Nuclear Dynamics and Threshold Phenomena

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Relativistic chiral nuclear force at leading order

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arXiv:1611.08475, 1612.08482

OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary and perspectives

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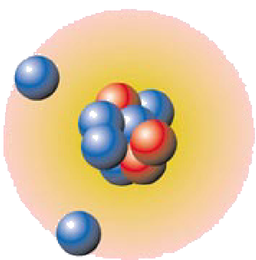
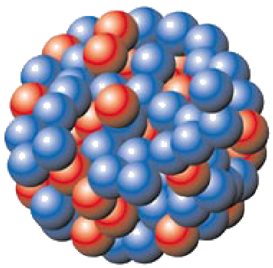
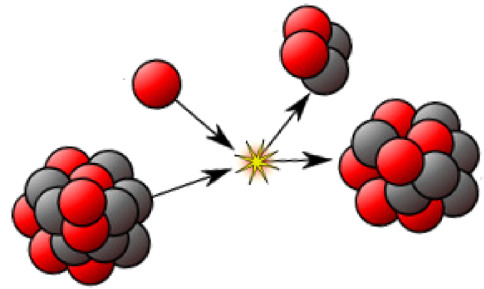
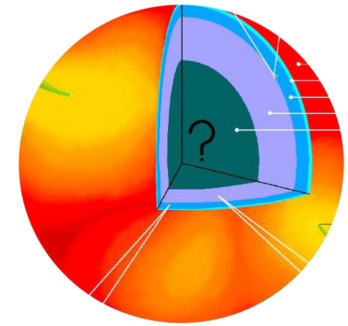
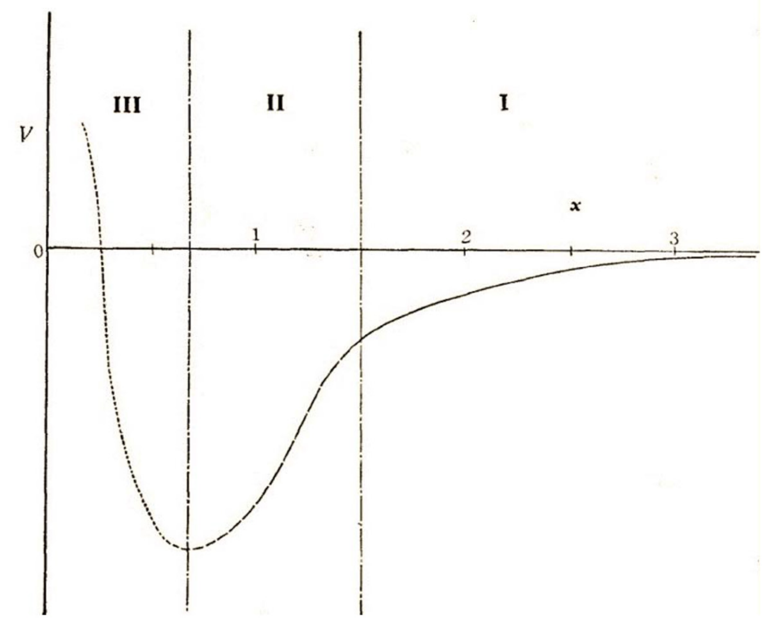
Basic for all nuclear physics

☐ Precise understanding of the nuclear force

☐ Complexity of the nuclear force (vs. electromagnetic force)

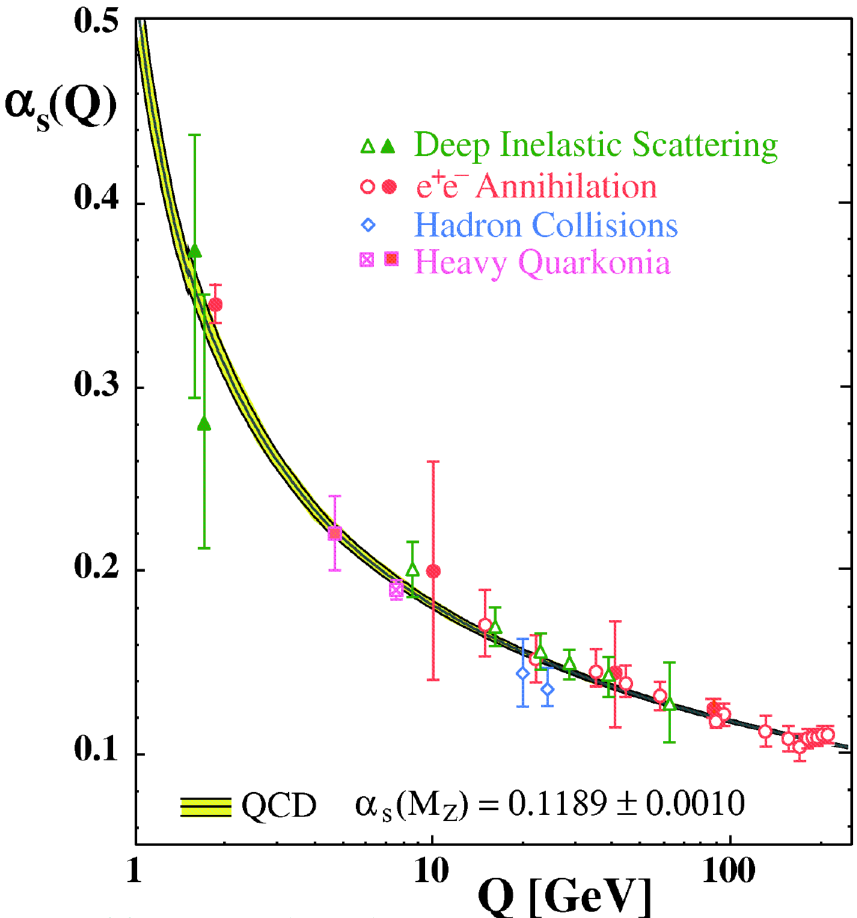
- Finite range
- Intermediate-range **attraction**
- Short-range **repulsion**-“hard core”
- Spin-dependent **non-central** force
  - Tensor interaction
  - Spin-orbit interaction
- Charge independent (approximate)



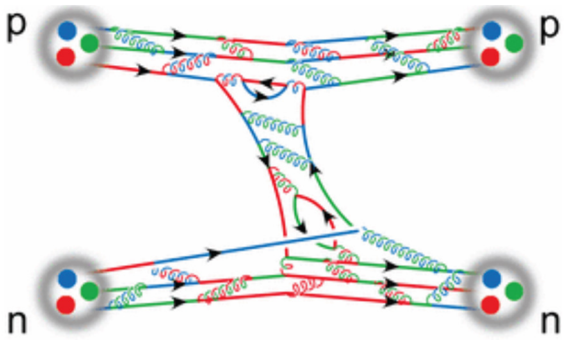
M. Taketani, Suppl.PTP3(1956)1

Nuclear force (NF) from QCD

- **Residual quark-gluon strong interaction**
- **Understood from QCD**



S.Bethke, PPNP(2013)



At low-energy region

- Running coupling constant $\alpha_s \geq 1$
- **Nonperturbative QCD -- unsolvable**

⇨
- **Phenomenological models**
- **Lattice QCD simulation**
- **Chiral effective field theory**

NF from phenomenological models

Phenomenological analysis

- Operator structures (allowed by symmetries)

$$
\begin{aligned}
V_{NN} = {} & V_0(r) + V_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_\tau(r)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + V_{\sigma\tau}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + V_{LS}(r)\boldsymbol{L}\cdot\boldsymbol{S} + V_{LS\tau}(r)(\boldsymbol{L}\cdot\boldsymbol{S})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + V_T(r)S_{12} + V_{T\tau}(r)S_{12}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& + V_Q(r)Q_{12} + V_{Q\tau}(r)Q_{12}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& + V_{PP}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{p})(\boldsymbol{\sigma}_2\cdot\boldsymbol{p}) + V_{PP\tau}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{p})(\boldsymbol{\sigma}_2\cdot\boldsymbol{p})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + \ldots\ldots
\end{aligned}
$$

