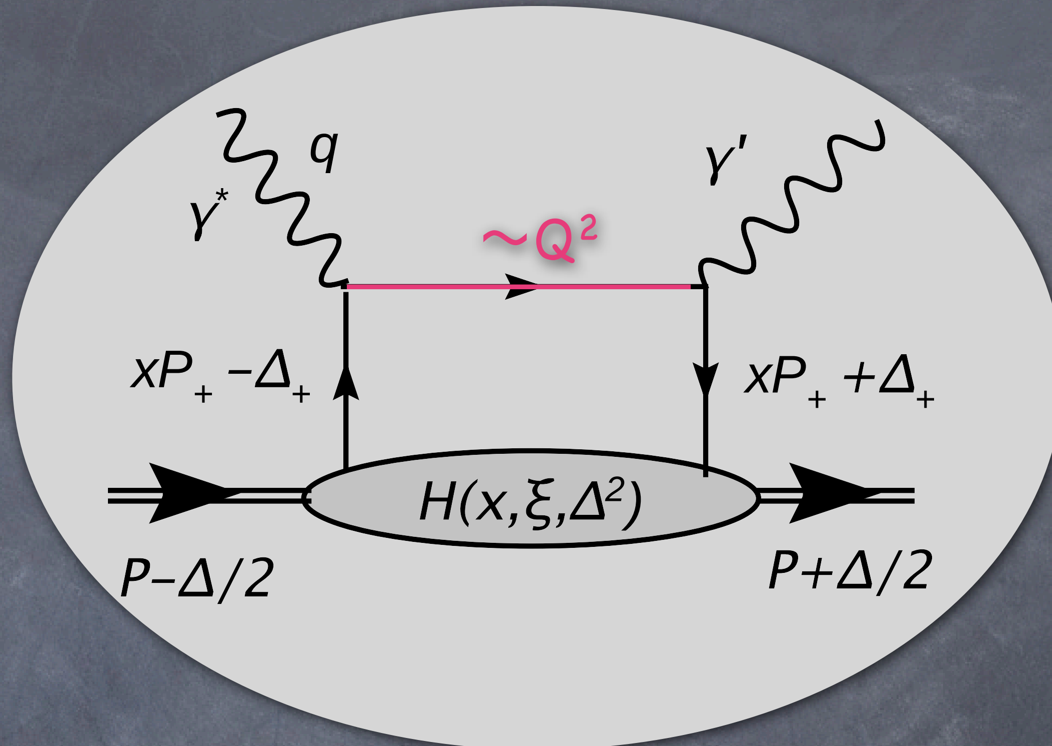
$q=(0,0,0,Q)$

$P_+ = E+P_z \sim Q$ large

$\Delta \simeq (\Delta_+ , \Delta_\perp, \Delta_-)$

$\Delta_+ \simeq \xi\, P_+$

$\Delta_\perp \sim 1/R_N \ll Q$



$$\mathcal{A} = \int dx \frac{1}{x-\xi+i0} H(x,\xi,\Delta^2)$$

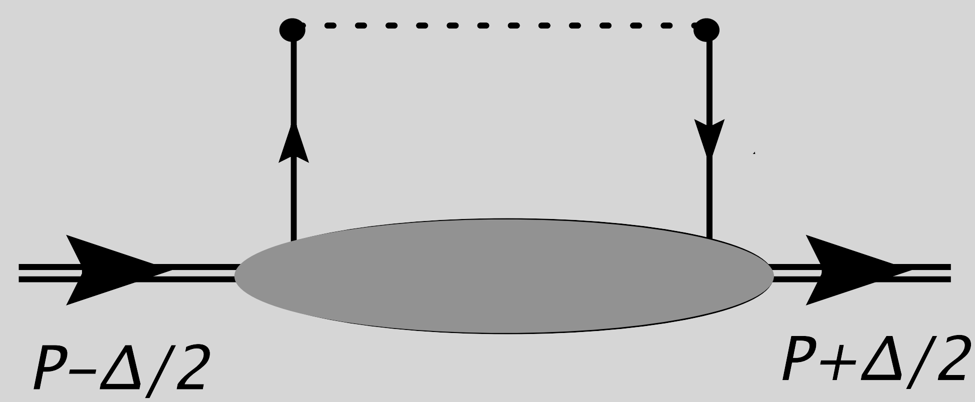
GPDs

n=(1,0,0,-1)

