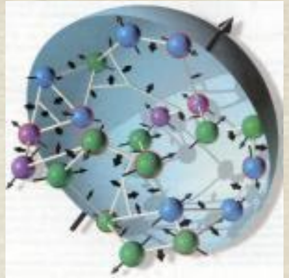
Some aspects of the chiral expansion for Generalized Parton Distributions



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in collaboration with

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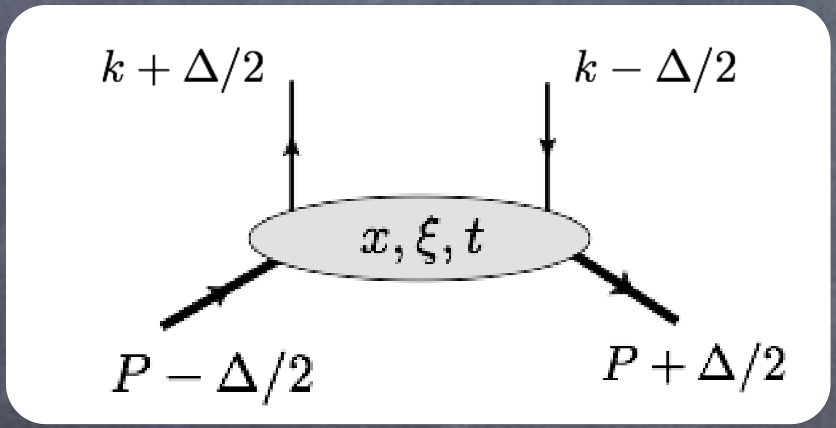
Hadrons and Chiral Symmetry

in honor of Professor Klaus Goeke

Generalized Parton Distribution

light-cone: $n^2=0$, $a_\mu n^\mu \equiv a_+$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x P_+} \langle P+\Delta/2|\bar{q}(\lambda n)\gamma_+ q(-\lambda n)|P-\Delta\rangle =$$
$$\bar{N}(P')\left[\gamma_+ H^q(x,\xi,t) + \frac{i\sigma^{+\nu}\Delta_{\perp\nu}}{2m_N} E^q(x,\xi,t)\right] N(p)$$



$$P_+ \to \infty$$

$$k = (k_+ = xP_+,\ k_\perp \sim 0,\ k_- \sim 0)$$

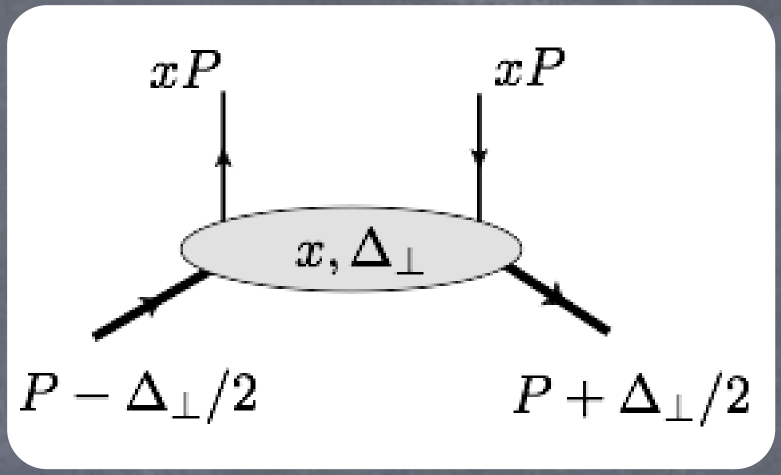
$$\Delta = (\Delta_+ = -2\xi P_+,\ \Delta_\perp \sim 0,\ \Delta_- \sim 0)$$