Gammel-Thaler (1957)

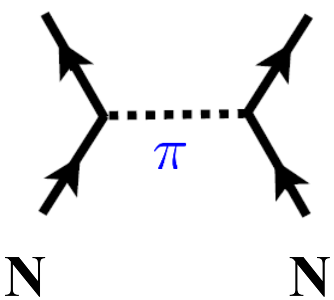
Hamada-Johnston (1962)

Reid 68, Argonne V14

Reid 93, Argonne V18

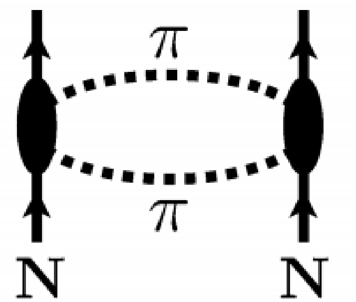
Meson "theory"

OPE



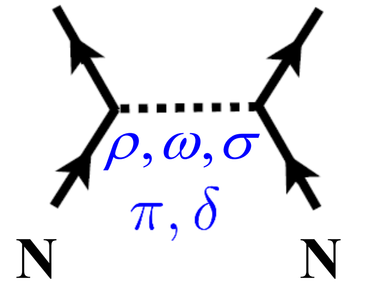
1935

TPE, MPE



1950's

OBE



1960's,1970's

Partovi-Lomon (1970)

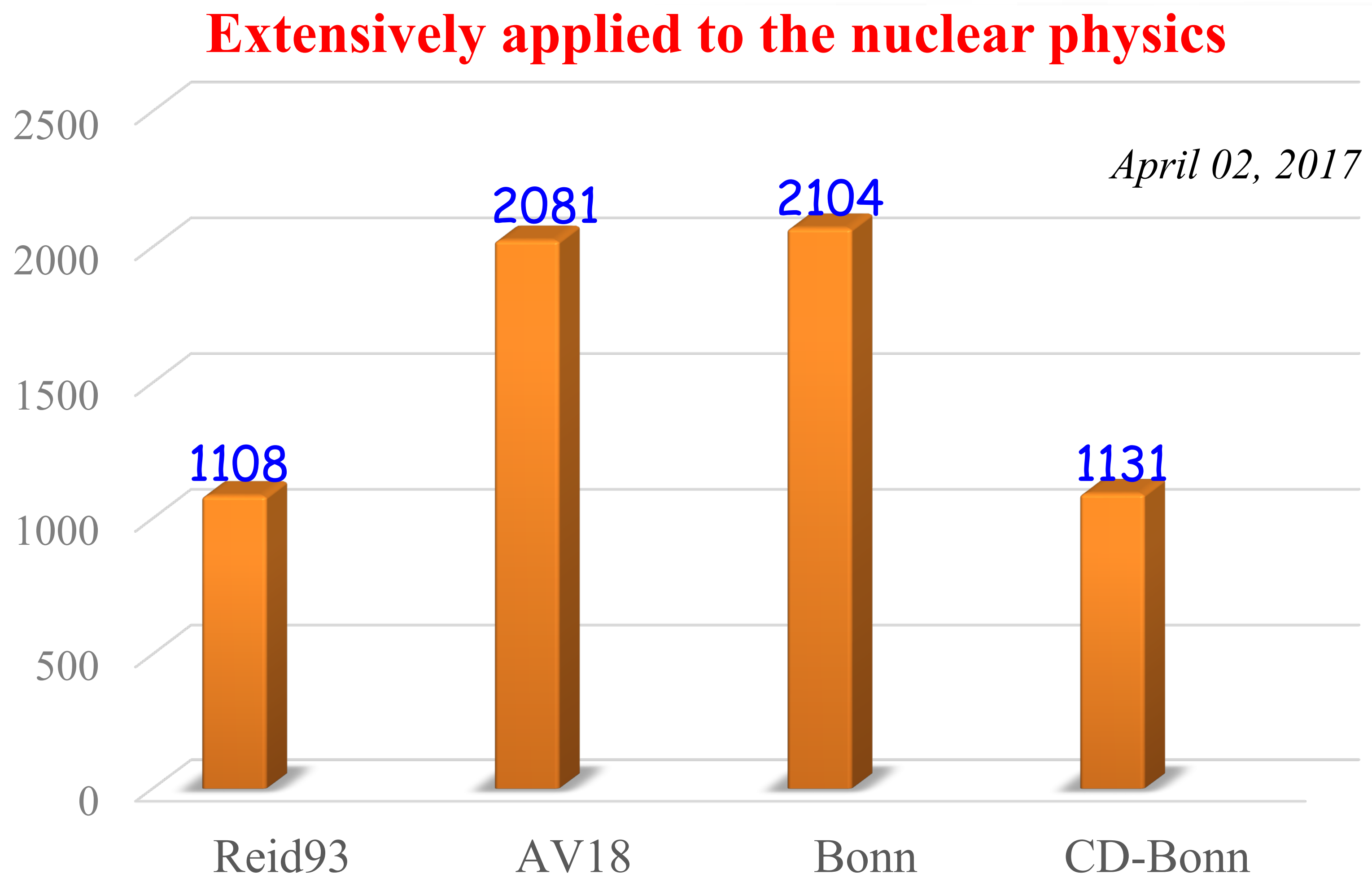
Stony Brook (1975)

Paris potential (1980)

