Exploring TMDs through Drell-Yan processes

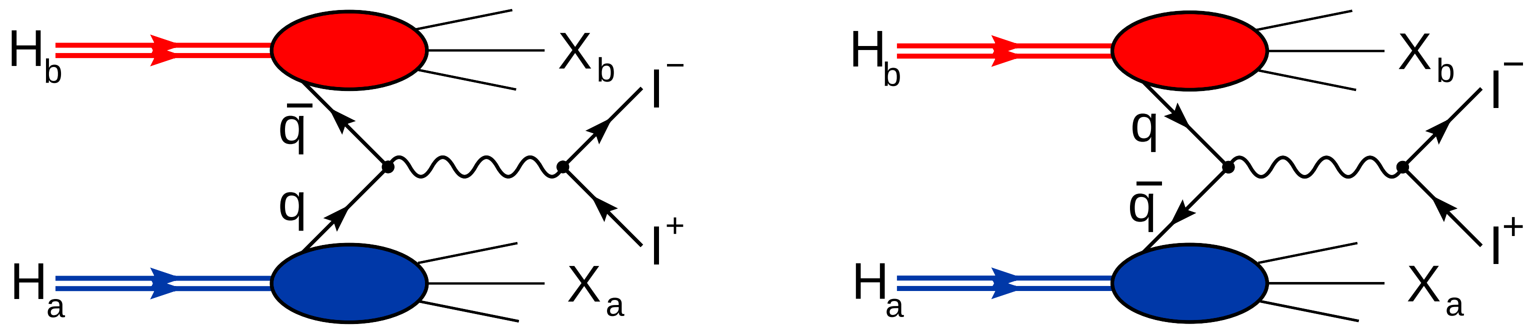
(Andreas Metz, Temple University, Philadelphia)

- Motivation
- Cross section and hadronic tensor
- Reference frames
- Angular distribution of the cross section
- Drell-Yan and TMDs
- Summary

Based on: arXiv:0809.2262, S. Arnold, A. Metz, M. Schlegel

Why dilepton production?

- Drell-Yan model (parton model) (Drell, Yan (1970))



$\longrightarrow$ access to parton distributions

- QCD-factorization for q_T-integrated cross section
  Bodwin (1984); Collins, Soper, Sterman (1985, 1988)

- QCD-factorization for q_T-dependent cross section (low q_T)
  Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2004); Collins, Metz (2004)
  $\longrightarrow$ access to transverse momentum dependent parton distributions (TMDs)
  $\longrightarrow$ DY is currently only known hadron-hadron process giving direct access to TMDs

- etc.

DY: theoretically cleanest hard hadron-hadron scattering process

Why new work on polarized dilepton production?

- Existing theoretical work
  1. Ralston, Soper (1979)
     $\longrightarrow$ formalism for double polarization, but only for q_T-integrated cross section
  2. Pire, Ralston (1983)
     $\longrightarrow$ contains basically full treatment for single polarization
  3. Tangerman, Mulders (1994)
     $\longrightarrow$ parton model calculation containing all T-even TMDs
     $\longrightarrow$ final results not very readable
  4. Boer (1999)
     $\longrightarrow$ parton model calculation containing all T-odd TMDs
     $\longrightarrow$ does not fully fit to work of Tangerman and Mulders

- Planned polarized DY experiments
  1. NN-DY (J-PARC, RHIC, JINR, IHEP)
  2. πN-DY (COMPASS)
  3. $N\bar{N}$-DY (GSI)

  Talks by Aronson, Goto, Lednicky, Saito, Steffens, Stoecker, Tyurin, ...

Complete formalism and comprehensive parton model calculation were needed

Cross section and hadronic tensor

- General cross section formula

$$\frac{l^0 l'^0\, d\sigma}{d^3\vec{l}\, d^3\vec{l'}} = \frac{\alpha_{em}^2}{F\, q^4}\, L_{\mu\nu}\, W^{\mu\nu} \qquad\qquad F \approx 2s$$

- Hadronic tensor

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = \frac{1}{(2\pi)^4} \int d^4x\, e^{iq\cdot x}\, \langle a, b \,|\, J^\mu_{em}(0) J^\nu_{em}(x) \,|\, a, b\rangle$$

