

Baryons as solitons a historical perspective

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Chiral symmetry

massless Left and Right fermions are „independent”

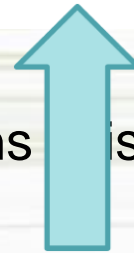
no massless fermions exist \rightarrow chiral symmetry is broken

explicit breaking by Higgs mechanism: $m \sim$ a few MeV

spontaneous breaking: $M \sim 300$ MeV



$$U = e^{i\vec{\tau} \cdot \vec{\pi}(x)/F_\pi}$$

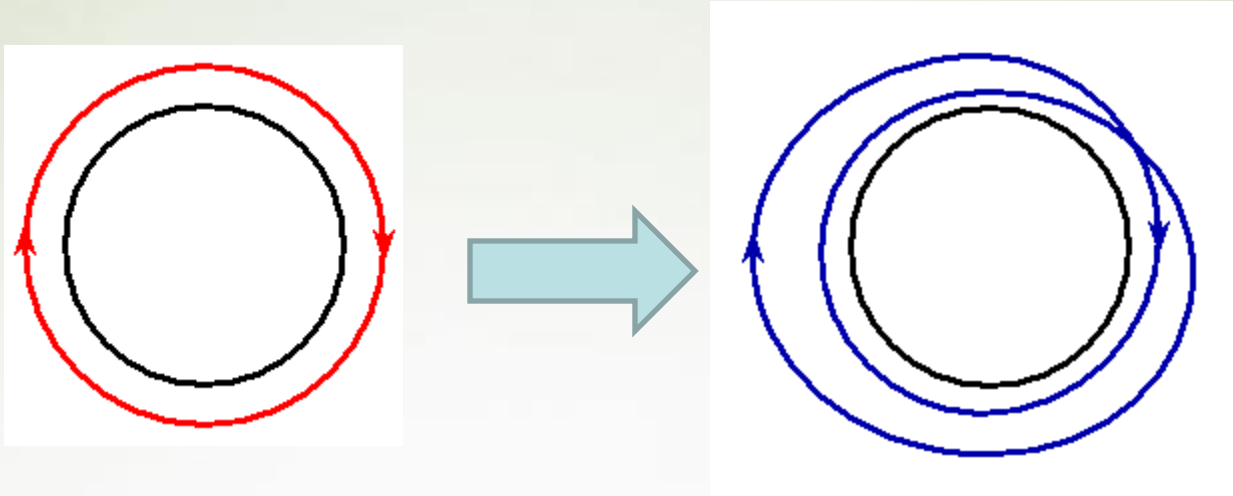


(nearly) massless Goldstone bosons exist: pions (kaons, eta)

this is a mapping of our space onto a group space

Something out of nothing^{*)}

winding number



Winding number can be defined for higher dimensional spheres. Of particular interest are mappings of 3-sphere (our 3 dim. space) onto an $SU(2)$ group which is also a 3 dimensional sphere

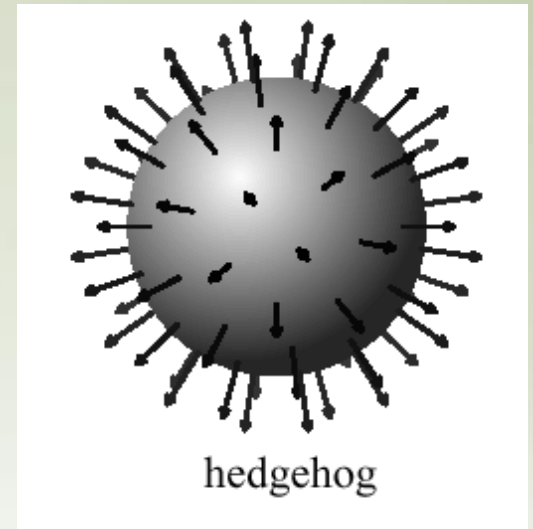
Winding number can be identified with a baryon number

^{*)} Andrzej Białas, opening speech at the Workshop on Skyrmions and Anomalies, 1987

Skyrme Model

$$U_0 = e^{i\vec{n}\cdot\vec{\tau}} P(r)$$

winding number is equal to $P(0)/\pi$



What all this has to do with physics?

Nonlinear sigma model describes selfinteraction of pions using U matrix

$$U = e^{i\vec{\tau}\cdot\vec{\pi}(x)/F_\pi}$$

Skyrme-Witten Lagrangian

$$\mathcal{L} = \frac{F_\pi^2}{4} \text{Tr} (\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} \left([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2 \right) + \Gamma_{\text{WZ}} + \dots$$

Stable soliton solutions exist, with function $P(r)$ that minimizes the energy

Chiral Quark Model

quark and meson fields coupled together, however there is no kinetic term for pions

$$\mathcal{L} = \bar{q}(x) (i\not{\partial} - M U \gamma^5(x)) q(x)$$

$$U = e^{i\vec{\tau} \cdot \vec{\pi}(x)/F_\pi}$$



$$\mathcal{L} = \frac{F_\pi^2}{4} \text{Tr} (\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} \left([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2 \right) + \Gamma_{\text{WZ}} + \dots$$

Chiral Quark Model - mesons

pi-pi scattering

Quark Models And Chiral Lagrangians.

[M. Praszalowicz](#), [G. Valencia](#)

Nucl.Phys.B341:27-49,1990.

The Low-energy expansion of the generalized SU(3) NJL model. [E. Ruiz Arriola](#)