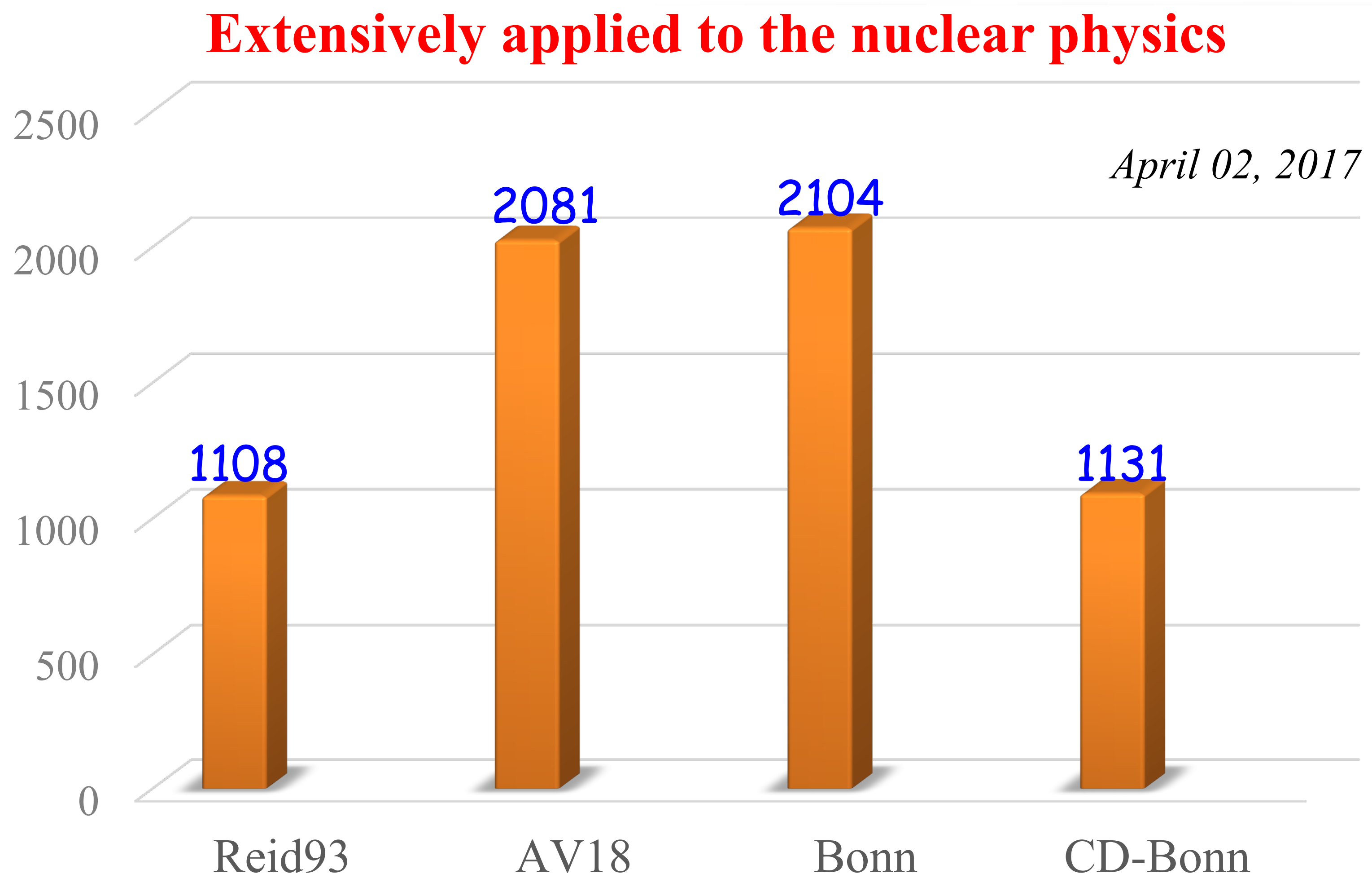
Bonn (1987),

CD-Bonn(2001)

High precision nuclear forces



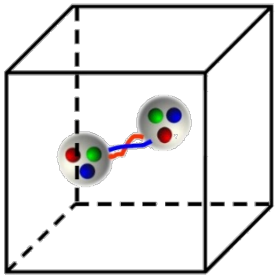
High precision nuclear forces



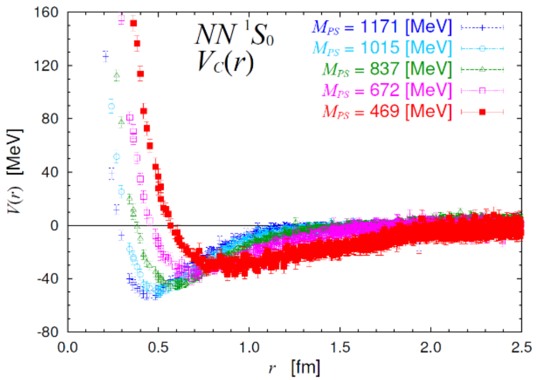
But, these potentials are not constructed directly from the fundamental theory.

NF from Lattice QCD

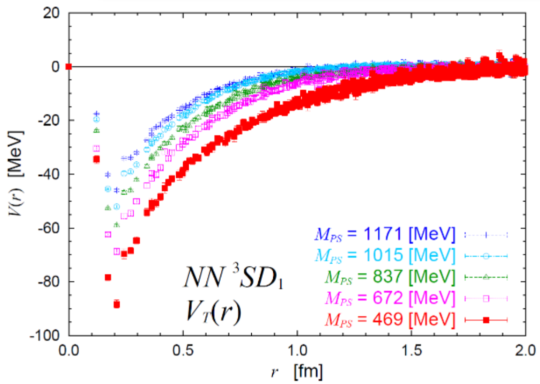
- Lattice QCD: numerical method of QCD *K.G. Wilson, PRD1974*
  - Discretized Euclidean space-time
  - Monte Carlo method



- Extract the nuclear force
  - **HAL QCD** coll. *T. Hatsuda, S. Aoki, et al.*
  - **NPLQCD** coll. *S. R. Beane, M. J. Savage, et al.*
    - **CalLat coll. / T. Yamazaki et al.**



The bulk properties of nuclear force can be produced from first principles



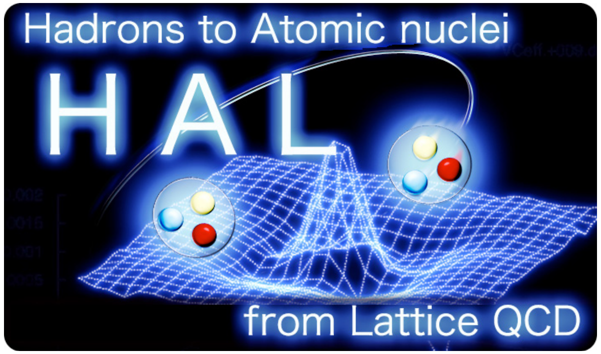
Input m_π=469 MeV is still larger than its physical value ~140 MeV

✓ **Repulsive core**
✓ **Attractive pocket**
✓ **Tensor force**

HAL QCD PRL(2007), arXiv: 1511.04871

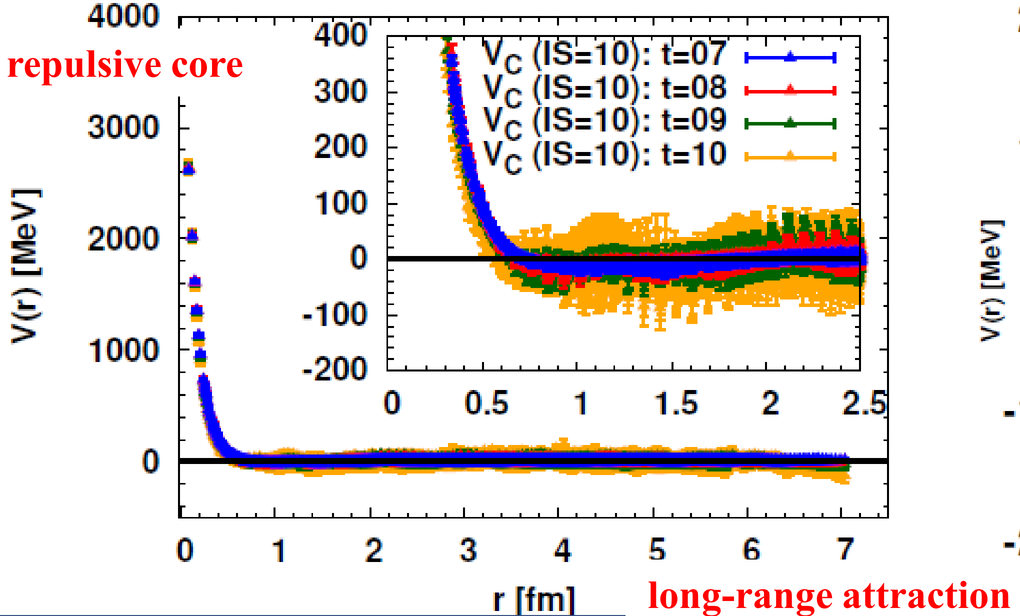
Preliminary results at physical point

- Lattice set-up
  - Pion mass: m_π ~= **145 MeV**
  - Lattice box size: L ~= 8 fm
  - Lattice spacing: $1/a$ ~= 2.3 GeV
- Central/Tensor forces for NN