$\Delta \simeq (\Delta_+ , \Delta_\perp)$

$\Delta_+ \simeq \xi P_+$

$$\langle N' | \, \bar{q}(\lambda n) \, \gamma_+ \, q(-\lambda n) \, | N \rangle$$



$$\langle N' | \, \bar{q}(\lambda n) \, \gamma_+ \, q(-\lambda n) \, | N \rangle \xrightarrow{\mathrm{FT}_{[d\lambda]}} H(x, \xi, \Delta^2)$$

GPDs to PDF:

$$H^q(x, 0, 0) = f_q(x)$$

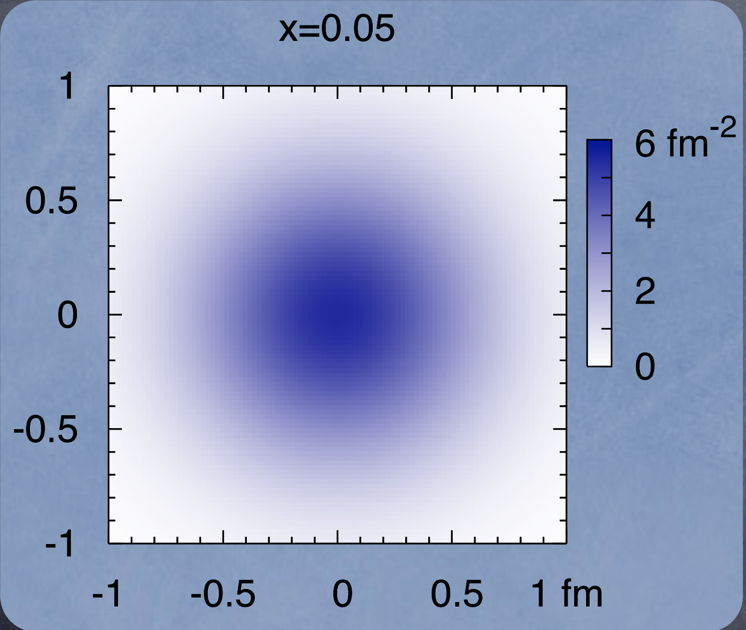
GPDs to FF:

$$\int dx H(x, \xi, \Delta^2) = F_{em}(\Delta^2)$$

Parton Distribution in Transverse Plane

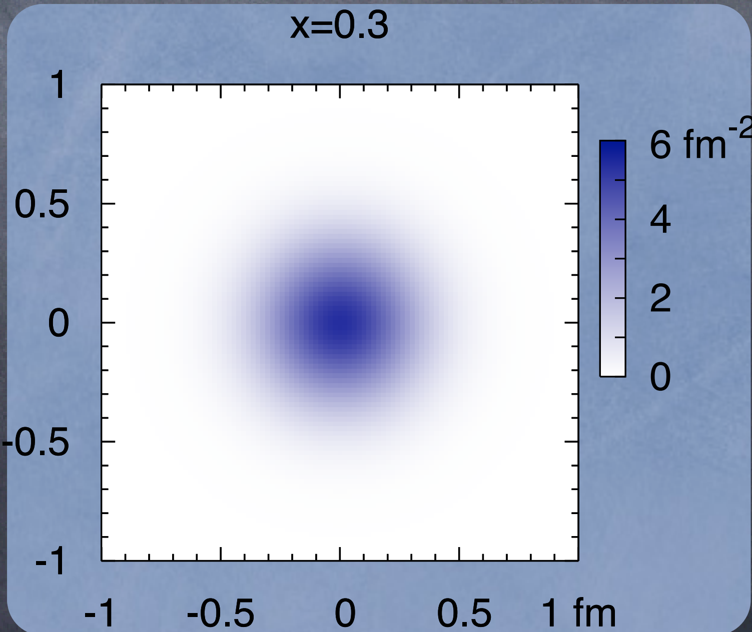
$$\int \frac{d\Delta}{2\pi} e^{i(\Delta_\perp b_\perp)} H(x, \xi = 0, \Delta_\perp^2) = q(x, b_\perp)$$

