Current conservation in effective field theories for the two-nucleon system

Master thesis presentation

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Introduction

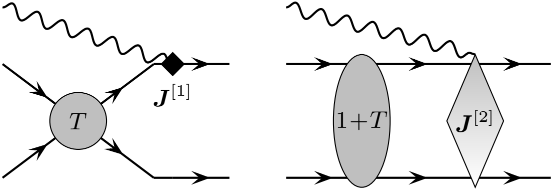
- QCD is widely believed to be the theory of strong interaction
- cannot be treated perturbatively at the low energies of interest in nuclear physics $\Rightarrow$ gives rise to low-energy EFTs
- The EFT preserving all relavant symmetries of QCD (including chiral invariance) is chiral perturbation theory (ChPT).
- In 1990 Weinberg proposed to use ChPT not only in the $\pi\pi$ and πN sector but also for the few-nucleon problem.

  S. Weinberg, *Physics Letters B* **251**, 288–292 (Nov. 15, 1990)
- Since then, his method has been extensively studied and calculations exist in ChPT up to high accuracy.

  E. Epelbaum *et al.*, *Reviews of Modern Physics* **81**, 1773–1825 (Dec. 21, 2009)
- Another and more recent testing ground of ChPT: Investigations on interactions of few-nucleon systems with external currents

Here: How to appropriately regularize the electromagnetic vector current

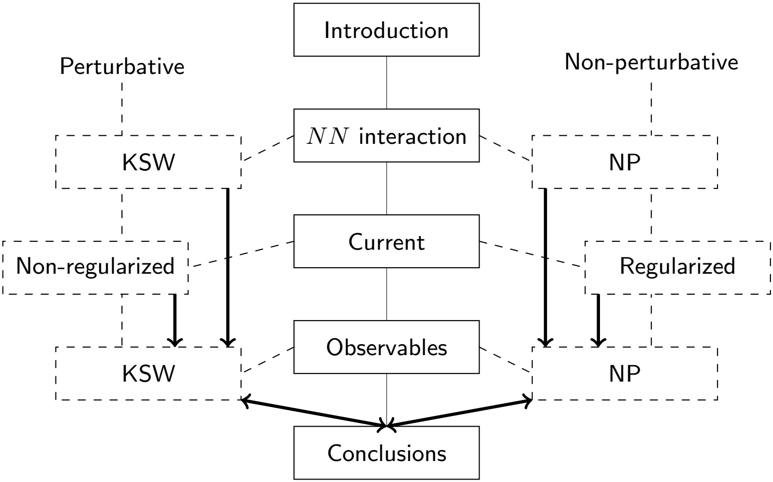
- Focus on particular $NN\gamma$ reaction at very low energies



Introduction

- work in pionless EFT (nucleon energies well below the pion production threshold) and stick to NN contact interactions
  $\Rightarrow NN$ potential is separable and the problem to a large extent solvable analytically
- Issue with NN compared to $\pi\pi$ and πN system is that the amplitude behaves non-perturbatively
  Two approaches:
  Weinberg (NP): Apply power counting to the NN potential and solve the Lippmann-Schwinger equation non-perturbatively for this potential
  S. Weinberg, *Physics Letters B* **251**, 288–292 (Nov. 15, 1990)
  S. Weinberg, *Nuclear Physics B* **363**, 3–18 (Sept. 30, 1991)
  Kaplan, Savage, Wise (KSW): Resum contribution of leading NN operators only and include the rest perturbatively (directly applied to the amplitude)
  D. B. Kaplan *et al.*, *Physics Letters B* **424**, 390–396 (Apr. 1998)
  D. B. Kaplan *et al.*, *Nuclear Physics B* **534**, 329–355 (Nov. 1998)
- Interested in the renormalization of the current operator in the NP approach
  KSW approach is used as a benchmark because power counting and renormalization are transparent

Structure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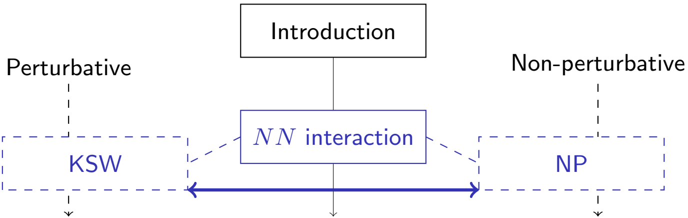
Structure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NN contact potential

Two nucleons in cms frame with initial and final momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$

$$V^{\mathrm{NNLO}} = \overbrace{\underbrace{V^{(0)}}_{V^{\mathrm{LO}}} + V^{(2)}}^{V^{\mathrm{NLO}}} + V^{(4)}$$

$$V^{(0)}(\boldsymbol{q},\boldsymbol{k}) = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \,,$$

$$V^{(2)}(\boldsymbol{q},\boldsymbol{k}) = C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{k}^2 + (C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{k}^2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \mathrm{i} C_5 \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot (\boldsymbol{q} \times \boldsymbol{k}) \,,$$
$$+ C_6 (\boldsymbol{q} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{q} \cdot \boldsymbol{\sigma}_2) + C_7 (\boldsymbol{k} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{k} \cdot \boldsymbol{\sigma}_2)$$

$$V^{(4)}(\boldsymbol{q},\boldsymbol{k}) = D_1 \boldsymbol{q}^4 + \dots \,, \qquad \text{with} \quad \boldsymbol{q} := \boldsymbol{p}' - \boldsymbol{p} \,, \ \boldsymbol{k} := \frac{\boldsymbol{p}' + \boldsymbol{p}}{2}$$

