Thermal Power Corrections in the Deconfined Phase

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Seminar on Hadrons and Chiral Symmetry in honour of Klaus Goeke

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Issues

1. QCD and Trace Anomaly
   - Trace Anomaly
   - Power temperature corrections
   - Scale invariance and confinement
2. Dimension Two Gluon Condensate
   - Polyakov loop and dimension two gluon condensate
   - Power temperature corrections
   - Non Perturbative model
   - Non Perturbative contributions in the Free Energy
   - Non perturbative contribution to the Trace Anomaly
3. Chiral Quark Models at Finite T
   - Standard Treatment
   - Coupling the Polyakov loop
   - Group averaging
4. Conclusions

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Trace Anomaly

QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f \, ;$$

In the limit of massless quarks ($m_f = 0$),

- Invariant under scale ($\mathbf{x} \longrightarrow \lambda \mathbf{x}$)
- Chiral **Left** $\leftrightarrow$ **Right** transformations.

Partition function (gluodynamics $m_f \to \infty$)

$$Z = \int \mathcal{D}\bar{A}_{\mu,a} \exp\left[-\frac{1}{4g^2} \int d^4x (\bar{G}^a_{\mu\nu})^2 \right]$$

$$\frac{\partial \log Z}{\partial g} = \frac{1}{2g^3}\left\langle \int d^4x (\bar{G}^a_{\mu\nu})^2 \right\rangle = \frac{1}{2g}\frac{V}{T}\langle (G^a_{\mu\nu})^2 \rangle$$

Free energy and Total Energy

$$F = -PV = -T \log Z \qquad \epsilon = \frac{E}{V} = \frac{T^2}{V}\frac{\partial \log Z}{\partial T} \tag{1}$$

$$\epsilon - 3P = T^5 \frac{\partial}{\partial T}\left(\frac{P}{T^4}\right) . \tag{2}$$

Renormalization scale μ

$$\frac{P}{T^4} = f(g(\mu), \log(\mu/2\pi T)). \tag{3}$$

$$\frac{\partial}{\partial \log T}\left(\frac{P}{T^4}\right) = \frac{\partial g}{\partial \log \mu}\frac{\partial}{\partial g}\left(\frac{P}{T^4}\right) \tag{4}$$

The trace anomaly

$$\epsilon - 3P = \frac{(g)}{2g}\langle (G^a_{\mu\nu})^2 \rangle \,,$$

where we have introduced the beta function

$$\beta(g) = \mu \frac{dg}{d\mu} = -\frac{11 N_c}{48\pi^2} g^3 + \mathcal{O}(g^5)\,. \tag{5}$$

Perturbation theory to two loops (J.I.Kapusta, NPB148 (1979)):

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} = \frac{N_c(N_c^2-1)}{1152\pi^2}\beta_0 g(T)^4 + \mathcal{O}(g^5)$$

where $1/g^2(\mu) = \beta_0 \log(\mu^2/\Lambda^2_{\rm QCD})$

- The free gluon gas has $\Delta = 0$.
- Perturbation Theory says that the trace of the Energy-Momentum-Tensor takes a nonzero value as a result of quantum corrections.
- Lattice data predicts a violent behaviour in powers of **T**. (Many groups: G. Boyd et al, NPB (1996), Y. Aoki et al (2006), ...).
- PT predicts a smooth dependence in T, because $g(T) \sim \log(T)$, so it is unable to reproduce this power behaviour, even if more and more orders are included (Andersen, Ann.Phys.317,2005).

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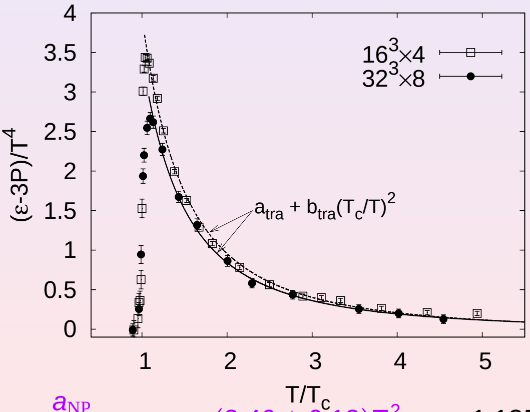
Power temperature corrections from Lattice data

Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).



$$\frac{\epsilon - 3P}{T^4} = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (3.46 \pm 0.13) T_c^2\,, \quad 1.13 T_c < T < 4.5 T_c\,.$$