Density of partons in transverse plane ($b_\perp$) with given momentum fraction x



u_v-quark

Kroll et al, 2004



Nucleon GPD at small momentum transfer

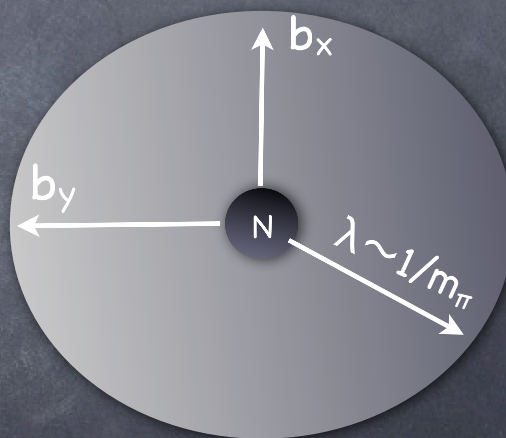
χPT : At large distances pions are the main dynamical degrees of freedom

Nucleon has large mass $m_N \gg |\Delta|, m_\pi$

$\Longrightarrow$ static source of pions (pion cloud)

$\Longrightarrow$ dominant contribution at small-t due to the pion cloud

$b_x \sim b_y \sim 1/m_\pi$



skewedness $\xi = \frac{\Delta_+}{\bar{P}_+} \sim m_\pi / m_N$

mom fraction

large $x \gg \frac{m_\pi}{m_N}$ light cone distance $\lambda \sim 1/m_N$

small $x \sim \frac{m_\pi}{m_N}$ $\lambda \sim 1/m_\pi$

Breakdown of χPT expansion at small-x

Kivel, Polyakov '07

Structure of chiral expansion

$$m_\pi = 0$$

$$a_\chi \equiv \frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ll 1$$

$SU(2)_L \times SU(2)_R$ ➡ $SU(2)_V$

$$H(x, \xi = 0, \Delta_\perp) \sim \mathring{q}(x) + \sum_{n>0} \omega_n \delta^{(n-1)}(x) a_\chi^n \ln^n[\mu_\chi^2/\Delta_\perp^2] + \ldots$$

If $x \sim \frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ll 1$ small

then the small parameter is compensated by δ-function

$$(a_\chi)^n \delta^{(n-1)}(x) = \delta^{(n-1)}(x/a_\chi) \sim \mathcal{O}(1)$$

Breakdown of χPT expansion at small-x

Kivel, Polyakov '07

Structure of chiral expansion

$$m_\pi = 0$$

$$a_\chi \equiv \frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ll 1$$

SU(2)$_L$xSU(2)$_R$ ➡ SU(2)$_V$

$$H(x, \xi = 0, \Delta_\perp) \sim \mathring{q}(x) + \sum_{n>0} \omega_n \delta^{(n-1)}(x) a_\chi^n \ln^n[\mu_\chi^2/\Delta_\perp^2] + \ldots$$

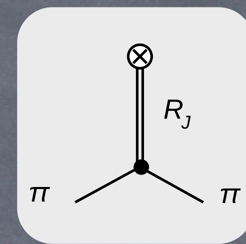
If $x \sim \frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ll 1$ small

Resummation $$\sum_{n>0} \omega_n \delta^{(n-1)}(x) \epsilon^n = q^{\text{sing}}\left(\frac{x}{\epsilon}\right)$$

LLog's accuracy: $\epsilon = a_\chi \ln[1/a_\chi]$

the sum of s- and t- channel resonances

$$\langle \pi | O(\lambda) | \pi \rangle \simeq \sum_{R_j} \langle 0 | O(\lambda) | R_J \rangle \frac{1}{M_R^2 - t} \langle \pi R_J | \pi \rangle = \sum_{R_j}$$



t-channel

s-channel

$$H(x,\xi,t) \simeq \sum_{R(l)} c_l \frac{G_{R\pi\pi}(M_R)^l}{M_R^2 - t} P_l\left(\frac{1}{\xi}\right) \theta(|x| \le \xi) f_R \varphi_R\left(\frac{x}{\xi}\right)$$

