Three bosons in two dimensions

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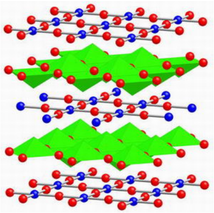
Outline

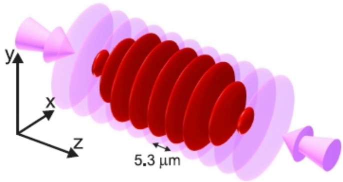
Introduction

Two-dimensional systems

- 2d systems can be found in many areas of physics
- Examples:
    - Surfaces
    - High-temperature superconductors
    - Graphen
    - ...
- Traps allow for lower-dimensional atomic gases



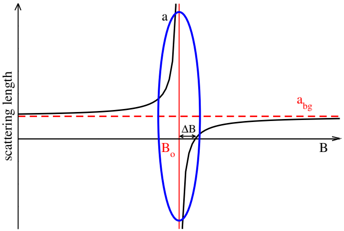
Argonne Nat. Lab.



Martiyanov et al. (2010)

Universality

- There is no Efimov effect in 2*d*
- However, 2*d* scattering length can be very large
- Feshbach resonance



Universal regime

- a_{2d} related to a_{3d}

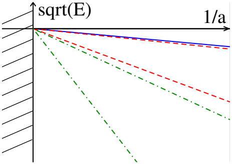
Universal $2d$ systems

- Shallow dimer

$$E_2 = 4e^{-2\gamma_E}\,\frac{\hbar^2}{m\,a^2}$$

- Trimer states Bruch, Tjon (1979); ...

$$E_3^{(0)} = 16.522688(1)\,E_2 \quad \text{and} \quad E_3^{(1)} = 1.2704091(1)\,E_2$$



- Tetramer states Platter, Hammer, Meißner (2004)

$$E_4^{(0)} = 197.3(1)\,E_2 \quad \text{and} \quad E_4^{(1)} = 25.5(1)\,E_2$$

- N-body droplets Hammer, Son (2004); Blume (2005); Lee (2006)

$$\frac{E_{N+1}}{E_N} \approx 8.567, \quad N \gg 1.$$

Effective field theory

Effective field theory

- Separation of scales: $r/a \ll 1$
- Effective range expansion Verhaar et al. (1984)

$$\cot\delta(k) = \frac{2}{\pi}\left\{\gamma_E + \ln\left(\frac{ka}{2}\right)\right\} + \frac{r^2}{2\pi}k^2 + \mathcal{O}(k^4)$$
$$= \frac{2}{\pi}\ln\left(\frac{k}{\kappa}\right) + \frac{r^2}{2\pi}(\kappa^2 + k^2) + \mathcal{O}(k^4)$$

- Binding momentum

$$\kappa \approx \frac{2e^{-\gamma_E}}{a}\sqrt{1 + 2e^{-2\gamma_E}\frac{r^2}{a^2}}$$

- Lagrangian

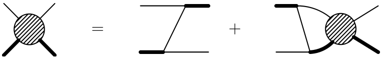
$$\boxed{\mathcal{L} = \Psi^\dagger\left(i\partial_t + \frac{\nabla^2}{2m}\right)\Psi + d^\dagger\left(\eta\left(i\partial_t + \frac{\nabla^2}{4m}\right) + \Delta\right)d - \frac{g}{4}(d^\dagger\Psi^2 + \Psi^{\dagger 2}d)}$$

Effective field theory

- Dimer propagator

$$iD(p) = -i\frac{32\pi}{mg^2}\left\{\ln\left[\frac{p^2/4 - mp_0 - i\epsilon}{\kappa^2}\right] + \frac{r^2}{2}\left(\kappa^2 + mp_0 - p^2/4\right)\right\}^{-1}$$

- Integral equation



$$\begin{aligned} T(p,k;E) &= \frac{16\pi\kappa^2/(2-\kappa^2 r^2)}{\sqrt{(p^2+k^2-mE)^2 - p^2k^2}} \\ &+ 4\int_0^\infty \frac{dq\, q\, T(q,k;E)}{\sqrt{(p^2+q^2-mE)^2 - p^2q^2}} \\ &\times \left(\ln\left[\frac{\frac{3}{4}q^2 - mE - i\epsilon}{\kappa^2}\right] + \frac{r^2}{2}\left(\kappa^2 + mE - \frac{3}{4}q^2\right)\right)^{-1} \end{aligned}$$