Constraints from gauge invariance, parity, hermiticity

$$\begin{aligned}
q_\mu W^{\mu\nu}(P_a, S_a; P_b, S_b; q) &= q_\nu W^{\mu\nu}(P_a, S_a; P_b, S_b; q) \;=\; 0 \\
W^{\mu\nu}(P_a, S_a; P_b, S_b; q) &= W_{\mu\nu}(\bar{P}_a, -\bar{S}_a; \bar{P}_b, -\bar{S}_b; \bar{q}) \\
W^{\mu\nu}(P_a, S_a; P_b, S_b; q) &= \left[W^{\nu\mu}(P_a, S_a; P_b, S_b; q)\right]^*
\end{aligned}$$

No constraint from time-reversal

Aim: construction of $W^{\mu\nu}$

- Construction of unpolarized hadronic tensor

1. Implementing parity

$$W_u^{\mu\nu} = \sum_{i=1}^{7} h_{u,i}^{\mu\nu}\, \tilde{V}_{u,i}(P_a \cdot q, P_b \cdot q, q^2) \qquad \text{with}$$

$$h_{u,1}^{\mu\nu}, \ldots, h_{u,7}^{\mu\nu} = \Big\{ g^{\mu\nu}\,,\; q^\mu q^\nu\,,\; P_a^\mu P_a^\nu\,,\; P_b^\mu P_b^\nu\,, \; q^\mu P_a^\nu + q^\nu P_a^\mu\,,\; q^\mu P_b^\nu + q^\nu P_b^\mu\,,\; P_a^\mu P_b^\nu + P_a^\nu P_b^\mu \Big\}$$

2. Implementing gauge invariance

$\longrightarrow$ use projector method proposed by Bardeen & Tung (1968)

$$P^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \qquad\qquad q_\mu P^{\mu\nu} = P^{\mu\nu} q_\nu = 0$$

$\longrightarrow$ replace

$$h_{u,i}^{\mu\nu} \quad \text{by} \quad P^\mu{}_\rho\, h_{u,i}^{\rho\sigma}\, P_\sigma{}^\nu$$

3. Final result

$$W_u^{\mu\nu} = \sum_{i=1}^{4} t_{u,i}^{\mu\nu}\, V_{u,i}(P_a \cdot q,\, P_b \cdot q,\, q^2) \qquad \text{with}$$

$$t_{u,1}^{\mu\nu}, \ldots, t_{u,4}^{\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},\ \tilde{P}_a^\mu \tilde{P}_a^\nu,\ \tilde{P}_b^\mu \tilde{P}_b^\nu,\ \tilde{P}_a^\mu \tilde{P}_b^\nu + \tilde{P}_a^\nu \tilde{P}_b^\mu \right\}$$

$$\tilde{P}_a^\mu = P_a^\mu - \frac{P_a \cdot q\, q^\mu}{q^2}, \qquad \tilde{P}_b^\mu = P_b^\mu - \frac{P_b \cdot q\, q^\mu}{q^2}$$

$\longrightarrow$ structure functions $V_{u,i}$ are frame-independent

$\longrightarrow$ cross section can be evaluated in any frame

$\longrightarrow$ relations between frame-specific structure functions can be obtained

- Construction of polarized hadronic tensor (include $S_a,\ S_b$ in parameterization)

1. Remove redundant structures

$$
\begin{aligned}
g^{\alpha\beta}\,\varepsilon^{\mu\nu\rho\sigma} &= g^{\mu\beta}\,\varepsilon^{\alpha\nu\rho\sigma} + g^{\nu\beta}\,\varepsilon^{\mu\alpha\rho\sigma} + g^{\rho\beta}\,\varepsilon^{\mu\nu\alpha\sigma} + g^{\sigma\beta}\,\varepsilon^{\mu\nu\rho\alpha} \\
g^{\alpha\beta}\,\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\bar\mu\bar\nu\bar\rho\bar\sigma} &= \left(g^{\mu\beta}\,\varepsilon^{\alpha\nu\rho\sigma} + g^{\nu\beta}\,\varepsilon^{\mu\alpha\rho\sigma} + g^{\rho\beta}\,\varepsilon^{\mu\nu\alpha\sigma} + g^{\sigma\beta}\,\varepsilon^{\mu\nu\rho\alpha}\right)\varepsilon^{\bar\mu\bar\nu\bar\rho\bar\sigma}
\end{aligned}
$$