$\xi \to 0$ $\qquad \sim \delta^{(l-1)}(x)$

$\xi \to 0$ **limit:**

$$H(x,\Delta_\perp) \simeq \sum_{R_j} c_j \frac{f_R G_{R\pi\pi} M_R^l}{M_R^2 + \Delta_\perp^2} \delta^{(l-1)}(x) + \ldots$$

QCD operator as a pion probe

SU(2)$_L$xSU(2)$_R$
➡ SU(2)$_V$

LO chiral Lagrangian

$$\mathcal{L}_2 = -\frac{1}{2}\pi^a\partial^2\pi^2 - \frac{1}{2}\sigma\partial^2\sigma + m^2 F_\pi \sigma \qquad \pi^2 + \sigma^2 = F_\pi^2$$

Light-cone operator

$$O^c(\lambda) = \bar{q}\left(\frac{1}{2}\lambda n\right)\tau^c\,\gamma_+ q\left(-\frac{1}{2}\lambda\, n\right)$$

$$\partial \sim \mathcal{O}(p), \quad \lambda \sim \mathcal{O}(p^{-1}), \quad x \sim \xi \sim \mathcal{O}(p^0)$$

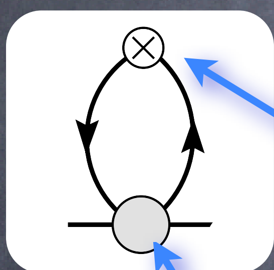
$$O^c(\lambda) \simeq \varepsilon_{abc}\, F(\alpha,\beta) * \pi^a(\alpha\,\lambda n)\partial_+\pi^b(\beta\,\lambda n)$$

LLog's accuracy

χPT case: $\dim[g_n] \sim (F_\pi)^{-2n}$

power divergencies

Lessons from 3-loop calc:



only 2-particle vertex

massless pions

$$a_\chi^n \ln^n[1/a_\chi]\delta^{(n-1)}(x)$$

maximal "spin" for fixed dimension

to LLog accuracy considerable simplifications

RG in renormalizable case:

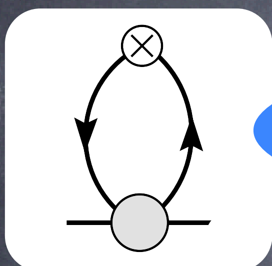
LLog's $\Leftrightarrow$ one loop calc of RG-functions

LLog's accuracy

χPT case: dim[g_n]∼$(F_\pi)^{-2n}$

power divergencies

Lessons from 3-loop calc:



$$a_\chi^n \ln^n[1/a_\chi]\delta^{(n-1)}(x)$$

to LLog accuracy considerable simplifications

RG works like in renormalizable case:

$$\mathcal{L} = -\frac{1}{2}\pi^a\partial^2\pi^a - \frac{1}{8F_\pi^2}\pi^2\partial^2\pi^2 + \frac{g_2}{8F_\pi^4}\pi^2\partial^4\pi^2 + \frac{g_3}{8F_\pi^6}\pi^2\partial^6\pi^2 + \ldots$$

1-loop β-functions of the couplings $g_{2,3}$ fix LLog coefficients

Chiral Leading Log resummation

χPT expansion

$$H^{sing}(x,\Delta_\perp) = \sum_{n>0} \frac{\omega_n}{n!} \langle x^{n-1}\rangle \delta^{(n-1)}(x)\epsilon^n + \ldots$$

small chiral parameter

$$\epsilon = -\frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ln \frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ll 1$$

PDF moments

$$\langle x^{n-1}\rangle = \int_{-1}^{1} d\beta \; \beta^{n-1} \; \mathring{q}(\beta)$$

Assume that $\dfrac{\omega_n^I}{n} = \displaystyle\int_0^1 dz \; z^{n-1} W(z)$

then

$$q^{\rm sing}(x) = -\theta(|x|<\epsilon) \int_{\frac{|x|}{\epsilon}}^{1} \frac{d\beta}{\beta} \; \mathring{q}(\beta) \; W\left(\frac{x}{\beta\epsilon}\right)$$

