Few nucleon forces with explicit Delta fields

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February 14, 2010, Strong interactions, Bad Honnef

With V. Bernard, E. Epelbaum, U.-G. Meißner

RUB

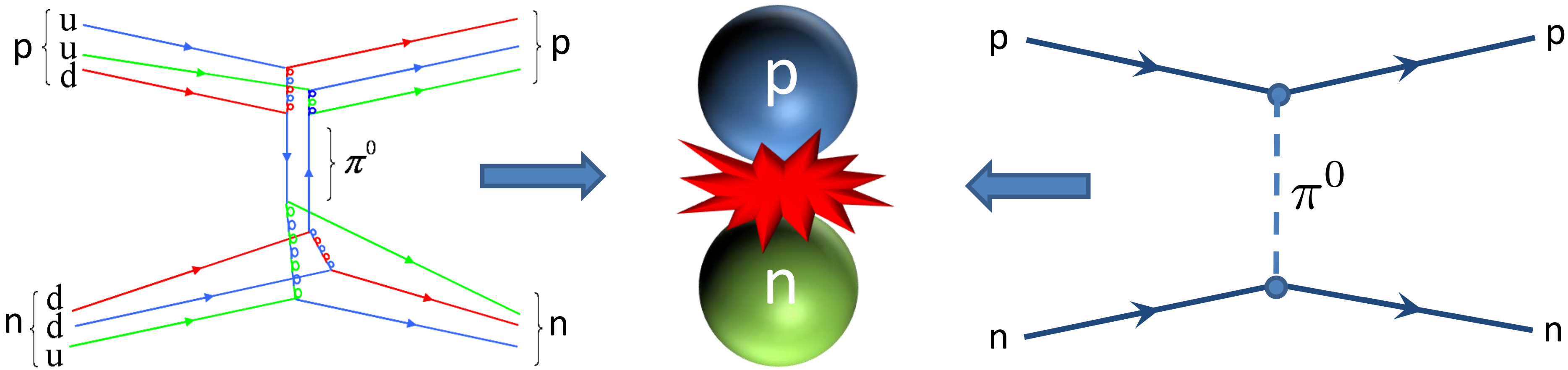
Outline

- Nuclear forces in chiral EFT
- Convergence of nuclear forces and the role of Δ-isobar
- N^3LO three-nucleon forces with Δ-dof.
- Summary & Perspectives

Nucleon-Nucleon forces

QCD (quark and gluon dof)

Chiral EFT (nucleon and pion dof)



$$\int [Dq][D\bar{q}][DG]\; e^{\int i d^4x\, \mathcal{L}_{\mathrm{QCD}}[q,\bar{q},G;\; J]} \;\rightarrow\; \int [D\Phi]\; e^{\int i d^4x\, \mathcal{L}_{\mathrm{eff}}[\Phi;\; J]}$$

Model independent treatment

- Underlying QCD symmetries implemented by construction
- At low energies NN force dominated by Goldstone Boson dynamics + short range int.
- Systematic perturbative description of few nucleon potentials

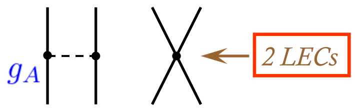
Nucleon-nucleon force up to N^{3}LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; …

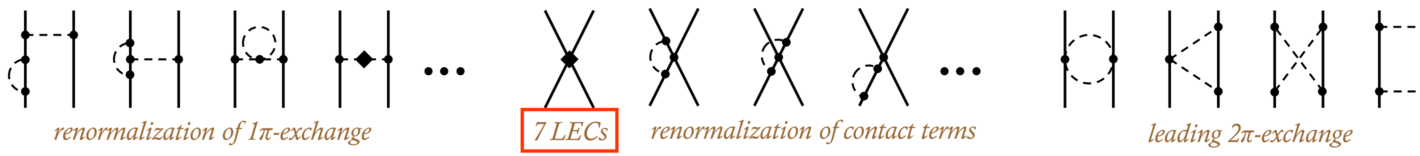
Chiral expansion for the 2N force:

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$$

- **LO:**

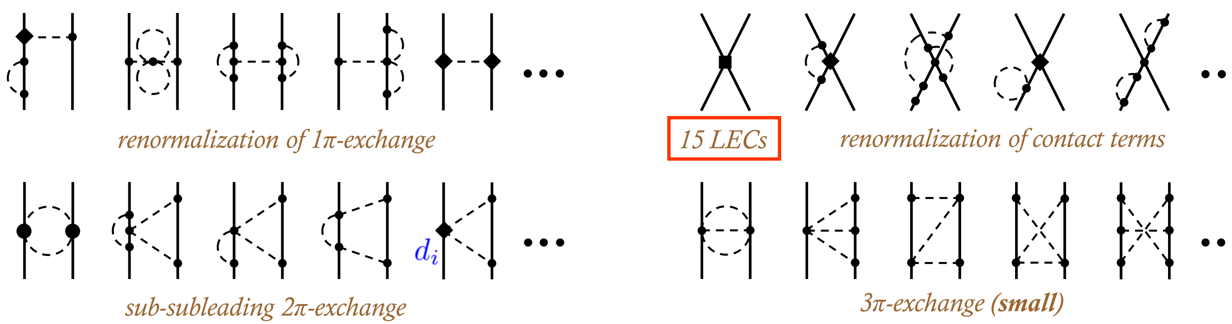


- **NLO:**



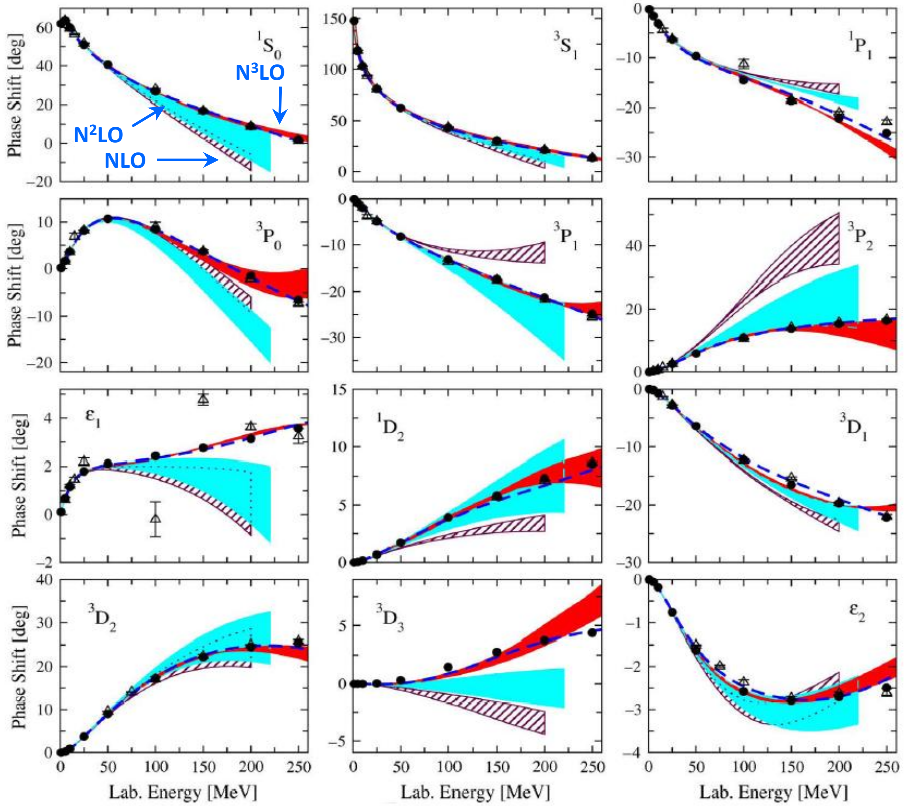
- **N^{2}LO:**

- **N^{3}LO:**

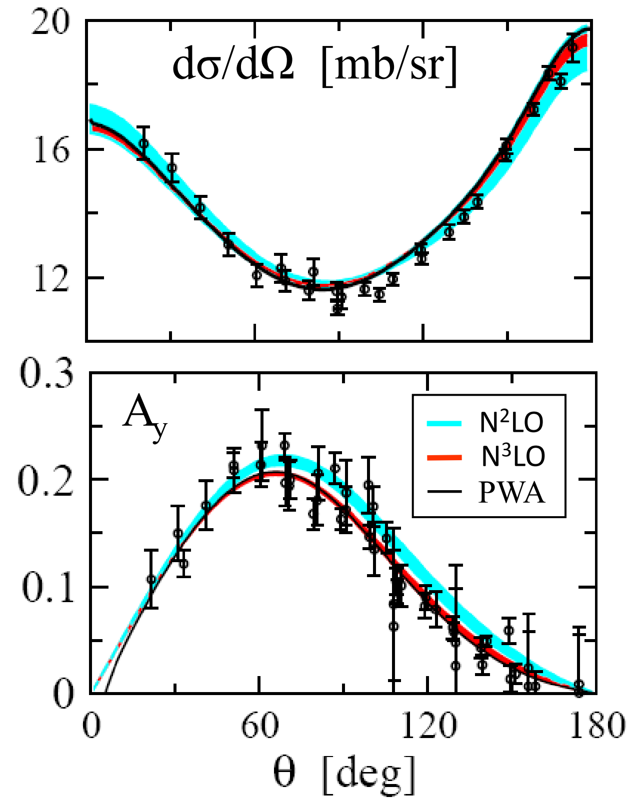


+ 1/m and isospin-breaking corrections…

Neutron-proton phase shifts up to N^3LO



np scattering at 50 MeV



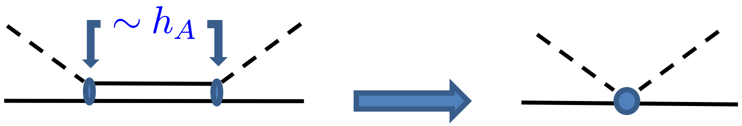
Deuteron binding energy & asymptotic normalizations A_s and η_d