2. Final result (for single polarization)

$$
\begin{aligned}
W_a^{\mu\nu} &= \sum_{i=1}^{8} t_{a,i}^{\mu\nu}\, V_{a,i}(P_a\cdot q,\, P_b\cdot q,\, q^2) \\
t_{a,1}^{\mu\nu},\ldots,t_{a,4}^{\mu\nu} &= \varepsilon^{S_a q P_a P_b}\left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\,,\ \tilde{P}_a^\mu \tilde{P}_a^\nu\,,\ \tilde{P}_b^\mu \tilde{P}_b^\nu\,,\ \tilde{P}_a^\mu \tilde{P}_b^\nu + \tilde{P}_a^\nu \tilde{P}_b^\mu \right\} \\
t_{a,5}^{\mu\nu},\ t_{a,6}^{\mu\nu} &= \left\{ S_a\cdot q\,,\ S_a\cdot P_b \right\}\left(\varepsilon^{\mu q P_a P_b}\,\tilde{P}_a^\nu + \varepsilon^{\nu q P_a P_b}\,\tilde{P}_a^\mu\right) \\
t_{a,7}^{\mu\nu},\ t_{a,8}^{\mu\nu} &= \left\{ S_a\cdot q\,,\ S_a\cdot P_b \right\}\left(\varepsilon^{\mu q P_a P_b}\,\tilde{P}_b^\nu + \varepsilon^{\nu q P_a P_b}\,\tilde{P}_b^\mu\right)
\end{aligned}
$$

- Total hadronic tensor

$$
\boxed{W^{\mu\nu} = W_u^{\mu\nu} + W_a^{\mu\nu} + W_b^{\mu\nu} + W_{ab}^{\mu\nu} \qquad (4+8+8+28=48)}
$$

Reference frames

- Lab-frame and cm-frame

  ⟶ closest to experimental situation

- Dilepton rest frames

  ⟶ lead to simple angular dependence of cross section

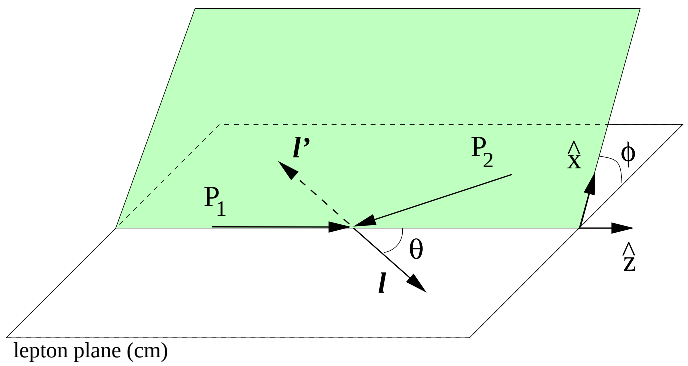
$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{2\,F\,q^4}\,L_{\mu\nu}W^{\mu\nu}$$

(corresponding formula in, e.g., cm-frame has angular dependence in denominator)

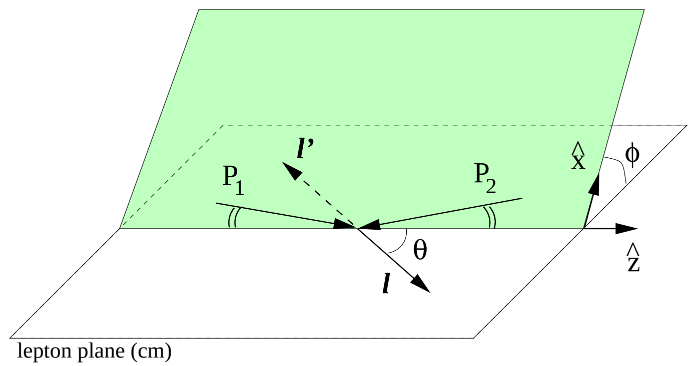
⟶ infinitely many dilepton rest frames

⟶ most frequently used choices

Gottfried-Jackson frame



Collins-Soper frame



plots from Boer, Vogelsang, hep-ph/0604177

Angular distribution of the cross section

- Unpolarized case

$$\frac{d\sigma_u}{d^4q\,d\Omega} \propto \Big\{(1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big\}$$

$\longrightarrow$ $\theta,\ \phi$ characterize orientation of dilepton pair (in dilepton rest frame)
$\longrightarrow$ structure of angular distribution holds in any dilepton rest frame
$\longrightarrow$ F-structure functions related to V-structure functions