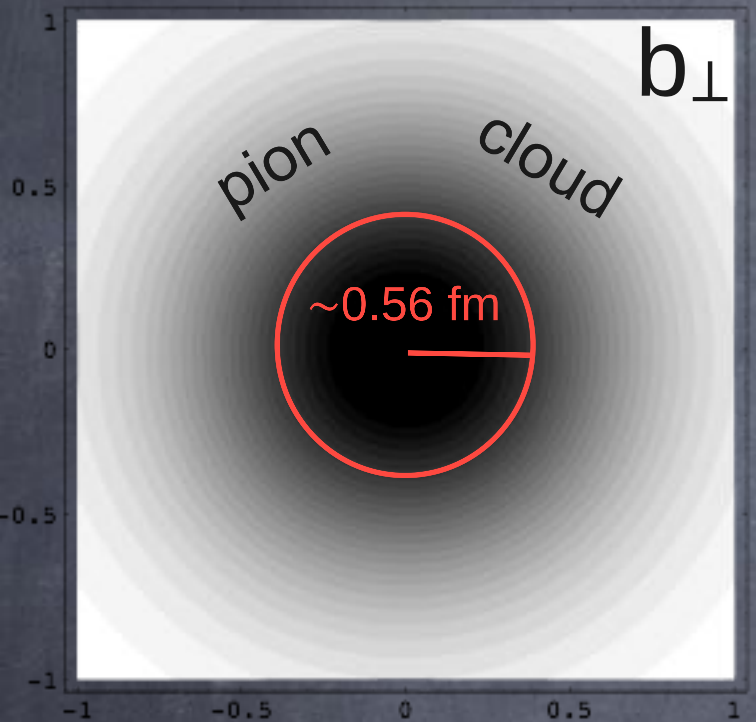
$$t = \Delta^2 < 0$$

Parton Distribution in Transverse Plane

$$\int \frac{d\Delta}{2\pi} e^{i(\Delta_\perp b_\perp)} H(x, \xi = 0, \Delta_\perp^2) = q(x, b_\perp)$$

Parton density
in $b_\perp$-plane (x dep.)





$$\int d^2 b_\perp q(x, b_\perp) = q(x)$$

longit. distrib

Burkardt '00, '02
Diehl '02

$$\langle r^2 \rangle = \frac{\int d^2 b b^2 q(x,b)}{\int d^2 b q(x,b)} = -4 \frac{1}{q(x)} \frac{d}{d\Delta_\perp^2} H(x, \Delta_\perp)|_{\Delta_\perp = 0}$$

Small values of $\Delta_\perp \leq$ 350MeV ↔ large $b_\perp \geq$ 0.56fm

$$\langle r^2 \rangle = \langle r^2 \rangle_{core} + \langle r^2 \rangle_{cloud} \simeq 0.31 \mathrm{fm}^2 + \langle r^2 \rangle_{cloud}$$

What can we learn about the large $b_\perp$-asymptotic from χPT approach?

QCD operator as a pion probe

$SU(2)_L \times SU(2)_R$
➡ $SU(2)_V$

Light-cone operator I=1

$$O^c(\lambda) = \bar{q}\left(\frac{1}{2}\lambda n\right)\tau^c\,\gamma_+ q\left(-\frac{1}{2}\lambda\, n\right)$$

Chiral matching

Double Distribution

$$O^c(\lambda) \simeq \varepsilon_{abc}\,F(\alpha,\beta) * \pi^a(\alpha\,\lambda n)\partial_+\pi^b(\beta\,\lambda n)$$

integral convolution $\{\alpha, \beta\}$

Chiral counting rules:

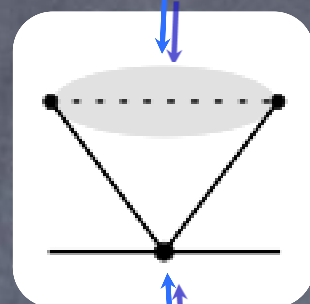
$$\partial \sim \mathcal{O}(p), \quad \lambda \sim \mathcal{O}(p^{-1}), \quad x \sim \xi \sim \mathcal{O}(p^0)$$

+ chiral Lagrangian describing pion dynamics

The point-like pion interactions ← singular contributions

$$m_\pi = 0$$

non-local operator $O(\lambda)$

FF  $\sim \omega_n \frac{\Delta_\perp^{2n}}{(4\pi F_\pi)^2}\delta^{(n-1)}(x) \sim \omega_n\, \delta^{(n-1)}(x/\epsilon)$

