Quark mass effects in QCD thermodynamics

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Bad Honnef, 13 Feb 2011

Motivation

how to check QCD vs Reality?

- just solve its eqs [$\longrightarrow$ see next slide]
  - ▷ by computer (lattice); tough; 'oracle'; understand?!
- consider models 'close to QCD'
  - ▷ fewer dims; different sy groups; diff particle content
- consider 'extreme' circumstances in which eqs simplify [$\longrightarrow$ see next^2 slide]
  - ▷ remainder of this talk

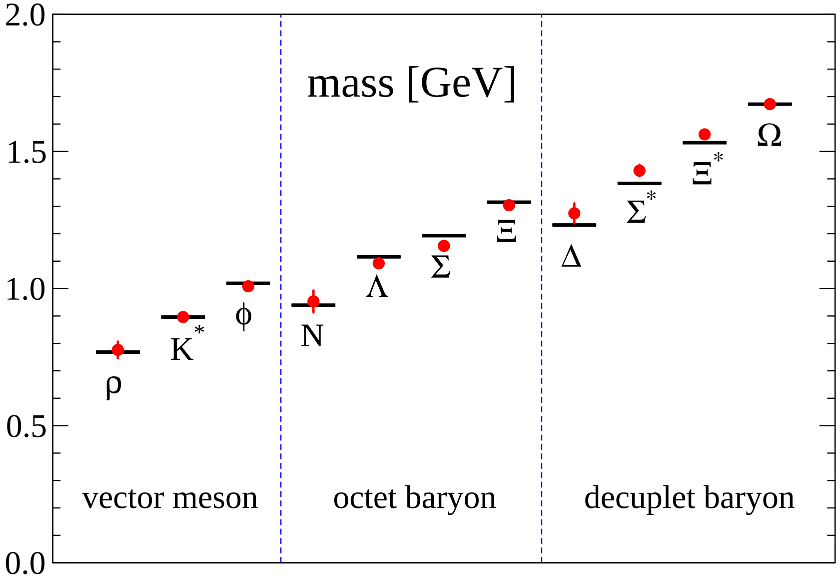
Why thermal QCD?

- study confinement and chiral symmetry breaking
- phenomenologically relevant for cosmology
- phenomenologically relevant for RHIC, LHC
- theoretical limit tractable with analytic methods
  - ▷ goal: no models - stay within QCD!
  - ▷ goal: possibility of systematic improvements

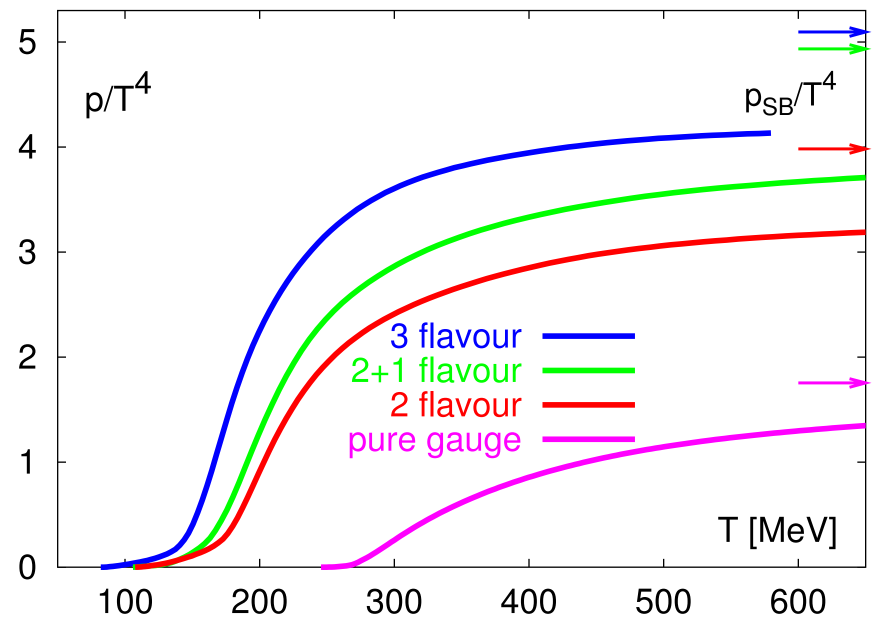
Motivation

solve QCD eqs by computer

- look at hadron spectrum
  [e.g. Aoki et al., PACS-CS 2008]



- look at QCD pressure
  [e.g. Karsch et al.]

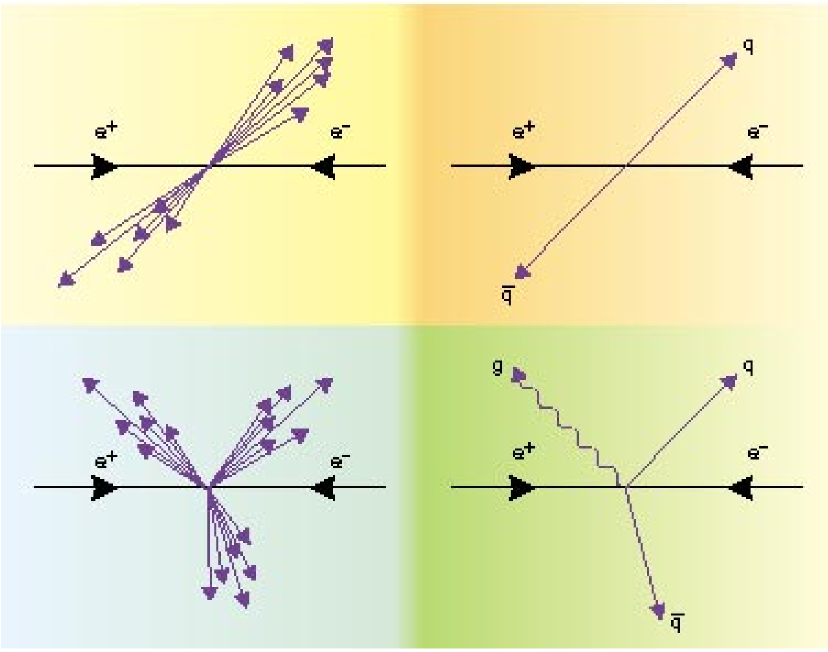


- punchline: QCD postdicts the low-lying hadron masses!
- teraflop speeds, worldwide effort
- confirms liberation of dofs: $\pi \longrightarrow$ qu + gl
- simple asymptotics: ideal gas