Wigner bound

- Integral equation

$$
\begin{aligned}
T(p,k;E) &= \frac{16\pi\kappa^2/(2-\kappa^2 r^2)}{\sqrt{(p^2+k^2-mE)^2-p^2k^2}} \\
&+ 4\int_0^\infty \frac{dq\, q\, T(q,k;E)}{\sqrt{(p^2+q^2-mE)^2-p^2q^2}} \\
&\times \left(\ln\left[\frac{\frac{3}{4}q^2-mE-i\epsilon}{\kappa^2}\right]+\frac{r^2}{2}\left(\kappa^2+mE-\frac{3}{4}q^2\right)\right)^{-1}
\end{aligned}
$$

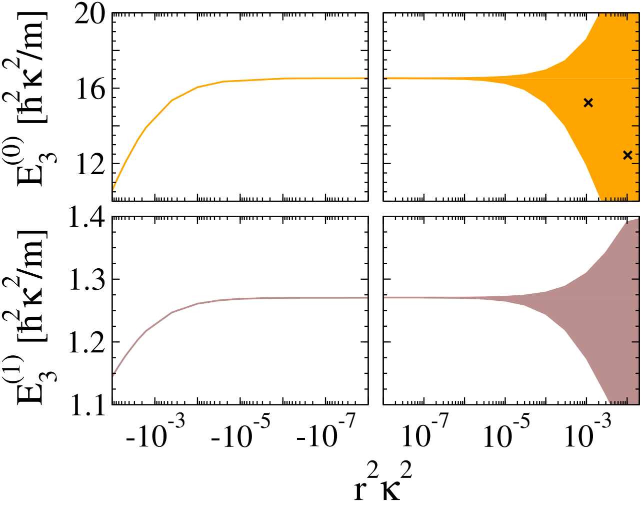
- Wigner bound Hammer, Lee (2009)

$$
r^2 \le \frac{2}{\Lambda^2}\left\{[\ln(\Lambda a)+1/2]^2+1/4\right\}
$$

Results for three bosons in 2d

KH and Hammer, arXiv:1101.1891

Three-body bound states



At LO:

$E_3^{(0)} \approx 16.5\, E_2$

$E_3^{(1)} \approx 1.27\, E_2$

Crosses: KORONA helium-helium potential Blume (2005)

Perturbative equations for E_3

- For $r^2\kappa^2 < 0$:

$$
\begin{aligned}
E_3^{(0)}/E_2 &= 16.522688(1) + 28000(5000)\, r^2\kappa^2 + \mathcal{O}(r^4\kappa^4) \\
E_3^{(1)}/E_2 &= 1.2704091(1) + 540(80)\, r^2\kappa^2 + \mathcal{O}(r^4\kappa^4)
\end{aligned}
$$

- For $r^2\kappa^2 > 0$:

$$
\begin{aligned}
E_3^{(0)}/E_2 &= 16.522688(1) - 28000(5000)\, r^2\kappa^2 + \mathcal{O}(r^4\kappa^4) \\
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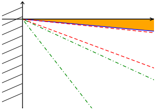
\end{aligned}
$$

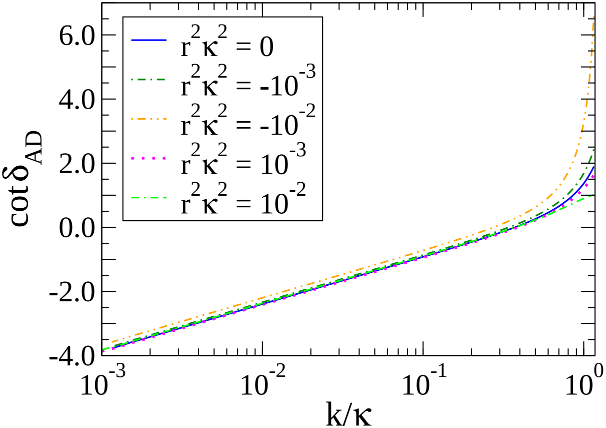
- Is there physics behind those large coefficients?
- What happens for $N > 3$?