- Single polarization (analogous for polarization of H_b)

$$\frac{d\sigma_a}{d^4q\,d\Omega} \propto \Big\{S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big) + |\vec{S}_{aT}|\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big) + |\vec{S}_{aT}|\cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big\}$$

$\longrightarrow$ components S_{aL} and $\vec{S}_{aT}$ can be understood in any frame
$\longrightarrow$ six independent (θ,ϕ)-structures

- Double polarization

$$
\begin{aligned}
\frac{d\sigma_{ab}}{d^4q\, d\Omega} \quad \propto \quad & \Big\{ S_{aL}\, S_{bL} \Big((1+\cos^2\theta)\, F_{LL}^{1} + (1-\cos^2\theta)\, F_{LL}^{2} \\
& \qquad + \sin 2\theta \cos\phi\, F_{LL}^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_{LL}^{\cos 2\phi} \Big) \\
& + S_{aL}\, |\vec{S}_{bT}| \Big[\cos\phi_b \Big((1+\cos^2\theta)\, F_{LT}^{1} + (1-\cos^2\theta)\, F_{LT}^{2} \\
& \qquad + \sin 2\theta \cos\phi\, F_{LT}^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_{LT}^{\cos 2\phi} \Big) \\
& \quad + \sin\phi_b \Big(\sin 2\theta \sin\phi\, F_{LT}^{\sin\phi} + \sin^2\theta \sin 2\phi\, F_{LT}^{\sin 2\phi} \Big) \Big] \\
& + |\vec{S}_{aT}|\, |\vec{S}_{bT}| \Big[\cos(\phi_a+\phi_b) \Big((1+\cos^2\theta)\, F_{TT}^{1} + (1-\cos^2\theta)\, F_{TT}^{2} \\
& \qquad + \sin 2\theta \cos\phi\, F_{TT}^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_{TT}^{\cos 2\phi} \Big) \\
& \quad + \cos(\phi_a-\phi_b) \Big((1+\cos^2\theta)\, \bar{F}_{TT}^{1} + (1-\cos^2\theta)\, \bar{F}_{TT}^{2} \\
& \qquad + \sin 2\theta \cos\phi\, \bar{F}_{TT}^{\cos\phi} + \sin^2\theta \cos 2\phi\, \bar{F}_{TT}^{\cos 2\phi} \Big) \\
& \quad + \sin(\phi_a+\phi_b) \Big(\sin 2\theta \sin\phi\, F_{TT}^{\sin\phi} + \sin^2\theta \sin 2\phi\, F_{TT}^{\sin 2\phi} \Big) \\
& \quad + \sin(\phi_a-\phi_b) \Big(\sin 2\theta \sin\phi\, \bar{F}_{TT}^{\sin\phi} + \sin^2\theta \sin 2\phi\, \bar{F}_{TT}^{\sin 2\phi} \Big) \Big] \Big\}
\end{aligned}
$$

q_T-integrated cross section in parton model

Leading twist, tree level $\longrightarrow$ just F_{UU}^{1}, F_{LL}^{1}, $F_{TT}^{\cos(2\phi-\phi_a-\phi_b)}$

$$\frac{d\sigma}{dx_a\, dx_b\, d\Omega} \propto \Big\{ (1+\cos^2\theta) \sum_q e_q^2 \Big(f_1^q(x_a)\, f_1^{\bar q}(x_b) + f_1^{\bar q}(x_a)\, f_1^q(x_b) \Big)$$

$$- S_{aL}\, S_{bL}\, (1+\cos^2\theta) \sum_q e_q^2 \Big(g_1^q(x_a)\, g_1^{\bar q}(x_b) + g_1^{\bar q}(x_a)\, g_1^q(x_b) \Big)$$

$$+ |\vec{S}_{aT}|\, |\vec{S}_{bT}|\, \sin^2\theta\, \cos(2\phi - \phi_a - \phi_b)$$

$$\times \sum_q e_q^2 \Big(h_1^q(x_a)\, h_1^{\bar q}(x_b) + h_1^{\bar q}(x_a)\, h_1^q(x_b) \Big) \Big\}$$

$\longrightarrow$ σ_{TT} promising for addressing transversity h_1

$\longrightarrow$ measurement of σ_{LL} and σ_{TT} in the same experiment desirable !