	NLO	N^2LO	N^3LO	Exp
E_d [MeV]	$-2.171\ldots-2.186$	$-2.189\ldots-2.202$	$-2.216\ldots-2.223$	$-2.224575(9)$
A_S [fm$^{-1/2}$]	$0.868\ldots0.873$	$0.874\ldots0.879$	$0.882\ldots0.883$	$0.8846(9)$
η_d	$0.0256\ldots0.0257$	$0.0255\ldots0.0256$	$0.0254\ldots0.0255$	$0.0256(4)$

Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05

Delta-less effective potential

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293 \text{ MeV}$
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$ *(Hemmert, Holstein & Kambor '98)*
- Delta contributions encoded in LECs
  (Bernard, Kaiser & Meißner '97)

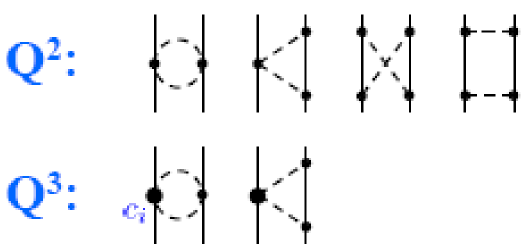


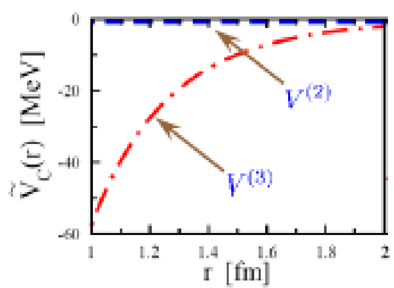
Delta-resonance saturation

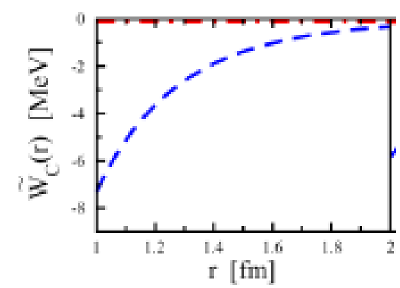
$$c_3 = -2c_4 = c_3(\not\Delta) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

- Convergence of EFT potential





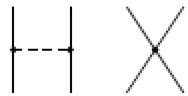
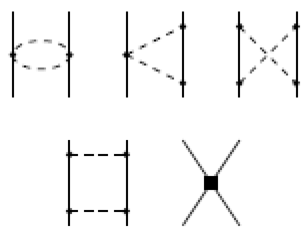
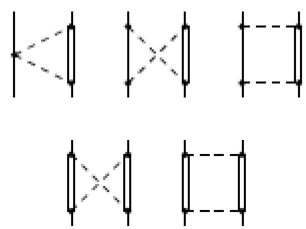
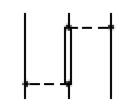
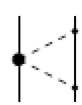
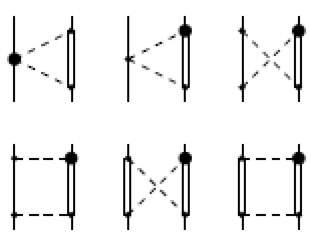
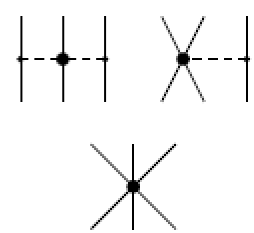


The subleading contribution is bigger than the leading one!

Expectation from inclusion of Δ explicitely
- more natural size of LECs
- better convergence
- applicability at higher energies

Few-nucleon forces with the Delta

Isospin-symmetric contributions

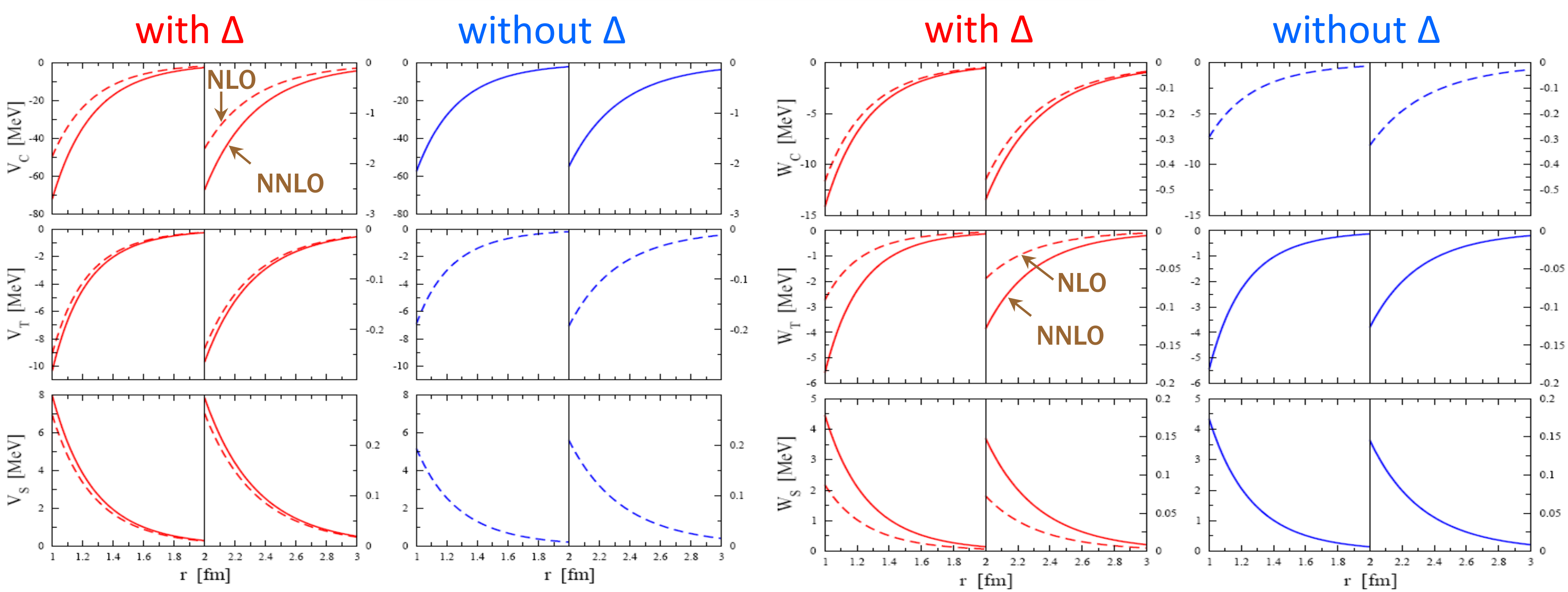
	Two–nucleon force		Three–nucleon force	
	Δ–less EFT	Δ–contributions	Δ–less EFT	Δ–contributions
LO		—	—	—
NLO		 Ordonez et al.'96, Kaiser et al. '98	—	
NNLO		 H.K., Epelbaum & Meißner '07		—

NN potential with explicit Δ

Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127

$$V_{\text{eff}} = V_C + W_C \vec{\tau}_1 \cdot \vec{\tau}_2 + [V_S + W_S \vec{\tau}_1 \cdot \vec{\tau}_2] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1 \cdot \vec{\tau}_2](3\,\vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