Local chiral vertex $\mathcal{L}_{2n}$

$$\epsilon \equiv \frac{\Delta_\perp^{2n}}{(4\pi F_\pi)^2} \ll 1$$

LO: pion PDF in the chiral limit

$$H(x,\Delta_\perp) = \mathring{q}(x) + \sum_{n>0}\omega_n \delta^{(n-1)}(x/\epsilon) = \mathring{q}(x) + H_s(x/\epsilon)$$

If $x \le \epsilon$ then the small parameter is compensated by δ-function:

$$\epsilon^n \delta^{(n-1)}(x) = \delta^{(n-1)}(x/\epsilon) \sim \mathcal{O}(1)$$

Pion loops and resummation

$$\epsilon \equiv \frac{\Delta_\perp^{2n}}{(4\pi F_\pi)^2} \ll 1$$

most singular term

$$H(x,\Delta_\perp) = \mathring{q}(x) + \sum_{n>0} \epsilon^n \delta^{(n-1)}(x) \left\{ \omega_n \ln^n[\Delta_\perp^2] + \omega'_n \ln^{n-1}[\Delta_\perp^2] + \ldots \right\}$$

Leading log

Next-to-Leading log

$$H_s(x/\epsilon, \ln\Delta_\perp^2) \simeq \sum_{n>0} (\epsilon \ln[\Delta_\perp^2])^n \delta^{(n-1)}(x)\omega_n$$

LL approximation

- parametrically justified as $\Delta_\perp^2 \to 0$
- model independent: chiral dynamics includes only F_π
- a lot of technical simplifications

Leading Log resummation: results

small chiral parameter $\epsilon = -\frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ln \frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ll 1$

$$H^{s}(x, \Delta_\perp) = -\theta(|x| \le \epsilon) \int_{|x|/\epsilon}^{1} \frac{d\beta}{\beta} \mathring{q}(x) W\left(\frac{x}{\beta\epsilon}\right)$$

$$\frac{\omega_n}{n} = \int_0^1 dz W(z) \qquad \omega_n = \sum_{A=0}^{[n/2]} \omega_{nA}$$

$$T_{\pi\pi}[s, \cos\theta] = \sum_l P_l(cos\theta) t_l(s)$$

$$t_{2A}(s) = \frac{s}{F_\pi^2} \sum_{n=2A}^{\infty} \omega_{nA} \left\{ \frac{-s \ln s}{(4\pi F_\pi)^2} \right\}^n$$

Recurrence relation: $\omega_{10} = 1$

$$\omega_{nA} = -\frac{1}{(n-1)} \sum_{i=1}^{n-1} \sum_{BC} \beta_{nA}[iB, (n-i)C] \, \omega_{iB} \, \omega_{(n-i)C}$$

$\beta_{nA}[iB, (n-i)C]$ one loop β-functions for the relevant couplings in the massless chiral Lagrangian

Simple realistic model for ω_n

$$\omega_n = a^n(1 + 0.306 e^{-(n-1)/b}), \quad a = 0.7657,\ b = 8$$

$$H_s(x, \Delta_\perp) \simeq -\frac{2}{3}(1 + 0.347)\theta(|x| \leq 3a\epsilon/8) \int_{\frac{8|x|}{3a\epsilon}}^{1} \frac{d\beta}{\beta} \mathring{q}(\beta) \sqrt{1 - \frac{8|x|}{3a\epsilon\beta}}$$

$$\epsilon = -\frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ln \frac{\Delta_\perp^2}{(4\pi F_\pi)^2} \ll 1$$

<u>Small-x:</u>

$$\mathring{q}(x) \sim 1/x^\omega, \quad \omega \sim 1/2$$

$$H(x, \Delta_\perp) \overset{x \to 0}{\sim} x^{-\omega}$$

Small-x: $\mathring{q}(x) \sim 1/x^{\omega}, \quad \omega \sim 1/2$

Small-t and small-x asymptotic (m$_\pi$=0)

$$H_s(x, \Delta_\perp) = \epsilon\delta(x) + ... \stackrel{\Sigma}{\rightarrow} \simeq const \left(\frac{\epsilon}{x}\right)^{\omega}$$

$$\epsilon \equiv \frac{\Delta_\perp^2}{\Lambda_\chi^2} \ln[\Lambda_\chi^2/\Delta_\perp^2] \ll 1$$

$$\mathring{q}(x) \sim x^{-\omega}$$

impact parameter $b_\perp \rightarrow \infty$

$$q(x, b_\perp) \sim \frac{\epsilon_b \delta(x)}{b_\perp^2} + ... \stackrel{\Sigma}{\rightarrow} \simeq \frac{const}{b_\perp^2} \left(\frac{\epsilon_b}{x}\right)^{\omega}$$