Motivation

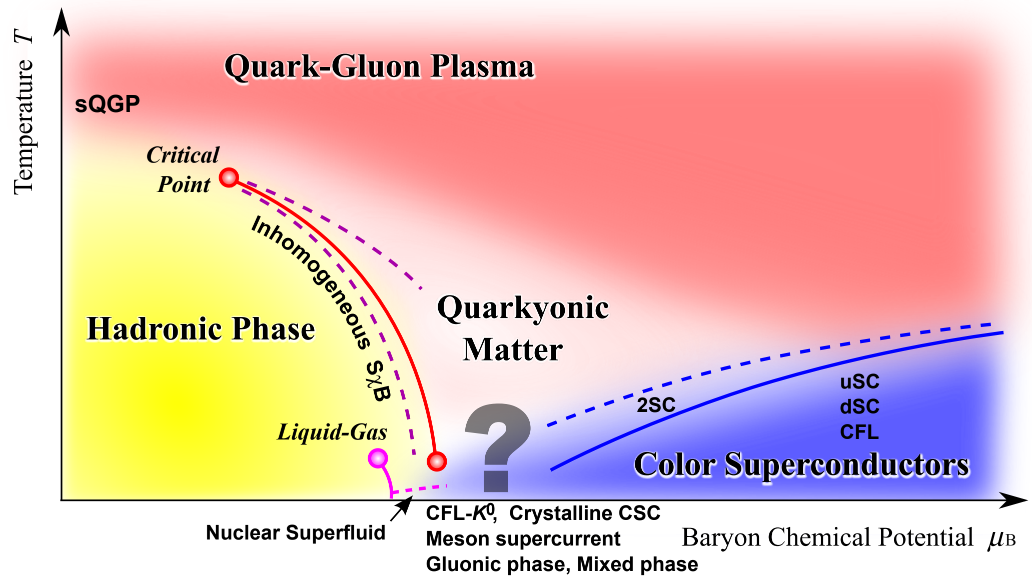
check QCD in extreme conditions

- $E\uparrow$: collider physics



- e.g. LEP, $e^+e^- \to X$
- check details of theory with jets
- nowadays: calc QCD background

- $T\uparrow$, $\mu\uparrow$: equilibrium phase diagram



[Fukushima/Hatsuda 2010]

- nature: early universe, n/qu stars
- $T_c \sim 170$ MeV $\sim 10\mu s$
- lab expt: SPS / RHIC / LHC HI / GSI

Motivation

Focus on equilibrium thermodynamics of QCD

- typical questions to be addressed
  - ▷ equation of state (EoS)
  - ▷ structure of QCD phase diagram
    transition lines, order of transitions, critical points
  - ▷ medium properties: spectral functions, correlation lengths, ...

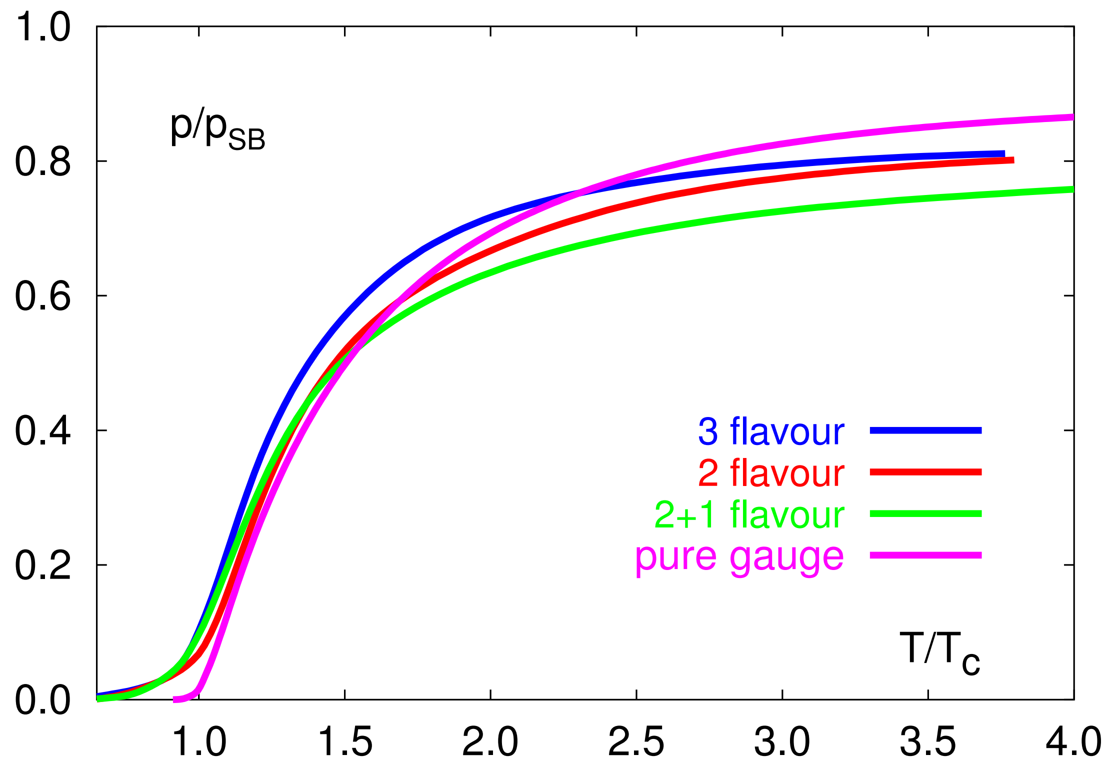
Interplay of methods

- QGP is strongly coupled system near $T_c \Rightarrow$ need e.g. LAT
- asymptotic freedom at high $T \Rightarrow$ weak-coupling approach in continuum
  - ▷ cave: strict loop expansion not well-defined
    IR divergences at higher orders [Linde 79; Gross/Pisarski/Yaffe 81]
- try to use best of both

Discuss

- effective theories (here: $\mu = 0$; $\mu \lesssim T$ similar)
- basic thermodynamic observable: pressure $p(T)$ [$\Leftarrow$ main playground]
- quark mass effects on EoS
- (strong consistency check: spatial string tension)

Once more $p(T)$ via (large) computer ($\mu_B = 0$)



[lattice data from Karsch et al.]

at $T \to \infty$, expect ideal gas: $p_{\text{SB}} = \left(16 + \frac{21}{2} N_f\right) \frac{\pi^2 T^4}{90}$

confirms simplicity: 3 dofs (π) $\to$ 52 ($3 \times 3 \times 2 \times 2$ qu + 8×2 gl)

Energy scales in hot QCD

Interactions make QCD a multi-scale system

At asymptotically high T, $g \ll 1 \Rightarrow$ clean separation of 3 scales

expansion parameter:

$$g^2\, n_b(|k|) = \frac{g^2}{e^{|k|/T} - 1} \overset{|k| \lesssim T}{\approx} \frac{g^2 T}{|k|}$$

- $|k| \sim \pi T$ aka 'hard': fully perturbative at high T
  thermal fluctuations; effective mass of non-static field modes
- $|k| \sim gT$ aka 'soft': dynamically generated; barely perturbative at high T
  inverse screening length of static color-electric fluctuations; thermal/Debye mass
- $|k| \sim g^2 T$ aka 'ultrasoft': dynamically generated; non-perturbative at high T
  inverse screening length of static color-magnetic fluctuations; 'magnetic mass'
- no smaller momentum scales / larger length scales due to confinement

treatment of a multi-scale system: effective field theory !