Elastic atom-dimer scattering



$$T(k,k;E) = \frac{3}{m} f_k = \frac{3}{m}\frac{1}{\cot\delta_{AD}(k) - i}$$

with $k = \sqrt{\frac{4m}{3}(E + E_2)}$



Elastic atom-dimer scattering

- Extracting atom-dimer scattering parameters

$$\cot \delta_{AD}(k) = \frac{2}{\pi}\left\{\gamma_E + \ln\left(\frac{k a_{AD}}{2}\right)\right\} + \frac{r_{AD}^2}{2\pi} k^2$$

- at $r = 0$:

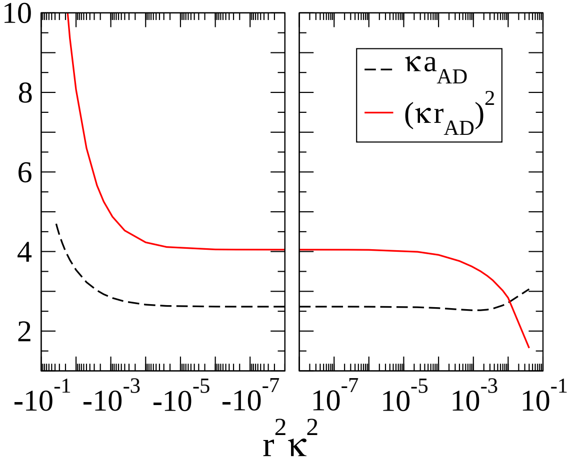
  $a_{AD} = 2.614(1)/\kappa = 2.328(1)\, a$ and
  $r_{AD}^2 = 4.0(2)/\kappa^2$

- in agreement with

  $\ln(a_{AD}/a) = 0.8451$ Kartavtsev, Malykh (2006)

  $a_{AD} = 2.95\, a$ Nielsen, Fedorov, Jensen (1999)

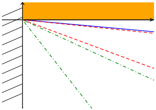
Elastic atom-dimer scattering



Perturbative equation for $\kappa^2 a_{AD}$:

$$\kappa^2 a_{AD} = 2.615(1) - 4100(500)\, r^2\kappa^2 + \mathcal{O}(r^4\kappa^4)$$

Three-body recombination rate

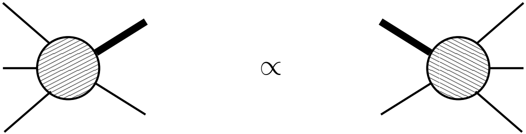


vanishes for $E \to 0$, $E \to \infty$, and $a \to \infty$

$$\frac{dn}{dt} = -3\alpha n^3$$

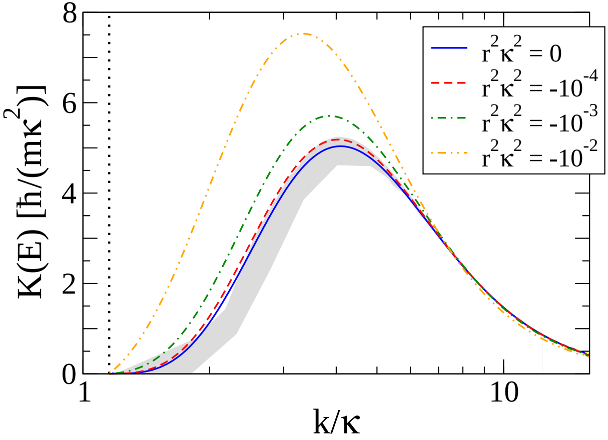
$$\alpha(T) = \frac{1}{6k_B^2 T^2} \int_0^\infty dE\, E\, e^{-E/(k_B T)} K(E)$$

$$K(E) = \frac{36\pi^2}{m^2 E} k\, \sigma_{AD}^{(\text{inel})}(E)$$



Three-body recombination rate

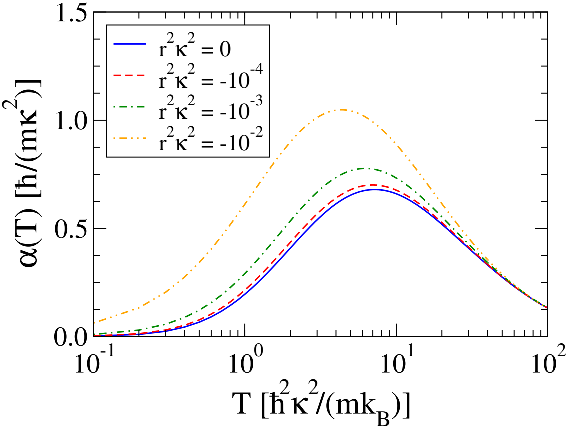
$$K(E) = \frac{36\pi^2}{m^2 E} k\, \sigma_{AD}^{(\mathrm{inel})}(E) \quad \text{with} \quad \sigma_{AD}^{(\mathrm{inel})}(E) = \frac{4}{k}(\mathrm{Im} f_k(0) - |f_k|^2)$$