E. Epelbaum *et al.*, *Nuclear Physics A* **747**, 362–424 (Jan. 2005)

Restrict on ${}^1S_0$ partial wave:

$$V_{{}^1S_0}(p',p) := \langle {}^1S_0 | V(\boldsymbol{p}',\boldsymbol{p}) | {}^1S_0 \rangle$$
$$V^{\mathrm{LO}}_{{}^1S_0}(p',p) = \tilde{C}_{{}^1S_0}$$
$$V^{\mathrm{NLO}}_{{}^1S_0}(p',p) = \tilde{C}_{{}^1S_0} + C_{{}^1S_0}(p^2 + p'^2)$$
$$V^{\mathrm{NNLO}}_{{}^1S_0}(p',p) = \tilde{C}_{{}^1S_0} + C_{{}^1S_0}(p^2 + p'^2) + 2D_{{}^1S_0} p^2 p'^2 + \tilde{D}_{{}^1S_0}(p^4 + p'^4)$$
$$\xrightarrow{\mathrm{EOM}} \tilde{C}_{{}^1S_0} + C_{{}^1S_0}(p^2 + p'^2) + 2D_{{}^1S_0} p^2 p'^2$$

KSW approach — Motivation

$$T_{^1S_0}(p) = \frac{\xi}{p\cot\delta_{^1S_0}(p) - \mathrm{i}p}\,, \quad \xi = -\frac{2}{\pi m}$$

ER expansion: $p\cot\delta_{^1S_0}(p) = -\frac{1}{a} + \frac{r}{2}p^2 + \sum_{i=2}^{\infty} v_i p^{2i}$

with

$a = -23.739\,\mathrm{fm}$	$r = 2.68\,\mathrm{fm}$	$v_2 = -0.48\,\mathrm{fm}^3$
$\lvert a\rvert \gg M_\pi^{-1}$	$\lvert r\rvert \sim M_\pi^{-1}$	$\lvert v_i\rvert \sim M_\pi^{1-2i}$

Unnatural large $|a|$ prevents expansion of $T_{^1S_0}$ in p from converging.
$\Rightarrow$ retain ap to all orders:

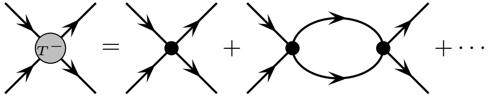
$$\begin{aligned} T_{^1S_0}(p) =& \,\xi\frac{1}{-1/a - \mathrm{i}p} - \xi\frac{r/2}{(-1/a-\mathrm{i}p)^2}p^2 \\ &+ \xi\left(\frac{r^2/4}{(-1/a-\mathrm{i}p)^3} - \frac{v}{(-1/a-\mathrm{i}p)^2}\right)p^4 + \mathcal{O}(p^2) \\ =:& \, T^{(-1)}_{^1S_0}(p) + T^{(0)}_{^1S_0}(p) + T^{(1)}_{^1S_0}(p) + \mathcal{O}(p^2) \end{aligned}$$

This shall be reproduced in an EFT.

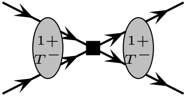
KSW approach — Derivation and renormalization of T

● $= \tilde{C}_{^1S_0}$ ■ $= C_{^1S_0}(p^2 + p'^2)$ ⬟ $= D_{^1S_0} p^2 p'^2$

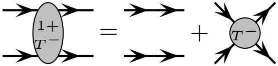
$-\mathbb{i}T^{(-1)}_{^1S_0} =$



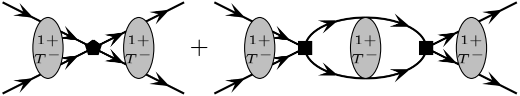
$-\mathbb{i}T^{(0)}_{^1S_0} =$



with



$-\mathbb{i}T^{(1)}_{^1S_0} =$



All emerging integrals are divergent and of the form

$$\mathbb{i}I_{2n}(p_{\mathrm{on}}) = \mathbb{i}m \int \frac{\mathrm{d}^3p}{4\pi} \frac{\boldsymbol{p}^{2n}}{p_{\mathrm{on}}^2 - \boldsymbol{p}^2 + \mathbb{i}\epsilon} \quad \text{with} \quad E_{\mathrm{cms}} = \frac{p_{\mathrm{on}}^2}{m} \, .$$