$p(T)$ via weak-coupling expansion

need to explain 20% deviation from ideal gas at $T \sim 4T_c$

- structure of pert series is non-trivial !
- $$p(T) \equiv \lim_{V\to\infty} \frac{T}{V} \ln \int \mathcal{D}[A_\mu^a, \psi, \bar{\psi}] \exp\left(-\frac{1}{\hbar}\int_0^{\hbar/T} d\tau \int d^{3-2\epsilon}x \, \mathcal{L}^E_{\text{QCD}}\right)$$
  $$= c_0 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4) g^4 + c_5 g^5 + (c_6' \ln g + c_6) g^6 + \mathcal{O}(g^7)$$

  [c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold/Zhai 94, c_5 Zhai/Kastening 95, Braaten/Nieto 96, c_6' KLRS 03]

- root cause of nonanalytic (in α_s) behavior well understood: above-mentioned dynamically generated scales
- clean separation best understood in effective field theory setup [here: $\mu = 0$]
  - ▷ generalizations, e.g. $\mu \neq 0$ [Vuorinen], standard model [Gynther/Vepsäläinen]
- other re-organizations possible, e.g. 2PI skeleton-expansion [eg Blaizot/Iancu/Rebhan]

Effective theory prediction for $p(T)$

$$\frac{p_{\mathsf{QCD}}(T)}{p_{\mathsf{SB}}} = \frac{p_{\mathsf{E}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{M}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{G}}(T)}{p_{\mathsf{SB}}} \quad , \quad p_{\mathsf{SB}} = \left(16 + \frac{21}{2} N_f\right) \frac{\pi^2 T^4}{90}$$

$$= 1 + g^2 \quad + g^4 \quad + g^6 \quad + \ldots \qquad \Leftarrow \text{4d QCD}$$

$$+ g^3 + g^4 + g^5 + g^6 + \ldots \qquad \Leftarrow \text{3d adj H}$$

$$+ \frac{1}{p_{\mathsf{SB}}} \frac{T}{V} \int \mathcal{D}[A_k^a] \exp\Big(-S_{\mathsf{M}}\Big) \qquad \Leftarrow \text{3d YM}$$

- this could be coined the *physical leading-order (!) approximation*
- collect contributions to $p(T)$ from all physical scales
  - ▷ weak coupling, effective field theory setup
  - ▷ faithfully adding up all Feynman diagrams
  - ▷ get long-distance input from clean lattice observable:

$$p_{\mathsf{G}}(T) \equiv \frac{T}{V} \ln \int \mathcal{D}[A_k^a] \exp\Big(-S_{\mathsf{M}}\Big) = T \# \, g_{\mathsf{M}}^6$$

only one non-perturbative (but computable!) coeff needed: 5×10^{16} flops

Open problem at LO

g^6 needs 4-loop sum-integrals

- a single one has already been computed
  - ▷ painfully disentangled (sub-)divergences by hand
  - ▷ constant term only numerically
  - ▷ gave the g^6 term in scalar ϕ^4 [GLSTV 08]
  - ▷ fermionic generalization for $g^6 N_f^3$ in QCD [Gynther et al. 09]

$$\sum\!\!\!\!\!\!\!\int_{PQRS} \frac{1}{P^2(P+S)^2\, Q^2(Q+S)^2\, R^2(R+S)^2} = \frac{T^4}{(4\pi)^4}\,\frac{1}{16\epsilon^2}\left[1+\epsilon t_{11}+\epsilon^2 t_{12}+\dots\right]$$

with $t_{11} = \frac{44}{5} - 4\gamma_E + 12\frac{\zeta'(-1)}{\zeta(-1)} - 4\zeta(3) - \zeta(2)$

- in QCD, need $\mathcal{O}(10^8)$ of them; reduction in progress

- masters: ideas to profit from algorithmic $T=0$ methods not fruitful (yet?)
  - ▷ as used - and tested extensively - for the 3d part
  - ▷ reduction with one generic index, difference eqs [Laporta 00]
  - ▷ do dimensional recurrences help? [Lee 10]
  - ▷ use sector decomposition in 3d piece; sum over 'masses' in the end
  - ▷ factorize sum and integrals via MB; generalized Zetas under MB ints

- find a smart duality to map the problem to sth simpler?

$p(T)$ beyond LO: $g^6 \to g^7 \to g^8$

$$\frac{p_{\mathsf{E}}}{p_{\mathsf{SB}}} = \#_{(0)} + \#_{(2)} g^2 + \#_{(4)} g^4 + \#_{(6)} g^6 + [\text{4d 5loop 0pt}]_{(8)} + \dots_{(10)}$$

$$g_{\mathsf{E}}^2 = T\left[g^2 + \#_{(6)} g^4 + \#_{(8)} g^6 + \#_{(10)} g^8 + \dots_{(12)}\right]$$

$$\lambda_{\mathsf{E}} = T\left[\#_{(6)} g^4 + \#_{(8)} g^6 + \dots_{(10)}\right]$$

$$m_{\mathsf{E}}^2 = T^2\left[\#_{(3)} g^2 + \#_{(5)} g^4 + [\text{4d 3loop 2pt}]_{(7)} + \dots_{(9)}\right]$$

$$\frac{p_{\mathsf{M}}}{p_{\mathsf{SB}}} = \frac{m_{\mathsf{E}}^3}{T^3}\left[\#_{(3)} + \frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\left(\#_{(4)} + \#_{(6)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right) + \left(\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\right)^2\left(\#_{(5)} + \#_{(7)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2} + \#_{(9)}\left(\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right)^2\right)\right.$$
$$+\left(\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\right)^3\left(\#_{(6)} + \#_{(8)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2} + \#_{(10)}\left(\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right)^2 + \#_{(12)}\left(\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right)^3\right)$$
$$\left.+[\text{3d 5loop 0pt}]_{(7)} + [\delta\mathcal{L}_{\mathsf{E}}]_{(7)} + [\text{3d 6loop 0pt}]_{(8)} + \dots_{(9)}\right]$$

$$g_{\mathsf{M}}^2 = g_{\mathsf{E}}^2\left[1 + \#_{(7)}\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}} + \left(\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\right)^2\left(\#_{(8)} + \#_{(10)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right) + \dots_{(9)}\right]$$

$$\frac{p_{\mathsf{G}}}{p_{\mathsf{SB}}} = \#_{(6)}\left(\frac{g_{\mathsf{M}}^2}{T}\right)^3 + [\delta\mathcal{L}_{\mathsf{M}}]_{(9)}$$