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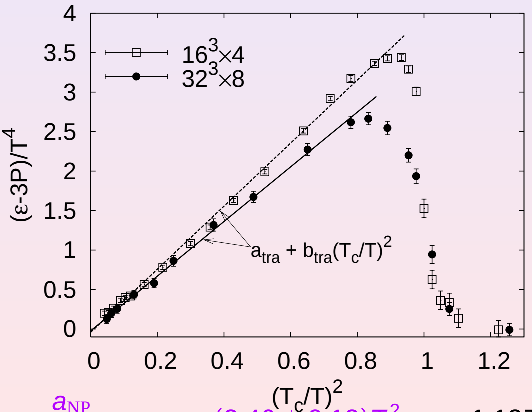

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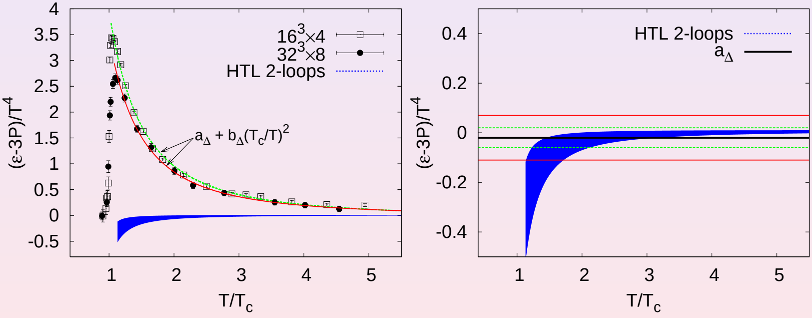
Perturbation theory vs Lattice data

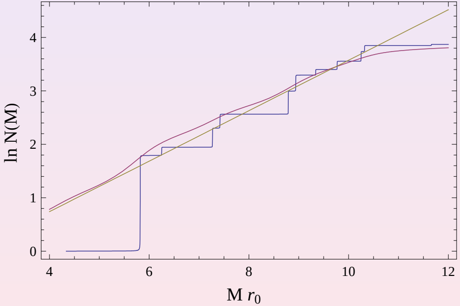
$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = a_{\Delta,P} + \frac{b_{\text{NP}}}{T^2}$$



Perturbation Theory and Hard thermal loops only yield a_Δ !!.

Glueball Hagedorn Spectrum

$$\rho(M) = \sum_i g_i \delta(M - M_i) = \frac{dN(M)}{dM},$$



$$\langle N_{\mathrm{lat}}(M) \rangle = \sum_i g_i \left(\frac{1}{\pi} \tan^{-1} \left[\frac{M - M_i}{\Delta M} \right] + \frac{1}{2} \right) \sim A e^{M/T_H} \qquad T_H \ll M_{0^{++}}$$

The fuzzy bag of Pisarski

Low temperature (confined) $\to$ glueball gas

$$P_{\rm glueball}(T) =\sim e^{-M_G/T} \qquad M_G \gg T_c \to P_{\rm glueball}(T_c) = 0$$

High temperature (deconfined) $\to$ free gluon gas

$$P_{\rm gluons}(T) = \frac{b_0}{2} T^4 \qquad b_0 = \frac{(N_c^2 - 1)\pi^2}{45}$$

Pisarski's (temperature dependent) fuzzy bag , PTP 2006

$$P(T) = P_{\rm gluons}(T) - B_{\rm fuzzy}(T), \qquad T > T_c\,, \qquad P(T_c) = P_{\rm glueballs}(T_c) = 0$$

$$B_{\rm fuzzy} = \frac{b_0}{2} T_c^2 T^2 \qquad \to \qquad P = \frac{b_0}{2}(T^4 - T^2 T_c^2)$$

Then

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = b_0 \left(\frac{T_c}{T}\right)^2 \qquad b_0 = 3.45(3.5 Fit!!!!) \tag{6}$$

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Scale invariance and confinement

Consider a rectangular Wilson loop:

$$W(\mathcal{C}) = \exp\left(ig \int_{\mathcal{C}} A_\mu dx^\mu \right)$$

It is related to the potential $V_{q\bar{q}}(R)$ acting between charges q and $\bar{q}$:

$$W(\mathcal{C}) \to \exp\left(-T V_{q\bar{q}}(R)\right)$$

Scale transformations: $T \to \lambda T$, $R \to \lambda R$,
The only scale invariant solution is the Coulomb Potential:

$$V_{q\bar{q}} \sim \frac{1}{R}$$

Running coupling and string tension break scale invariance:

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_s(R)}{R} + \sigma R \, .$$

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Polyakov loop and dimension two gluon condensate

E.Megías et al, JHEP 0601 (2006).