Phys.Lett.B253:430-435,1991

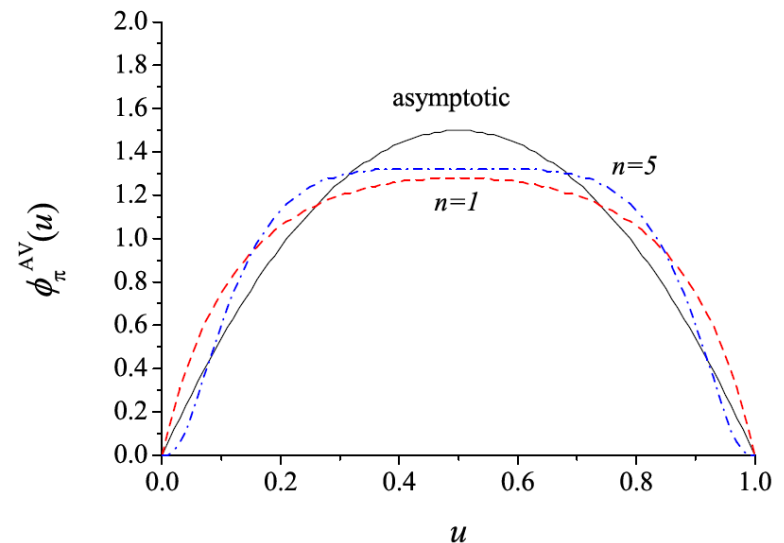
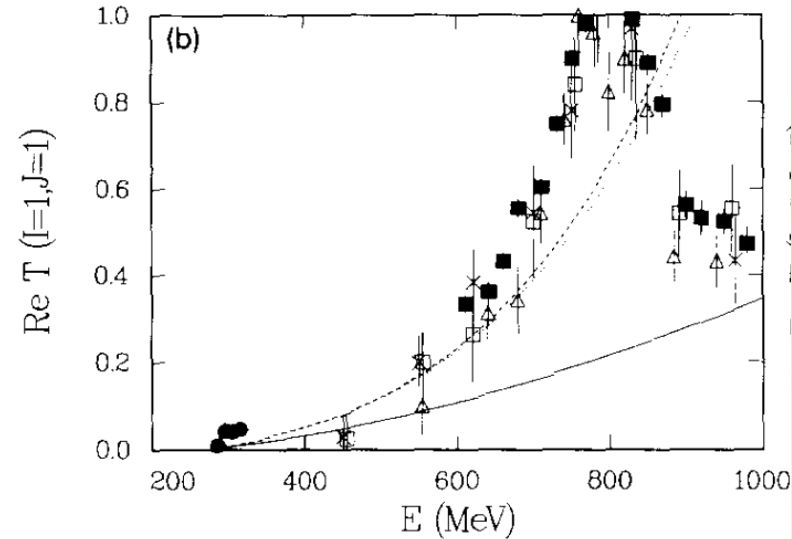
pion distribution amplitudes

Pion wave function from the instanton vacuum model. [V.Yu. Petrov](#), [P.V. Pobylitsa](#)
hep-ph/9712203

Pion light cone wave function in the nonlocal NJL model. Phys.Rev.D64:074003,2001

[M. Praszalowicz](#), [A. Rostworowski](#)

Stefanis, Bakulev, Mukhailov,
Broniowski, Arriola, Dorokhov ...

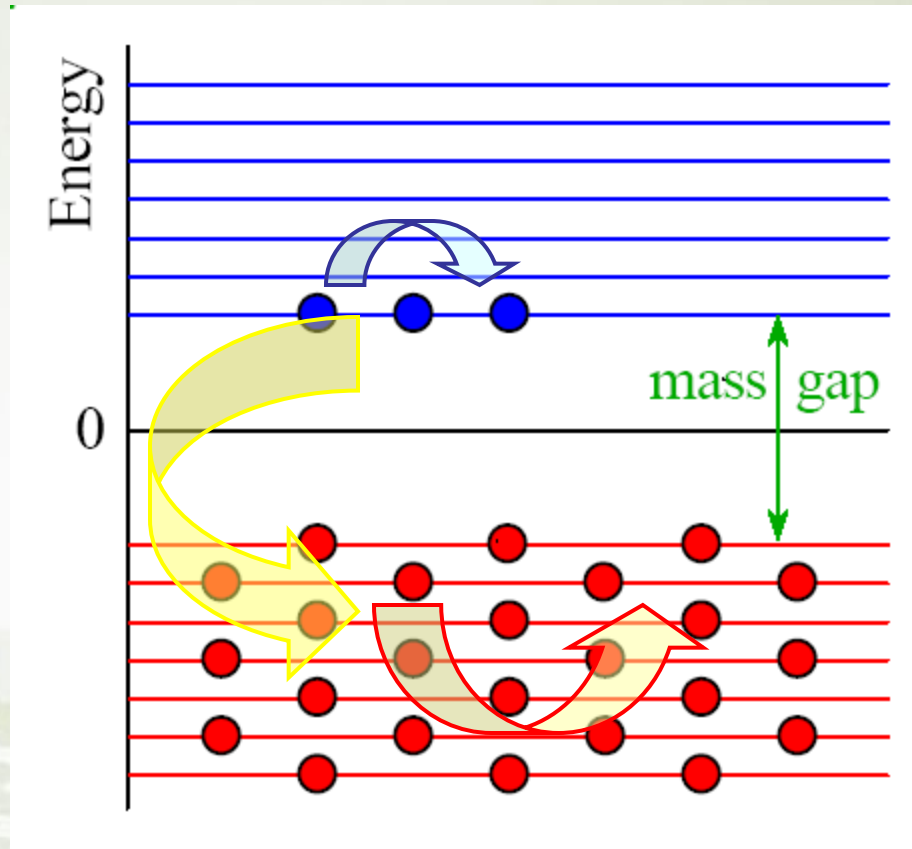


Chiral Quark Model - baryons

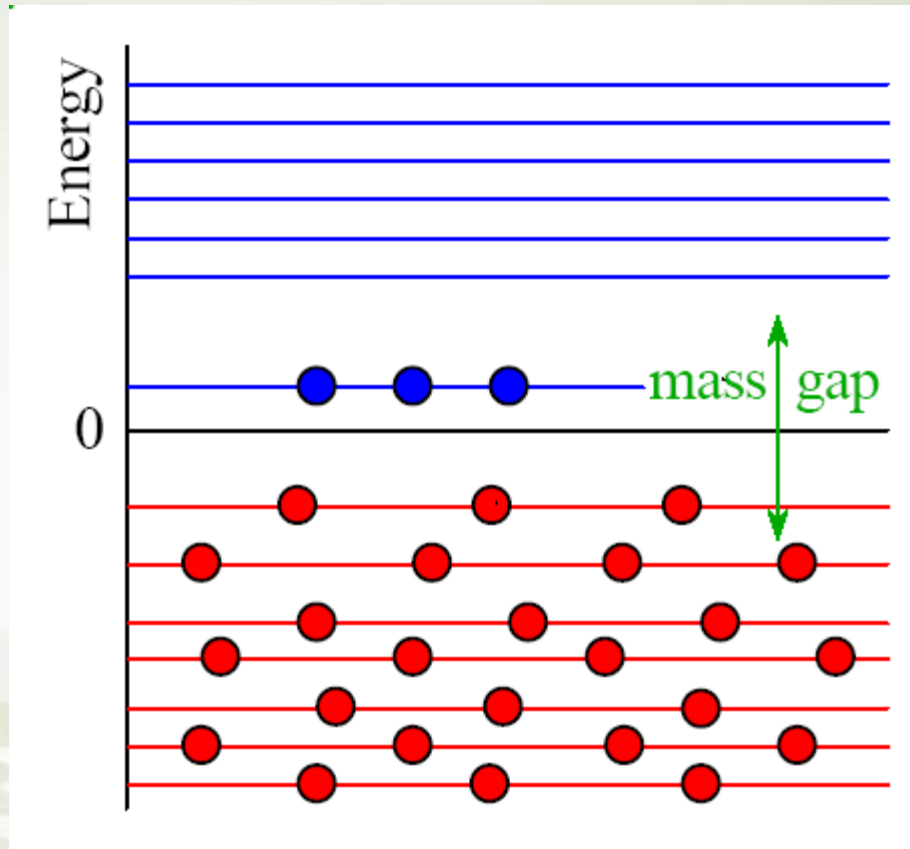
consider spectrum of the Dirac operator
in the presence of the topologically nontrivial meson field