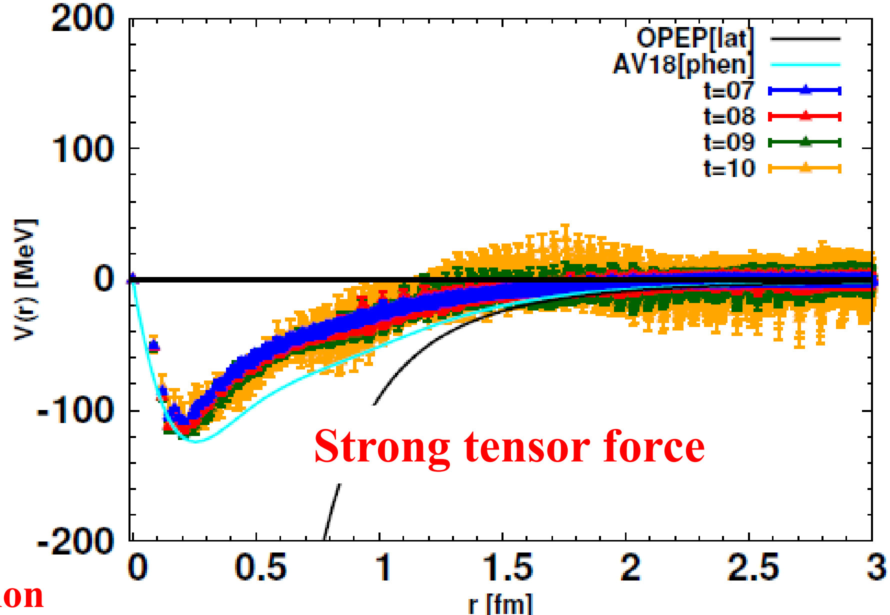


T. Doi, Lattice2016

1S0: center force



3S1-3D1: tensor force



NF from Chiral EFT

- Chiral effective field theory *S. Weinberg, Phys. A1979*
  - Effective field theory (EFT) of **low-energy QCD**
  - **Model independent** to study the nuclear force *S. Weinberg, PLB1990*
- Main advantages of chiral nuclear force
  - **Self-consistently include** many-body forces

$$V = V_{2N} + V_{3N} + \cdots + V_{iN} + \cdots$$

  - **Systematically improve** NF order by order

$$V_{iN} = V_{iN}^{\rm LO} + V_{iN}^{\rm NLO} + V_{iN}^{\rm NNLO} + \cdots$$

  - **Systematically estimate** theoretical uncertainties

$$|V_{iN}^{\rm LO}| > |V_{iN}^{\rm NLO}| > |V_{iN}^{\rm NNLO}| > \cdots$$

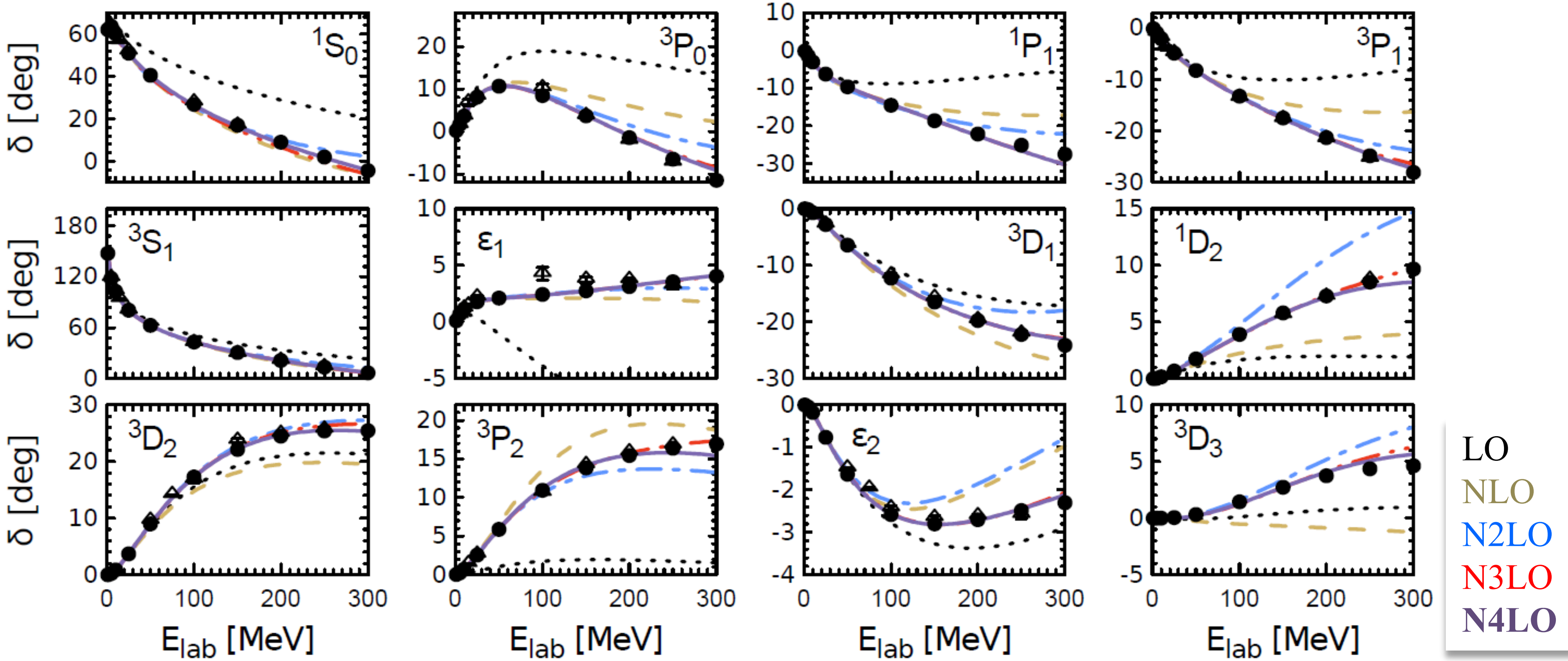
Current status of chiral NF

Nonrelativistic (NR) chiral NF

- NN interaction
  - up to NLO *U. van Kolck et al., PRL, PRC1992-94; N. Kaiser, NPA1997*
  - up to NNLO *E. Epelbaum, et al.,NPA2000; U. van Kolck et al.,PRC1994*
  - up to **N^3LO** *R. Machleidt et al., PRC2003; E. Epelbaum et al., NPA2005*
  - up to **N^4LO** *E. Epelbaum et al., PRL2015, D.R. Entem, et al., PRC2015*
  - up to **N^5LO** (dominant terms) *D.R. Entem, et al., PRC2015*
- 3N interaction
  - up to NNLO *U. van Kolck, PRC1994*
  - up to N^3LO *S. Ishikwas, et al, PRC2007; V. Bernard et al, PRC2007*
  - up to **N^4LO** *H. Krebs, et al., PRC2012-13*
- 4N interaction
  - up to N^3LO *E. Epelbaum, PLB 2006, EPJA 2007*

E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773
R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

Chiral Force up to N4LO



E. Epelbaum, H. Krebs, & Ulf-G. Meißner, PRL 115, 122301 (2015)

A high precision description of NN phase shifts is achieved!

Current status of chiral NF

Nonrelativistic (NR) chiral NF

	Phenomenological forces			NR Chiral nuclear force				
	Reid93	**AV18**	**CD-Bonn**	LO	NLO	NNLO	**N^3LO**	**N^4LO**
No. of para.	**50**	**40**	**38**	2+2	9+2	9+2	**24+2**	**24+3**
χ^2/datum **(np data)**	**1.03**	**1.04**	**1.02**	94	36.7	5.28	**1.23, 1.27**	**1.14, 1.10**

P.Reinert's talk

D.Entem, et al., arXiv:1703.05454

Chiral Nuclear Force in the precision era!