After dimensional regularization and power divergence subtraction:

$$\mathbb{i}I^{\mu}_{2n}(p_{\mathrm{on}}) := \frac{\mathbb{i}}{\xi} p_{\mathrm{on}}^{2n} (\mu + \mathbb{i}p_{\mathrm{on}})$$

Renormalization: μ-dependence can be absorbed into the LECs order by order by fitting to the respective terms in $T^{(-1)}_{^1S_0}(p) + T^{(0)}_{^1S_0}(p) + \cdots$

Non-perturbative approach — Solution of the LS equation

Regularization of the LS eq. in momentum space:

$$T(\boldsymbol{p}', \boldsymbol{p}; \boldsymbol{p}_{\text{on}}) = V(\boldsymbol{p}', \boldsymbol{p}) + m \int \frac{\mathrm{d}^3 p''}{4\pi} \frac{V(\boldsymbol{p}', \boldsymbol{p}'') T(\boldsymbol{p}'', \boldsymbol{p}; \boldsymbol{p}_{\text{on}})}{\boldsymbol{p}_{\text{on}}^2 - \boldsymbol{p}''^2 + \mathrm{i}\epsilon}$$

$V(\boldsymbol{p}', \boldsymbol{p}) \to g^\Lambda(\boldsymbol{p}') V^\Lambda(\boldsymbol{p}', \boldsymbol{p}) g^\Lambda(\boldsymbol{p})$ and same for $T(\boldsymbol{p}', \boldsymbol{p}; \boldsymbol{p}_{\text{on}})$

Partial wave projection:

$$T^\Lambda_{^1S_0}(p', p; p_{\text{on}}) = V^\Lambda_{^1S_0}(p', p) + m \int_0^\infty p''^2 \mathrm{d}p'' \frac{V^\Lambda_{^1S_0}(p', p'') T^\Lambda_{^1S_0}(p'', p; p_{\text{on}})}{p_{\text{on}}^2 - p''^2 + \mathrm{i}\epsilon} g^\Lambda(p'')^2$$

Solution for the separable potential:

$$V^\Lambda_{^1S_0}(p', p) = \tilde{C}^\Lambda_{^1S_0} + C^\Lambda_{^1S_0}(p^2 + p'^2) + 2D^\Lambda_{^1S_0} p^2 p'^2 = (1, p'^2) \underbrace{\begin{pmatrix} \tilde{C}^\Lambda_{^1S_0} & C^\Lambda_{^1S_0} \\ C^\Lambda_{^1S_0} & 2D^\Lambda_{^1S_0} \end{pmatrix}}_{=:\lambda^\Lambda} \begin{pmatrix} 1 \\ p^2 \end{pmatrix}$$

$$T^\Lambda_{^1S_0}(p', p; p_{\text{on}}) = (1, p'^2) \left(\mathbb{1}_4 - \lambda^\Lambda \begin{pmatrix} I^\Lambda_0(p_{\text{on}}) & I^\Lambda_2(p_{\text{on}}) \\ I^\Lambda_2(p_{\text{on}}) & I^\Lambda_4(p_{\text{on}}) \end{pmatrix} \right)^{-1} \lambda^\Lambda \begin{pmatrix} 1 \\ p^2 \end{pmatrix}$$

Non-perturbative approach — Renormalization

$$I_{2n}^{\Lambda}(p_{\text{on}}) := m \int_0^{\infty} p''^2 \mathrm{d}p'' \frac{p''^{2n} g^{\Lambda}(p'')^2}{p_{\text{on}}^2 - p''^2 + \mathrm{i}\epsilon}, \qquad g^{\Lambda}(p) := \exp\left(-\frac{p^2}{\Lambda^2}\right)$$

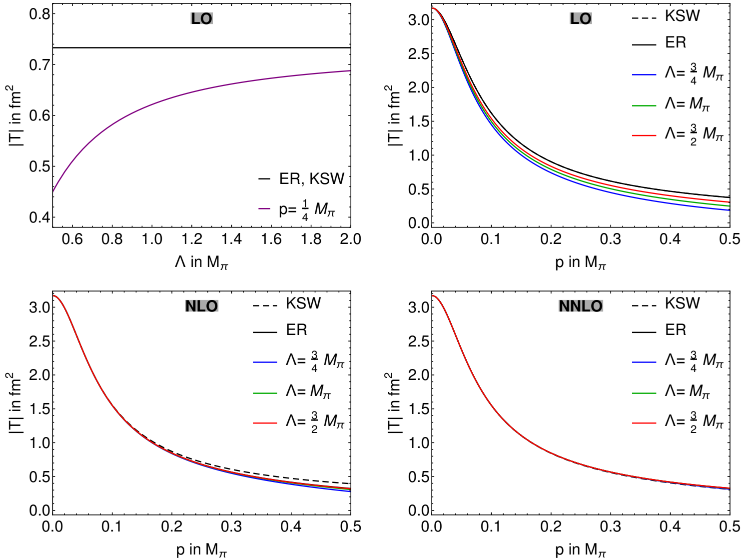
$$I_{2n}^{\Lambda}(p_{\text{on}}) = \frac{1}{\xi}\left[\sqrt{\frac{2}{\pi}} \sum_{i=0}^{n} p_{\text{on}}^{2i} \left(\frac{\Lambda}{2}\right)^{2(n-i)+1} |2(n-i)-1|!! \right.$$
$$\left. + \mathrm{i} p_{\text{on}}^{2n+1} \exp\left(-2\frac{p_{\text{on}}^2}{\Lambda^2}\right) - p_{\text{on}}^{2n+1} \exp\left(-2\frac{p_{\text{on}}^2}{\Lambda^2}\right) \operatorname{erfi}\left(\sqrt{2}\frac{p_{\text{on}}}{\Lambda}\right)\right]$$