Spectrum of the Dirac operator



Spectrum of the Dirac operator



for winding number = 1 one valence level appears,
however it is in a singlet flavor and spin state
→ no other quantum numbers apart from baryon number

Soliton with Valence Quarks in the Chiral Invariant Sigma Model.

[S. Kahana](#), [G. Ripka](#), [V. Soni](#) Nucl.Phys.A415:351-364,1984.

A Chiral Theory of Nucleons.

[Dmitri Diakonov](#), [V.Yu. Petrov](#), [P.V. Pobylitsa](#) Nucl.Phys.B306:809,1988

Sea Quark Effects In Nontopological Chiral Soliton Models And The Nambu-Jona-Lasinio Approach Phys.Lett.B214:312-316,1988

[T. Meissner](#), [E. Ruiz Arriola](#), [F. Grummer](#), [H. Mavromatis](#), [K. Goeke](#)

The Soliton of the Effective Chiral Action.

[H. Reinhardt](#), [R. Wunsch](#) Phys.Lett.B215:577,1988

Nucleon Mass And Nucleon Sigma Term.

[Dmitri Diakonov](#), [V.Yu. Petrov](#), [M. Praszalowicz](#) Nucl.Phys.B323:53,1989.

Solitons In The Nambu-Jona-Lasinio Model.

[T. Meissner](#), [F. Grummer](#), [K. Goeke](#) Phys.Lett.B227:296-300,1989.

A Chiral quark model of the nucleon.

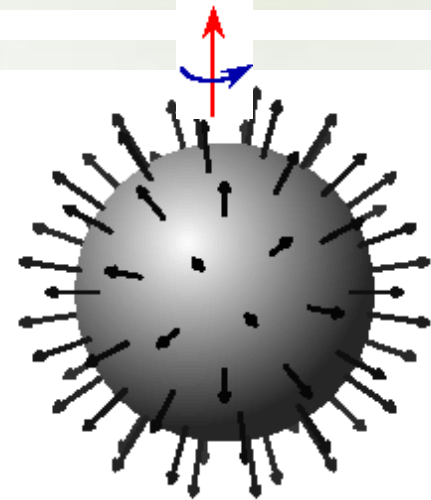
[M. Wakamatsu](#), [H. Yoshiki](#) Nucl.Phys.A524:561-600,1991.

Other quantum numbers

$U(x)$ and $AU(x)A^\dagger$ have the same energy

promote $A \rightarrow A(t)$ and quantize rotations as a symmetric top

$$\mathcal{H} = M_{sol} + \frac{1}{2I_1} J(J + 1)$$



hedgehog

A Chiral Theory of Nucleons.

[Dmitri Diakonov](#), [V.Yu. Petrov](#), [P.V. Pobylitsa](#)

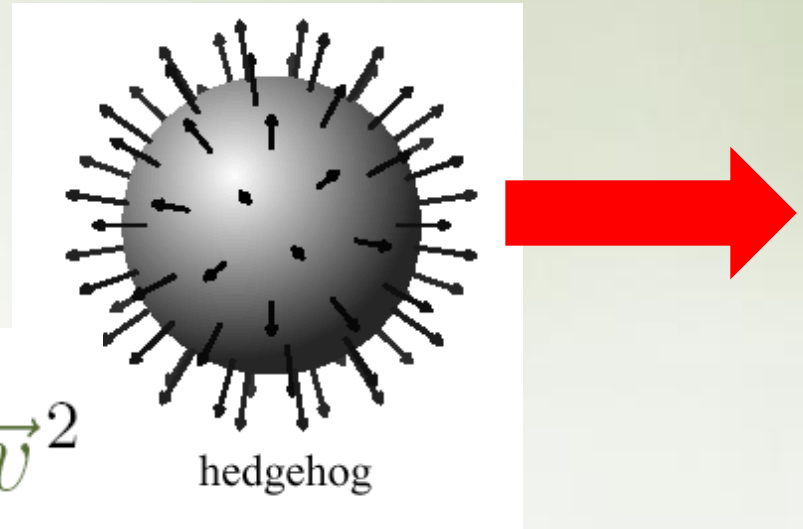
Nucl.Phys.B306:809,1988

Quantization of the Nambu-Jona-Lasinio soliton and the nucleon Delta splitting.

[K. Goeke](#), [A.Z. Gorski](#), [F. Gruemmer](#), [T. Meissner](#), [H. Reinhardt](#), [R. Wunsch](#)

Phys.Lett.B256:321-324,1991.

Other quantum numbers



$$\mathcal{L} = -M_{sol} + \frac{1}{2}M_{sol}\vec{v}^2$$

Pushing the Nambu-Jona-Lasinio soliton and the zero point energy.

[P.V. Pobylitsa](#), [E. Ruiz Arriola](#), [T. Meissner](#), [F. Grummer](#), [K. Goeke](#), [W. Broniowski](#)

J.Phys.G18:1455-1466,1992.

SU(3) case

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}} P(r) & 0 \\ 0 & 1 \end{bmatrix}$$

← commutes with λ_8

$$\mathcal{L} = -M_{sol} + \frac{I_1}{2} \sum_{i=1}^3 \Omega_i^2$$

Baryons as Solitons and Mass Formulae.

[E. Guadagnini](#) Nucl.Phys.B236:35,1984.

SU(3) Extension of the Skyrme Model.