Nuclear lattice effective field theory has made remarkable achievements in nuclear structure and reaction studies.

S. Elhatisari, B.N. Lu's talk

E. Epelbaum, et al., PRL 106(2011) 192501, PRL109(2012) 252501, PRL110(2013) 112502
E. Epelbaum, et al., PRL 112(2014) 102501, S. Elhatisari, et al., Nature 528 (2015) 111, PRL117 (2016)132501…

Limitations of current chiral NF

- **Not "renormalization group invariance"**
  - Dependent on the UV cutoff
  - Diverse opinions on this issue
    - **Renormalized formulation (EG approach)**

      E. Epelbaum & J. Gegelia,PLB(2012); E. Epelbaum et al.,EPJA(2015), J.Behrendt,et al., EPJA(2016),...
- **Based on heavy baryon ChEFT**
  - **Cannot be used directly in covariant nuclear structure studies**

⇨ **Relativistic nuclear force based on covariant ChEFT?**

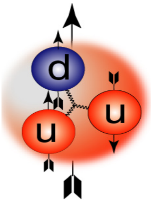
Motivation for the relativistic formulation

- Relativistic effects in nuclear physics
  - **Kinematical effect**: safely neglected or perturbatively treated

NR approximation: $$\sqrt{p^2 + m_N^2} = m_N\sqrt{1 + 0.102}$$

  - **Dynamical effect**: nucleon spin, spin-orbit splitting, anti-nucleon …

NR approximation:



$$f(r)\boldsymbol{S}\cdot\boldsymbol{L}$$

- **Relativistic (dynamical) effects are important**
  - Nuclear system:
    - **Covariant density functional theory (CDFT)**

P. Ring, PPNP (1996), D.Vretenar et al.,Phys.Rept.(2005), J. Meng, IRNP(2016)

  - One-nucleon system:
    - **Covariant ChEFT with extended-on-mass-shell (EOMS) scheme**

J. Gegelia,PRD(1999), T. Fuchs,PRD(2003)

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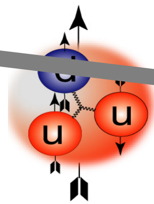
NR approximation:



~~$f(r)\boldsymbol{S}\cdot\boldsymbol{L}$~~

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J. Gegelia,PRD(1999), T. Fuchs,PRD(2003)

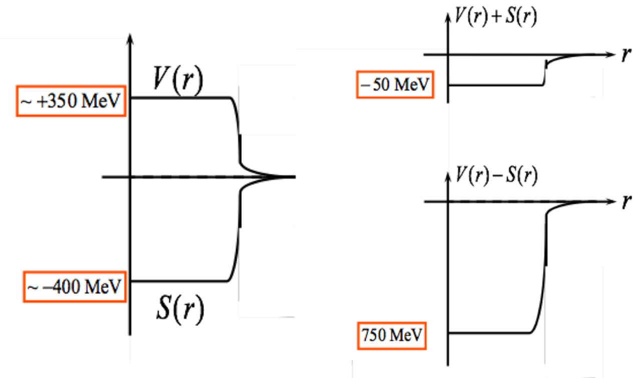
Covariant density functional theory

From Prof. Meng's talk

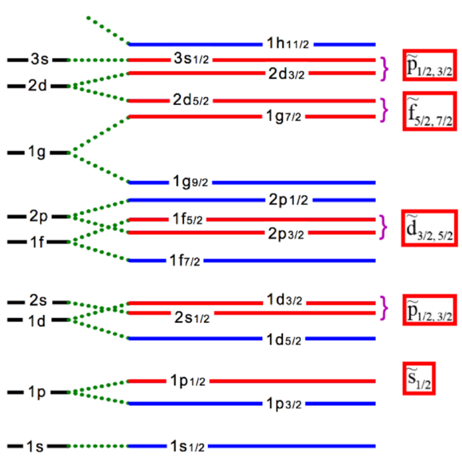
Why Covariant?

P. Ring Physica Scripta, T150, 014035 (2012)

- ✓ Spin-orbit automatically included
- ✓ Lorentz covariance restricts parameters
- ✓ Pseudo-spin Symmetry
- ✓ Connection to QCD: big V/S ~ ±400 MeV
- ✓ Consistent treatment of time-odd fields
- ✓ Relativistic saturation mechanism
- ✓ … Liang, Meng, Zhou, Physics Reports **570** : 1-84 (2015).



Pseudospin symmetry



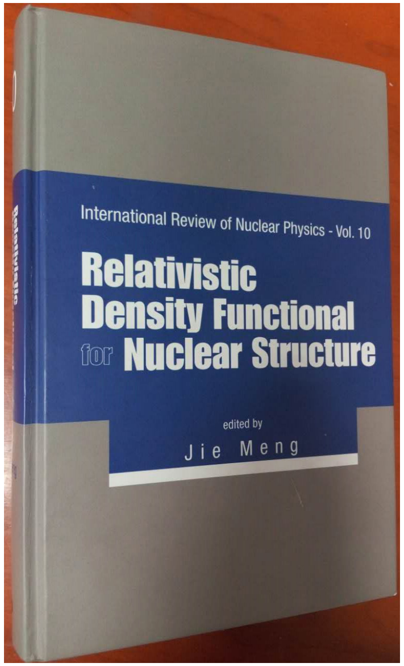
$$\begin{array}{lll} n-1, & l+2, & j=l+3/2 \\ n, & l, & j=l+1/2 \end{array}$$

$$\begin{array}{lll} \text{pseudo-orbit} & : & \tilde{l}=l+1 \\ \text{pseudo-spin} & : & \tilde{s}=1/2 \end{array}$$