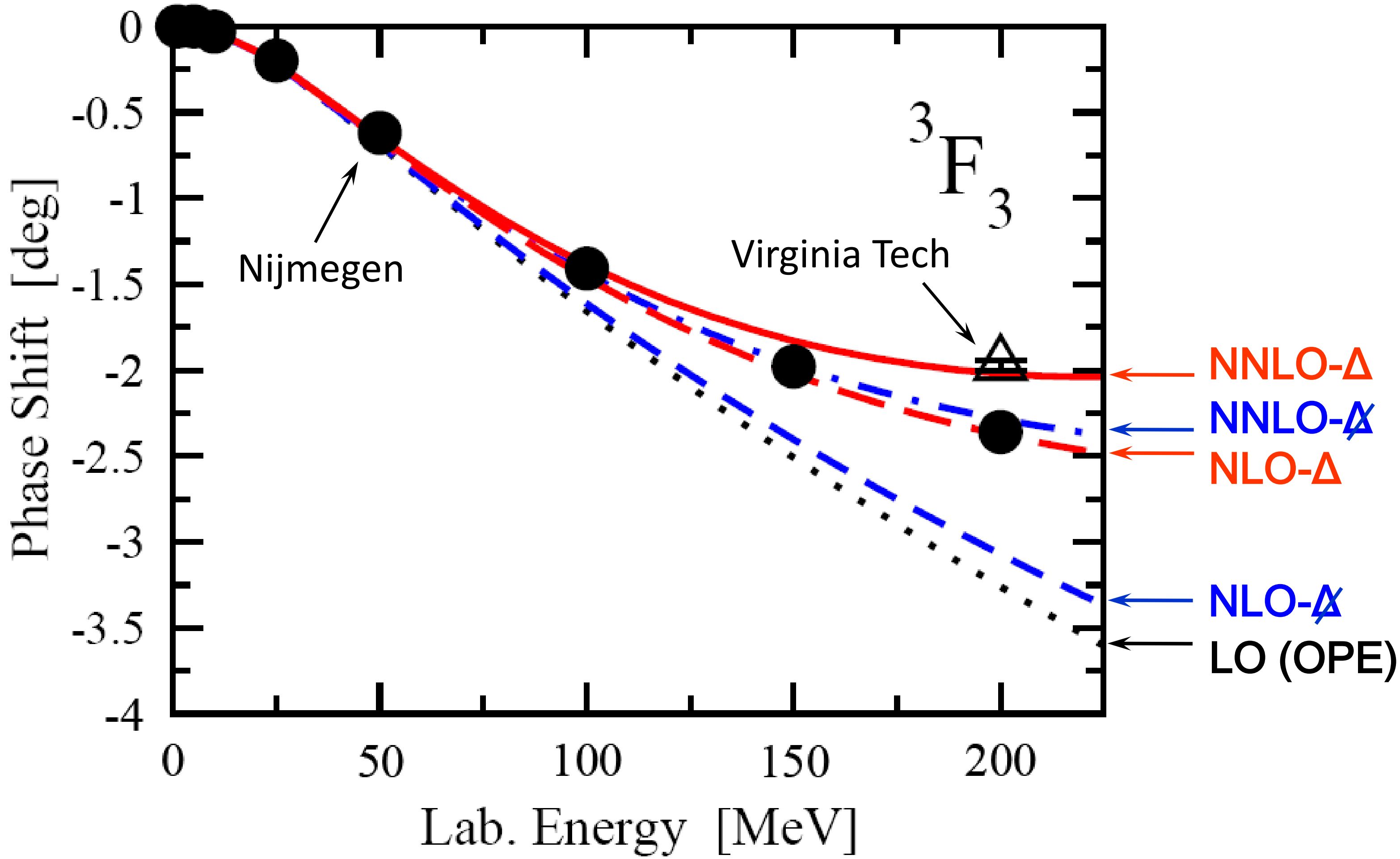
Chiral 2π- exchange potential up to NNLO



Advantages when Δ is included explicitly

- Dominant contributions already at NLO
- Much better convergence in all potentials

$^3F_3$ partial waves up to NNLO with and without Δ

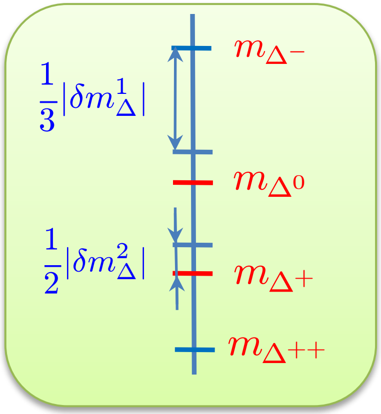


(calculated in the first Born approximation)

Δ-mass splitting in chiral EFT

Epelbaum, H.K., Meißner, Nucl. Phys. A806 (2008) 65
Tiburzi, Walker-Loud, Nucl. Phys. A 764 (2006) 274 (strong splitting)

$$\mathcal{L}^{\rm LO}_{\Delta,\rm mass} = -\bar{T}^\mu_i \left[-\delta m^1_\Delta \frac{1}{2}\tau^3\delta_{ij} - \delta m^2_\Delta \frac{3}{4}\delta_{i3}\delta_{j3} \right] g_{\mu\nu} T^\nu_j \,.$$



Equidistant splitting: strong & em

$$\delta m^1_\Delta = -4 M_\pi^2 \epsilon c_5^\Delta - F_\pi^2 e^2 f_2^\Delta$$

Non-equidistant splitting: em

$$\delta m^2_\Delta = -\frac{4}{3} F_\pi^2 e^2 f_2^\Delta$$

- Most recent data from the PDG: $m_{\Delta^{++}} = 1230.80 \pm 0.30$ MeV, $m_{\Delta^0} = 1233.45 \pm 0.35$ MeV

  In addition, PDG's recommended value for the average mass:

$$m_\Delta = \frac{1}{4}\left(m_{\Delta^{++}} + m_{\Delta^+} + m_{\Delta^0} + m_{\Delta^-}\right) = \tilde{m}_\Delta + \frac{1}{4}\delta m^2_\Delta = 1231 \ldots 1233 \,\text{MeV}$$

  On the other hand: $m_\Delta = 1233.4 \pm 0.4$ MeV *(Arndt et al. '06)* ⟹ use: $m_\Delta = 1233$ MeV

  ⟹ $\tilde{m}_\Delta = 1233.4 \pm 0.7$ MeV, $\delta m^1_\Delta = -5.3 \pm 2.0$ MeV, $\delta m^2_\Delta = -1.7 \pm 2.7$ MeV

- Alternatively, use $m_{\Delta^{++}}/m_{\Delta^0}$ & the QM relation: $m_{\Delta^+} - m_{\Delta^0} = m_p - m_n$ *(Rubinstein et al. '67)*

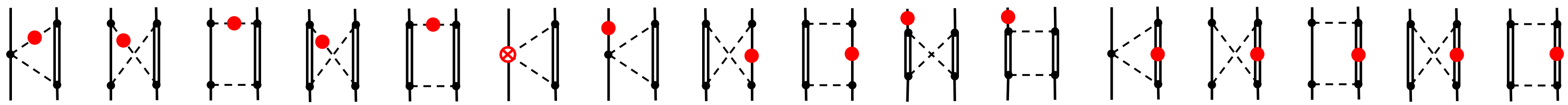
  ⟹ $\tilde{m}_\Delta = 1232.7 \pm 0.3$ MeV, $\delta m^1_\Delta = -3.9$ MeV, $\delta m^2_\Delta = 0.3 \pm 0.3$ MeV

LO Isospin-breaking NN potential

Epelbaum, H.K., Meißner, Phys. Rev. C77 (2008) 034006

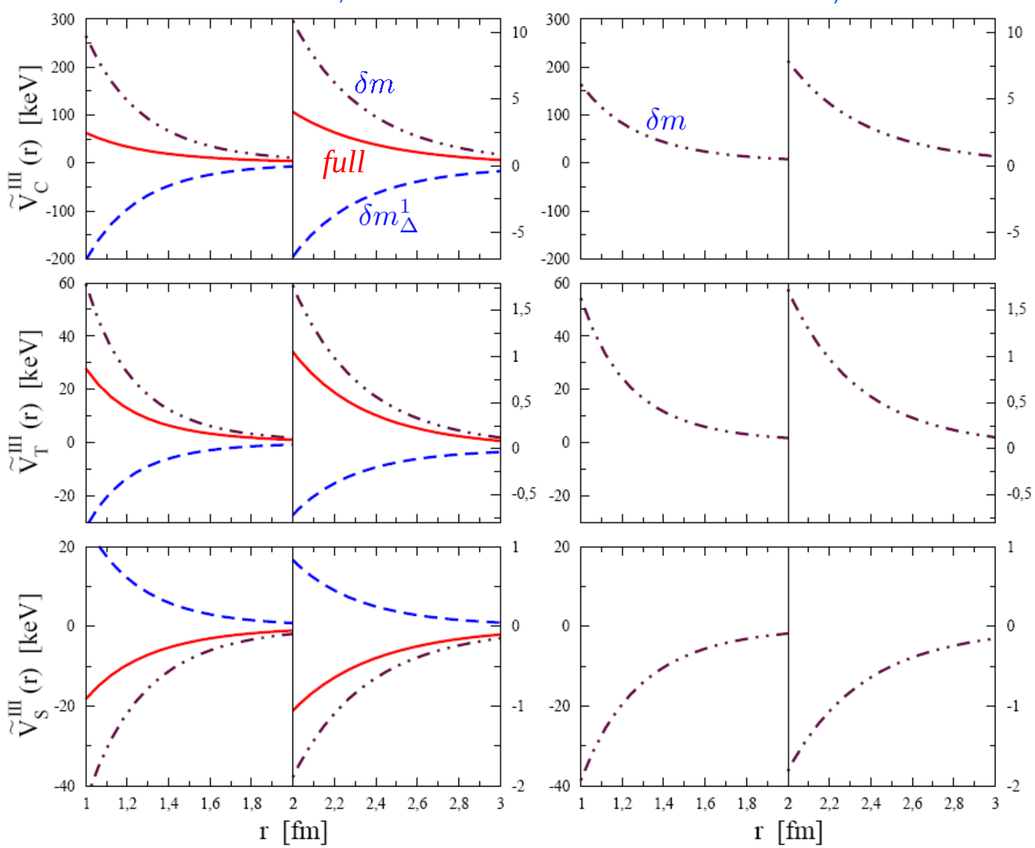
2π – exchange contributions with explicit Δ

$$V = (\tau_1^3 + \tau_2^3)\left[V_C^{\rm III} + V_S^{\rm III}\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T^{\rm III}\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}\right] + \ldots$$