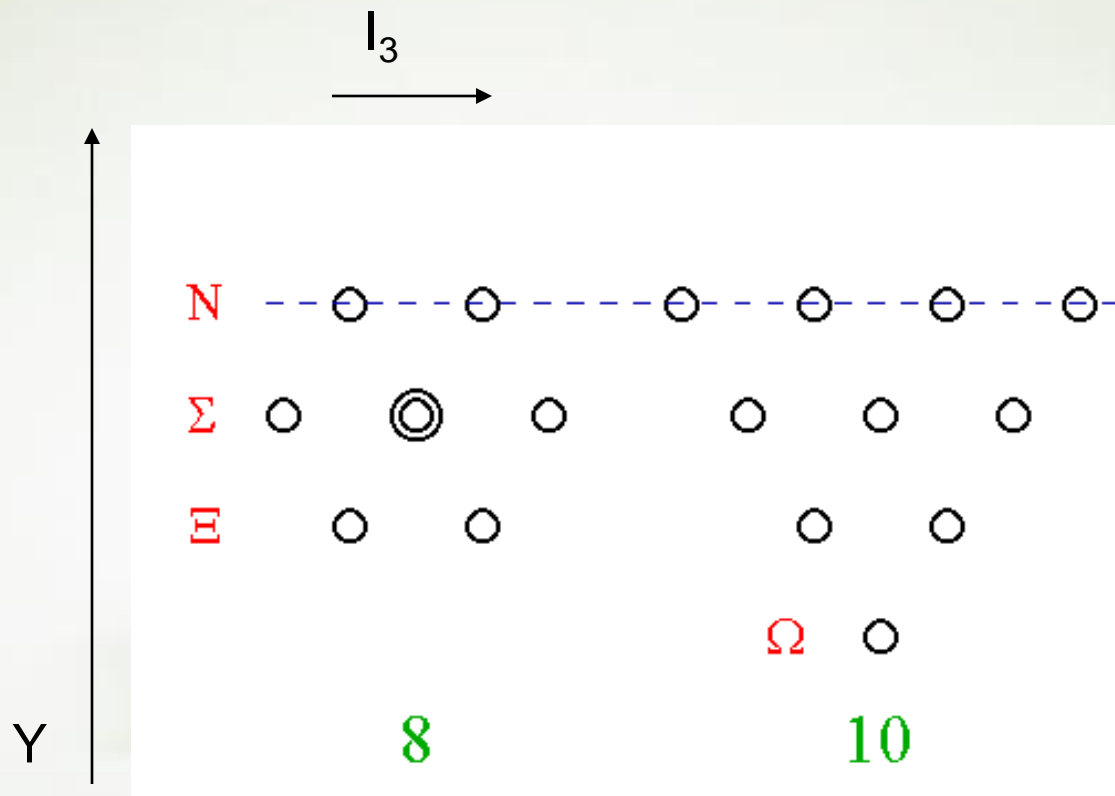
[Pawel O. Mazur](#), [Maciej A. Nowak](#), [Michal Praszalowicz](#) Phys.Lett.B147:137,1984.

Large N Baryons: Collective Coordinates Of The Topological Soliton In SU(3) Chiral Model.

[Sanjay Jain](#), [Spenta R. Wadia](#) Nucl.Phys.B258:713,1985

SU(3) case

Only representations containing hypercharge $Y = 1 = N_c / 3$ are allowed



The Strange chiral soliton of the Nambu-Jona-Lasinio model.

[H. Weigel](#), [Reinhard Alkofer](#), [H. Reinhardt](#) Phys.Lett.B284:296-302,1992.

Strange baryons as chiral solitons of the Nambu-Jona-Lasinio model.

[H. Weigel](#), [Reinhard Alkofer](#), [H. Reinhardt](#) Nucl.Phys.B387:638-674,1992.

The SU(3) Nambu-Jona-Lasinio soliton in the collective quantization formulation.

[A. Blotz](#), [Dmitri Diakonov](#), [K. Goeke](#), [N.W. Park](#), [V. Petrov](#), [P.V. Pobylitsa](#)

Nucl.Phys.A555:765-792,1993.