Hecht & Adler
NPA137(1969)129

Arima, Harvey & Shimizu
PLB 30(1969)517

Ginocchio PRL 78, 436

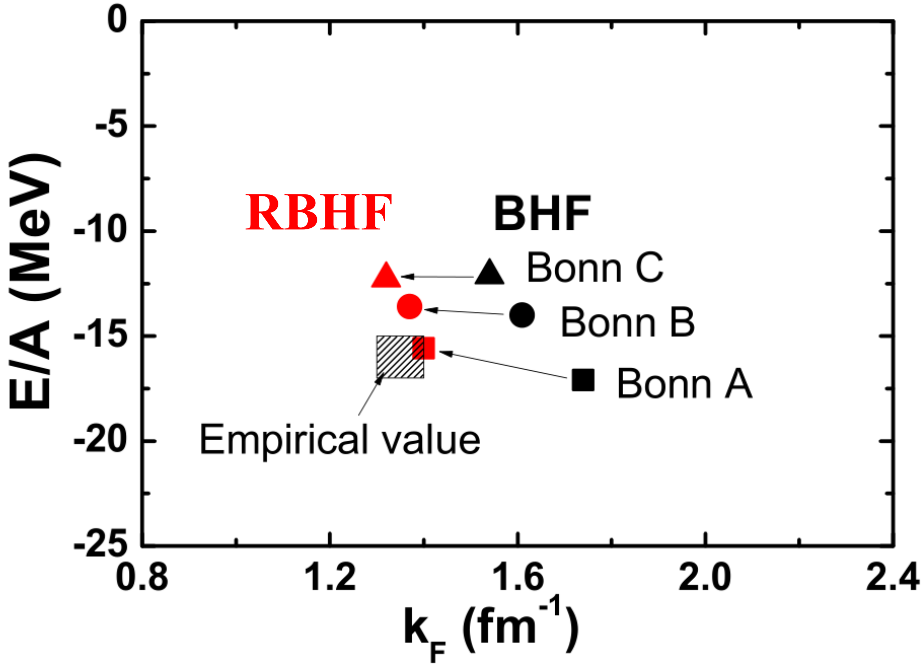


Relativistic Brueckner Hartree-Fock

- **Key input:** relativistic Bonn A, B, C potentials

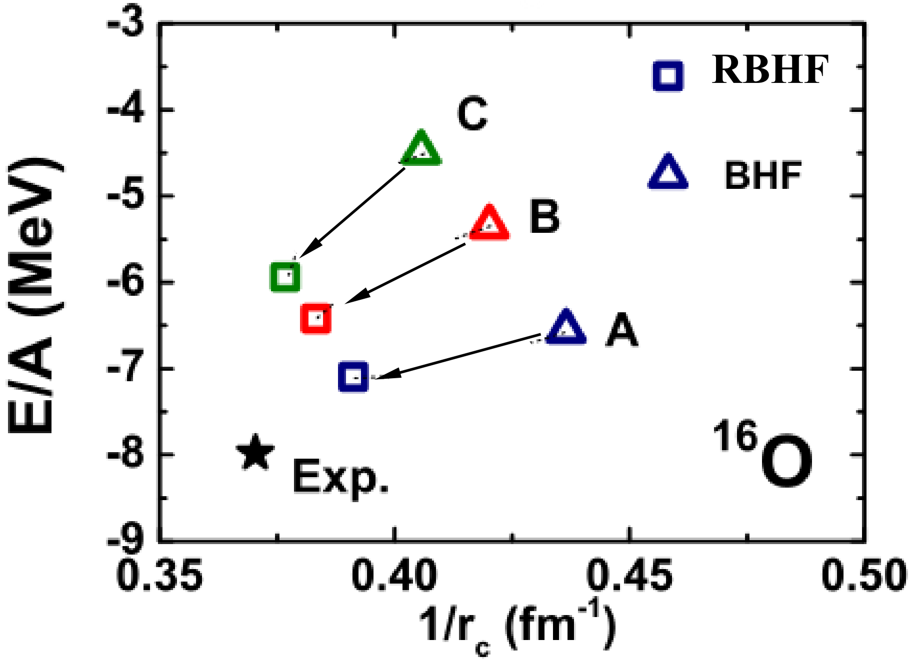
See Prof. Meng's talk

Nuclear Matter



R. Brockmann & R. Machleidt, PRC(1990)

Finite Nuclei



S.H. Shen, et al., CPL(2016)

Relativistic NF based on ChEFT is needed !

BChEFT: HB vs. IR vs. EOMS

Heavy baryon (HB) *E.E. Jenkins et al., PLB(1991)*

- non-relativistic scheme
- breaks analyticity of loop amplitudes
- converges slowly (particularly in three-flavor sector)
- strict PC and simple nonanalytical results

Infrared *T. Becher et al., EPJC(1999)*

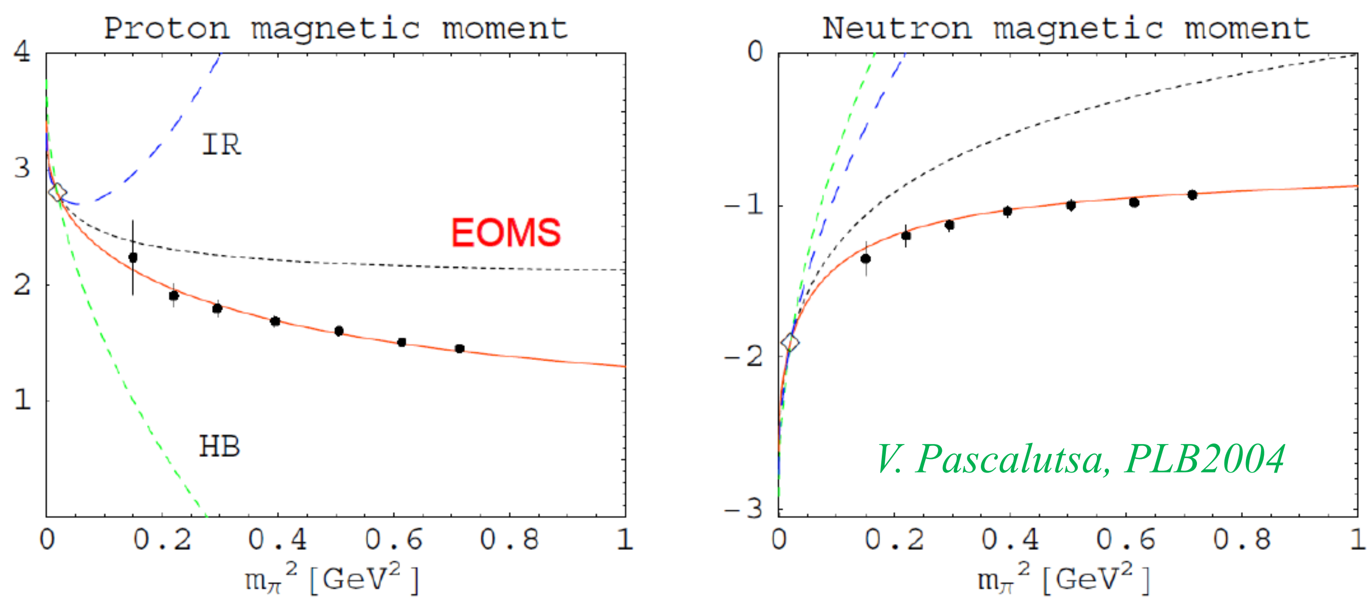
- breaks analyticity of loop amplitudes
- converges slowly (particularly in three-flavor sector)
- analytical terms the same as HBChEFT

Extended-on-mass-shell (EOMS) *J. Gegelia et al., PRD(1999), T. Fuchs et al., PRD(2003)*

- satisfies all symmetry and analyticity constraints
- **converges relatively faster** --- an appealing feature

Successful applications of EOMS BChEFT

- Nucleon magnetic moments, polarizabilities



- Pion-Nucleon scattering

  J.M. Alarcon, et al., PRD2012, Y.-H. Chen, et al., PRD2013, D. Siemens, et al., PRC2014, PRC2016
  E. Epelbaum, et al., EPJC2015, D.-L. Yao, et al., JHEP2016

- Octet baryon masses, axial and vector form factors

  J.M.Camalich, et al., PRD2010; L.S.Geng et al. PRD2011, PRD2014;
  XLR, et al., JHEP2012;PRD2013;PRD2014;EJPC2014;PRD2015;PLB2017

NF from EOMS ChEFT may have a faster convergence!

In this work

We try to develop a relativistic nuclear force up to leading order based on covariant ChEFT

- Construct the kernel potential in **covariant power counting**
  - Employ the Lorentz invariant chiral Lagrangains
  - Retain the complete form of Dirac spinor

$$u(\vec{p}, s) = N_p \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{\epsilon_p} \end{pmatrix} \chi_s, \quad N_p = \sqrt{\frac{\epsilon_p}{2M_N}}. \quad \begin{array}{l} E_p = \sqrt{M_N^2 + \vec{p}^2} \\ \epsilon_p = E_p + M_N \end{array}$$

  - Use naïve dimensional analysis to determine the chiral dimension

- Employ the 3D-reduced **Bethe-Salpeter** equation, such as **Kadyshevsky/Blankenbecler-Sugar** equation, to resum the potential.