The vacuum expectation value of the Polyakov loop serves as an order parameter for the deconfinement phase transition in gluodynamics:

$$\mathbf{L} = \frac{1}{N_c}\langle \mathrm{tr}_c \Omega \rangle \equiv \frac{1}{N_c}\left\langle \mathrm{tr}_c \mathcal{P}\left(e^{ig\int_0^{1/T} dx_0 A_0(\vec{x},x_0)}\right)\right\rangle .$$

$\mathcal{P}$ denotes path ordering. In the Polyakov gauge ($\partial_0 A_0(\vec{x},x_0) = 0$) a gaussian approximation is possible:

$$\mathbf{L} = \frac{1}{N_c}\left\langle \mathrm{tr}_c\, e^{igA_{0,a}T_a/T}\right\rangle \longrightarrow \exp\left[-\frac{g^2\langle A_{0,a}^2\rangle}{4N_cT^2}\right] .$$

- Cumulant expansion and vacuum saturation of condensates ($\langle A_0^{2k}\rangle = (2k-1)!!\langle A_0^2\rangle^k +$ n.v.c.) are applied.
- Contribution from $\langle A_0^4\rangle$ starts at $\mathcal{O}(g^6)$. So, it is valid up to $\mathcal{O}(g^5)$.

The dynamics of $A_0(\vec{x})$ can be described by the **dimensional reduced effective theory** of QCD (S.Nadkarni PRD27 (1983)):

$$\mathcal{L}'_{QCD} = -\frac{1}{T}\mathrm{tr}([D_i, A_0]^2) + \frac{m_D^2}{T}\mathrm{tr}(A_0^2) + \cdots$$

$D_{00}(\vec{k})\delta_{ab}$ is the propagator of the canonical fields $T^{-1/2}A_{0,a}(\vec{x})$. The integration of the propagator is related to the vacuum expectation value of the gluon fields (the dimension two gluon condensate):

$$\langle A_{0,a}^2 \rangle = (N_c^2 - 1)T \int \frac{d^3k}{(2\pi)^3} D_{00}(\vec{k})\,.$$

Perturbative contribution (at leading order):

$$D_{00}^{\mathrm{P}}(\vec{k}) = \frac{1}{\vec{k}^2 + m_D^2}\,; \qquad \langle A_{0,a}^2 \rangle_{\mathrm{P}} = -\frac{(N_c^2 - 1)Tm_D}{4\pi} \sim T^2\,.$$

Leading order of E.Gava PLB105 (1981). It reproduces lattice data above $\sim 6T_D$. Below $6T_D$ non perturbative effects become important.

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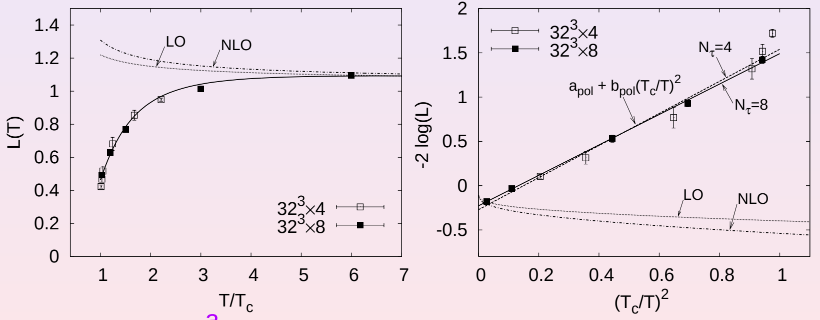
Power temperature corrections in the Polyakov loop

Renormalized Polyakov Loop $N_c = 3, N_f = 0$
O. Kaczmarek et al. PLB543 (2002).



$$-2\log(L) = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (1.81 \pm 0.13) T_c^2\,, \qquad 1.03 T_c < T < 6 T_c\,.$$

Perturbative result fails to reproduce lattice data in this regime.