Charge-symmetry-breaking 2π–exchange potential

Δ-full EFT, LO   Δ-less EFT, NLO



- Similar $\sim \delta m$ contr. to $\tilde{V}_{\rm S,T}^{\rm III}$ in the Δ-less and Δ-full EFT
- Sizeable deviation in $\sim \delta m$ contr. for $\tilde{V}_{\rm C}^{\rm III}$
- Strong cancelations between $\sim \delta m$ and $\sim \delta m_\Delta^1$ terms

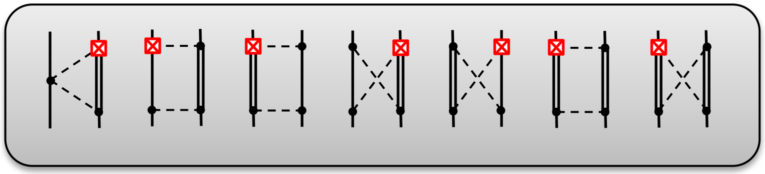
↓

Big contributions beyond the subleading corrections in the Δ-less EFT

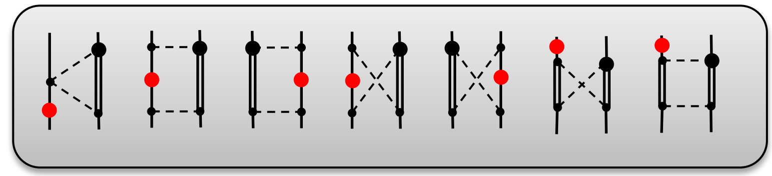
NLO Isospin-breaking NN potential

Epelbaum, H.K. , Meißner : forthcoming

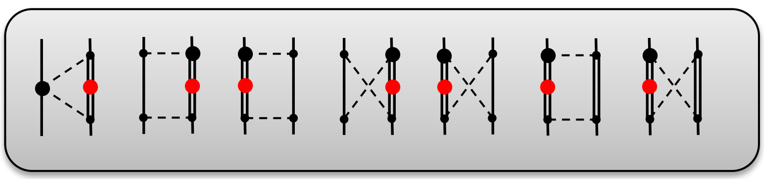
NLO diagram classes for 2π – exchange with explicit Δ



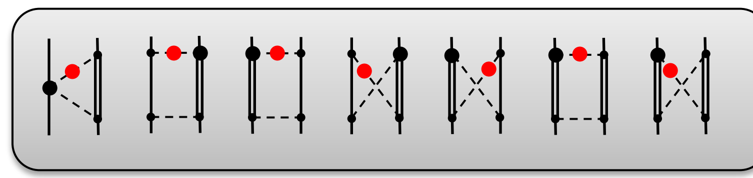
With 3 unknown LECs



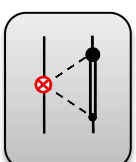
δm + isospin-inv. Vertex corr.



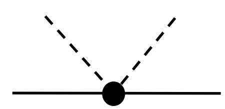
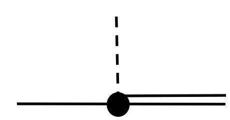
δm_Δ + isospin-inv. Vertex corr.



δM_π^2 + isospin-inv. Vertex corr.



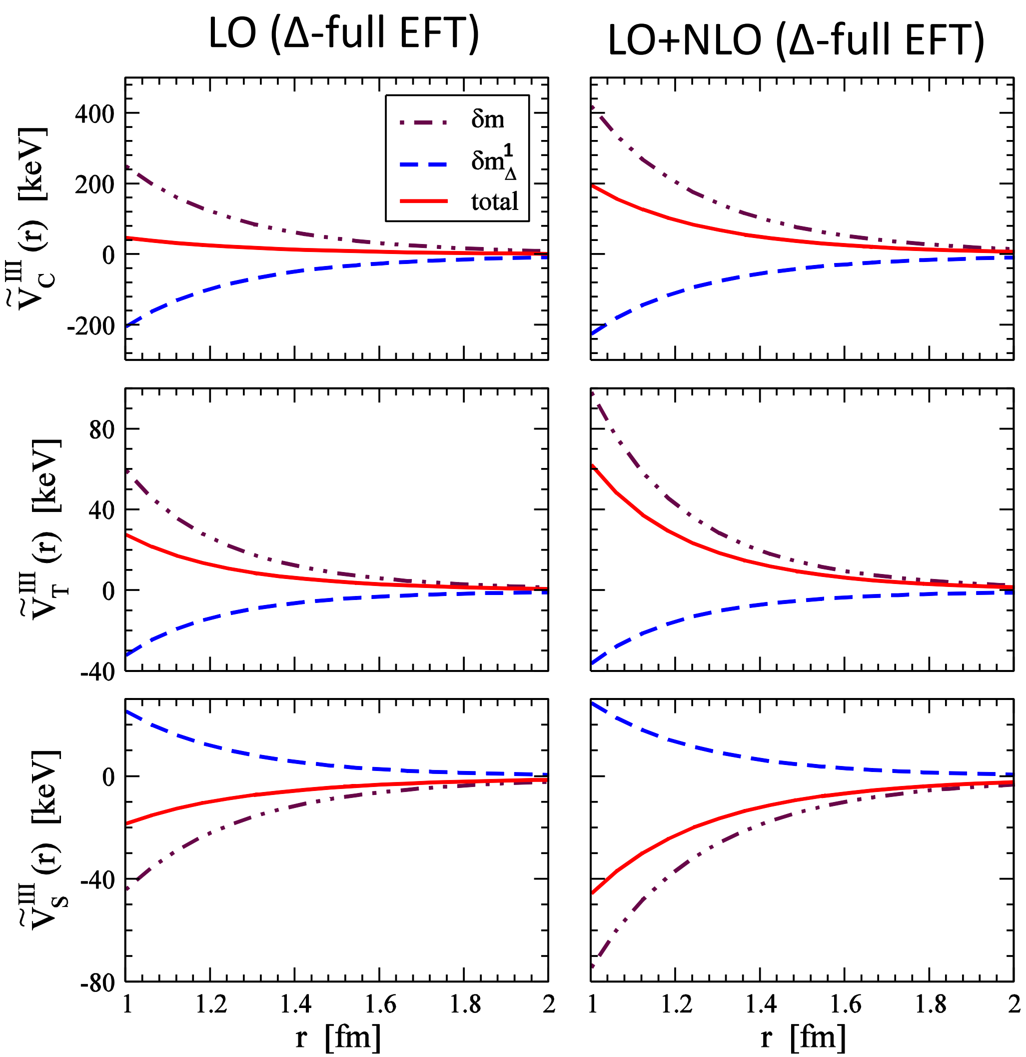
Isospin-breaking $\pi\pi\mathrm{N}$ vertex ($\sim \delta m$)+ isospin-inv. Vertex corr.

- Isospin-invariant vertex corrections:  $\sim c_i$  $\sim b_3 + b_8$
- For the following numerical considerations we set 3 unknown LECs to zero

NLO Isospin-breaking NN potential

Charge Symmetry Breaking (CSB) 2π–exchange potential

$$V = (\tau_1^3 + \tau_2^3)\left[V_C^{\rm III} + V_S^{\rm III}\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T^{\rm III}\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}\right] + \ldots$$