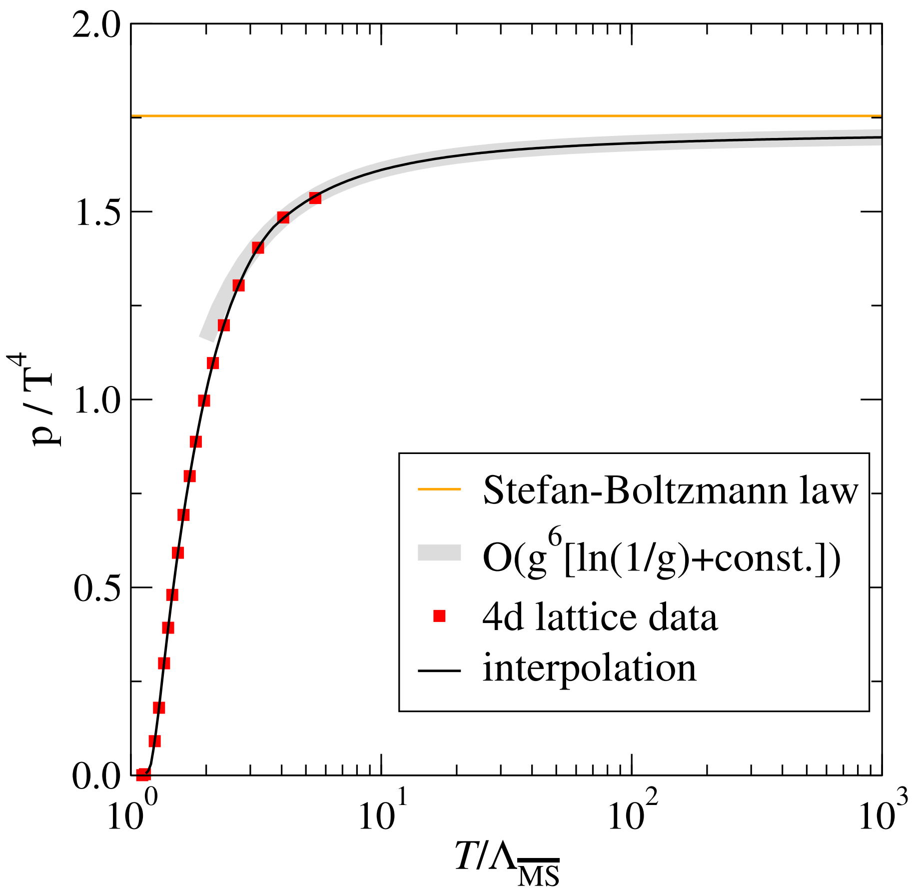
notation: $\#_{(n)}$ **enters** p_{QCD} **at** g^n

[cave: no $\frac{1}{\epsilon} + 1 + \epsilon$, no IR/UV, and no logs shown above]

Results: estimating $p(T, N_f{=}0)$ at LO

while working on the open problems at LO ...

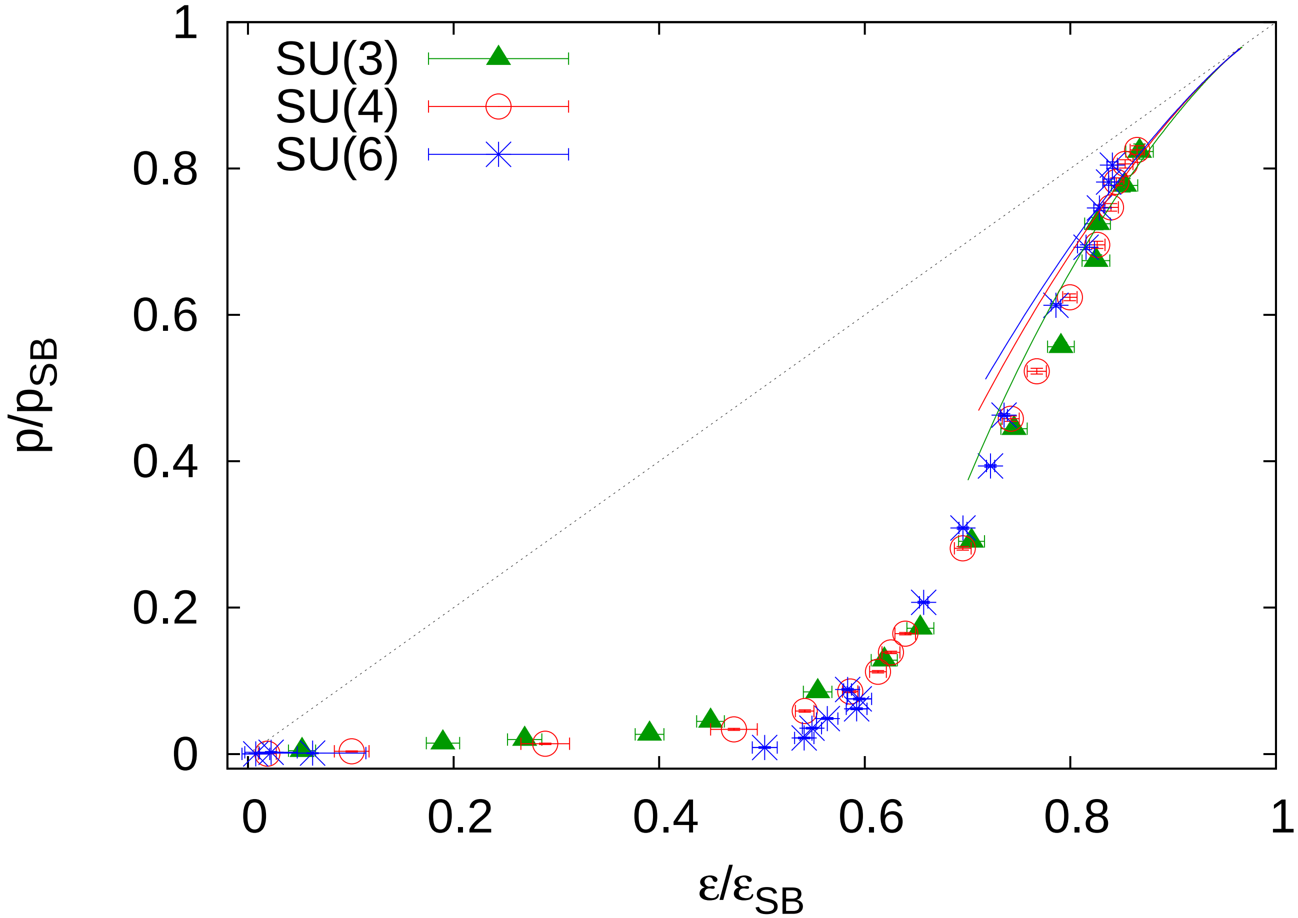
- want to show results / tackle simpler problems / do phenomenology
- strive for best possible description of pure-glue sector