Renormalization: $$\frac{\xi}{T_{{}^1S_0}(p,p;p)} + \mathrm{i}p \overset{!}{=} -\frac{1}{a} + \frac{r}{2}p^2 + \sum_{i=2}^{\infty} v_i p^{2i}$$

Λ dependence is absorbed into the LECs for a given order of $V_{{}^1S_0}$.

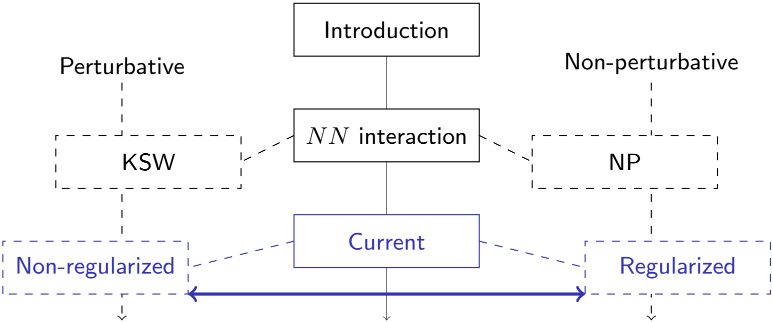
Order	LECs $\downarrow$ Λ $\rightarrow$	$\frac{1}{2}M_\pi$	$\frac{3}{4}M_\pi$	$1M_\pi$	$\frac{3}{2}M_\pi$	$2M_\pi$
LO	$\tilde{C}^{\Lambda}_{{}^1S_0}$ [fm^2]	-0.7277	-0.5253	-0.4110	-0.2864	-0.2197
NLO	$\tilde{C}^{\Lambda}_{{}^1S_0}$ [fm^2]	-0.3865	-0.3062	-0.2725	-0.2503	-0.2497
	$C^{\Lambda}_{{}^1S_0}$ [fm^4]	-5.5516	-1.5186	-0.5378	-0.0655	0.0349
NNLO	$\tilde{C}^{\Lambda}_{{}^1S_0}$ [fm^2]	-0.4847	-0.3659	-0.3124	-0.2865	-0.2868
	$C^{\Lambda}_{{}^1S_0}$ [fm^4]	-2.9135	-0.8174	-0.2682	0.0581	0.1298
	$D^{\Lambda}_{{}^1S_0}$ [fm^6]	-35.442	-4.1135	-0.9113	-0.2113	-0.1214

Non-perturbative approach — Observables



⇒ Convergence with larger cutoffs improves with higher orders.

Structure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Two-body current — Non-regularized

From $N\gamma$-vertex of $\mathcal{L}^{(1)}_{\pi N}$: $\begin{pmatrix}\rho^{[1]}\\ \boldsymbol{J}^{[1]}\end{pmatrix} := \frac{e}{2}(1+\tau^3)\begin{pmatrix}v^0\\ \boldsymbol{v}\end{pmatrix}, \begin{pmatrix}v^0\\ \boldsymbol{v}\end{pmatrix} = \begin{pmatrix}1\\ \boldsymbol{0}\end{pmatrix}$

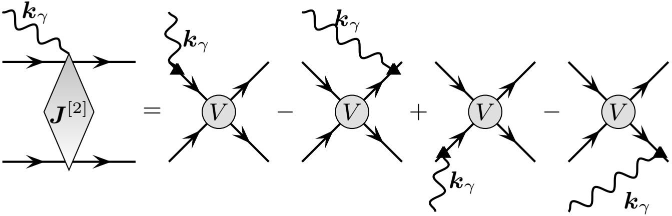
V. Bernard *et al.*, *International Journal of Modern Physics E* **04**, 193–344 (June 1995)

Siegert's hypothesis:

At low energies there is no two-body contribution $\rho^{[2]}$ from the charge.
$\Rightarrow \boldsymbol{J}^{[2]}$ can be determined from the potential via a continuity equation:

$$\boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[2]}(\boldsymbol{p}', \boldsymbol{p}) = [V(\boldsymbol{p}', \boldsymbol{p}), \rho^{[1]}(\boldsymbol{p}', \boldsymbol{p})]$$