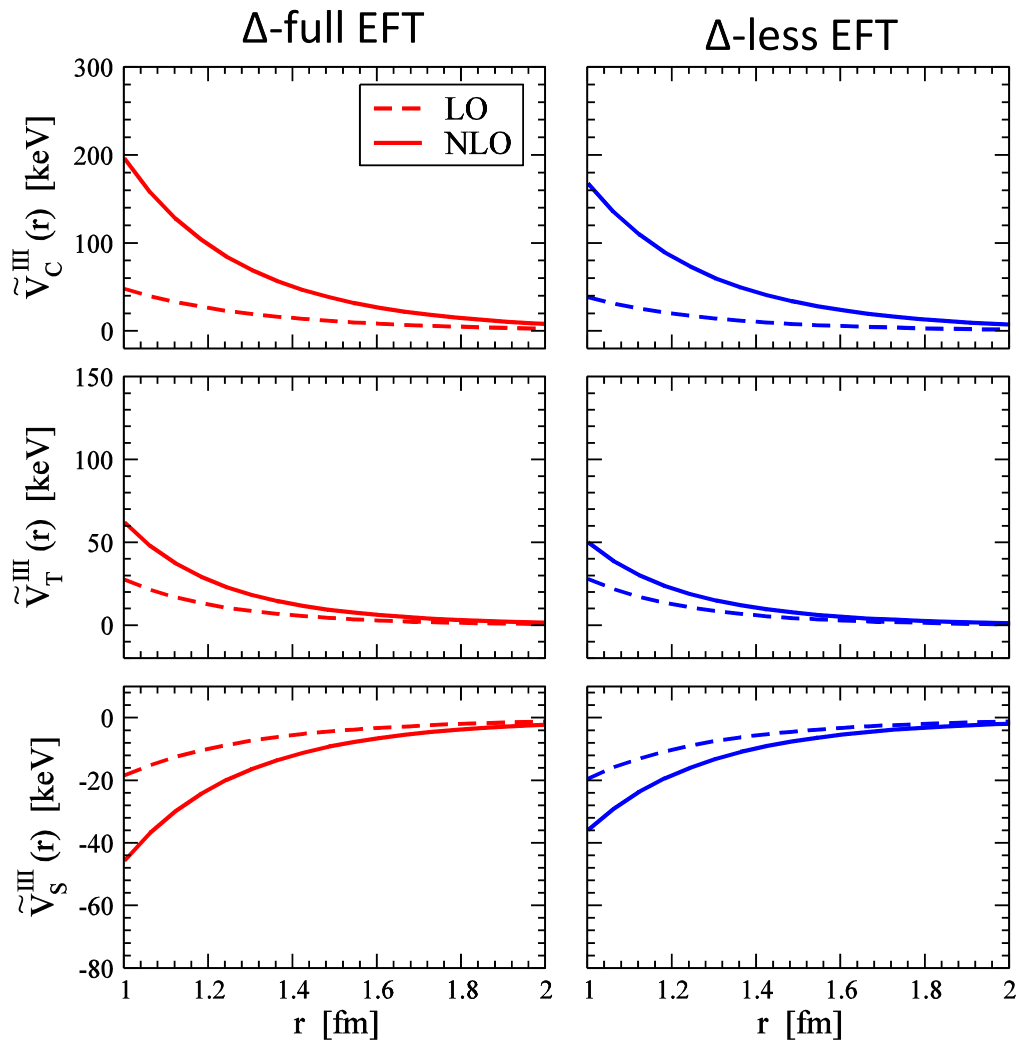
Individual contributions to CSB force in Δ-full EFT

- Strong cancelations between $\sim\delta m$ and $\sim\delta m_\Delta^1$ terms both at LO and NLO
- In all CSB forces $\sim\delta m$ terms are by almost factor 2 larger than $\sim\delta m_\Delta^1$ terms
- NLO corrections appear to be of natural size

NLO Isospin-breaking NN potential

Charge Symmetry Breaking (CSB) 2π–exchange potential

$$V = (\tau_1^3 + \tau_2^3)\left[V_C^{\rm III} + V_S^{\rm III}\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T^{\rm III}\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}\right] + \ldots$$



CSB forces in Δ-full and Δ-less EFT

- Overall results in both EFTs are remarkably close

Big contributions beyond the subleading corrections in the Δ-less EFT are completely compensated at NLO

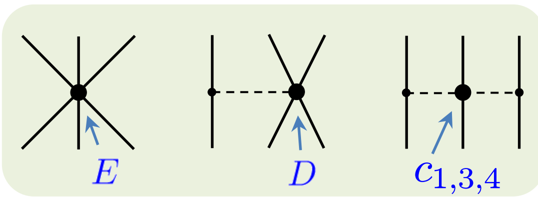
⇓

Equivalent results for CSB forces in Δ-less and Δ-full EFT

Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at NNLO

U. van Kolck '94; Epelbaum et al. '02; Nogga et al. '05; Navratil et al. '07



$c_{1,3,4}$ from the fit to πN-scattering data

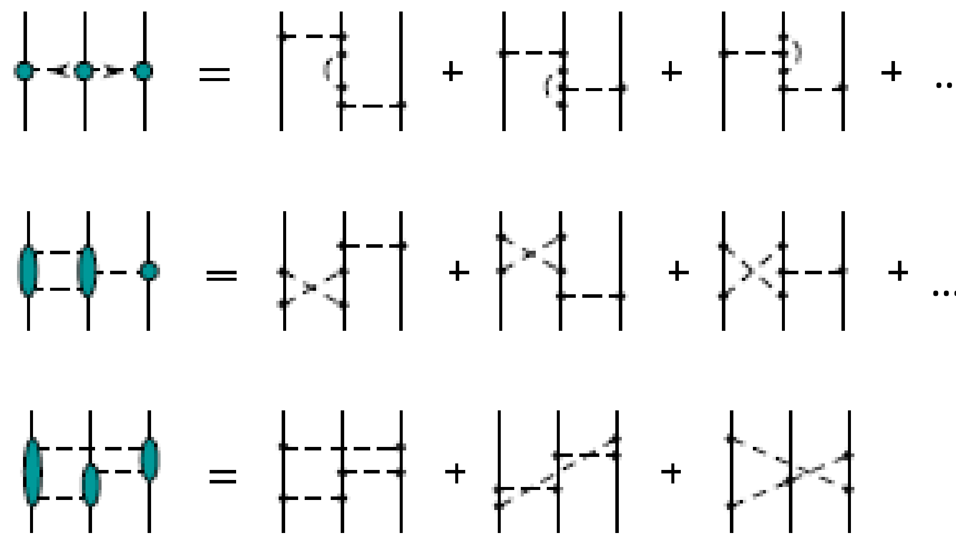
D, E from $^3H, ^4He, ^{10}B$ binding energy + coherent nd scattering length

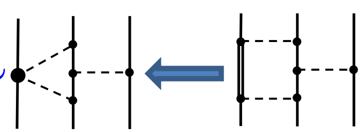
- Three-nucleon forces at N^{3}LO

Long range contributions

Bernard, Epelbaum, H.K. , Meißner '08; Ishikawa, Robilotta '07

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. are important



Large → $c_i \sim$ 

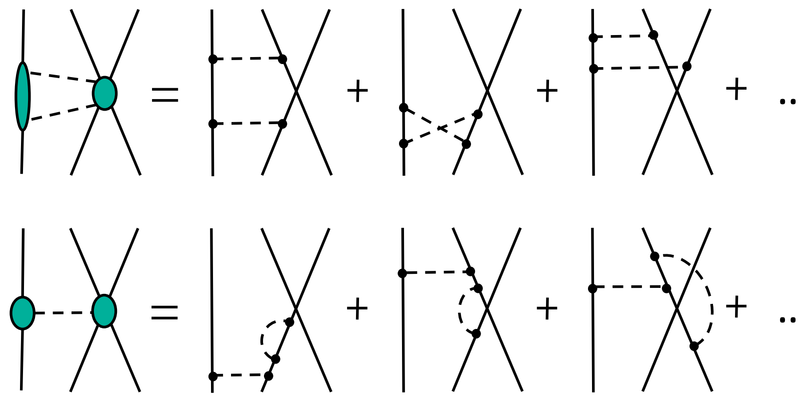
N^{4}LO-contr.

Kaiser '00/'01, see also Machleidt, Entem '10

Shorter range contributions

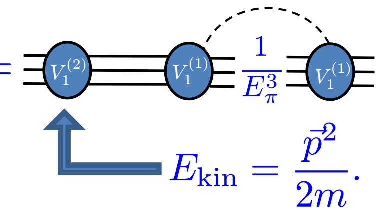
Bernard, Epelbaum, H.K. , Meißner : forthcoming

- LECs needed for shorter range contr. g_A, F_π, M_π, C_T
- Central NN contact interaction $\sim C_S$ does not contribute (note $C_S \gg C_T$)
- Smaller N^{3}LO shorter range contr. expected (approx. Wigner sym.)



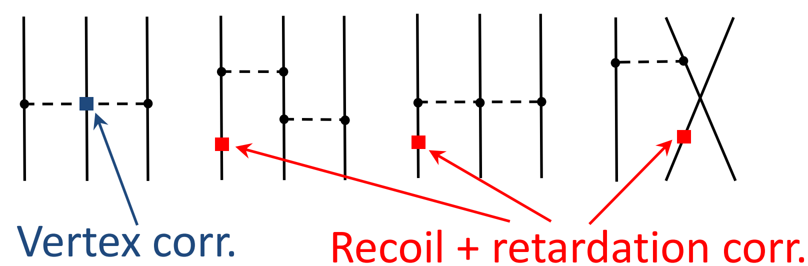
Relativistic 1/m corrections

- Additional unitary transformation: $U = \exp(\alpha_7(S_7 - S_7^\dagger))$ $\quad S_7 =$