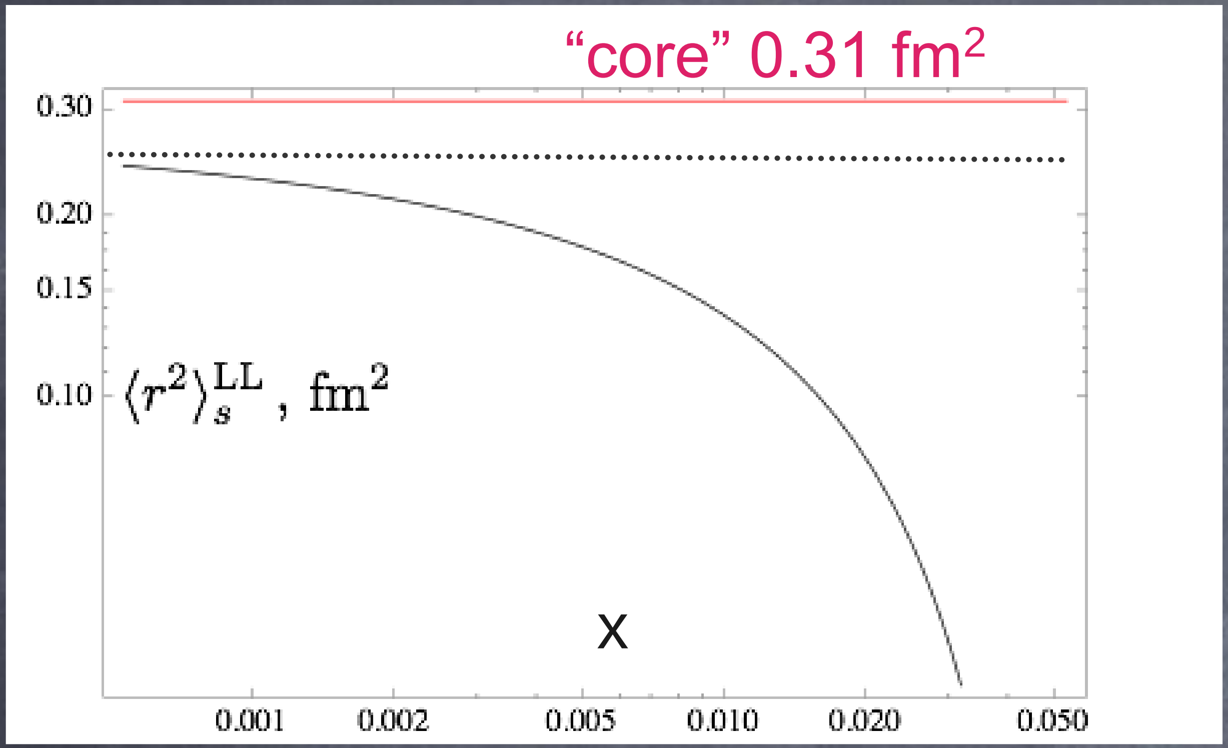
$$\epsilon_b \equiv \frac{\ln[\Lambda_\chi^2 b_\perp^2]}{b_\perp^2 \Lambda_\chi^2} \ll 1$$

