Pion-mass dependence of light nuclei.

Johannes Kirscher יוהנס קירשר

N. Barnea, D. Gazit, U. v. Kolck

Proper references in arXiv:1509.07697 [nucl-th]



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האוניברסיטה העברית בירושלים The Hebrew University of Jerusalem

MOTIVATION: FUNDAMENTAL, ELEGANT, AND SIMPLE THEORY OF NUCLEI.

arXiv:1509.07697 [nucl-th]

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{8} W^b_{\mu\nu} W^{b,\mu\nu} - \frac{1}{2} G^a_{\mu\nu} G^{a,\mu\nu}$$

$$+ (\overline{\nu}_L, \overline{e}_L) \overline{\sigma}^{\mu} i D_{\mu} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \overline{e}_R \sigma^{\mu} i D_{\mu} e_R + \overline{\nu}_R \sigma^{\mu} i D_{\mu} \nu_R + (\text{h.c.})$$

$$- \frac{\sqrt{2}}{v} \left[(\overline{\nu}_L, \overline{e}_L) \phi M^e e_R + \overline{e}_R \overline{M}^e \overline{\phi} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right]$$

$$- \frac{\sqrt{2}}{v} \left[(-\overline{e}_L, \overline{\nu}_L) \phi^* M^\nu \nu_R + \overline{\nu}_R \overline{M}^\nu \phi^T \begin{pmatrix} -e_L \\ \nu_L \end{pmatrix} \right]$$

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Parametrization of

- i) shell structure (relatively deep α nucleus)
- ii) spectral peculiarities (drip line, particle-unstable nuclei)
- iii) nuclear response to external probes (electro-weak, gravitation)

Motivation: Fundamental, elegant, and simple theory of nuclei.

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ARXIV:1509.07697 [NUCL-TH]

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A hadron prepared at the source

 $\overline{N}_{\text{source}}^{\alpha}(\mathbf{0}, t_0) = \epsilon_{abc}(u^{a,T}C\gamma_5 d^b)u^{c,\alpha}(\mathbf{0}, t_0)$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_{\mu} \mathcal{O}e^{-\int d^{4}x(\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\sum_{f}\log(\text{Det}M_{f}))}$$





A hadron **prepared** at the source

 $\overline{N}_{\text{source}}^{\alpha}(\mathbf{0}, t_0) = \epsilon_{abc}(u^{a,T}C\gamma_5 d^b)u^{c,\alpha}(\mathbf{0}, t_0)$

$$N_{\rm sink}^{\alpha}(\mathbf{x},t) = \epsilon_{abc} (u^{a,T} C \gamma_5 d^b) u^{c,\alpha}(\mathbf{x},t)$$

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 m_{π} [MeV]



















Lattice QCD measurements of hadron amplitudes at $m_{\pi} > 140$ MeV.



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An effective theory for nuclei in a $m_\pi > 140~{\rm MeV}$ universe.

$$m_N \gg Q_{\text{typ}} \quad \curvearrowright \quad \mathcal{L} = N^+ \left[i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right] N$$

non-relativistic spin/isospin- $\frac{1}{2}$ particles





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 \Rightarrow local $C_0(N^{\dagger}N)^2$ and $C_2(\nabla N^{\dagger}\nabla N)(N^{\dagger}N)$ interactions

$$\begin{split} \mathcal{L} &= N^{\dagger} \left[i \partial_{0} + \frac{\boldsymbol{\nabla}^{2}}{2m_{N}} + \mathcal{O}(m_{N}^{-3}) \right] N \\ &+ \left(C_{0}^{(0)} + C_{0}^{(1)} + \ldots \right) (N^{T}N)^{2} + \left(C_{0}^{\prime(0)} + C_{0}^{\prime(1)} + \ldots \right) (N^{T}\boldsymbol{\sigma}N)^{2} \\ &+ \left(D_{1}^{(0)} + D_{1}^{(1)} + \ldots \right) (N^{T}N)^{3} \\ &+ C_{2}^{(1)} \left[(NN)^{\dagger} (N \overleftrightarrow{\boldsymbol{\nabla}}N) + \mathbf{h.c.} \right] \end{split}$$

$$\mathcal{L} = N^{\dagger} \begin{bmatrix} i\partial_{0} + \frac{\nabla^{2}}{2m} + \mathcal{O}(m_{N}^{-3}) \end{bmatrix} N \\ + \begin{pmatrix} C_{0}^{(0)} + C_{0}^{(1)} + \dots \end{pmatrix} (N^{T} \sigma N)^{2} \\ + \begin{pmatrix} C_{0}^{(0)} + C_{0}^{(1)} + \dots \end{pmatrix} (N^{T} N)^{2} + \begin{pmatrix} C_{0}^{(0)} + C_{0}^{'(1)} + \dots \end{pmatrix} (N^{T} \sigma N)^{2} \\ + \begin{pmatrix} D_{1}^{(0)} + D_{1}^{(1)} + \dots \end{pmatrix} (N^{T} N)^{3} \\ + C_{2}^{(1)} \begin{bmatrix} (NN)^{\dagger} (N^{\overleftarrow{\nabla}} N) + h.c. \end{bmatrix}$$