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Non Perturbative model

Consider **new phenomenological pieces** in the gluon propagator to take into account for non perturbative contributions (E.Megías JHEP0601(2006), see also K.G.Chetyrkin et al, NPB550 (1999)):

$$D_{00}(\vec{k}) = \underbrace{D_{00}^{\rm P}(\vec{k})}_{\sim 1/k^2} + \underbrace{D_{00}^{\rm NP}(\vec{k})}_{\sim 1/k^4}\,; \qquad \mathbf{D_{00}^{\rm NP}(k) = \frac{m_G^2}{(k^2+m_D^2)^2}}\,, \qquad \mathbf{m_G^2 > 0}\,.$$

It produces a non perturbative contribution to the gluon condensate:

$$\langle A_{0,a}^2\rangle = \underbrace{\langle A_{0,a}^2\rangle_{\rm P}}_{\sim T^2} + \underbrace{\langle A_{0,a}^2\rangle_{\rm NP}}_{\sim T^0}\,; \qquad \mathbf{\langle A_{0,a}^2\rangle_{\rm NP} = \frac{(N_c^2-1)Tm_G^2}{8\pi m_D} \sim T^0}\,.$$

Adding perturbative and non perturbative contributions:

$$-2\log \mathbf{L} = \frac{g^2\langle A_{0,a}^2\rangle_{\rm NP}}{2N_cT^2} = \underbrace{-\frac{N_c^2-1}{2N_c}\frac{g^2 m_D}{4\pi T}}_{\rm Pert.\sim\log(T)} + \underbrace{\frac{g^2\langle A_{0,a}^2\rangle_{\rm NP}}{2N_cT^2}}_{\textbf{Non Pert.}\sim 1/T^2} \equiv a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,.$$

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Non Perturbative contributions in the Free Energy

Correlation functions of Polyakov loops define the free energy of a heavy $\bar{q}q$ pair (O.Kaczmarek et al, PLB543(2002)):

$$e^{-F_{q\bar{q}}(\vec{x},T)/T+c(T)} = \frac{1}{N_c^2}\langle \mathrm{tr}_c \Omega(\vec{x})\, \mathrm{tr}_c \Omega^\dagger(\vec{0})\rangle \,.$$

Pert. evaluation of Free Energy $\Rightarrow$ Expand Ω in powers of gA_0 and compute correlation functions:

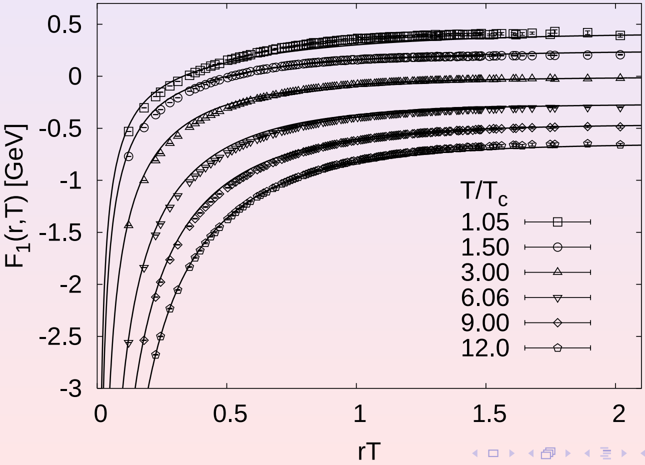
$$\langle A_{0,a}(\vec{x}) A_{0,b}(\vec{y})\rangle = \delta_{ab} T \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \underbrace{D_{00}(\vec{k})}_{D_{00}^{\rm P}+D_{00}^{\rm NP}} \,.$$

At leading order ($\mathcal{O}(g^2)$) and next to leading order ($\mathcal{O}(g^3)$):

$$F_1(r,T) = -\frac{N_c^2-1}{2N_c}\left(\frac{g^2}{4\pi r} + \frac{1}{N_c^2-1}\frac{g^2\langle A_{0,a}^2\rangle^{\rm NP}}{T}\right) e^{-\mathbf{m_D r}}$$

$$-\frac{N_c^2-1}{2N_c}\frac{g^2 m_D}{4\pi} + \frac{1}{2N_c}\frac{g^2\langle A_{0,a}^2\rangle^{\rm NP}}{T}$$

Singlet Free Energy $N_c = 3, N_f = 0$
Lattice data (O. Kaczmarek PRD70 (2004)) vs NP model



Assymptotic limits

Taking the assymptotic limits:

- $\underline{\mathbf{T \to 0}}$: $F_1(r,T) \overset{T\to 0}{\sim} -\frac{N_c^2-1}{2N_c}\frac{g^2}{4\pi}\frac{1}{r} + \underbrace{\frac{g^3\langle A_{0,a}^2\rangle^{NP}}{2N_c}}_{\equiv\sigma} r \equiv V_{q\bar{q}}(r)\,.$

  $V_{q\bar{q}}(r)$ well known from lattice: S.Necco NPB622(2002).