$\longrightarrow$ transversity drops out upon ϕ-integration

Drell-Yan and TMDs

- 8 leading twist quark TMDs
  $\longrightarrow$ 6 T-even: f_1, g_{1L}, g_{1T}, h_1, $h_{1T}^{\perp}$, $h_{1L}^{\perp}$
  $\longrightarrow$ 2 T-odd: $f_{1T}^{\perp}$, $h_1^{\perp}$

- TMDs appear in q_T-dependent DY cross section for $q_T \sim \Lambda$, $q_T \ll M_{l\bar{l}}$

- Here: no α_S-correction in hard part, no soft gluon emission

- Restriction to twist-2

- Currently no valid twist-3 TMD factorization formula
  Gamberg, Hwang, Metz, Schlegel (2006)
  Bacchetta, Boer, Diehl, Mulders (2008)

Leading twist DY cross section

Results (symbolically, dependence on ϕ_a, ϕ_b suppressed)

$$\sigma_{UU} \propto f_1 f_1 + \cos 2\phi \, h_1^\perp h_1^\perp$$

$$\sigma_{LU} \propto \sin 2\phi \, h_{1L}^\perp h_1^\perp$$

$$\sigma_{TU} \propto f_{1T}^\perp f_1 + \sin 2\phi \, h_1 h_1^\perp + \sin 2\phi \, h_{1T}^\perp h_1^\perp$$

$$\sigma_{LL} \propto g_{1L} g_{1L} + \cos 2\phi \, h_{1L}^\perp h_{1L}^\perp$$

$$\sigma_{TL} \propto g_{1T} g_{1L} + \cos 2\phi \, h_1 h_{1L}^\perp + \cos 2\phi \, h_{1T}^\perp h_{1L}^\perp$$

$$\sigma_{TT} \propto f_{1T} f_{1T} + g_{1T} g_{1T} + \cos 2\phi \, h_1 h_1 + \cos 2\phi \, h_1 h_{1T}^\perp + \cos 2\phi \, h_{1T}^\perp h_{1T}^\perp$$

$\longrightarrow$ 16 independent structure functions

$\longrightarrow$ TMDs appear either twice or four-times

$\longrightarrow$ complete experiment possible

$\longrightarrow$ cross checks possible

$\longrightarrow$ chiral-odd TMDs drop out upon ϕ-integration

Leading twist SIDIS cross section

Results (symbolically)

$$\sigma_{UU} \propto f_1 D_1 + \cos 2\phi_h \, h_1^{\perp} H_1^{\perp}$$

$$\sigma_{UL} \propto \sin 2\phi_h \, h_{1L}^{\perp} H_1^{\perp}$$

$$\sigma_{UT} \propto \sin(\phi_h - \phi_S) \, f_{1T}^{\perp} D_1 + \sin(\phi_h + \phi_S) \, h_1 H_1^{\perp} + \sin(3\phi_h - \phi_S) \, h_{1T}^{\perp} H_1^{\perp}$$

$$\sigma_{LL} \propto g_{1L} D_1$$

$$\sigma_{LT} \propto \cos(\phi_h - \phi_S) \, g_{1T} D_1$$

$\longrightarrow$ 8 independent structure functions

$\longrightarrow$ TMDs appear exactly once

$\longrightarrow$ complete experiment possible

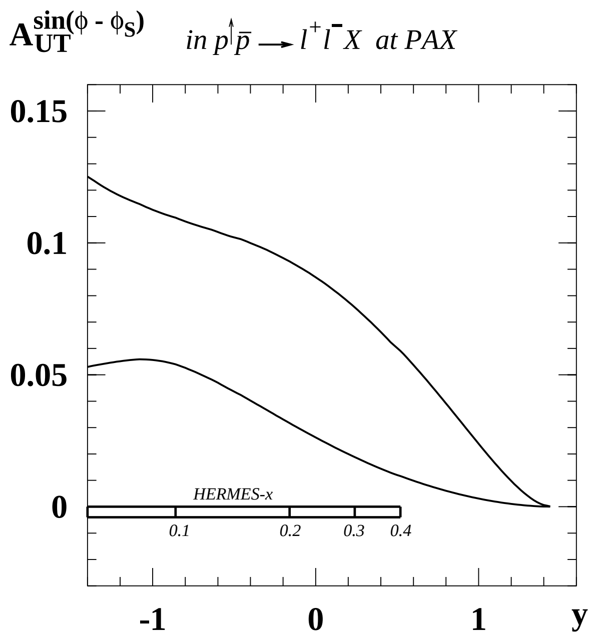
$\longrightarrow$ no cross checks possible (for parton distributions)