- fix unknown perturbative $\mathcal{O}(g^6)$ coeff
- match to lattice data [Boyd et al. 96] at intermediate T $\sim$ 3-5T_c
- translate via $T_c/\Lambda_{\overline{\rm MS}} \approx 1.20$

Results: estimating $p(T, N_f{=}0)$ at LO

test the approach to conformality for pure YM theory



- diag line: conformal theories
- lines: weak-coupling results

⇒ no window where a strongly coupled conformal theory describes SU(N) thermodynamics?!

[Datta/Gupta 2010]

Results: Quark mass dependence

analyze quark mass dependence to NLO

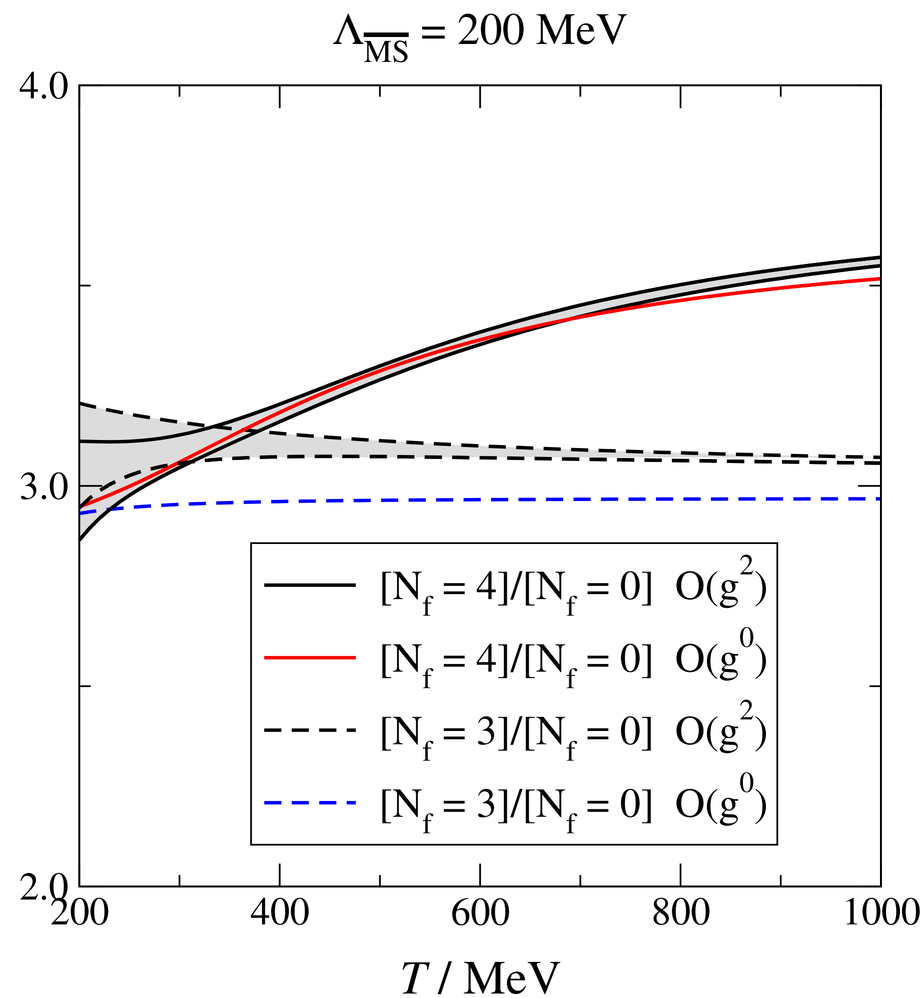
- strategy: 'unquenching'
  start from $N_f = 0$, i.e. $m_q = \infty$
  lower N_f quark masses to $m_{q,phys}$
  p at any T increases

- estimate this 'correction factor'

- aproach is systematic
  LO: $c_0(N_f)/c_0(0)$
  NLO: $[c_0 + g^2 c_2](N_f)\ /\ [c_0 + g^2 c_2](0)$