The asymptotic is non-analytic with respect to chiral small parameter

The size of the pion

the eff. size of pion

$$\langle r^2\rangle_s = \frac{\int d^2b\, b^2 q(x,b)}{\int d^2b\, q(x,b)} = -4\frac{1}{q(x)}\frac{d}{d\Delta_\perp^2}H(x,\Delta_\perp)|_{\Delta_\perp=0}$$



$$\langle r^2\rangle_s^{\rm LL} \simeq \theta(x<x_0)\frac{\mathring{q}\left(\frac{x}{x_0}\right)}{\mathring{q}(x)}\times 0.82\,{\rm fm}^2$$

$$x_0 = 0.75\frac{m_\pi^2\ln[\Lambda_\chi^2/m_\pi^2]}{(4\pi F_\pi)^2}\simeq 0.052$$

$$\mathring{q}(x)\sim x^{-1/2}$$

$$\langle r^2\rangle_s^{\rm LL} \le r_{max}^2 \simeq 0.25\,{\rm fm}^2$$

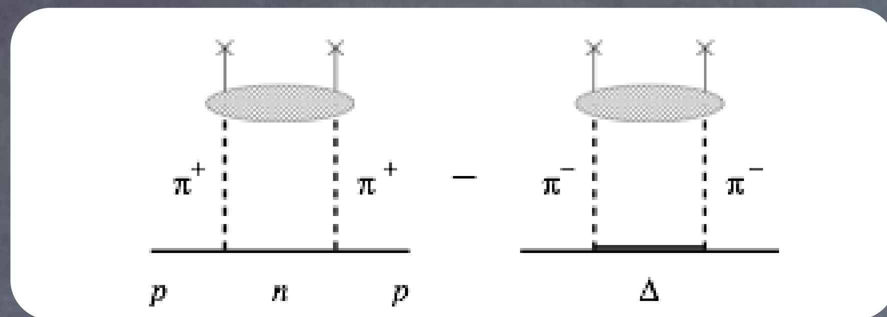
Charge radius

$$\langle r^2\rangle_V^{1loop} = -\frac{6l_6^r(\mu)}{F_\pi^2} + \underline{\langle r^2\rangle_V^{LL}} = 0.36{\rm fm}^2 + \underline{0.06{\rm fm}^2}$$

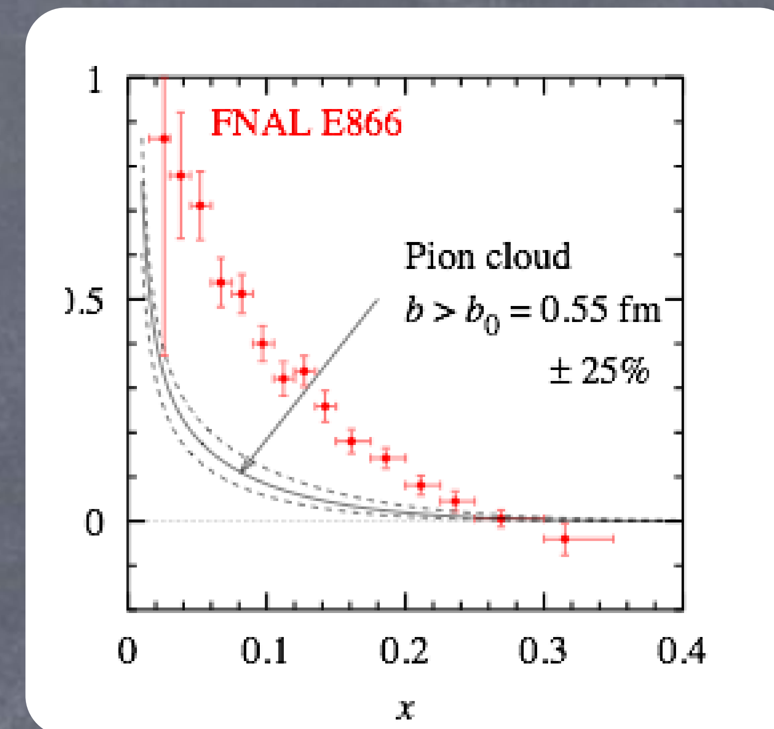
14%

Transverse distribution in nucleon

C.Weiss M.Strikman, '03/'08



$\bar{d} - \bar{u}$



Pion cloud is important at $x \ll m_\pi / M_N \quad x \sim 10^{-2}$

$b_\perp > b_\perp^{core} \simeq 0.55\text{fm}$

but at small values $x \sim \frac{m_\pi}{M_N} \frac{m_\pi^2}{\Lambda_\chi^2}$ the transverse size of pion is important!

then one has to perform the resummation discussed for the pion GPDs