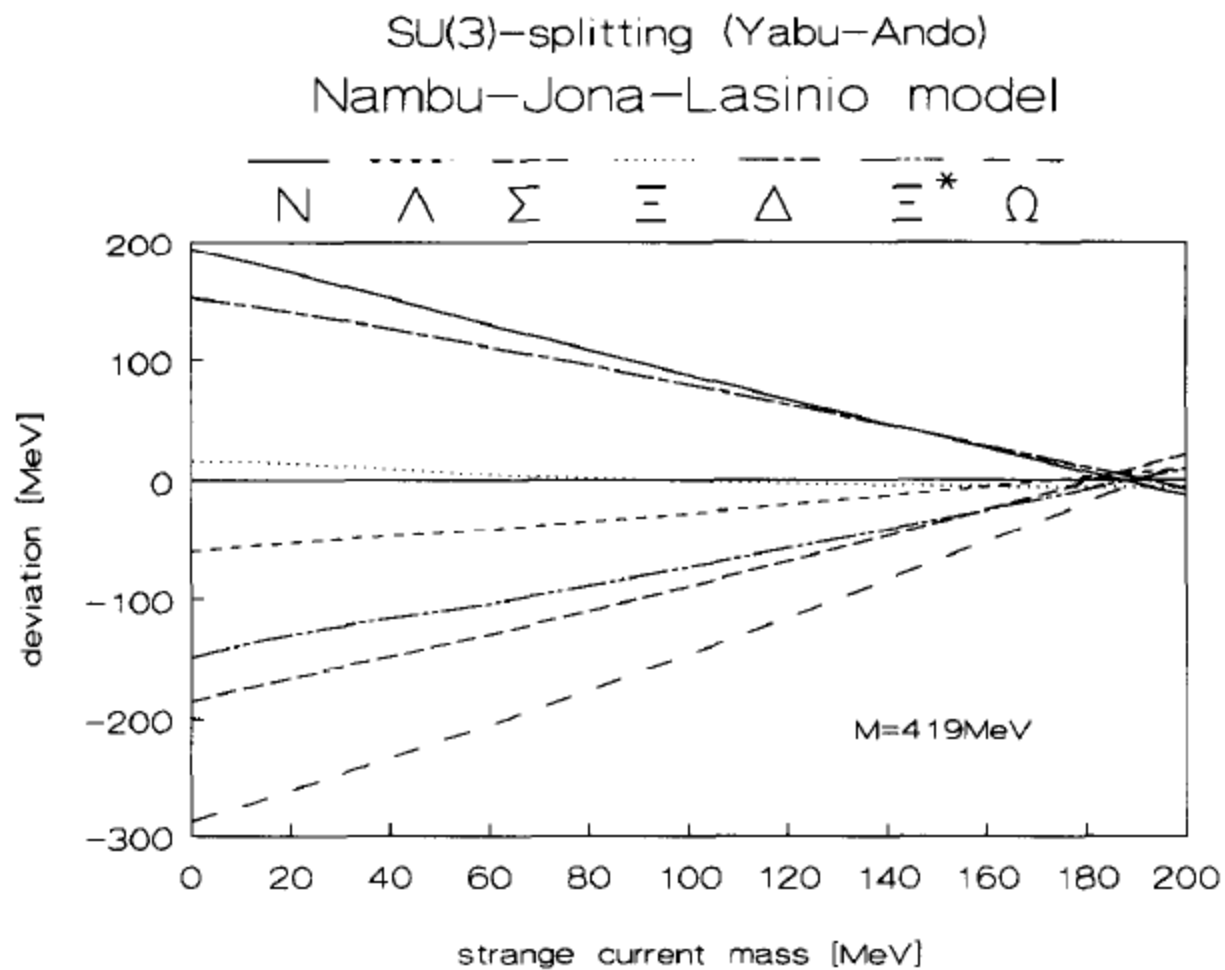


Fig. 5. The deviation of the theoretical mass from the experimental one is shown for the Yabu-Ando treatment and a constituent quark mass of $M = 419$ MeV.

Problem with g_A

The Axial form-factor in the Nambu-Jona-Lasinio model.
[T. Meissner](#), [K. Goeke](#) Z.Phys.A339:513-522,1991.

The value of g_A is $g_A = 0.8$ and hence too low in comparison with experiment. This is like in the Skyrme model and might change if one takes higher order cranking corrections into account. This point is presently under consideration.

Problem with g_A

The $g(A)$ problem in hedgehog soliton models revisited.

[M. Wakamatsu](#), [T. Watabe](#) Phys.Lett.B312:184-190,1993.

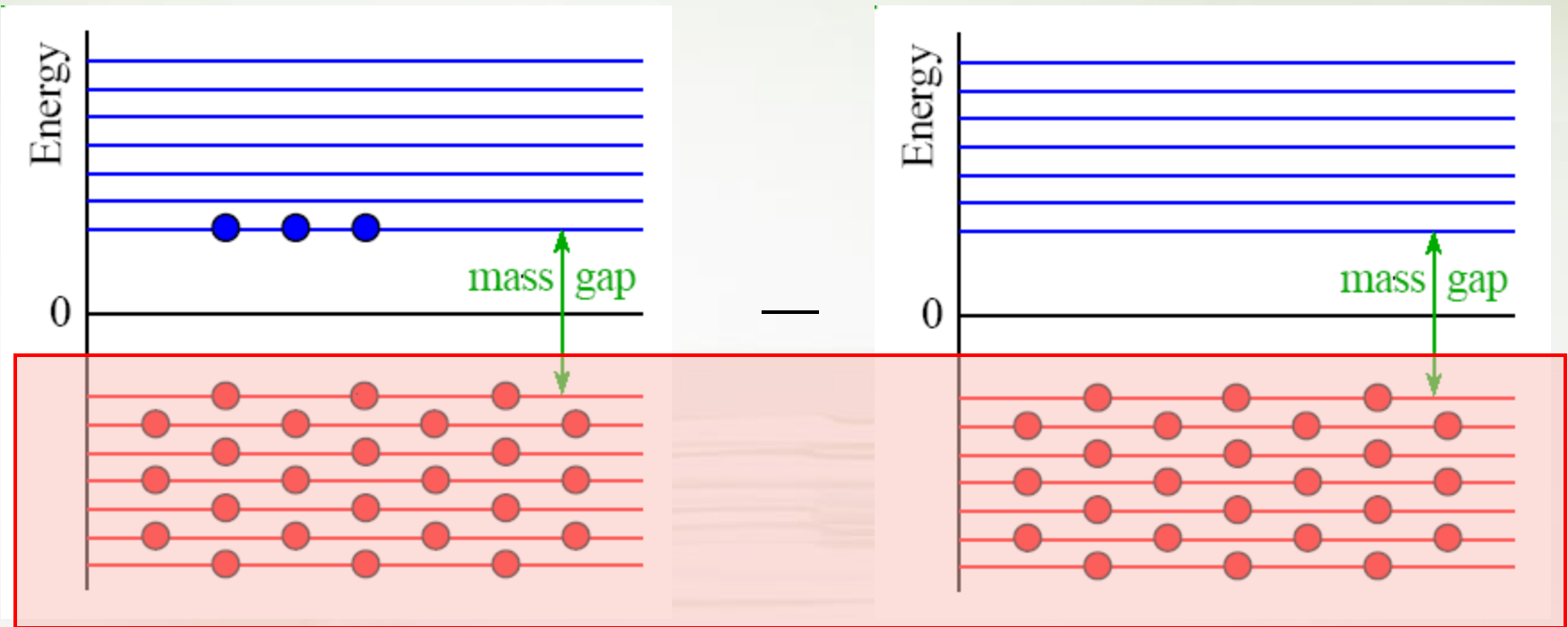
1/N(c) rotational corrections to $g(A)$ and isovector magnetic moment of the nucleon.

[C.V. Christov](#), [A. Blotz](#), [K. Goeke](#), [P. Pobylitsa](#), [V. Petrov](#), [M. Wakamatsu](#), [T. Watabe](#)
Phys.Lett.B325:467-472,1994.

$$\begin{aligned} \mathcal{O}_\alpha^\Gamma &= N_c D_{\alpha\beta} \sum_m \langle m | \lambda_\beta \Gamma | m \rangle \\ &- \frac{N_c}{4} \{D_{\alpha\rho}, \Omega_\beta\} \sum_{m,n} \frac{\langle m | \lambda_\rho \Gamma | n \rangle \langle n | \lambda_\beta | m \rangle + \langle m | \lambda_\beta | n \rangle \langle n | \lambda_\rho \Gamma | m \rangle}{\varepsilon_n - \varepsilon_m} \\ &- \frac{N_c}{4} [D_{\alpha\rho}, \Omega_\beta] \sum_{m,n} \frac{\langle m | \lambda_\rho \Gamma | n \rangle \langle n | \lambda_\beta | m \rangle - \langle m | \lambda_\beta | n \rangle \langle n | \lambda_\rho \Gamma | m \rangle}{\varepsilon_n - \varepsilon_m} \end{aligned}$$