- computed $c_{0,2}(T, N_c, N_f, m_i, \mu_i)$

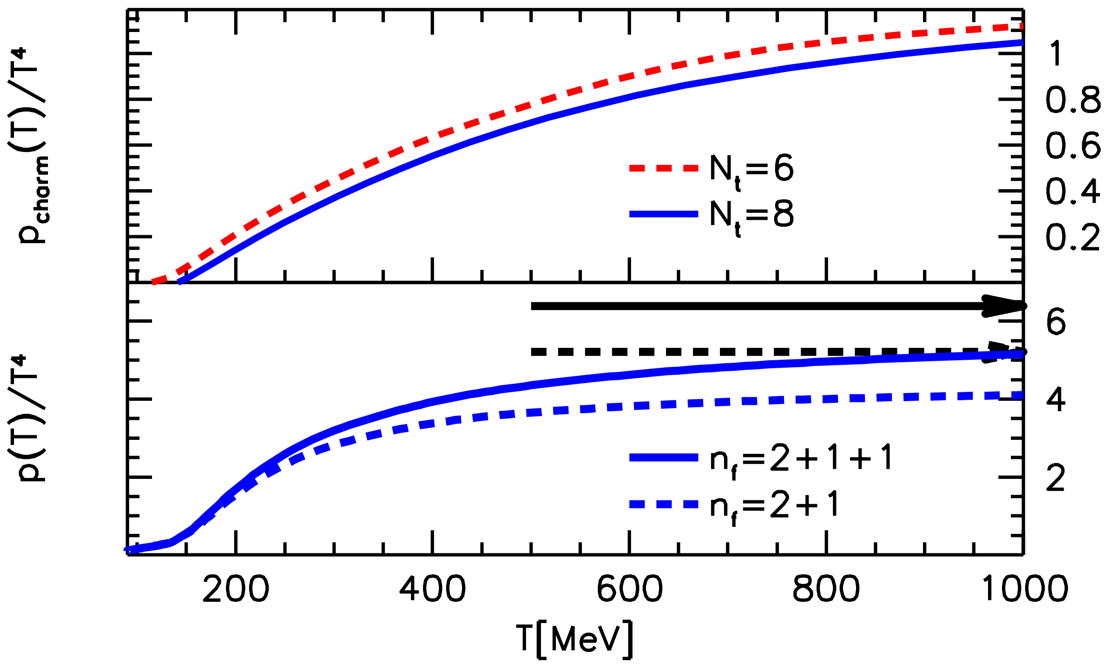
- good convergence LO$\longrightarrow$NLO
  - $N_f = 3$: 5% effect
  - $N_f = 4$: even better



$\Rightarrow$ charm quark contributes already at low $T \sim 350 MeV$

Results: Quark mass dependence

Charm contribution: Lattice estimate for $N_f = 2{+}1{+}1$ EoS [Borsanyi et al. 10]



- upper: charm contribution to pressure (for 2 different lattice spacings)
- lower: pressure with and w/o charm (on $N_t = 8$ lattices; $m_c/m_s = 11.85$)

$\Rightarrow$ confirmation of early onset of charm quark contribution

Results: Quark mass dependence

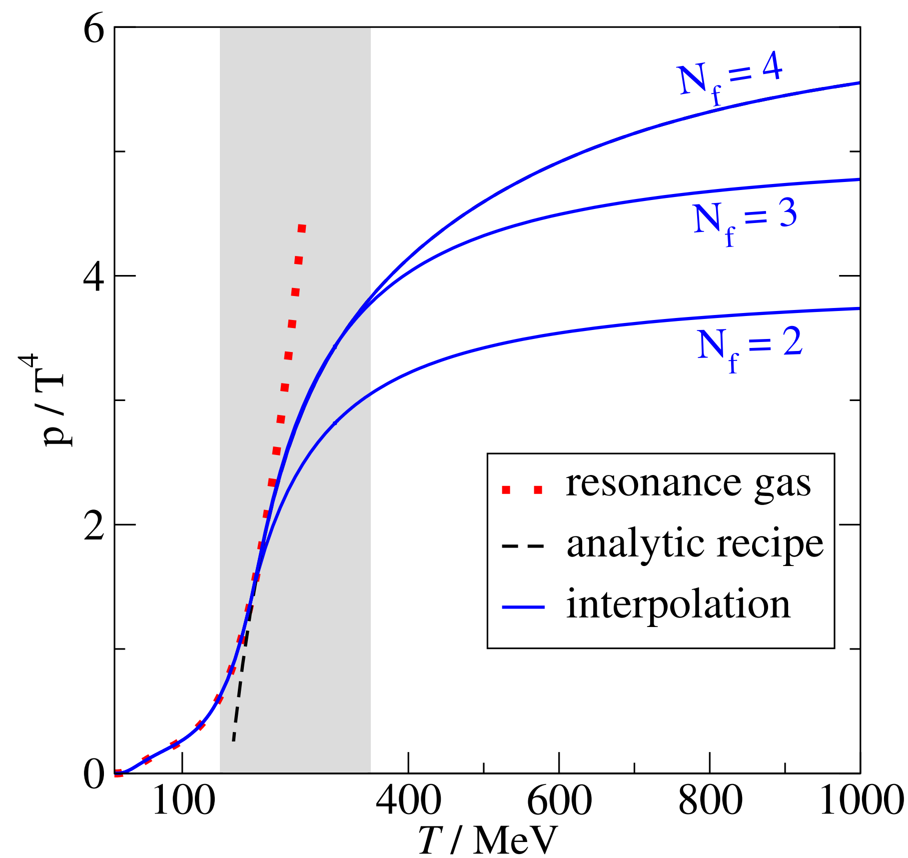
now ready to estimate thermodynamic quantities

multiply best $N_f = 0$ result with correction factor

$$g^2(\bar{\mu}) = \frac{24\pi^2}{(11C_A - 2N_f)\ln(\bar{\mu}/\Lambda_{\overline{\rm MS}})}\,, \quad m_i(\bar{\mu}) = m_i(\bar{\mu}_{\rm ref})\left[\frac{\ln(\bar{\mu}_{\rm ref}/\Lambda_{\overline{\rm MS}})}{\ln(\bar{\mu}/\Lambda_{\overline{\rm MS}})}\right]^{\frac{9C_F}{11C_A-2N_f}}$$