LLog Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \sum_{i=1}^{\infty} V_i$$

$$g_{10} = \frac{1}{F_\pi^2}$$

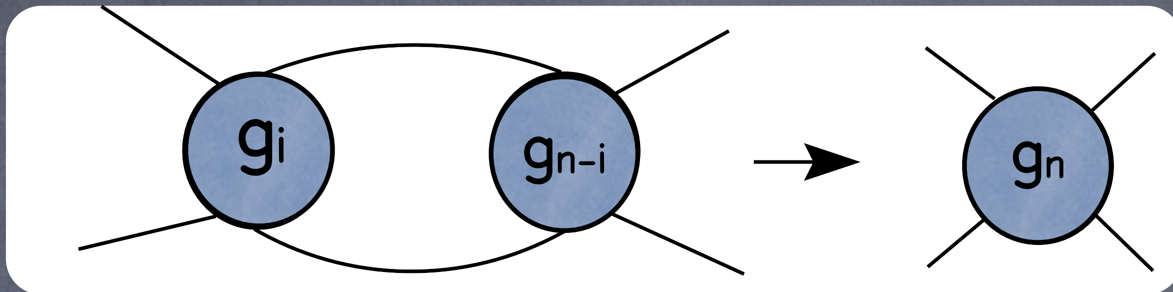
$$V_i = -\frac{1}{8} \sum_{k=0}^{[i/2]} g_{ik} \pi^a \nabla_{\mu_1 \ldots \mu_k} \pi^a \partial^{2(i-2k)} \pi^b \nabla_{\mu_1 \ldots \mu_k} \pi^b$$

- renormalizable for massless pions (no tadpoles)
- 1-loop β-functions of the couplings g_{ik} fix the LLogs coefficients

$$\mu \frac{d}{d\mu} g_{nr}(\mu) = \beta_{nr}(g_{ik}), \quad i < n$$

Results

$$V_i = -\frac{1}{8} \sum_{k=0}^{[i/2]} g_{ik} \pi^a \, \nabla_{\mu_1 \ldots \mu_k} \pi^a \partial^{2(i-2k)} \pi^b \nabla_{\mu_1 \ldots \mu_k} \, \pi^b$$



$$H^{sing}(x, \Delta_\perp) = \sum_{n>0} \frac{\omega_n}{n!} \langle x^{n-1} \rangle \delta^{(n-1)}(x) \epsilon^n + \ldots$$

$$\omega_n = \sum_{r=0}^{[n/2]} \omega_{nr}$$

$$\omega_{10} = 1$$

$$\omega_{nr} = -\frac{1}{(n-1)} \sum_{i=1}^{n-1} \sum_{(jm)} \beta_{nr}[ij, (n-i)m] \, \omega_{ij} \, \omega_{(n-i)m}.$$

$$\omega_{nr} = -\frac{1}{(n-1)} \sum_{i=1}^{n-1} \sum_{(jm)} \beta_{nr}[ij,(n-i)m]\, \omega_{ij}\, \omega_{(n-i)m}.$$

β-functions:

$$\beta_{nr}[ji,(n-i)m] = \frac{3}{2}\frac{\delta_{rj}\delta_{rm}}{4r+1} + 8\delta_{rm}\Omega_{2r}^{i(2j)} + 8(4r+1)\sum_{k=0}^{n}(2k+1)\Omega_k^{i(2j)}\Omega_k^{(n-i)(2m)}\Omega_{(2r)}^{nk}$$

$$\Omega_j^{nm} = 2^{-n-1}\int_{-1}^{1} dx\, P_j(x)\,(x-1)^n\, P_m\left[\frac{(x+3)}{(x-1)}\right]$$

$P_i(x)$ Legendre polynomials

Checks: reproduce correctly LLogs in all higher order calculations existing in literature

- 2-loop LLogs for pion-pion scattering
- 2-loop LLogs for em. form factor
- 4-loop LLogs for the scalar form factor