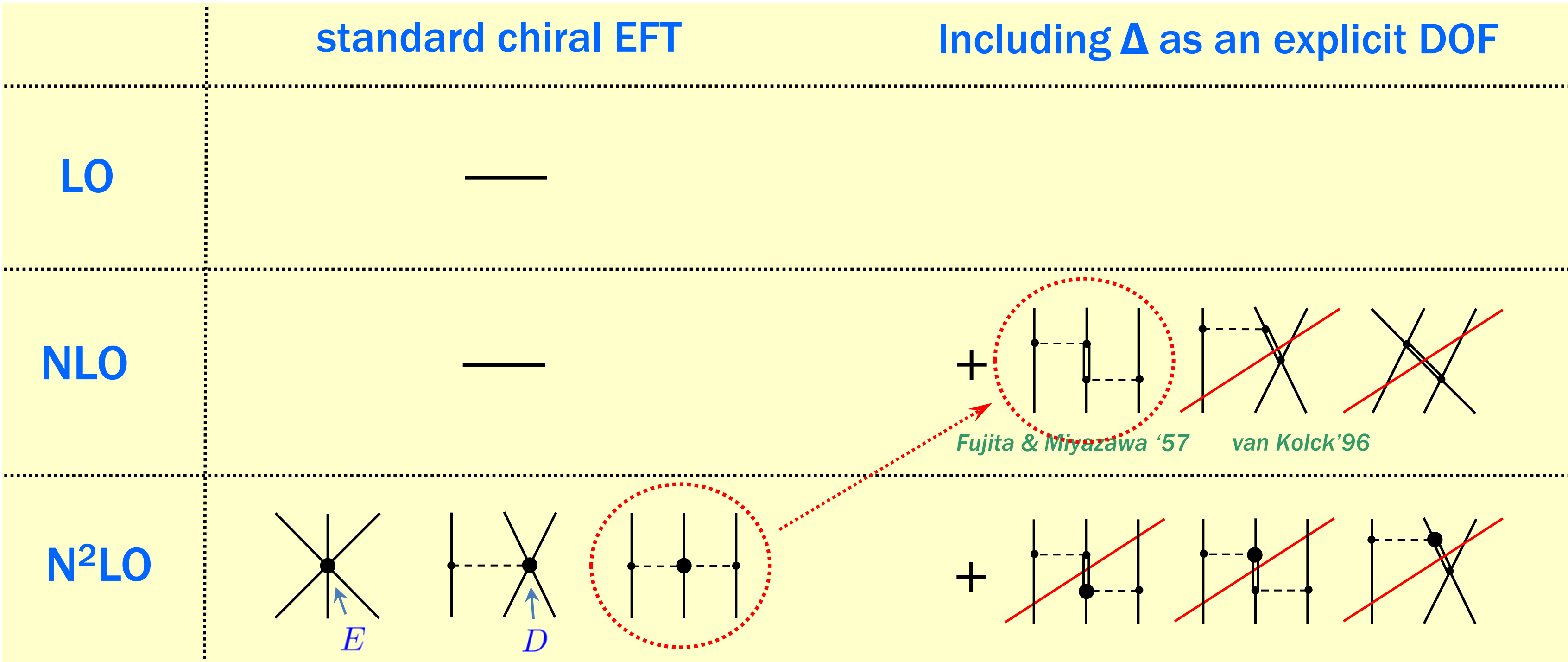
Adam et al. '92, Friar & Coon '94, Friar'99

- $\alpha_7 = \frac{1}{4}$ consistent with N^{3}LO NN potential
  Epelbaum, Glöckle, Meißner '05



Delta excitations and the three-nucleon force

Epelbaum, H.K., Meißner, Nucl. Phys. A806 (2008) 65



- The LO NNNΔ contact interaction $\bar{T}_i^{\mu} N \bar{N} S_{\mu} \tau^i N + \text{ h.c.}$ vanishes due to the Pauli principle
  ⟹ the LECs D and E are not saturated by the delta.
- No contributions from subleading 2π –exchange due to ∂^0 at the $b_3 + b_8$ vertex.
- The entire effect of the Δ is given by a partial shift of the N²LO TPE 3NF to NLO…

Computational strategy

- d-dim one loop tensor integrals by Passarino-Veltman reduction

$$\int \frac{d^dl}{(2\pi)^d}\frac{l_{\mu_1}\ldots l_{\mu_n}}{[l^2-M^2][(l+p)^2-M^2]} = T^{(1)}_{\mu_1\ldots\mu_n}(p) f_1(p^2)\int\frac{d^dl}{(2\pi)^d}\frac{1}{[l^2-M^2][(l+p)^2-M^2]} + T^{(2)}_{\mu_1\ldots\mu_n}(p) f_2(p^2)\int\frac{d^dl}{(2\pi)^d}\frac{1}{[l^2-M^2]}$$

Tensors in p

$f_1(p^2)$ and $f_2(p^2)$ include in general non-physical singularities which cancel in final result

- Dimensional-shift reduction Davydychev '91

Combinatorial factors

$$\int \frac{d^dl}{(2\pi)^d}\frac{l_{\mu_1}\ldots l_{\mu_n}}{[l^2-M^2][(l+p)^2-M^2]} = \sum_{ij} T^{(i)}_{\mu_1\ldots\mu_n}(p)\int\frac{d^{d+2i}l}{(2\pi)^{d+2i}}\frac{c_{ij}}{[l^2-M^2]^{n_{ij}}[(l+p)^2-M^2]^{m_{ij}}}$$

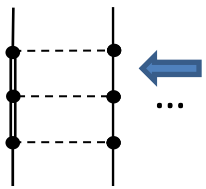
- Partial integration techniques provide recursion relations

$$\int\frac{d^dl}{(2\pi)^d}\frac{\partial}{\partial l_\mu} f(l) = 0$$ ⟸ Connection betw. Dimensional-shift and Passarino-Veltman red.

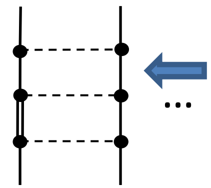
Implement Heavy-Baryon extension of these techniques in Mathematica/FORM

N^{3}LO potential with explicit Δ

- N^{3}LO two-nucleon force with explicit Δ - dof. not yet available (part of ERC project)

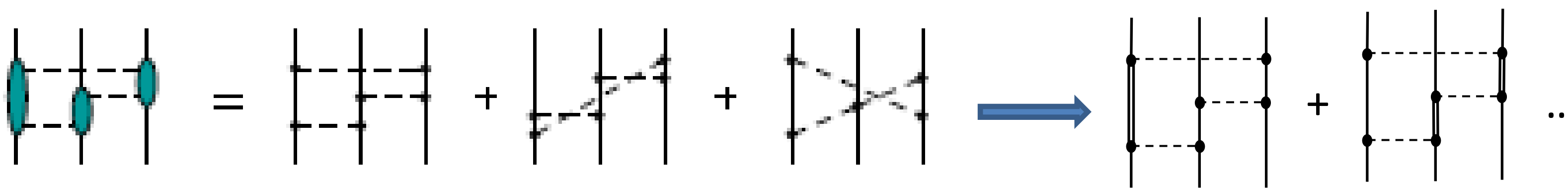


Straightforward calc. of irred. diagrams using 4-dim repr.



Only 3-dim repr. of red. diagrams by unitary transf.

- N^{3}LO three-nucleon force with explicit Δ - dof. (long range part)



- Replace inner nucleon lines by Δ - lines in all possible ways

Additional Δ - scale makes 3-dim. calculations inconvenient

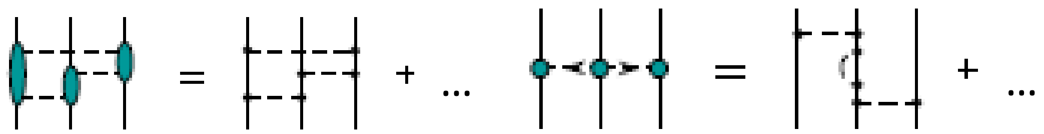
Is it possible to give a 4-dim. representation for N^{3}LO three-nucleon force ?

- Irreducible diagrams are naturally described as 4-dim Heavy Baryon loops

$$\frac{1}{l_0 + i\epsilon} \longrightarrow \text{Inclusion of } \Delta \longrightarrow \frac{1}{l_0 - \Delta + i\epsilon}$$

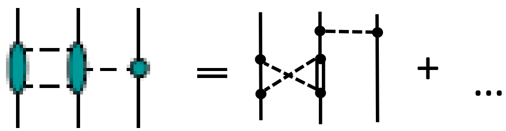
4-dim. matching of reducible diagrams

- Topology classes where 4-dim. matching is possible for all diagrams



Unitary transformations chosen to make the force renormalizable

- Topology classes where not all diagrams do match to 4-dim. loop structures



Introduce additional one dimensional integration to match the whole topology class

$$\frac{1}{q_3^2+M^2}\int\frac{dz}{2\pi}\frac{d^dl}{(2\pi)^d}\frac{1}{l^2-M^2}\frac{1}{(l-q_1-zv)^2-M^2}\frac{1}{l_0-\Delta}\frac{1}{z+i\epsilon}$$

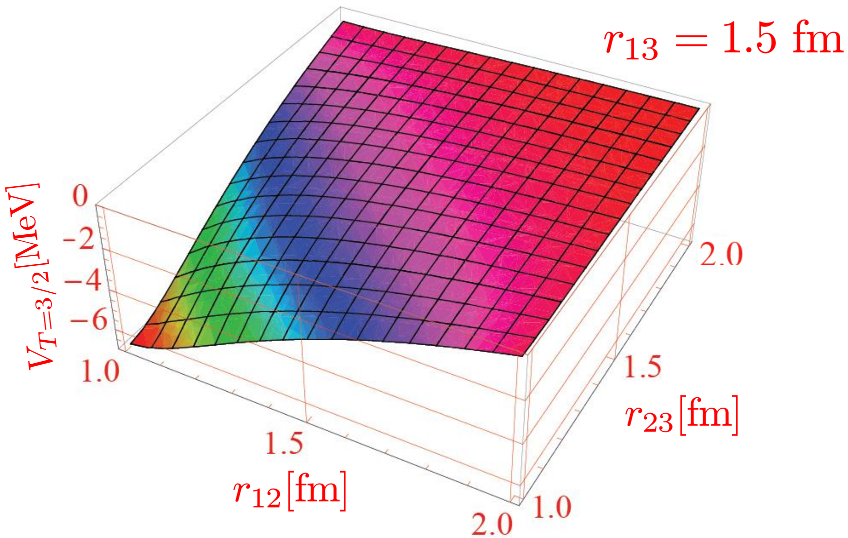
Sample matched integral

Additional integration → New master integrals given by elliptic integrals

New master integrals do not appear in the final result but cancel after all contributions have been summed up