A. J. F. Siegert, *Physical Review* **52**, 787–789 (1937)
H. Arenhövel, *Chinese Journal of Physics* **1**, 17–95 (Feb. 1992)



$$\boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[2]}(\boldsymbol{p}', \boldsymbol{p}) = V\left(\boldsymbol{p}', \boldsymbol{p} + \frac{\boldsymbol{k}_\gamma}{2}\right)\rho^{[1]}_1 - \rho^{[1]}_1 V\left(\boldsymbol{p}' - \frac{\boldsymbol{k}_\gamma}{2}, \boldsymbol{p}\right) + V\left(\boldsymbol{p}', \boldsymbol{p} - \frac{\boldsymbol{k}_\gamma}{2}\right)\rho^{[1]}_2 - \rho^{[1]}_2 V\left(\boldsymbol{p}' + \frac{\boldsymbol{k}_\gamma}{2}, \boldsymbol{p}\right)$$

Two-body current — Non-regularized

- No contribution of LO potential due to momentum independence: $\boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[2]}_{\mathrm{LO}} = \tilde{C}_{^1S_0}\rho_1^{[1]} - \rho_1^{[1]}\tilde{C}_{^1S_0} + \tilde{C}_{^1S_0}\rho_2^{[1]} - \rho_2^{[1]}\tilde{C}_{^1S_0} = 0$
- Practical agreement of $\boldsymbol{J}^{[2]}_{\mathrm{NLO}}$ with the result obtained with the method of unitary transformation in S. Kölling *et al.*, *Physical Review C* **84** (Nov. 28, 2011)
- Interested in the $^1S_0 \to {^1P_1}$ transition
- Partial wave projected end result can be written in terms of the projected LECs

$$\langle {^1P_1} | \boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[2]}_{\mathrm{NNLO}}(\boldsymbol{p}', \boldsymbol{p}) | {^1S_0} \rangle$$
$$= \underbrace{\frac{e}{8\sqrt{3}} k_\gamma p' \left(8C_{^1S_0} + 12C_{^1P_1}\right.}_{=\langle {^1P_1} | \boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[2]}_{\mathrm{NLO}}(\boldsymbol{p}', \boldsymbol{p}) | {^1S_0} \rangle} \left. + 8D_{^1S_0} p^2 + D_{^1P_1}(3k_\gamma^2 + 20p^2 + 12p'^2) \right)$$

$\to$ can directly be used with KSW approach (μ dependence of the LECs is known)

Two-body current — Regularized

Naive substitution $\boldsymbol{J}^{[2]}(\boldsymbol{p}', \boldsymbol{p}) \to g^{\Lambda}(\boldsymbol{p}')\boldsymbol{J}^{[2]^{\Lambda}}(\boldsymbol{p}', \boldsymbol{p})g^{\Lambda}(\boldsymbol{p})$ is not reasonable, instead:

$$\boldsymbol{k}_{\gamma} \cdot \boldsymbol{J}^{[2]^{\Lambda}}(\boldsymbol{p}', \boldsymbol{p}) = \left[g^{\Lambda}(\boldsymbol{p}')V^{\Lambda}(\boldsymbol{p}', \boldsymbol{p})g^{\Lambda}(\boldsymbol{p}), \rho^{[1]}(\boldsymbol{p}', \boldsymbol{p})\right]$$

with $\quad g^{\Lambda}(p) := \exp\left(-\frac{p^2}{\Lambda^2}\right)$

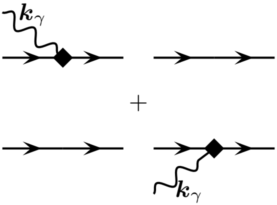
- Expressions for the regularized current are significantly more complicated than for the non-regularized one
- ${\boldsymbol{J}^{[2]}_{\mathrm{LO}}}^{\Lambda} \neq 0$ now, due to momentum dependence of the regulator functions
- At all orders the limit $\Lambda \to \infty$ yields consistently the non-regularized result,
  in particular $\lim_{\Lambda\to\infty} {\boldsymbol{J}^{[2]}_{\mathrm{LO}}}^{\Lambda} = 0$

One-body current

From $N\gamma$-vertex of $\mathcal{L}_{\pi N}^{(2)}$: $$\begin{pmatrix}\rho^{[1]} \\ \boldsymbol{J}^{[1]}\end{pmatrix} := \frac{e}{4m}(1+\tau^3)\begin{pmatrix}p_{\text{in}}^0 + p_{\text{out}}^0 \\ \boldsymbol{p}_{\text{in}} + \boldsymbol{p}_{\text{out}}\end{pmatrix}$$