Quark Model limit

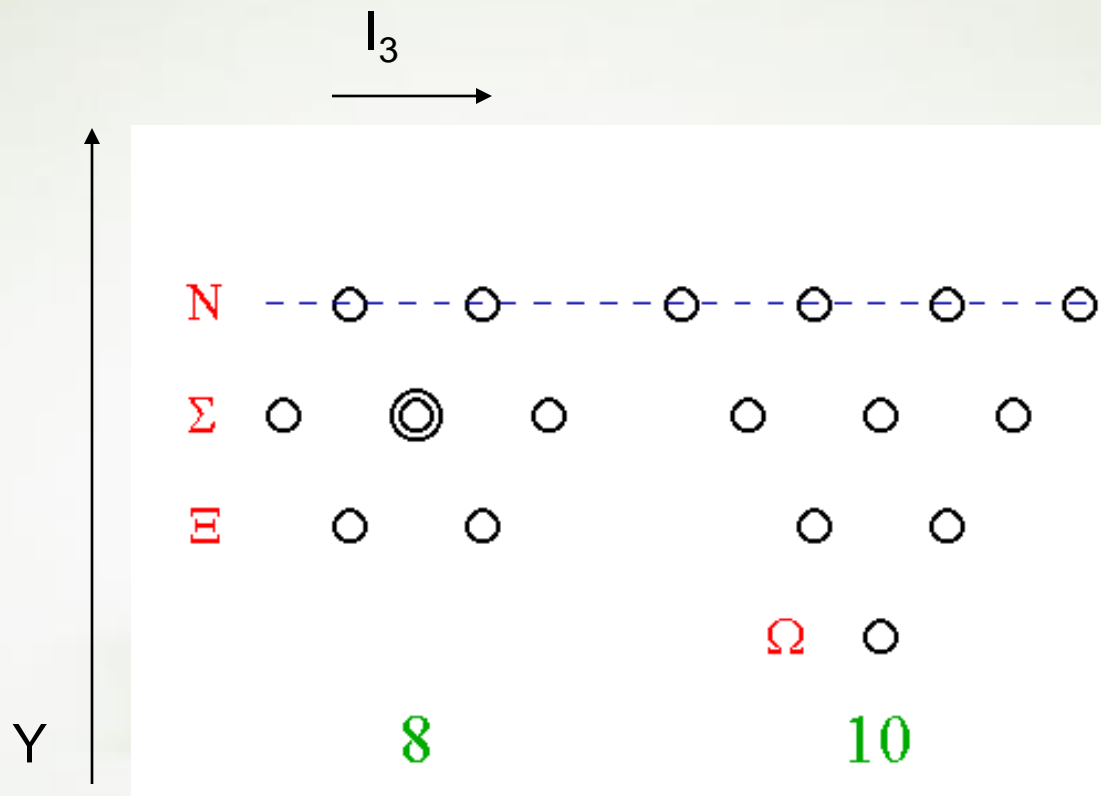
$$g_A = \frac{N_c + 2}{3}, \quad g_A^{(0)} = 1, \quad \frac{\mu_p}{\mu_n} = -\frac{3}{2}$$



in the NRQM limit only valence level contributes

SU(3) case

Only representations containing hypercharge $Y = 1 = N_c / 3$ are allowed



Theta +

Biedenharn, Dothan (1984):

$\Delta_{10^{-8}}^- \sim 600$ MeV from Skyrme model

MP (1987):

$M_{\Theta} = 1535$ MeV from Skyrme model
in model independent approach,
second order

Diakonov, Petrov, Polyakov (1997):

χ QM - model independent approach,
 $1/N_c$ corrections $\rightarrow M_{\Theta} = 1530$ MeV
small width < 15 MeV ! 