Properties ω_n

$$H^{sing}(x,\Delta_\perp) = \sum_{n>0} \frac{\omega_n}{n!} \langle x^{n-1} \rangle \delta^{(n-1)}(x) \epsilon^n$$

$$\omega_n = \sum_{r=0}^{[n/2]} \omega_{nr}$$

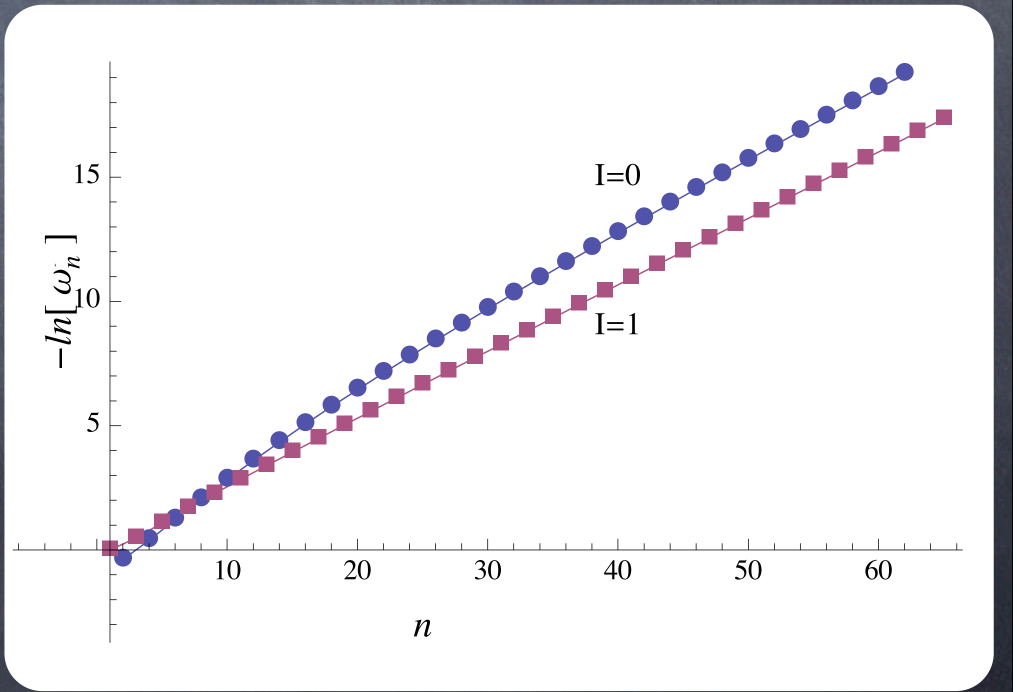
$$\omega_{n(r+1)}/\omega_{nr} \sim 1/10 \quad \Longrightarrow \quad \omega_n = \sum_{r=0}^{[n/2]} \omega_{nr} \simeq \omega_{n0}$$

Large-n:

$$\omega_n \sim \left(\frac{3a}{2}\right)^n, \quad a < 1$$

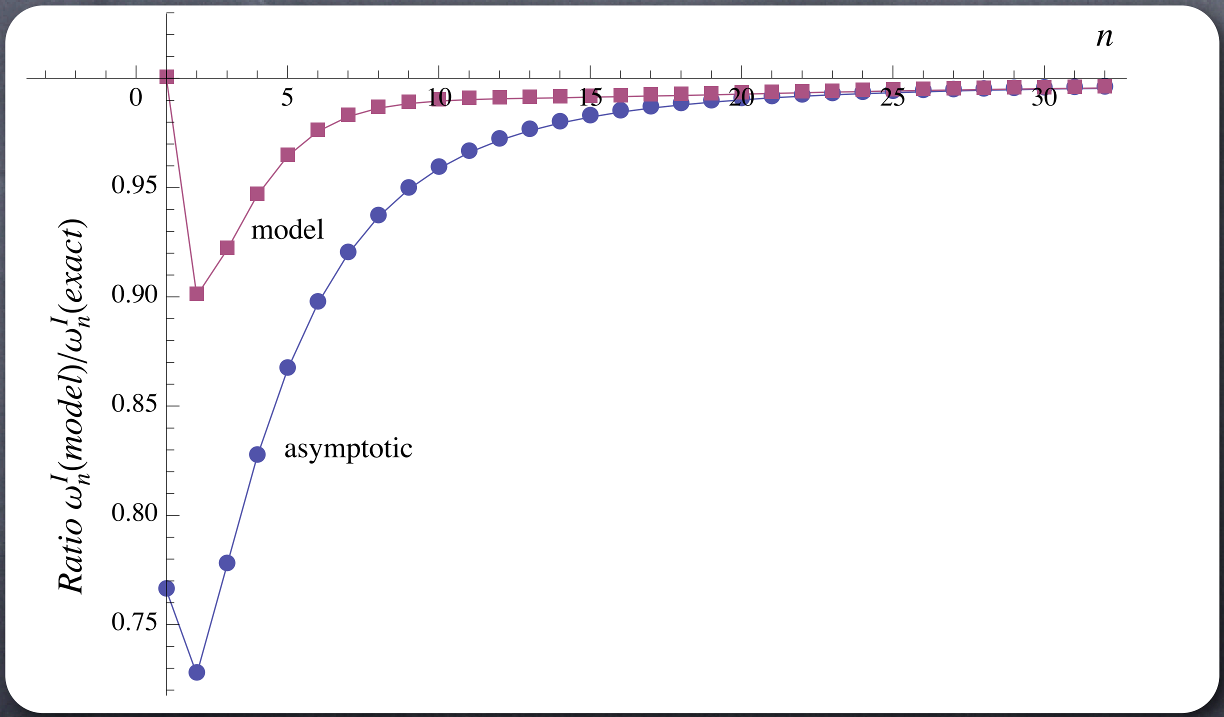
I=1 a = 0.766, n=1,3,...