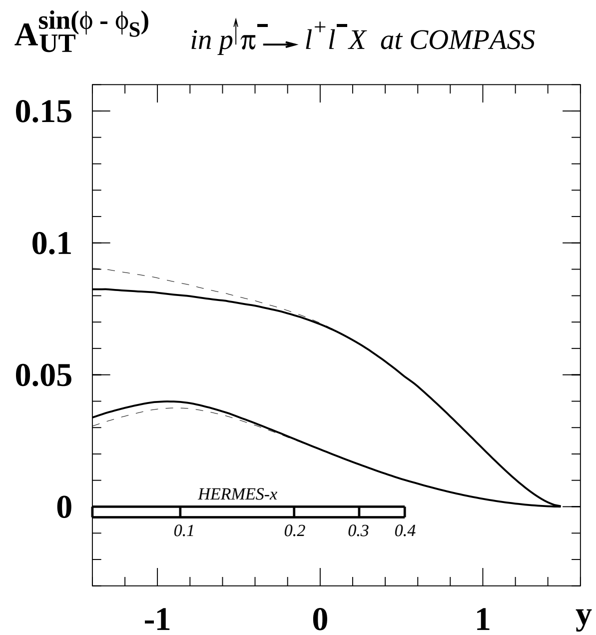
Sivers function in Drell-Yan

(Collins et al., 2005)

- Sivers asymmetry at GSI and COMPASS

$$y = \frac{1}{2} \ln \frac{x_1}{x_2} \qquad f_{1T}^{\perp(1)\bar{q}}(x) = \pm \frac{f_1^{\bar{u}}(x) + f_1^{\bar{d}}(x)}{f_1^{u}(x) + f_1^{d}(x)} f_{1T}^{\perp(1)q}(x)$$





GSI $p^{\uparrow}\,\bar{p} \rightarrow l^+\,l^-\,X$

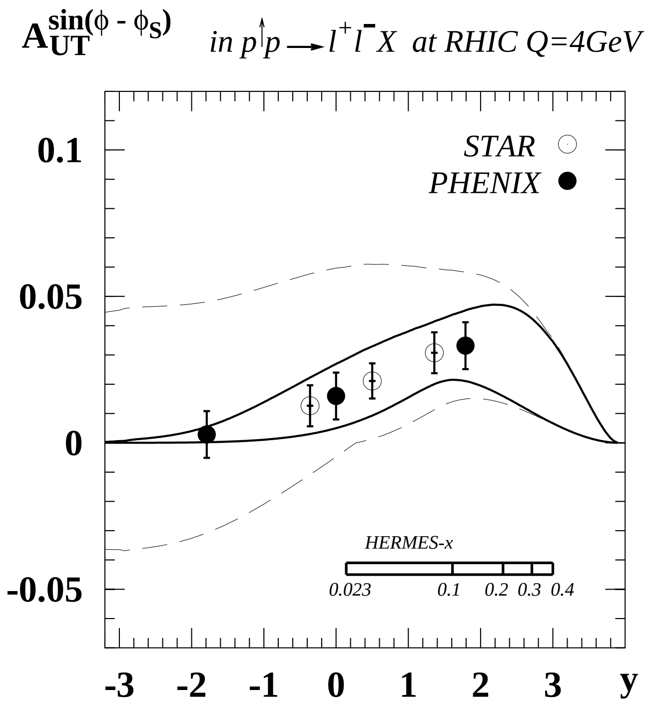
$s = 45\,\mathrm{GeV}^2 \quad Q^2 = 6.25\,\mathrm{GeV}^2$