- $\underline{\mathbf{r \to \infty}}$: $F_\infty(T) = F_1(r\to\infty, T) = -\frac{N_c^2-1}{2N_c}\frac{g^2 m_D}{4\pi} + \frac{g^2\langle A_{0,a}^2\rangle^{NP}}{2N_c T}\,.$

  $L(T) = e^{-F_\infty(T)/2T}$ also known from lattice: O.Kaczmarek.

From a fit of $V_{q\bar{q}}$ at $T = 0$ ($F_1(r, T = 0)$):

$$\sigma = (0.42(1)\,\mathrm{GeV})^2 \Longrightarrow g^2\langle A_{0,a}^2\rangle^{NP} = (0.82(2)\,\mathrm{GeV})^2\,.$$

From a fit of the Polyakov loop ($F_1(r=\infty, T)$):

$$a_{\mathrm{NP}} = (0.49(4)\,\mathrm{GeV})^2 \Longrightarrow g^2\langle A_{0,a}^2\rangle^{NP} = (0.84(4)\,\mathrm{GeV})^2\,.$$

These values agree with $\frac{1}{4}g^2\langle A_{\mu,a}^2\rangle_{T=0} = (0.8 - 1.8\,\mathrm{GeV})^2$.

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Non perturbative contribution to the Trace Anomaly

Our model assumes the leading NP contribution to be encoded in the $A_{0,a}$ field. Taking $A_{i,a} = 0$:

$$\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle^{NP} = 2 \langle \partial_i A_{0,a} \partial_i A_{0,a} \rangle^{NP} = -6 m_D^2 \langle A_{0,a}^2 \rangle^{NP} \sim T^2 \, .$$

It reproduces the thermal behaviour of the interaction measure:

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G^a_{\mu\nu})^2 \rangle = \underbrace{(\mathrm{Pert.})}_{\sim T^4} - \underbrace{3g^2 \langle A_{0,a}^2 \rangle^{NP} \frac{\beta(g)}{g} T^2}_{\sim T^2} \, .$$

Values of the dimension two gluon condensate from a fit of:

Observable	$\mathbf{g^2 \langle A_{0,a}^2 \rangle_{NP}}$
Polyakov loop	$(3.22 \pm 0.07 T_c)^2$
Heavy $\overline{q}q$ free energy	$(3.33 \pm 0.19 T_c)^2$
Trace Anomaly	$(2.86 \pm 0.24 T_c)^2$

Chiral Quark Models at Finite T

- Chiral Quark Models $\longrightarrow$ Dynamics of QCD at low energies (low temperatures).
- Chiral Perturbation Theory $\longrightarrow$ Suppose the non-vanishing of chiral condensate. It cannot describe the QCD phase transition.
- K. Fukushima, PLB591, 277 (2004). W. Weise et al. PRD73, 014019 (2006), S.K. Ghosh et al. PRD73, 114007 (2006) Minimal coupling of Polyakov loop (analogy with chemical potential). **Mean field approximation**.
- E.Megías, E.Ruiz Arriola and L.L.Salcedo, **PRD74**: 065005 (2006). **Quantum and local polyakov loop**

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Standard Treatment of Chiral Quark Models at Finite T

E. Ruiz Arriola, Chr. Christov, K. Goeke, PLB 1989, APP 1990
The **standard rule** (to pass from $T=0$ to $T\neq 0$) is:

$$\int \frac{dk_0}{2\pi} F(\vec{k},k_0) \to iT \sum_{n=-\infty}^{\infty} F(\vec{k}, i\omega_n) \, .$$

The chiral condensate at one loop is:

$$\langle \overline{q}q \rangle^* = 4MT \mathrm{Tr}_c \sum_{\omega_n} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + k^2 + M^2} \, , \quad \omega_n = 2\pi T(n+\frac{1}{2}) \, .$$