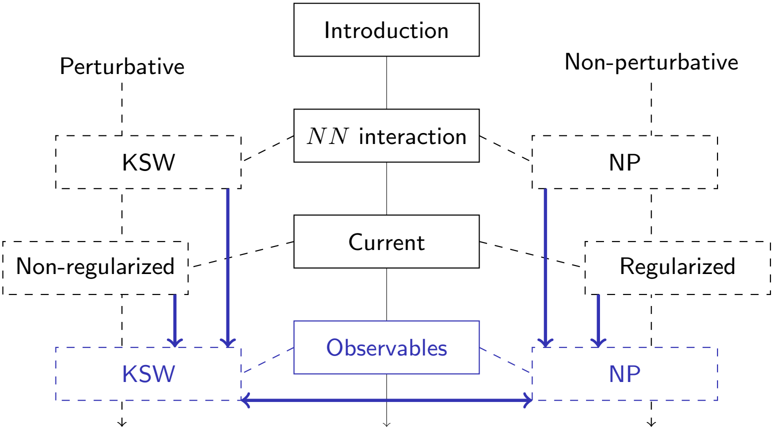
V. Bernard *et al.*, *International Journal of Modern Physics E* **04**, 193–344 (June 1995)

Include symmetry between nucleons in the same way as done for $\boldsymbol{J}^{[2]}$:



$$\langle {}^1P_1 | \boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[1]}(\boldsymbol{p}', \boldsymbol{p}) | {}^1S_0 \rangle = \frac{e}{\sqrt{3}m} k_\gamma p'$$

Structure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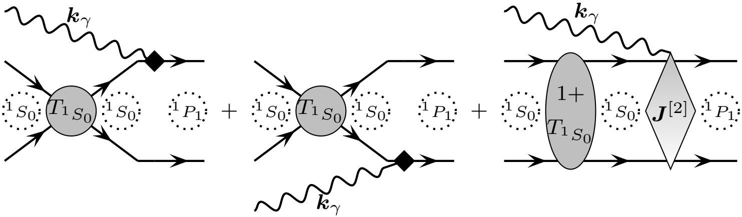
Electromagnetic process considered

${}^1S_0 \to {}^1P_1$ transition of $NN\gamma \to NN$ reaction



Energies are assumed low enough that the final nucleon pair does not interact

$\Rightarrow C_{{}^1P_1} \to 0$ and $D_{{}^1P_1} \to 0$

$$
\begin{aligned}
M(p', p_{\text{int}}, p, k_\gamma) = {} & m \frac{T_{{}^1S_0}(p_{\text{int}}, p)}{p^2 - p_{\text{int}}^2 + \mathbb{i}\epsilon} \langle {}^1P_1 | \boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[1]}(\boldsymbol{p}_{\text{int}}, \boldsymbol{p}) | {}^1S_0 \rangle \\
& + \langle {}^1P_1 | \boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[2]}(\boldsymbol{p}', \boldsymbol{p}) | {}^1S_0 \rangle \\
& + m \int p''^2 \mathrm{d}p'' \frac{\langle {}^1P_1 | \boldsymbol{k}_\gamma \cdot \boldsymbol{J}^{[2]}(\boldsymbol{p}', \boldsymbol{p}'') | {}^1S_0 \rangle \, T_{{}^1S_0}(p'', p)}{p^2 - p''^2 + \mathbb{i}\epsilon}
\end{aligned}
$$

KSW approach

M calculable perturbatively order by order

$$M_{\text{LO}}(p', p_{\text{int}}, p, k_\gamma) = \frac{e}{\sqrt{3}} k_\gamma p' \frac{T^{(-1)}_{{}^1S_0}(p)}{p^2 - p_{\text{int}}^2 + \mathbb{i}\epsilon}$$

$$M_{\text{NLO}}(p', p_{\text{int}}, p, k_\gamma) = \frac{e}{\sqrt{3}} k_\gamma p' \Bigg[\frac{T^{(-1)}_{{}^1S_0}(p) + T^{(0)}_{{}^1S_0}(p_{\text{int}}, p)}{p^2 - p_{\text{int}}^2 + \mathbb{i}\epsilon} + C^{\mu}_{{}^1S_0} \left(1 + T^{(-1)}_{{}^1S_0}(p) I^{\mu}_0(p)\right) \Bigg]$$

$$= \frac{e}{\sqrt{3}} k_\gamma p' \frac{T^{(-1)}_{{}^1S_0}(p) + T^{(0)}_{{}^1S_0}(p)}{p^2 - p_{\text{int}}^2 + \mathbb{i}\epsilon}$$

$$T^{(0)}_{{}^1S_0}(p_{\text{int}}, p) = T^{(0)}_{{}^1S_0}(p) - C^{\mu}_{{}^1S_0}(p^2 - p_{\text{int}}^2)\left(1 + T^{(-1)}_{{}^1S_0}(p) I^{\mu}_0(p)\right)$$

$$M_{\text{NNLO}}(p', p_{\text{int}}, p, k_\gamma) = \frac{e}{\sqrt{3}} k_\gamma p' \frac{T^{(-1)}_{{}^1S_0}(p) + T^{(0)}_{{}^1S_0}(p) + T^{(1)}_{{}^1S_0}(p)}{p^2 - p_{\text{int}}^2 + \mathbb{i}\epsilon}$$

- Delicate cancellation between off-shell one-body contribution and two-body contribution
- M is μ-independent at all orders