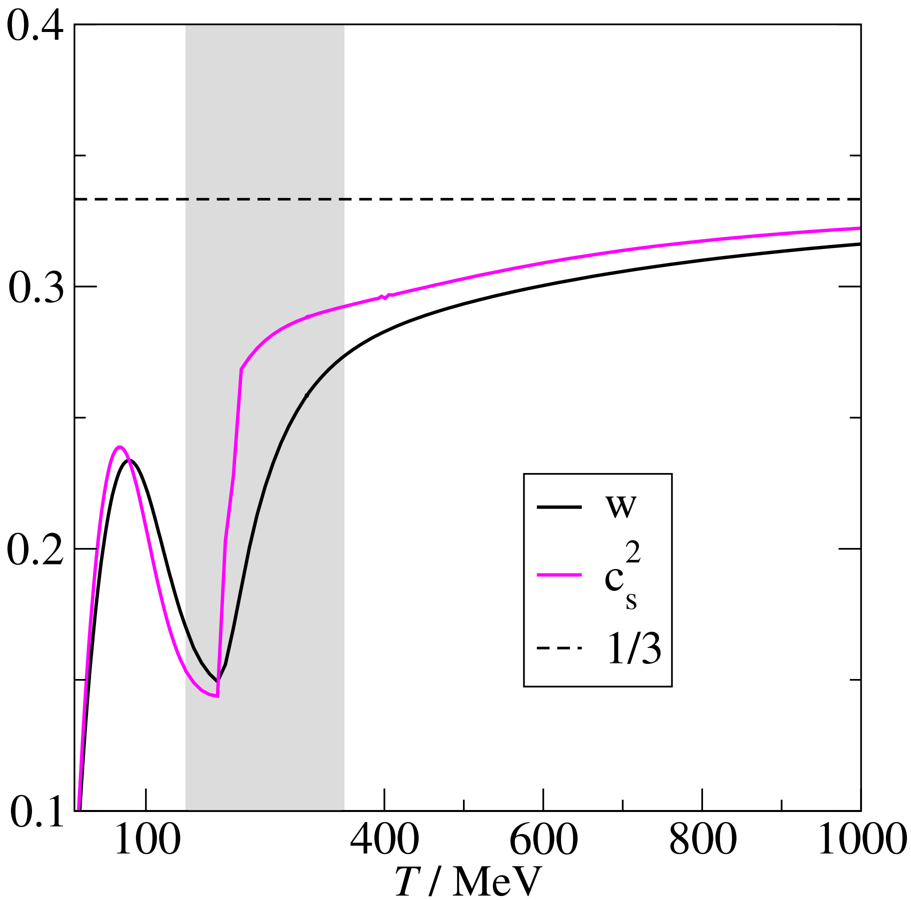
Setting the scale

- need to fix $\Lambda_{\overline{\rm MS}}$ in physical units!
- strategy: matching
  take p of hadronic resonances
  match p and p' to our recipe
- obtain $\Lambda^{(\rm eff)}_{\overline{\rm MS}} \approx 175...180$ MeV
- shaded: lattice simulations needed!



Results: EoS with physical quark masses at NLO

now use the recipe $\boxed{p(N_f{=}0)\times \text{corr.fct}}$ and plot dimensionless ratios



- equation of state

$$w(T) \equiv \frac{p(T)}{e(T)} = \frac{p(T)}{Tp'(T) - p(T)}$$

- sound speed (squared)

$$c_s^2(T) \equiv \frac{p'(T)}{e'(T)} = \frac{p'(T)}{Tp''(T)} = \frac{s(T)}{c(T)}$$

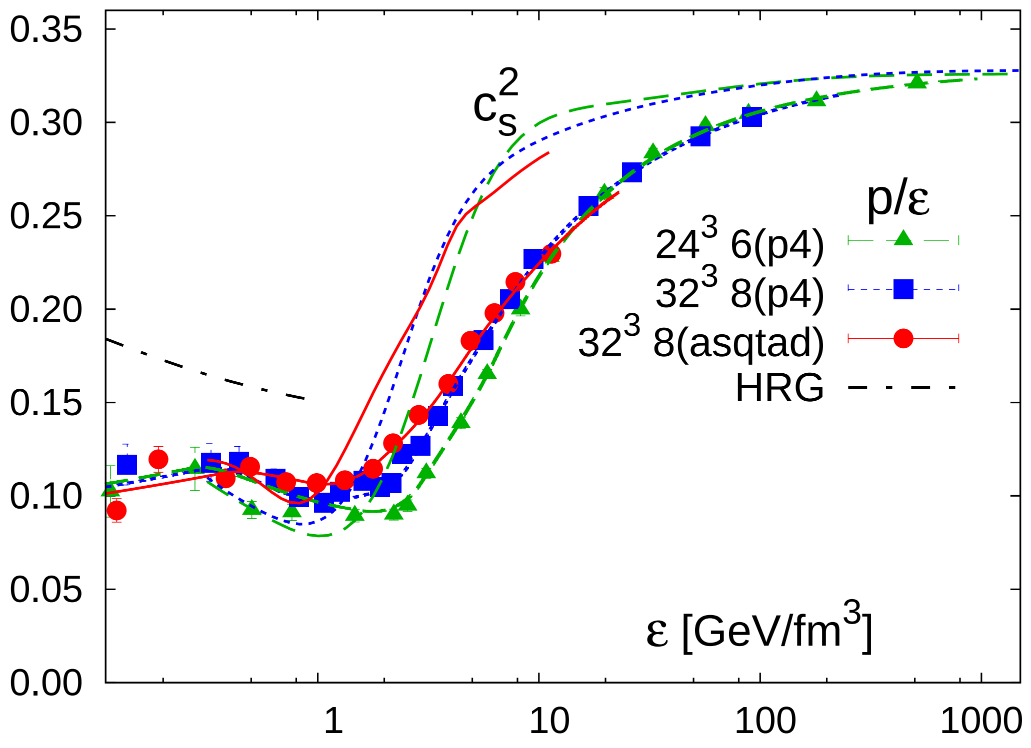
- $\left(\frac{1}{3} - w(T)\right) \propto$ 'trace anomaly'
- observe significant structure
- at 2nd order phase transition $c(T) \sim (T - T_c)^{-\gamma}$

peak around 70MeV not (yet) visible in lattice simulations

Results: EoS with physical quark masses at NLO

recent lattice data

[Bazavov et al. 09]



- HotQCD 2009
- $N_f = 2+1$
  m_s physical
  light quarks not
- $N_\tau = 8$
  two (staggered) actions