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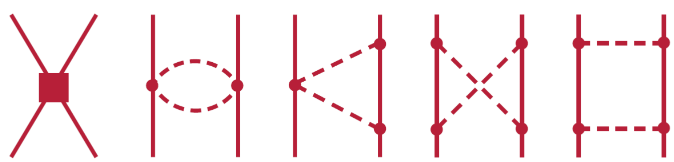
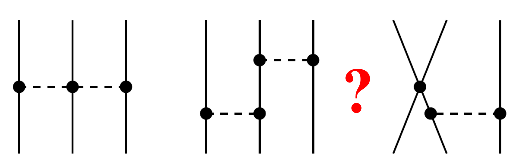
Covariant power counting

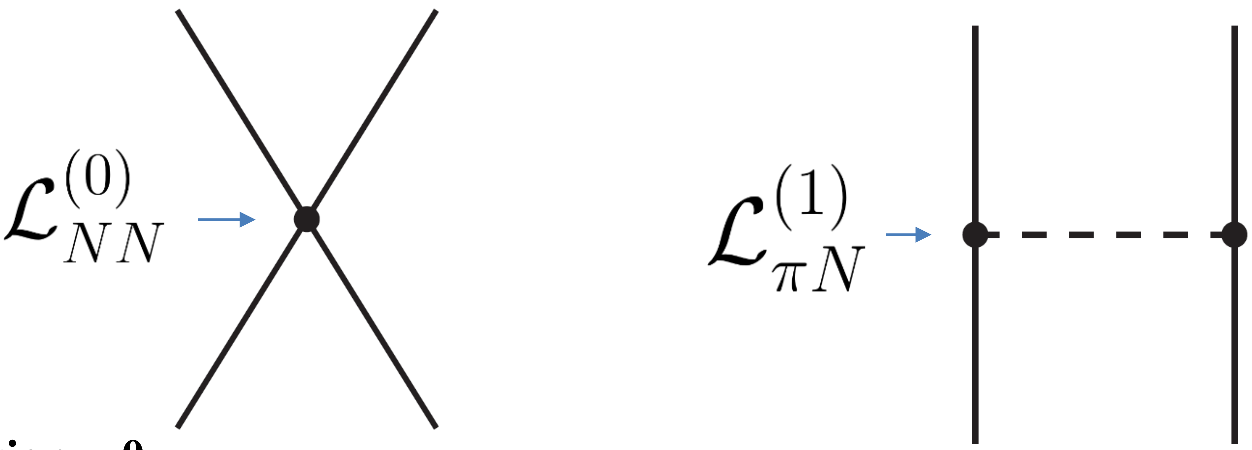
- Degrees of freedom: pions (GBs) : π^+, π^0, π^- , nucleons: *p*, *n*
  - Retain the complete form of Dirac spinor $$u(\vec{p}, s) = N_p \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{\epsilon_p} \end{pmatrix} \chi_s.$$
- Energy scales: light --- $Q \sim p, m_\pi,$ heavy --- $\Lambda_\chi \sim 1 \text{ GeV}$
  - **Perturbative expansion:** $(Q/\Lambda_\chi)^{n_\chi}$
  - **Chiral dimension (NDA):** $n_\chi = 4L - 2N_\pi - N_n + \sum_k kV_k$
  - **Hierarchy of chiral nuclear force:**

	NN force	*NNN* force
$(Q/\Lambda_\chi)^0$		------
$(Q/\Lambda_\chi)^2$		

Relativistic chiral NF up to LO

$$V = V_{2N}^{\mathrm{LO}}$$

$$= V_{\mathrm{CTP}} + V_{\mathrm{OPEP}}$$



Chiral dimension = 0

$$4L(=0) - 2N_\pi(=0) - N_n(=0) + V_k \qquad 4L(=0) - 2N_\pi(=1) - N_n(=0) + 2V_k$$

Covariant chiral Lagrangians

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)}.$$

- Pion-pion interaction:

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + (U + U^\dagger) m_\pi^2 \rangle.$$

$$U = 1 + i\frac{\Phi}{f_\pi} - \ldots$$
$$\Phi = \tau_\sigma \pi^\sigma$$
$$f_\pi = 92.4 \text{ MeV}$$

- Pion-nucleon interaction:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}(i\not{\partial} - M_N)\Psi + \frac{g_A}{2}\bar{\Psi}\gamma^\mu\gamma^5 u_\mu \Psi.$$

$$u_\mu = -\frac{1}{f_\pi}\partial_\mu \Phi + \ldots$$
$$\Psi = (p, n)^\dagger$$
$$g_A = 1.26$$

- Nucleon-nucleon interaction: *D.Djukanovic, et al., FBS(2007)*

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2}\left[C_S(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + C_A(\bar{\Psi}\gamma_5\Psi)(\bar{\Psi}\gamma_5\Psi) + C_V(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) + C_{AV}(\bar{\Psi}\gamma_5\gamma_\mu\Psi)(\bar{\Psi}\gamma_5\gamma^\mu\Psi) + C_T(\bar{\Psi}\sigma_{\mu\nu}\Psi)(\bar{\Psi}\sigma^{\mu\nu}\Psi).\right]$$

5 unknown low-energy constants (LECs)

Contact potential

$$u_i(\vec{p}, s) = \sqrt{\frac{E_N + M_N}{2M_N}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}_1 \cdot \vec{p}}{\epsilon_p} \end{pmatrix} \chi_{s,i}$$

- Covariant form (momentum space):

$$\begin{aligned} V_{\text{CTP}} &= C_S(\bar{u}_4 u_2)(\bar{u}_3 u_1) + C_A(\bar{u}_4 \gamma_5 u_2)(\bar{u}_3 \gamma_5 u_1) \\ &+ C_V(\bar{u}_4 \gamma_\mu u_2)(\bar{u}_3 \gamma^\mu u_1) + C_{AV}(\bar{u}_4 \gamma_\mu \gamma_5 u_2)(\bar{u}_3 \gamma^\mu \gamma_5 u_1) \\ &+ C_T(\bar{u}_4 \sigma_{\mu\nu} u_2)(\bar{u}_3 \sigma_{\mu\nu} u_1). \end{aligned}$$

- Relativistic 3D form:

$$\begin{aligned} V_{\text{CTP}} = \sum_{i=S,A,V,AV,T} C_i \Big[& V_C^i(E_N) + V_\sigma^i(E_N) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_{SO}^i(E_N) \frac{i}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{k} \times \boldsymbol{q}) \\ & + V_{\sigma q}^i(E_N) \boldsymbol{\sigma}_1 \cdot \boldsymbol{q} \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} + V_{\sigma k}^i(E_N) \boldsymbol{\sigma}_1 \cdot \boldsymbol{k} \boldsymbol{\sigma}_2 \cdot \boldsymbol{k} \\ & + V_{\sigma L}^i(E_N) \boldsymbol{\sigma}_1 \cdot (\boldsymbol{q} \times \boldsymbol{k}) \boldsymbol{\sigma}_2 \cdot (\boldsymbol{q} \times \boldsymbol{k}) \Big]. \end{aligned}$$

All allowed spin operators

- Non-relativistic expansion:

$$V_{\text{CTP}}^{\text{NonRel.}} = \underbrace{(C_S + C_V)}_{C_S^{\text{HB}}} - \underbrace{(C_{AV} - 2C_T)}_{C_T^{\text{HB}}} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \mathcal{O}\left(\frac{1}{M_N}\right).$$

S. Weinberg, PLB1990

One-pion exchange potential

- Covariant form (momentum space):

$$V_{\mathrm{OPEP}} = \frac{g_A^2}{4f_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\frac{(\bar{u}_1\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_2\gamma^\nu\gamma_5 q_\nu u_2)}{\boldsymbol{q}^2+m_\pi^2}.$$