COMPASS $p^{\uparrow}\,\pi^- \rightarrow l^+\,l^-\,X$

$s = 400\,\mathrm{GeV}^2 \quad Q^2 = 20\,\mathrm{GeV}^2$

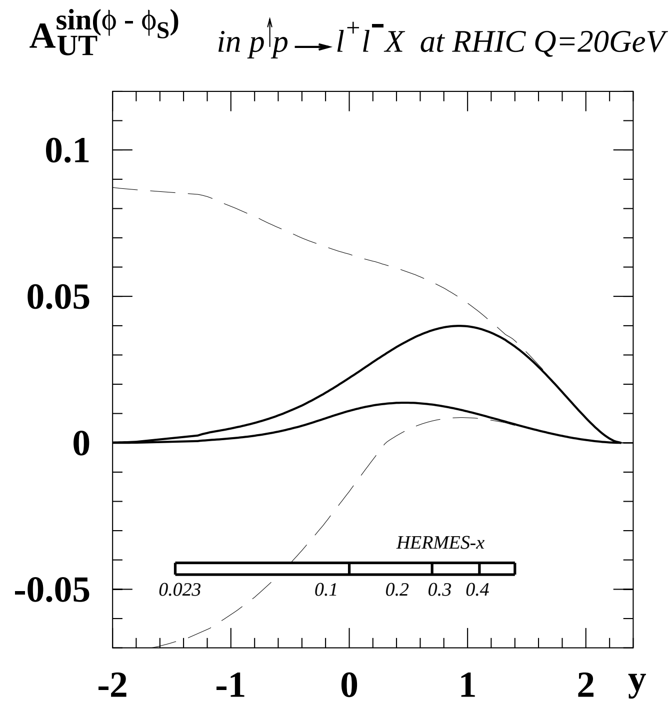
$\longrightarrow$ Prediction $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{DIS}$ may be checked !

- Sivers asymmetry at RHIC



RHIC $p^\uparrow p \to l^+ l^- X$

$\sqrt{s} = 200\,\mathrm{GeV} \quad Q^2 = 16\,\mathrm{GeV}^2$



RHIC $p^\uparrow p \to l^+ l^- X$

$\sqrt{s} = 200\,\mathrm{GeV} \quad Q^2 = 400\,\mathrm{GeV}^2$

$\longrightarrow$ In pp-collisions harder to check sign reversal, but strong sensitivity to $f_{1T}^{\perp \bar{q}}$

(More recent calculation by Anselmino et al. confirms our general conclusions)

Lam-Tung relation

- Unpolarized case

$$\frac{d\sigma_{UU}}{d^4q\, d\Omega} \quad \propto \quad \Big\{(1+\cos^2\theta)\, F_{UU}^1 + (1-\cos^2\theta)\, F_{UU}^2 + \sin 2\theta \cos\phi\, F_{UU}^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_{UU}^{\cos 2\phi}\Big\}$$

$\longrightarrow$ LT-relation: $F_{UU}^2 = 2F_{UU}^{\cos 2\phi} + \mathcal{O}(\alpha_S^2)$
$\longrightarrow$ violated in πN Drell-Yan (CERN, FermiLab)
$\longrightarrow$ violation may be due to transverse parton motion: $F_{UU}^{\cos 2\phi} \propto h_1^\perp h_1^\perp$ (Boer, 1999)
$\longrightarrow$ not violated in $N D$ Drell-Yan (FermiLab)

- Double longitudinal polarization

$$\frac{d\sigma_{LL}}{d^4q\, d\Omega} \quad \propto \quad S_{aL} S_{bL} \Big\{(1+\cos^2\theta)\, F_{LL}^1 + (1-\cos^2\theta)\, F_{LL}^2 + \sin 2\theta \cos\phi\, F_{LL}^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_{LL}^{\cos 2\phi}\Big\}$$

$\longrightarrow$ LT-relation: $F_{LL}^2 = 2F_{LL}^{\cos 2\phi} + \mathcal{O}(\alpha_S^2)$
(Berger, Qiu, Rodriguez-Pedraza (2007); Vogelsang (2008))
$\longrightarrow$ transverse parton motion: $F_{LL}^{\cos 2\phi} \propto h_{1L}^\perp h_{1L}^\perp$

Summary

1. Hadronic tensor for double-polarized DY derived
2. General form of cross section in any frame can be obtained
3. Angular distribution of cross section in dilepton rest frame given
4. DY can serve as important process to get (complementary) information on TMDs
5. Plenty of phenomenological studies possible
6. Important points
   - Experimental check of $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{SIDIS}$
   - Understanding of the azimuthal dependence of the (unpolarized) cross section
7. Polarized DY measurements would allow systematic studies of QCD resummation (q_T-resummation, threshold resummation)

Looking forward to the first polarized DY data