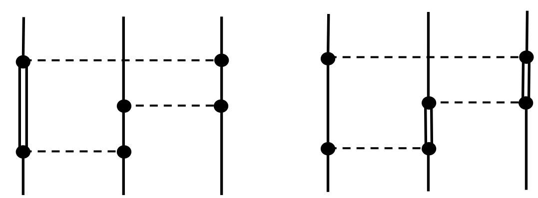
Illinois vs Chiral Ring Diagrams
(Preliminary)

- Illinois ring diagrams contr. with Δ-dof *(Pieper et al. PRC64 (2002) 014001)*



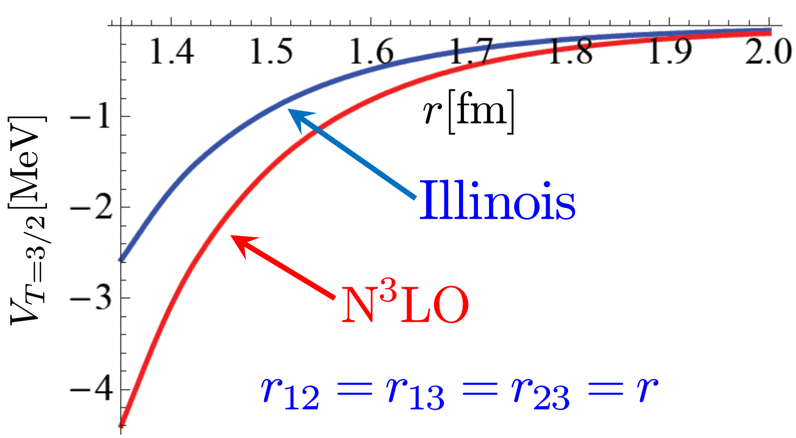
$$V^{3\pi,\Delta R} = S^I_\tau V_{T=3/2} + \ldots$$

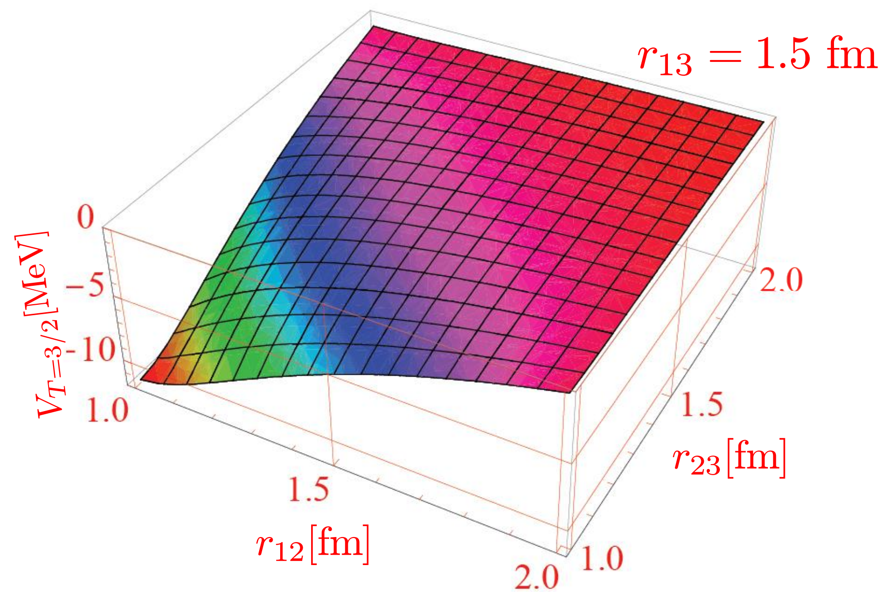
$$S^I_\tau = 2 + \frac{2}{3}(\tau_1 \cdot \tau_2 + \tau_1 \cdot \tau_3 + \tau_2 \cdot \tau_3) = 4P_{T=3/2}$$



- No contr. $\sim 1/\Delta$ taken into account

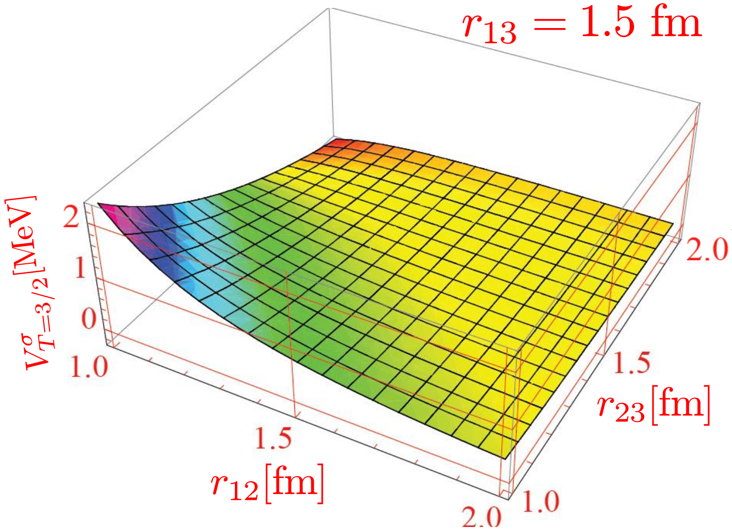
- Chiral ring diagrams contr. with Δ-dof



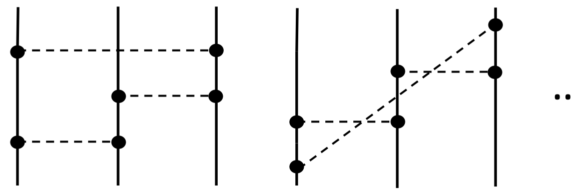


N^3LO Chiral Ring Diagrams
(Preliminary)

- Dominant nucleon contributions to N^3LO ring diagrams

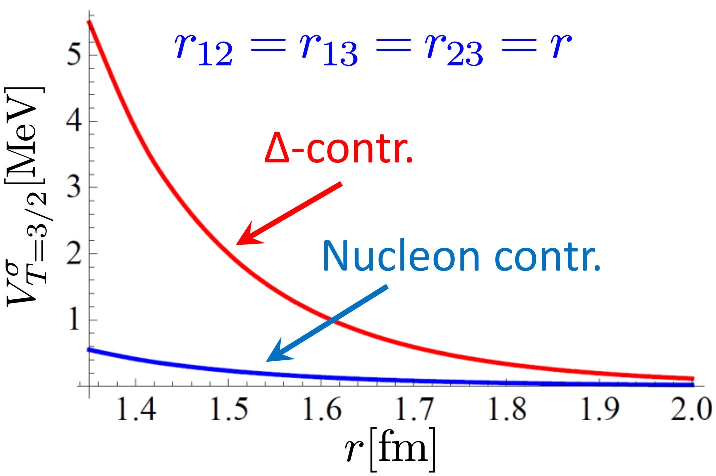


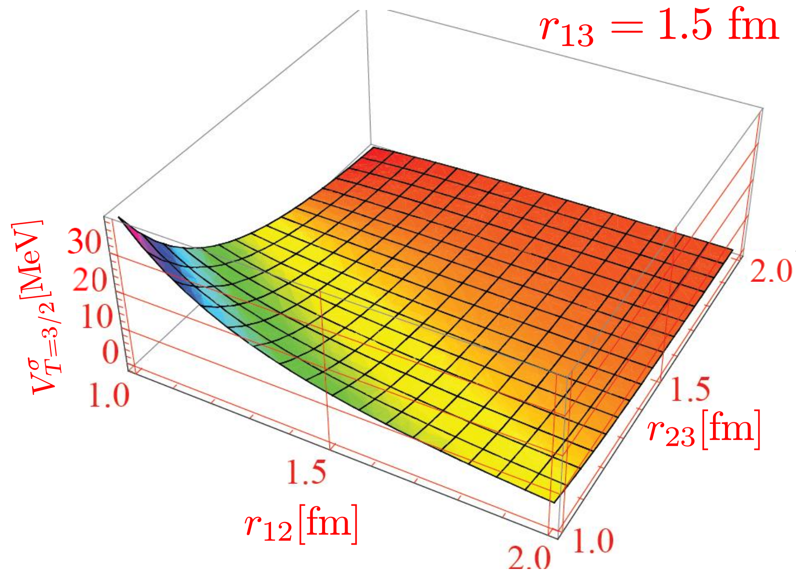
$$V^{3\pi,R} = S^I_\tau \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \vec{\sigma}_3 \cdot (\hat{r}_{13} \times \hat{r}_{23}) V^{\sigma}_{T=3/2} + \ldots$$



- No contr. to the central force

- Δ – contributions to N^3LO ring diagrams





Dominant N^3LO ring diagram contributions

(Preliminary)