 \bigcirc

 $\leftrightarrow \frac{\vec{\nabla}^2}{2m} + \frac{\vec{\nabla}^4}{8m^3} + ...$

$V = C_0 + C_{0,0} + C_2 q^2 + \dots$ 'Useless for external momenta $\gtrsim m_{\pi}$;

- Useful for external momenta $\approx \aleph \sim \sqrt{m_N B(2)}$;
- 1st ordering scheme amongst an ∞ number of terms
 ↔ relativistic, multipole, and nucleon-number expansion;
- mostly natural low-energy (Wilson) coefficients

$$C_{2n} = rac{4\pi \mathcal{O}(1)}{m \aleph (M \aleph)^n} \quad C_{2n}' = rac{4\pi \mathcal{O}(1)}{m M^{2n+1}} ;$$

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- ii) Effective-mass plots for hadrons with $A \leq 4$ are available (HAL, NPLQCD, Yamazaki).
- iii) Universal volume dependence of the 2-nucleon spectrum \Rightarrow effective-range parameters (Lüscher):

$$k \cot \delta(k) = \frac{1}{L\pi} \lim_{\lambda \to \infty} \left(\sum_{j=1}^{\lambda} \frac{1}{|j|^2 - (Lk/2\pi)^2} - 4\pi\lambda \right) = -\frac{1}{a} + \frac{1}{2}rk^2 + \dots$$

The n-p amplitude with quarks & gluons:



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- ii) Effective-mass plots for hadrons with $A \le 4$ are available (HAL, NPLQCD, Yamazaki).
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$$k \cot \delta(k) = \frac{1}{L\pi} \lim_{\lambda \to \infty} \left(\sum_{j=1}^{\lambda} \frac{1}{|j|^2 - (Lk/2\pi)^2} - 4\pi\lambda \right) = -\frac{1}{a} + \frac{1}{2}rk^2 + \dots$$



The n-p amplitude with quarks & gluons:



Regularization of the few-body Schrödinger equation?





























Predictive power with 3 parameters!





Predictive power with 5 parameters?





$$\langle \vec{r} | (n-p) \rangle = \sum_{a,d} \left\{ c_a \left[|S=1\rangle e^{-\beta_a r^2} \mathcal{Y}_0(\vec{r}) \right]^{J=1} + c_d \left[|S=1\rangle e^{-\beta_d r^2} \mathcal{Y}_2(\vec{r}) \right]^J \right\}$$

Ritz variation \Rightarrow bound states

Kohn-Hulthén variation \Rightarrow S-matrix



John Wheeler's idea:

=1

[...] It was as if, at a party, all the tall people clustered together at one moment, with all the short people in another cluster; then at the next moment [...] four groups formed, consisting of guests from the north, east, west, and south parts of the city; and so on, [...]



















A = 2, 3



ii) Low-energy constants \approx SU(4) symmetric.

i) Low-energy constants scale natural.



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Observations:

- i) No bound ${}^{4}S_{3}$ 3-nucleon state.
- ii) Scattering lengths run non monotonous with m_{π} .

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At physical m_{π} , scattering and bound state are correlated (Phillips).

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What happens at larger m_{π} ?

A = 2, 3



E.
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- Peculiar correlation even at larger m_{π} .
- EFT uncertainty insignificant relative to uncertainty in input data

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- [•] Peculiar correlation even at larger m_{π} .
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i) At physical *m*_π, the 3- and 4-nucleon ground states are correlated.
ii) This correlation is preserved at higher *m*_π.





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[•] Relative α "deepness" insensitive to structural features (m_{π} , Λ).

ישאלה : Effect of a long-range interaction?

A = 4



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י שאלה: Effect of a long-range interaction?



i) Probing lattice nuclei electromagnetically (E. Pazy, J. Drachman, N. Barnea, JK).



WHAT'S NEXT?

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iii) Extrapolation of $A \ge 3$ observables from $m_{\pi} \sim 400$ MeV to 140 MeV.



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iii) Extrapolation of $A \ge 3$ observables from $m_{\pi} \sim 400$ MeV to 140 MeV. Laboratory to assess validity/consistency of the various χ EFTs, *e.g.* perturbative π 's in bound nuclei analogous to Coulomb A_0 's.





iii) Extrapolation of A ≥ 3 observables from m_π ~ 400 MeV to 140 MeV.
 Laboratory to assess validity/consistency of the various χEFTs,
 Guiding LQCD to the critical pion masses.





iv) Exploration of the strange sector (M. Elyahu, N. Barnea).



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Fundamental understanding of the *strangeness* of the strange sector. Extrapolation most useful here! (insufficient of real-world data)







Weak reactions with upcoming lattice measurements in axial background fields. pp fusion, triton β decay (see H. Deleon, D. Gazit physical m_{π})



Weak reactions with upcoming lattice measurements in axial background fields. pp fusion, triton β decay (see H. Deleon, D. Gazit physical m_{π}) Relative stability of the "magic" α , ⁸Be, ¹⁶O, ⁴⁰Ca nuclei, *i.e.* how does the shell model react to changes in m_{π} ? {2, 8, 16, 20, 28, 50, 82, and 126} = $f(m_{\pi})$



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סיכום: Analysis of lattice "experiments" as cool as ...





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