I=0 a = 0.750, n=2,4,...



Approximate solutions

$$\omega_n \simeq a^n \left(1 + 0.306\; e^{-(n-1)/b}\right), \quad a = 0.7657,\; b = 8,$$



Approximate solution

$$q^{\text{sing}}(x) = -\theta(|x| < \epsilon) \int_{\frac{|x|}{\epsilon}}^{1} \frac{d\beta}{\beta}\, \mathring{q}(\beta)\, W\left(\frac{x}{\beta\epsilon}\right)$$

Model

$$\omega_n \simeq a^n \left(1 + 0.306\, e^{-(n-1)/b}\right), \quad a = 0.7657,\; b = 8,$$

$$q^{\text{sing}}(x) \simeq -\frac{2}{3}\,\theta(|x| < a\,\epsilon) \int_{|x|/a\epsilon}^{1} \frac{d\beta}{\beta}\, \mathring{q}(\beta) - 0.231\,\theta(|x| < a'\,\epsilon) \int_{|x|/a'\epsilon}^{1} \frac{d\beta}{\beta}\, \mathring{q}(\beta)$$

a = 0.766, a′=0.676

Properties of the solution

Small-x: $\mathring{q}(x) \sim 1/x^{\omega}, \quad \omega \sim 1/2$

Small-t and small-x asymptotic (m$_\pi$=0)

$$H^{sing}(x, \Delta_\perp) \sim \frac{1}{x^\omega}\left[\frac{\Delta_\perp^2}{(4\pi F_\pi)^2}\ln\left[\frac{(4\pi F_\pi)^2}{\Delta_\perp^2}\right]\right]^{\omega}$$

$$H^{reg}(x, \Delta_\perp) \sim \frac{1}{x^\omega}\frac{\Delta_\perp^4}{(4\pi F_\pi)^4}\ln\left[\frac{(4\pi F_\pi)^2}{\Delta_\perp^2}\right]$$

Phenomenology Regge motivated model

$$H(x, \Delta_\perp) \sim q(x) x^{-\alpha' \Delta_\perp^2} \sim x^{-\omega - \alpha' \Delta_\perp^2}$$

Properties of the solution

Small-x: $\mathring{q}(x) \sim 1/x^{\omega}, \quad \omega \sim 1/2$

Distribution of partons in transverse plane (m_π=0)

$$\int \frac{d\Delta}{2\pi} e^{i(\Delta_\perp b_\perp)} H(x, \xi = 0, \Delta_\perp^2) = q(x, b_\perp)$$

$b_\perp \to \infty$

$$q(x, b_\perp) \sim \frac{1}{b_\perp^2} \frac{1}{x^\omega} \left[\frac{\ln[b_\perp^2 F_\pi^2]}{b_\perp^2 F_\pi^2}\right]^\omega$$

"naive" NLO result:

$$q(x, b_\perp) \sim \frac{1}{b_\perp^4}\, \delta(x)\,.$$