- Dominant contribution of ring diagrams with Δ - dof. at $r_{12} = r_{13} = r_{23} = 1.5$ fm

$$S_\tau^I = 2 + \frac{2}{3}(\tau_1 \cdot \tau_2 + \tau_1 \cdot \tau_3 + \tau_2 \cdot \tau_3),\ S_{\tau,312}^D = \frac{2}{3}\tau_1 \cdot \tau_2,\ A_\tau^I = \frac{i}{3}\tau_1 \cdot (\tau_2 \times \tau_3)$$

$$S_\sigma^1 = \vec\sigma_2 \cdot (\hat r_{12} \times \hat r_{23})\vec\sigma_3 \cdot (\hat r_{13} \times \hat r_{23}),\ S_\sigma^2 = i\vec\sigma_1 \cdot (\hat r_{12} \times \hat r_{13})$$
$$S_\sigma^3 = \vec\sigma_1 \cdot (\hat r_{12} \times \hat r_{13})\vec\sigma_2 \cdot (\hat r_{12} \times \hat r_{23})\vec\sigma_3 \cdot (\hat r_{13} \times \hat r_{23})$$

	1	S_σ^1	S_σ^2	S_σ^3
S_τ^I	-1.559 MeV	2.004 MeV	0	0
$S_{\tau,312}^D$	0.271 MeV	-0.472 MeV	-0.037 MeV	0
A_τ^I	0	0	-1.219 MeV	1.045 MeV

- Important to make a partial wave analysis

Find a phase-space point in 3N continuum which is most sensitive to large N^3LO contr.

Planed pd-break up experiment at COSY with proton beam energy betw. 30 and 50 MeV by PAX Collaboration

Summary

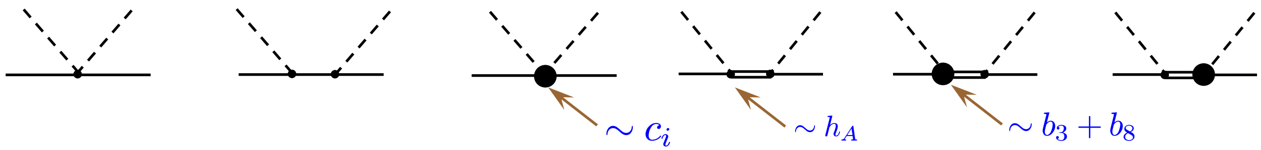
- Few-nucleon forces within chiral EFT are analyzed upto N^3LO
- Better convergence of nuclear forces if Δ-isobar is included explicitly
- Sizeable contributions from N^3LO three nucleon forces with explicit Δ-isobar

Perspectives

- Partial wave analysis of N^3LO three body forces
- Complete numerical studies with 3NF and 4NF upto N^3LO
- Electroweak reactions with few-nucleon systems

Fit of LECs to πN scattering data

Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127



+ crossed graphs

Two possible values for h_A

- fit 1: $h_A = \frac{3 g_A}{2\sqrt{2}} \sim 1.34$ *(SU(4), large N_c)*
- fit 2: $h_A = 1.05$ *(Fettes & Meißner '01)*

LECs	Q^2, no Δ	Q^2, fit 1	Q^2, fit 2
c_1	-0.57	-0.57	-0.57
c_2	2.84	-0.25	0.83
c_3	-3.87	-0.79	-1.87
c_4	2.89	1.33	1.87
h_A	–	$1.34^\star$	$1.05^\star$
$b_3 + b_8$	–	1.40	2.95

Results of the fit

- Improved description of P-wave parameters when Δ is included
- Strongly reduced values for c_i
- Resulting c_i depend strongly on h_A while the thresh. param. do not

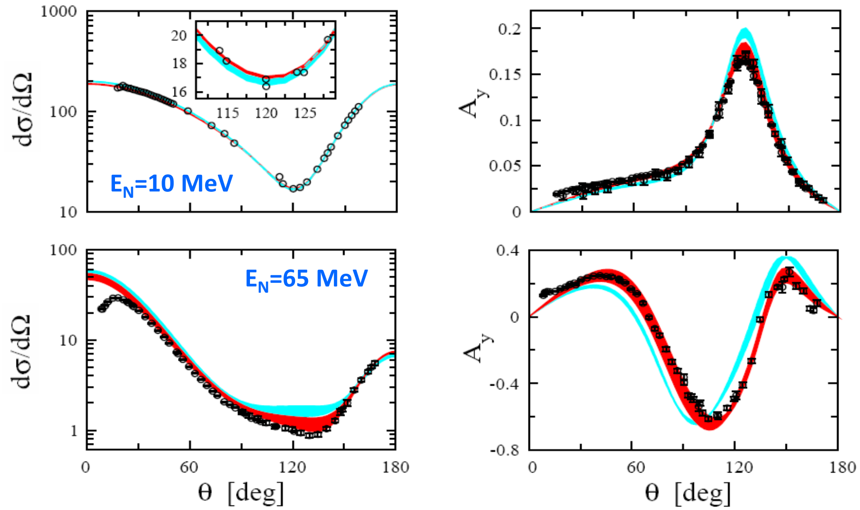
S- and P-wave threshold parameters

	Q^2, no Δ	Q^2 fits 1, 2	EM98
a_{0+}^+	0.41	0.41	0.41 ± 0.09
b_{0+}^+	-4.46	-4.46	-4.46
a_{0+}^-	7.74	7.74	7.73 ± 0.06
b_{0+}^-	3.34	3.34	1.56
a_{1-}^-	-0.05	-1.32	-1.19 ± 0.08
a_{1-}^+	-2.81	-5.30	-5.46 ± 0.10
a_{1+}^-	-6.22	-8.45	-8.22 ± 0.07
a_{1+}^+	9.68	12.92	13.13 ± 0.13

Nd elastic scattering

Cross section & vector analyzing power

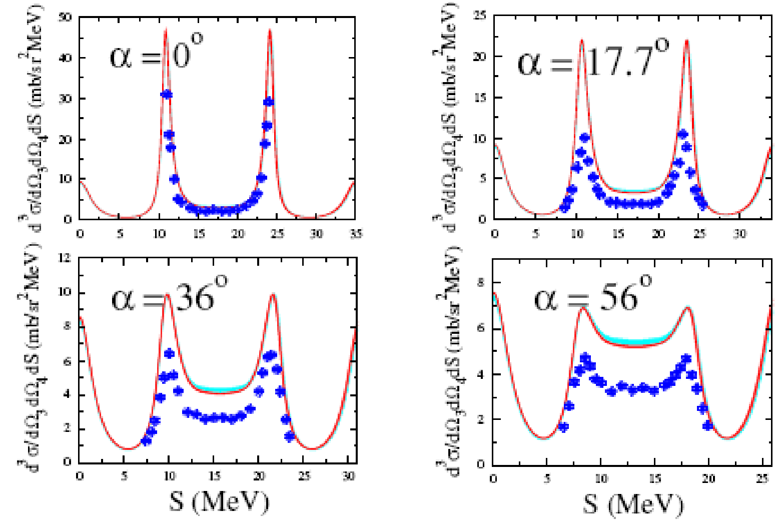
E.pelbaum, PPNP 57 (2006) 654



Deuteron break-up

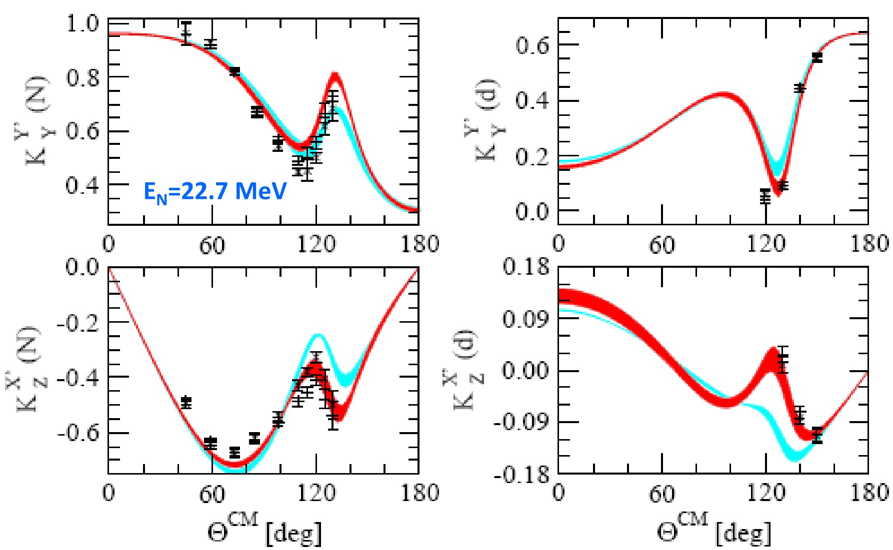
SCRE configuration at E_d=19 MeV

Ley et al., PRC 73 (2006) 064001



Polarization transfer coefficients

Witała et al., PRC 73 (2006) 044004



- Promising NNLO results for Nd elastic scattering
- Satisfactory A_y description related to overprediction of triplet P-waves
- Systematic overestimation of deuteron break-up data
- Hope for improvement at N^3LO