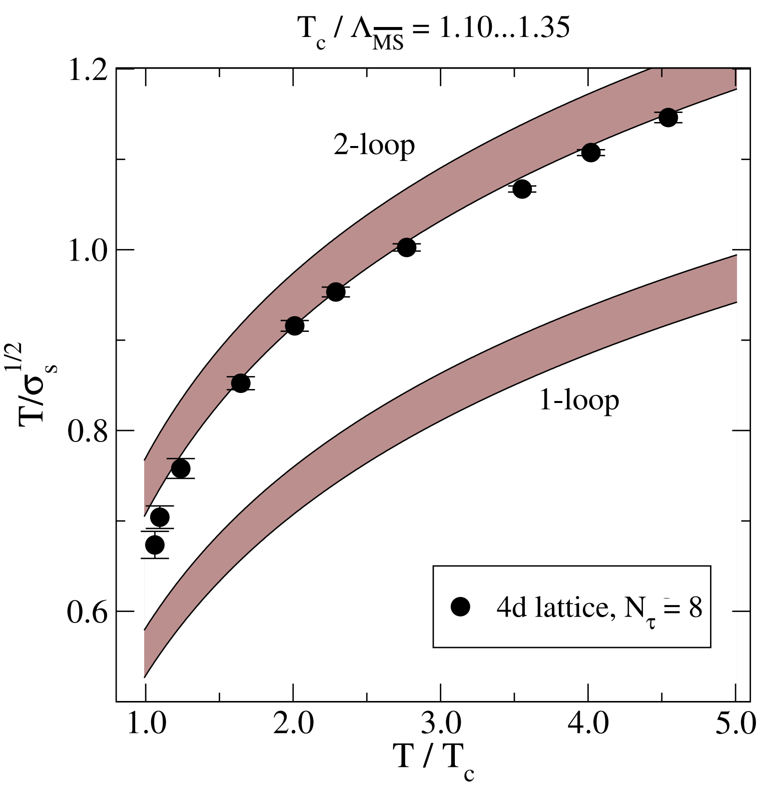
Summary

- thermodynamic quantities of QCD are relevant for cosmology and heavy ion collisons
- these quantities can be determined
  - ▷ numerically at $T \sim 200$ MeV; analytically at $T \gg 200$ MeV
  - ▷ multi-loop sports, eff. theories convenient $\longrightarrow$ systematic improvement
- 3d effective field theory opens up tremendous opportunities
  - ▷ analytic treatment of fermions (cf. LAT problems!)
  - ▷ universality, superrenormalizabilty
  - ▷ ideal playground for multi-loop methods
- e.g. QCD pressure: not even known at 'physical leading order'
  - ▷ (mild) open problem: LAT-continuum matching for general N_c
  - ▷ (hard) open problem: 4-loop sum-integrals
  - ▷ shows friendly functional behavior with fitted unknown coefficient
- e.g. Quark mass dependence in EoS; Spatial string tension
  - ▷ show good convergence
  - ▷ successful test of effective theory setup
  - ▷ even higher precision under investigation

Results: spatial string tension σ_s at NNLO

Define: $W_s(R_1, R_2) = \exp(-\sigma_s R_1 R_2)$ at large R_1, R_2

- SU(3), 4d lat: $\frac{\sqrt{\sigma_s}}{T} = \text{fct}\left(\frac{T}{Tc}\right)$; $T_c \approx 1.2\Lambda_{\overline{\text{MS}}}$
- SU(3), 3d MQCD: $\frac{\sqrt{\sigma_s}}{T} = \# \frac{g_M^2}{g_E^2}\frac{g_E^2}{T} = \text{fct}\left(\frac{T}{\Lambda_{\overline{\text{MS}}}}\right)$; $\# = 0.553(1)$ [Teper/Lucini 02]



- 4d lattice data from [Boyd et al. 96]
  (cave: no cont. extrapolation)
- parameter-free comparison

$\Rightarrow$ support for hard/soft+ultrasoft picture of thermal QCD

- NNNLO (3-loop) appears doable

Effective theory setup: QCD $\to$ EQCD

high T: QCD dynamics contained in 3d EQCD

integrate out $|p| \gtrsim 2\pi T$: ψ, $A_\mu(n \neq 0)$

$$p_{\mathsf{QCD}}(T) \equiv p_{\mathsf{E}}(T) + \frac{T}{V}\ln \int \mathcal{D}[A_k^a, A_0^a] \exp\Big(-\int d^{3-2\epsilon}x\, \mathcal{L}_{\mathsf{E}}\Big)$$

$$\mathcal{L}_{\mathsf{E}} = \frac{1}{2} Tr\, F_{kl}^2 + Tr\,[D_k, A_0]^2 + m_{\mathsf{E}}^2 Tr\, A_0^2 + \lambda_{\mathsf{E}}^{(1)} (Tr\, A_0^2)^2 + \lambda_{\mathsf{E}}^{(2)} Tr\, A_0^4 + \ldots$$

five matching coefficients [E. Braaten, A. Nieto, 95; KLRS 02; M. Laine, YS, 05]

$$p_{\mathsf{E}} = T^4\left[\# + \# g^2 + \# g^4 + \# g^6 + \ldots\right],\; m_{\mathsf{E}}^2 = T^2\left[\# g^2 + \# g^4 + \ldots\right],$$

$$g_{\mathsf{E}}^2 = T\left[g^2 + \# g^4 + \# g^6 + \ldots\right],\; \lambda_{\mathsf{E}}^{(1),(2)} = T\left[\# g^4 + \ldots\right].$$

higher order operators do not (yet) contribute [S. Chapman, 94; Kajantie et al, 97, 02]

$$\frac{\delta p_{\mathsf{QCD}}(T)}{T} \sim \delta \mathcal{L}_{\mathsf{E}} \sim g^2 \frac{D_k D_l}{(2\pi T)^2} \mathcal{L}_{\mathsf{E}} \sim g^2 \frac{(gT)^2}{(2\pi T)^2} (gT)^3 \sim g^7 T^3$$

Effective theory setup: QCD $\to$ EQCD $\to$ MQCD

the IR of 3d EQCD is contained in 3d MQCD

integrate out $|p| \gtrsim gT$: A_0

$$p_{\mathsf{QCD}}(T) \equiv p_{\mathsf{E}}(T) + p_{\mathsf{M}}(T) + \frac{T}{V}\ln\int\mathcal{D}[A_k^a]\exp\Big(-\int d^{3-2\epsilon}x\,\mathcal{L}_{\mathsf{M}}\Big)$$

$$\mathcal{L}_{\mathsf{M}} = \frac{1}{2}Tr\,F_{kl}^2 + \ldots$$

two matching coefficients [KLRS 03; P. Giovannangeli 04, M. Laine/YS 05]

$$p_{\mathsf{M}} = Tm_{\mathsf{E}}^3\left[\# + \#\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}} + \#\frac{g_{\mathsf{E}}^4}{m_{\mathsf{E}}^2} + \#\frac{g_{\mathsf{E}}^6}{m_{\mathsf{E}}^3} + \ldots\right],\; g_{\mathsf{M}}^2 = g_{\mathsf{E}}^2\left[1 + \#\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}} + \#\frac{g_{\mathsf{E}}^4}{m_{\mathsf{E}}^2} + \ldots\right].$$

higher order operators do not (yet) contribute

$$\frac{\delta p_{\mathsf{QCD}}(T)}{T} \sim \delta\mathcal{L}_{\mathsf{M}} \sim g_{\mathsf{E}}^2\frac{D_kD_l}{m_{\mathsf{E}}^3}\mathcal{L}_{\mathsf{M}} \sim g_{\mathsf{E}}^2\frac{(g^2T)^2}{m_{\mathsf{E}}^3}(g^2T)^3 \sim g^9T^3$$