NP approach

- Because $T_{^1S_0}$ is non-perturbative, some residual cutoff dependence is expected and numerical treatment necessary.
- Momenta are connected by energy conservation.
  Expressing p' and p_{int} in terms of p and k_γ yields:

$$p' = \sqrt{p^2 + mk_\gamma - \frac{k_\gamma^2}{4}}\,,$$

$$p_{\text{int}} = -\frac{k_\gamma \cos\theta}{2} + \sqrt{\frac{k_\gamma^2 \cos^2\theta}{4} + p^2 + mk_\gamma - \frac{k_\gamma^2}{2}}$$

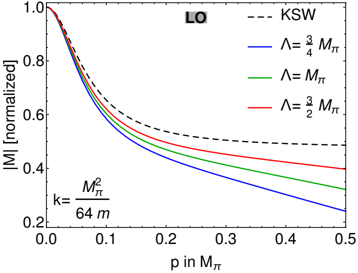
  θ: angle between $\boldsymbol{p}$ and $\boldsymbol{k_\gamma}$ (found to contribute not much but a scale), set to $\theta \to \pi/2$
- Since $p, p' \ll \Lambda$, it is required that $p \sim p'$.
  $\Rightarrow k_\gamma \sim \dfrac{p^2}{m}$
- Choose $|M|$ as observable and plot vs. initial nucleon momentum p for fixed k_γ.

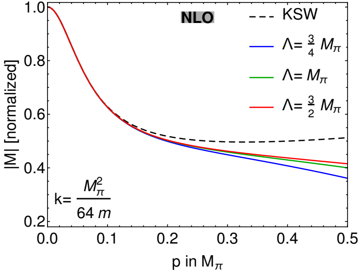
NP approach — Observables at different orders

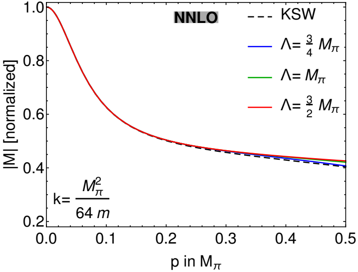
Normalize $|M|$ to KSW at $p = 0$

Plot $p \in [0, M_\pi/2]$ in units of the pion mass

Fix $k_\gamma = \frac{(M_\pi/8)^2}{m} = \frac{M_\pi^2}{64m}$





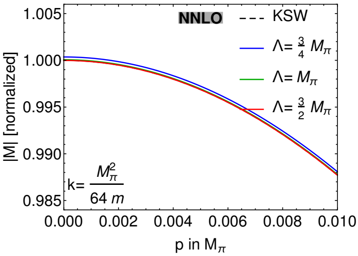


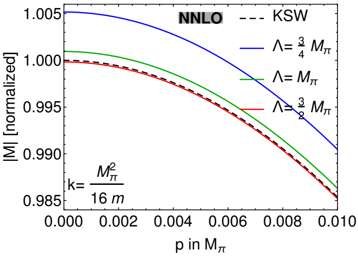
$\Rightarrow$ Good convergence with larger cutoffs and higher orders, consistent with KSW

NP approach — Threshold region and k_γ dependence

Cutoff dependence at $p = 0$ becomes more obvious when zooming into the threshold region and increasing the photon energy

$$k_\gamma = \frac{(M_\pi/8)^2}{m} = \frac{M_\pi^2}{64m} \qquad\qquad k_\gamma = \frac{(M_\pi/4)^2}{m} = \frac{M_\pi^2}{16m}$$

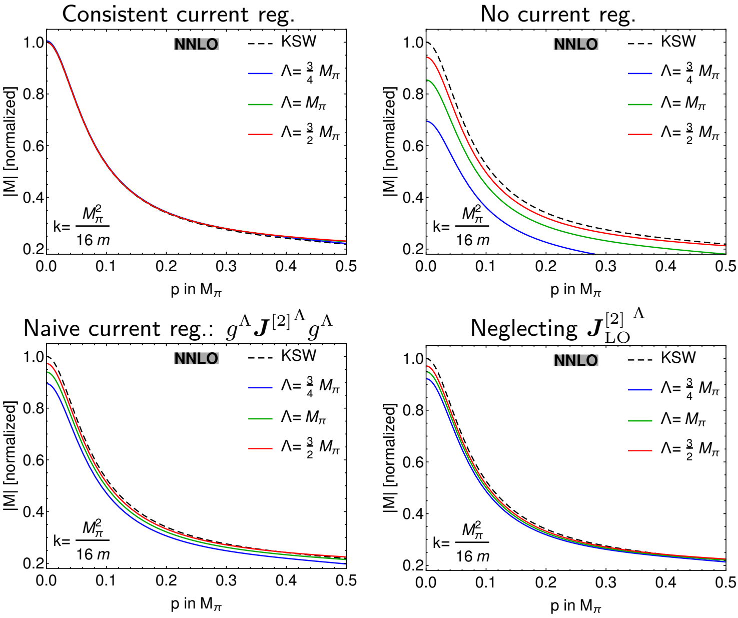




⇒ There is some small but visible cutoff dependence even at $p = 0$ that scales with k_γ.
Nevertheless, the regularization done yields better results then various alternatives.