today we know that if Θ exists $\Gamma_{\Theta} < 1$ MeV

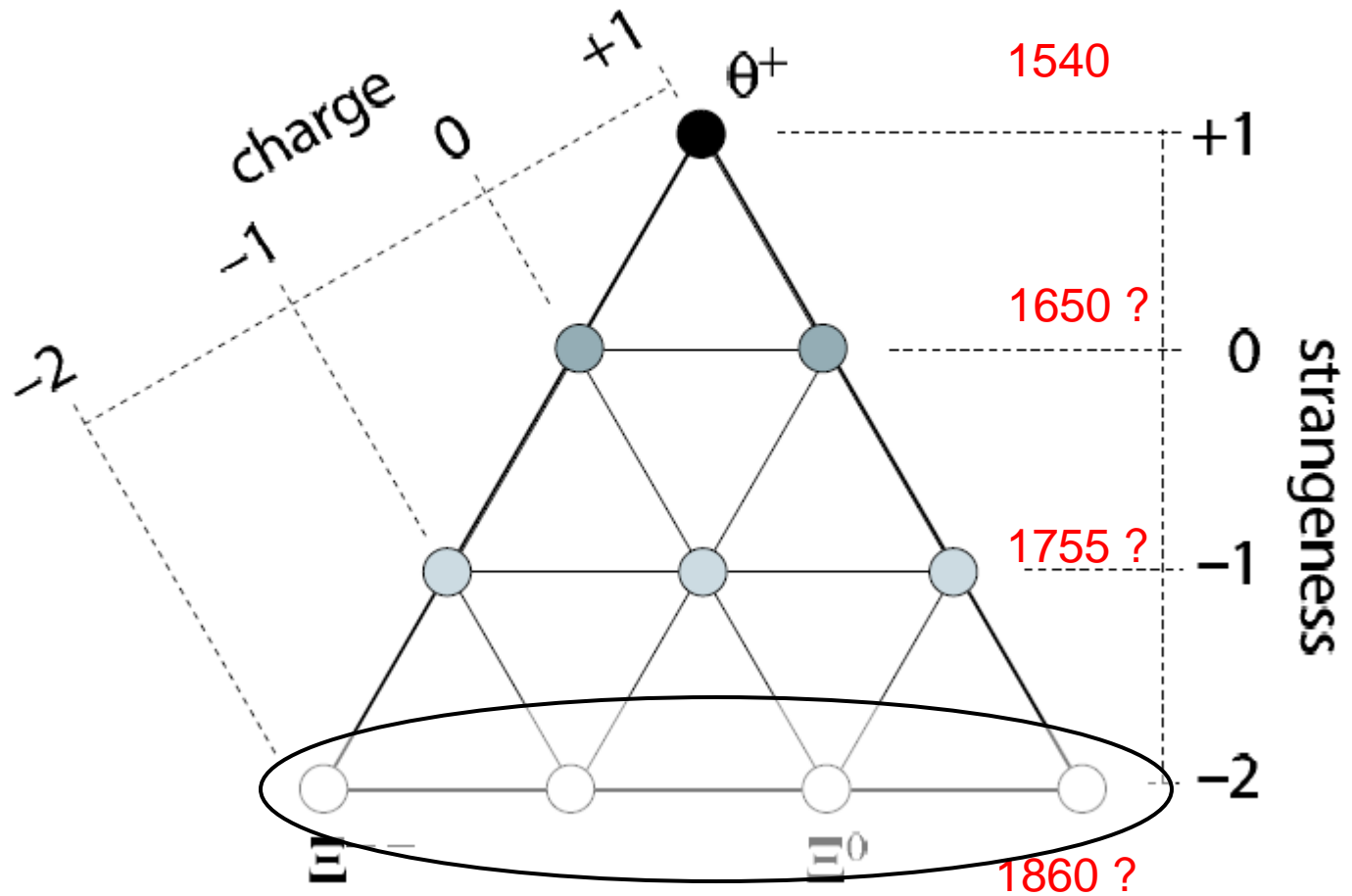
Antidecuplet decay width

$$\Gamma_{B_1 \rightarrow B_2 \varphi}^{(\overline{10})} = \frac{G_{\overline{10}}^2}{8\pi M_1 M_2} \times C^{(\overline{10})}(B_1, B_2, \varphi) \times p_\varphi^3$$

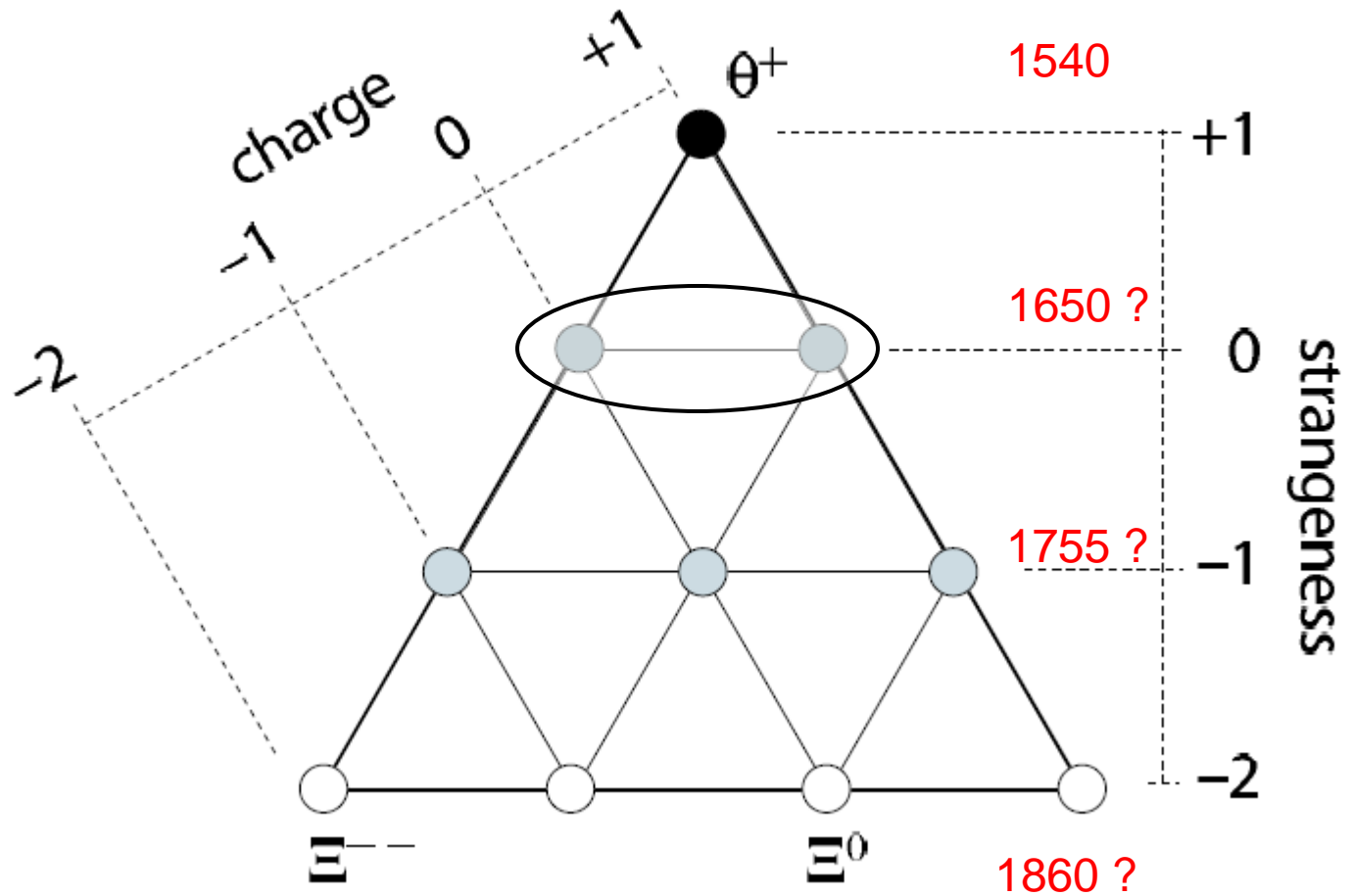
In NRQM limit:

$$G_{\overline{10}} = G_0 - G_1 - \frac{1}{2}G_2 = 0$$



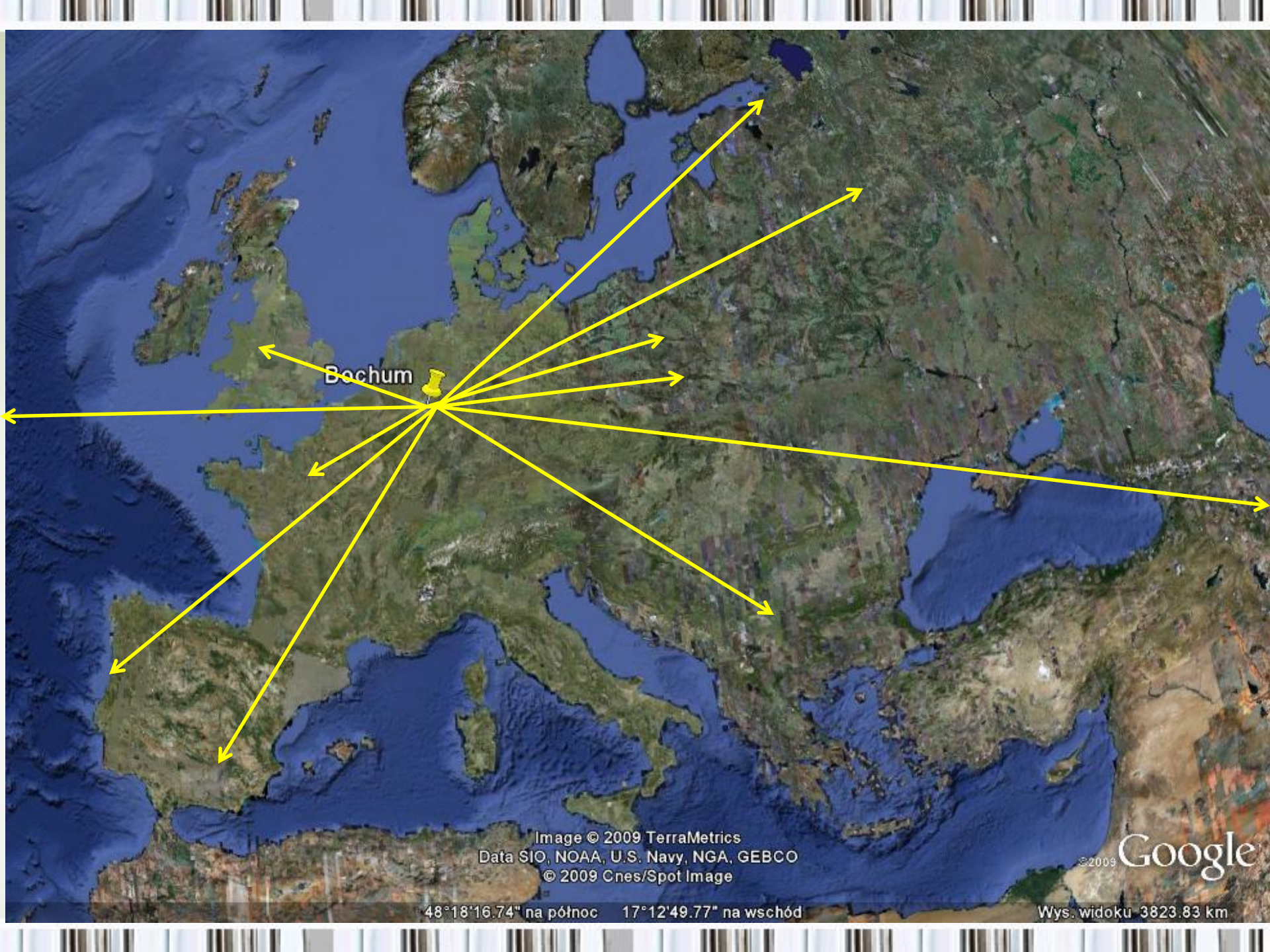


NA 49
NA 61 ?



Chiral soliton in Bochum

- **vector mesons:** Arriola, Schuren, Doring
 - **in medium properties:** Arriola, Christov, Nikolov, Broniowski, Schaldach,
 - **form factors:** Grummer, Gorski, T. Meissner, Nikolov, Bergmann, Christov, Watabe, Silva, Urbano
 - **strangeness:** Blotz, Pobylitsa, Diakonov, Kim, Sieber, Schneider
 - **spin:** Blotz, Polyakov, Metz, Schweitzer, Weiss, Lee, Menzel, S.Messner, Schlegel
 - **gradient expansion:** Arriola, Kim, Franz
 - **mesons:** Ripka, Nikolov, Broniowski, Rostworowski, Arriola, Petrov, Pobylitsa, Ruskov, Lee
 - **structure functions:** Diakonov, Petrov, Weiss, Dressler, Watabe, Bornig, Urbano
 - **GPDs:** Polyakov, Pobylitsa, Penttinen, Ossman, Kirch
 - **pentaquarks:** Polyakov, Diakonov, Petrov, Pobylitsa, Kim, Yang, Ledwig, Silva
- other subjects are also present: Stefanis et. al., Kivel, Manaschov,



Bochum

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Data SIO, NOAA, U.S. Navy, NGA, GEBCO
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48°18'16.74" na północ 17°12'49.77" na wschód

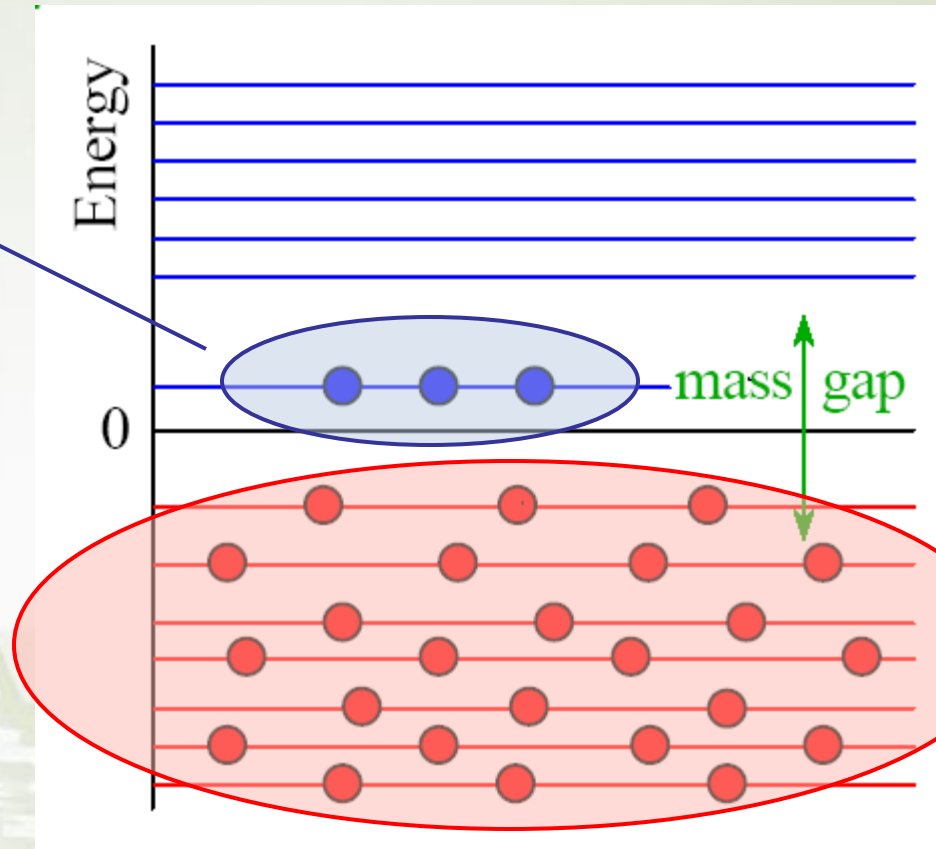
Wys. widoku 3823.83 km



Spectrum of the Dirac operator

valence
level:

energy
decreases



sea
levels:

energy
increases

system stabilizes