Grey band:
$r^2\kappa^2 = 10^{-4}$

Three-body recombination rate

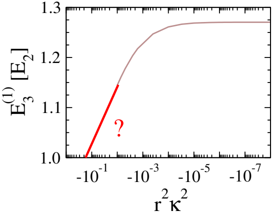
$$\alpha(T) = \frac{1}{6k_B^2 T^2} \int_0^\infty dE\, E\, e^{-E/(k_B T)} K(E)$$



- Maximum at $5 - 7\, E_2/k_B$
- Peak $\approx \hbar^3/(E_2 m^2)$

Summary and outlook

- $2d$ systems are experimentally available
- They show interesting effects
- Next-to-leading order calculations for three-body observables
- Comparison to experiment interesting
- Large coefficients in ERE remain a puzzle
- Experimentally possible to tune r^2:



Summary and outlook

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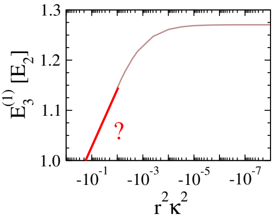


Thank you for your attention

Supplement

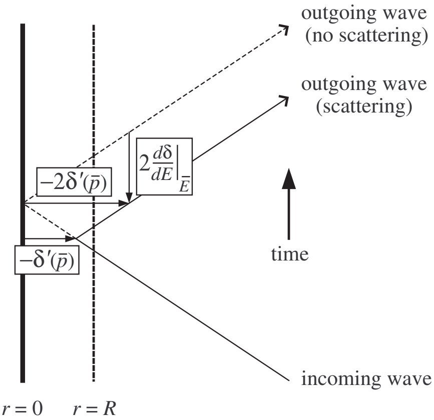
- Perturbative method

$$T(p,k;E) = T_0(p,k;E) + T_2(p,k;E) + \ldots$$

$$T(k,k;E) = \left(1 + \frac{r^2\kappa^2}{2}\right) T_0(p,k;E)$$
$$- \quad \frac{mr^2}{4\pi\kappa^2}\int_0^\infty dq\, q\left(k^2 + mE - \frac{3}{4}q^2\right)\left\{\frac{T_0(k,q;E)}{\ln\left[\left(\frac{3}{4}q^2 - mE - i\epsilon\right)/\kappa^2\right]}\right\}^2$$

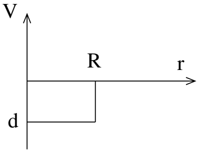
Supplement

- Wigner bound Hammer, Lee (2010)



Supplement

- Example: step potential in $2d$ Hammer, Lee (2010)



$$\tilde{\kappa}^2 = -2\mu d$$

$$|d| \ll \frac{1}{2\mu R^2} \quad \leftrightarrow \quad \tilde{\kappa} R \ll 1$$

$$
\begin{aligned}
r^2 &\approx R^2 \left[\frac{1}{2} + \frac{2}{(\tilde{\kappa} R)^2} \right] \\
a &\approx R \exp\left\{ \frac{2}{(\tilde{\kappa} R)^2} - \frac{1}{4} \right\} \\
\kappa &\approx \frac{2 e^{-\gamma_E}}{R} \exp\left\{ - \left(\frac{2}{(\tilde{\kappa} R)^2} - \frac{1}{4} \right) \right\} \\
\Rightarrow \kappa^2 r^2 &\approx 4 e^{-2\gamma_E} \exp\left\{ -2 \left(\frac{2}{(\tilde{\kappa} R)^2} - \frac{1}{4} \right) \right\} \left(\frac{1}{2} + \frac{2}{(\tilde{\kappa} R)^2} \right) \to 0
\end{aligned}
$$