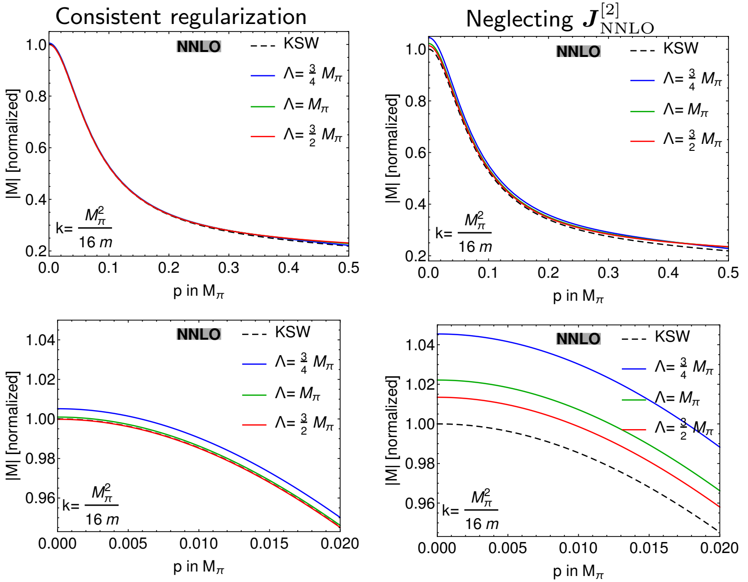
Keep $k_\gamma = \dfrac{M_\pi^2}{16m}$ in the following.

NP approach — Comparison to inconsistent regularization



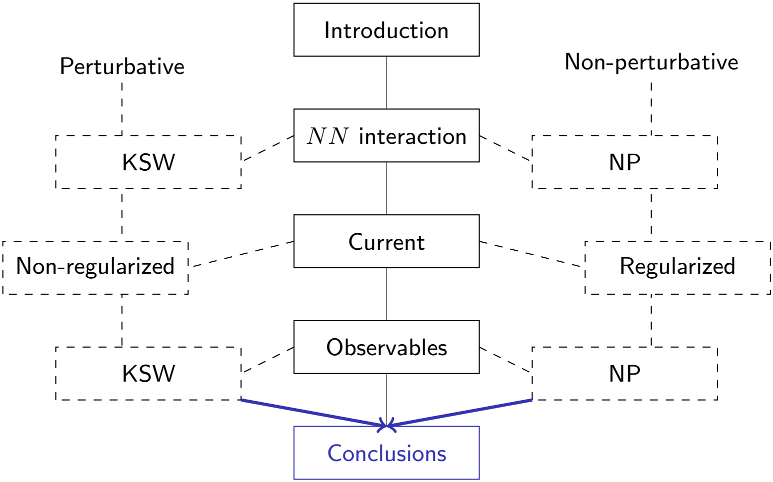
⇒ Consistent current regularization is needed to produce reliable results.

NP approach — Comparison to inconsistent current reg.



⇒ Potential must be included to the same order in the current as in the amplitude.

Structure


Introduction
Perturbative
Non-perturbative
NN interaction
KSW
NP
Current
Non-regularized
Regularized
Observables
KSW
NP
Conclusions

Conclusions and Outlook

- Determined and compared NN contact scattering amplitude in KSW and NP approach
  $\rightarrow$ consistent behavior
- Derived current operator from the NN potential via current conservation and regularized it
- Investigated electromagnetic processes in the two-nucleon sector for both approaches
  $\rightarrow$ proper treatment in NP approach produces results consistent with KSW:
  - The right way of deriving the regularized current is to apply current conservation to the regularized potential
  - Potential must be included up to the same order in the current derivation as in the LS equation

Outlook

Use the results as a basis for realistic calculations in chiral effective field theory

References

S. Weinberg, *Physics Letters B* **251**, 288–292 (Nov. 15, 1990).

E. Epelbaum, H.-W. Hammer, U.-G. Meißner, *Reviews of Modern Physics* **81**, 1773–1825 (Dec. 21, 2009).

S. Weinberg, *Nuclear Physics B* **363**, 3–18 (Sept. 30, 1991).

D. B. Kaplan, M. J. Savage, M. B. Wise, *Physics Letters B* **424**, 390–396 (Apr. 1998).

D. B. Kaplan, M. J. Savage, M. B. Wise, *Nuclear Physics B* **534**, 329–355 (Nov. 1998).

E. Epelbaum, W. Glöckle, U.-G. Meißner, *Nuclear Physics A* **747**, 362–424 (Jan. 2005).

V. Bernard, N. Kaiser, U.-G. Meißner, *International Journal of Modern Physics E* **04**, 193–344 (June 1995).

A. J. F. Siegert, *Physical Review* **52**, 787–789 (1937).

H. Arenhövel, *Chinese Journal of Physics* **1**, 17–95 (Feb. 1992).

S. Kölling, E. Epelbaum, H. Krebs, U.-G. Meißner, *Physical Review C* **84** (Nov. 28, 2011).