After application of the Poisson's summation formula:

$$\langle \overline{\mathbf{q}}\mathbf{q} \rangle^* \stackrel{\text{Low T}}{\sim} \langle \overline{q}q \rangle - \frac{N_c}{2} \sum_{n=1}^{\infty} (-1)^n \left(\frac{2MT}{n\pi} \right)^{3/2} e^{-nM/T} \, .$$

Interpretation: Consider the fermionic propagator at low T

$$S(\vec{x},T) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot x}}{\not{k} - M} \sim e^{-M/T}$$

(exponential suppression of a single quark).
$\langle \overline{q}q \rangle^*$ can be written in terms of Boltzmann factors with $M_n = nM$:

$$\langle \overline{q}q \rangle^* = \langle \overline{q}q \rangle + \mathbf{O}_q e^{-M/T} + \mathbf{O}_{qq} e^{-2M/T} + \dots$$

Problem: When temperature rises every 1,2,3, ... quark state is generated.
Another problem: In Chiral Perturbation Theory:

$$\langle \overline{\mathbf{q}}\mathbf{q} \rangle^* |_{\text{ChPT}} = \langle \overline{q}q \rangle \left(1 - \frac{T^2}{8f_\pi^2} - \frac{T^4}{384 f_\pi^4} + \cdots \right) , \quad f_\pi^2 \sim N_c \,.$$

Finite temperature corrections are N_c **suppressed**.

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Minimal coupling of the Polyakov loop

Constituent Quark model:

$$\mathcal{L}_{\mathrm{QC}} = \overline{q}\, \mathbf{D}\, q \,, \qquad \mathbf{D} = \not{\partial} + \not{V}^f + \not{A}^f + MU^{\gamma_5} + \hat{m}_0$$

Consider the minimal coupling of the gluons in the model:

$$V_\mu^f \longrightarrow V_\mu^f + gV_\mu^c \,, \quad V_\mu^c = \delta_{\mu 0} V_0^c$$

Covariant derivative expansion (E. Megías et al. PLB563(2003), PRD69(2004), Oswald and Dyakonov PRD (2004)).

$$\mathcal{L}(x) = \sum_n \mathrm{tr}[f_n(\Omega(x))\mathcal{O}_n(x)] \,, \qquad \Omega(\vec{x}, x_0) = \mathbb{P}\, e^{i\int_{x_0}^{x_0+\beta} dx_0' V_0^c(\vec{x}, x_0')}$$

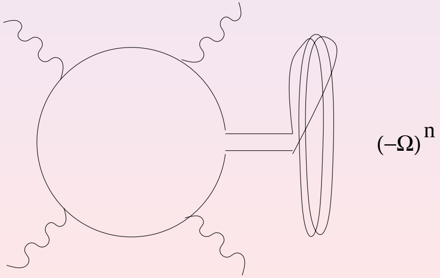
Ω enters in: $\hat{\omega}_{\mathbf{n}} = \mathbf{2}\pi\mathbf{T}(\mathbf{n} + \mathbf{1}/\mathbf{2} + \hat{\nu}) \,, \qquad \Omega = e^{i2\pi\hat{\nu}}$

The rule to pass from $T = 0$ to $T \neq 0$ is:

$$\tilde{F}(x;x) \to \sum_{n=-\infty}^{\infty} (-\Omega(\vec{x}))^n \tilde{F}(\vec{x}, x_0 + n\beta; \vec{x}, x_0) \,.$$

The quark condensate writes:

$$\langle \overline{q} q \rangle^* = \sum_n \frac{1}{N_c} \langle \mathrm{tr}_c (-\Omega)^n \rangle \langle \overline{q}(n\beta) q(0) \rangle \,.$$



Issues

1. QCD and Trace Anomaly
   - Trace Anomaly
   - Power temperature corrections
   - Scale invariance and confinement
2. Dimension Two Gluon Condensate
   - Polyakov loop and dimension two gluon condensate
   - Power temperature corrections
   - Non Perturbative model
   - Non Perturbative contributions in the Free Energy
   - Non perturbative contribution to the Trace Anomaly
3. Chiral Quark Models at Finite T
   - Standard Treatment
   - Coupling the Polyakov loop
   - Group averaging
4. Conclusions

Peierls-Yoccoz projection on color singlets

- We introduce a colour source (Polyakov loop).
- We obtain the projection onto the color neutral states by integrating over the A_0 field.
- In Quenched approximation: Group integration in SU(N_c).

$$\langle \mathrm{tr}_c(-\Omega)^n \rangle \equiv \int_{\mathrm{SU}(N_c)} D\Omega \, \mathrm{tr}_c(-\Omega)^n = \begin{cases} N_c \,, & n = 0 \\ -1 \,, & n = \pm N_c \\ 0 \,, & \text{otherwise} \end{cases}$$

There is only contribution from $n = 0, \pm N_c$.

$$\langle \bar{\mathbf{q}}\mathbf{q} \rangle^* \overset{\text{Low T}}{\sim} \langle \bar{q}q \rangle + 4 \left(\frac{MT}{2\pi N_c} \right)^{3/2} e^{-N_c M/T} \,.$$

The N_c suppression is consistent with ChPT.

- Beyond the Quenched approximation:

$$Z = \int DUD\Omega\, e^{-\Gamma_G[\Omega]}\, e^{-\Gamma_Q[U,\Omega]}$$

For any observable: $\langle \mathcal{O} \rangle^* = \frac{1}{Z} \int DUD\Omega\, e^{-\Gamma_G[\Omega]}\, e^{-\Gamma_Q[U,\Omega]}\, \mathcal{O}$.

1. $\int$ **DU**: Saddle point approximation.
2. $\int$ **D**Ω:
   - Analytically $\longrightarrow$ Expand the exponents and compute correlation functions of Polyakov loops:

$$\int D\Omega\, \mathrm{tr}_c \Omega(\vec{x})\, \mathrm{tr}_c \Omega^{-1}(\vec{y}) = e^{-\sigma|\vec{x}-\vec{y}|/T}$$

   - Numerically $\longrightarrow$ Consider the Polyakov gauge.

Analytical results in the Unquenched Theory

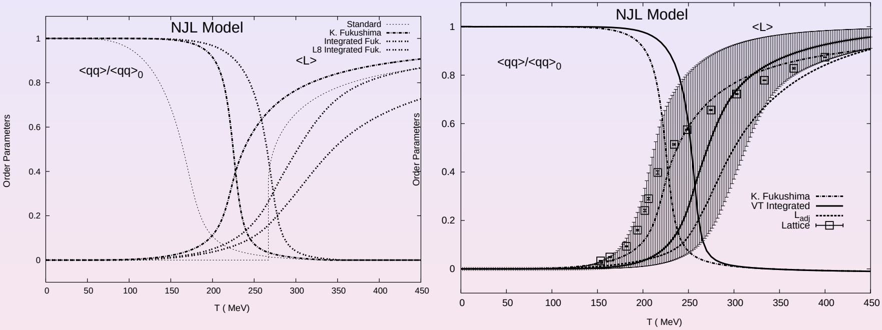
In the NJL model with Polyakov loop:

$$\langle \overline{q}q \rangle^* \overset{\text{Low T}}{\sim} \langle \overline{q}q \rangle + \frac{N_f V}{\pi^3} (MT)^3 e^{-2M/T} + \mathcal{O}(e^{-N_c M/T})$$

$$L \equiv \left\langle \frac{1}{N_c} \mathrm{tr}_c \Omega \right\rangle \overset{\text{Low T}}{\sim} \frac{N_f}{N_c} \frac{V}{T} \sqrt{\frac{M^3 T^5}{2\pi^3}} e^{-M/T} + \mathcal{O}(e^{-2M/T})$$

Taking into account the quark binding effects:

$$\mathcal{O}_q^* = \mathcal{O}_q + \sum_{m_\pi} \mathcal{O}_{m_\pi} \frac{1}{N_c} e^{-m_\pi/T} + \sum_B \mathcal{O}_B \, e^{-M_B/T} + \dots$$



Polyakov “cooling” : The condensate does not change at low temperatures.

Conclusions:

- Trace anomaly, like other thermal observables in QCD (Polyakov loop, heavy $\overline{q}q$ free energy, pressure, energy density, entropy density), has a non perturbative behaviour near and above T_c characterized by power corrections in T.
- We propose a simple model to describe this behaviour. Non perturbative contributions come from the dimension two gluon condensate $\langle A_0^2 \rangle_{\mathrm{NP}}$. $\langle A_0^2 \rangle_{\mathrm{NP}}$ can be choosen to fit thermal observables. Its value agrees for all of them.
- Chiral quark models at finite temperature have much better properties when the Polyakov loop (colour source) is projected a la Peierls-Yoccoz onto singlet colour states.