- Relativistic 3D form:

$$\begin{aligned}V_{\mathrm{OPEP}} \; = \; & \frac{g_A^2}{4f_\pi^2}\frac{1}{\boldsymbol{q}^2+m_\pi^2+i\epsilon}[V_{\sigma q}(E_N)\boldsymbol{\sigma_1}\cdot\boldsymbol{q}\boldsymbol{\sigma_2}\cdot\boldsymbol{q} \\ & + V_C(E_N) + U_\sigma\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_{SO}(E_N)\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}\times\boldsymbol{q}) \\ & + V_{\sigma k}(E_N)\boldsymbol{\sigma}_1\cdot\boldsymbol{k}\boldsymbol{\sigma}_2\cdot\boldsymbol{k} + V_{\sigma L}(E_N)\boldsymbol{\sigma}_1\cdot(\boldsymbol{q}\times\boldsymbol{k})\boldsymbol{\sigma}_2\cdot(\boldsymbol{q}\times\boldsymbol{k})]\end{aligned}$$

All allowed spin operators

- Non-relativistic expansion:

$$V_{\mathrm{OPEP}}^{\mathrm{NonRel.}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}^2+m_\pi^2+i\epsilon}+\mathcal{O}(\frac{1}{M_N}).$$

S. Weinberg, PLB1990

Relativistic potential in LSJ basis

$\langle p'|V_{\rm LO}|p\rangle$ → (rotation invariant; conservation of total spin) → $\langle L'SJ|V_{\rm LO}|LSJ\rangle$

All partial waves with J = 0, 1

$$
\begin{aligned}
V_{1S0} &= \xi_N\left[\boldsymbol{C_{1S0}}\left(1+R_p^2R_{p'}^2\right)+\boldsymbol{\hat{C}_{1S0}}\left(R_p^2+R_{p'}^2\right)\right],\\
V_{3P0} &= -2\xi_N\boldsymbol{C_{3P0}}R_pR_{p'},\\
V_{1P1} &= -\frac{2\xi_N}{3}\boldsymbol{C_{1P1}}R_pR_{p'},\\
V_{3P1} &= -\frac{4\xi_N}{3}\boldsymbol{C_{3P1}}R_pR_{p'},\\
V_{3S1} &= \frac{\xi_N}{9}\left[\boldsymbol{C_{3S1}}\left(9+R_p^2R_{p'}^2\right)+\boldsymbol{\hat{C}_{3S1}}\left(R_p^2+R_{p'}^2\right)\right],\\
V_{3D1} &= \frac{8\xi_N}{9}\boldsymbol{C_{3S1}}R_p^2R_{p'}^2,\\
V_{3S1-3D1} &= \frac{2\sqrt{2}\xi_N}{9}\left(\boldsymbol{C_{3S1}}R_p^2R_{p'}^2+\boldsymbol{\hat{C}_{3S1}}R_p^2\right),\\
V_{3D1-3S1} &= \frac{2\sqrt{2}\xi_N}{9}\left(\boldsymbol{C_{3S1}}R_p^2R_{p'}^2+\boldsymbol{\hat{C}_{3S1}}R_{p'}^2\right).
\end{aligned}
$$

$$\xi_N = 4\pi N_p^2N_{p'}^2,\ R_p = |\vec{p}|/\epsilon_p, \text{ and } R_{p'} = |\vec{p}'|/\epsilon_{p'}.$$

$$
\begin{aligned}
C_{1S0} &= (C_S + C_V + 3C_{AV} - 6C_T),\\
\hat{C}_{1S0} &= (3C_V + C_A + C_{AV} + 6C_T).
\end{aligned}
$$

$$C_{3P0} = (C_S - 4C_V + C_A - 4C_{AV}).$$

$$C_{1P1} = (C_S + C_A).$$

$$C_{3P1} = (C_S - 2C_V - C_A + 2C_{AV} + 4C_T).$$

$$
\begin{aligned}
C_{3S1} &= (C_S + C_V - C_{AV} + 2C_T),\\
\hat{C}_{3S1} &= 3(C_V - C_A - C_{AV} + 2C_T).
\end{aligned}
$$

7 combinations, only 5 independent.

Hint at a more efficient formulation

- **V_{1S0}: $1/m_N$ expansion**

$$V_{1S0} = 4\pi\left[C_{1S0} + (C_{1S0} + \hat{C}_{1S0})\left(\frac{\vec{p}^2 + \vec{p}'^2}{4M_N^2} + \cdots\right)\right]$$

$$+\frac{\pi g_A^2}{2f_\pi^2}\int_{-1}^{1}\frac{dz}{\vec{q}^2 + m_\pi^2}\left[\vec{q}^2 - \left(\frac{(\vec{p}^2 - \vec{p}'^2)^2}{4M_N^2} + \cdots\right)\right].$$

- Relativistic corrections are suppressed
- One has to be careful with **the new contact term, the momentum dependent term**, which is desired to achieve a reasonable description of the phase shifts of 1S0 channel.

J. Soto et al., PRC(2008), B. Long, PRC (2013)

T-matrix and phase shift

$$V = V_{\text{CTP}} + V_{\text{OPEP}} \equiv F(C_{S,V,A,AV,T})$$

Determine unknown LECs →

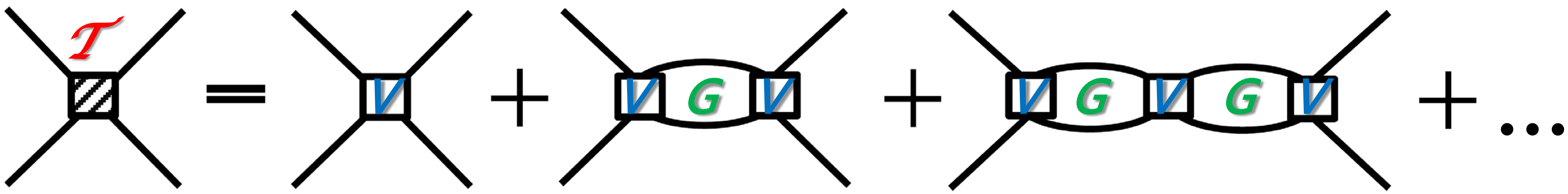
Observables

e.g. NN scattering

phase shifts δ

$T = e^{i\delta} \sin \delta$

- **Nuclear force is nonperturbative** (e.g. deuteron)
  - Scattering equation: $T = V + VGT$



- **Relativistic Kadyshevsky Eq.** (3D-reduced Bethe-Salpeter Eq.)

$$T(p',p) = V(p',p) + \int_0^{+\infty} \frac{k^2 dk}{(2\pi)^3} V(p',k) \frac{2\pi M_N^2}{(k^2+M_N^2)(\sqrt{p^2+M_N^2} - \sqrt{k^2+M_N^2} + i\epsilon)} T(k,p).$$

V. Kadyshevsky, NPB (1968).

The "on-mass-shell" approximation is employed for the kernel potential

$$E_p = \sqrt{M_N^2 + \vec{p}^2}$$

Numerical details

- 5 LECs $C_{S,A,V,AV,T}$ are determined by fitting
  - **NPWA**: ***p-n*** scattering phase shifts of Nijmegen 93

    V. Stoks et al., PRC48(1993)792
  - **7** partial waves: ***J=0, 1*** $^1S_0,\ ^3P_0,\ ^1P_1,\ ^3P_1,\ ^3D_1,\ ^3S_1,\ \epsilon_1$
  - **42** data points: 6 data points for each partial wave

    $(E_{\rm lab} = 1,\ 5,\ 10,\ 25,\ 50,\ 100\ \text{MeV})$
  - **Fit-$\tilde{\chi}^2$**:

$$\tilde{\chi}^2 = \sum_i \left(\delta_i^{\rm Theory} - \delta_i^{\rm Nij93}\right)^2.$$

- Cutoff renormalization for scattering equation
  - Potential in scattering equation:

$$V(p', p) \to V(p', p)\boldsymbol{f(p', p)}.$$

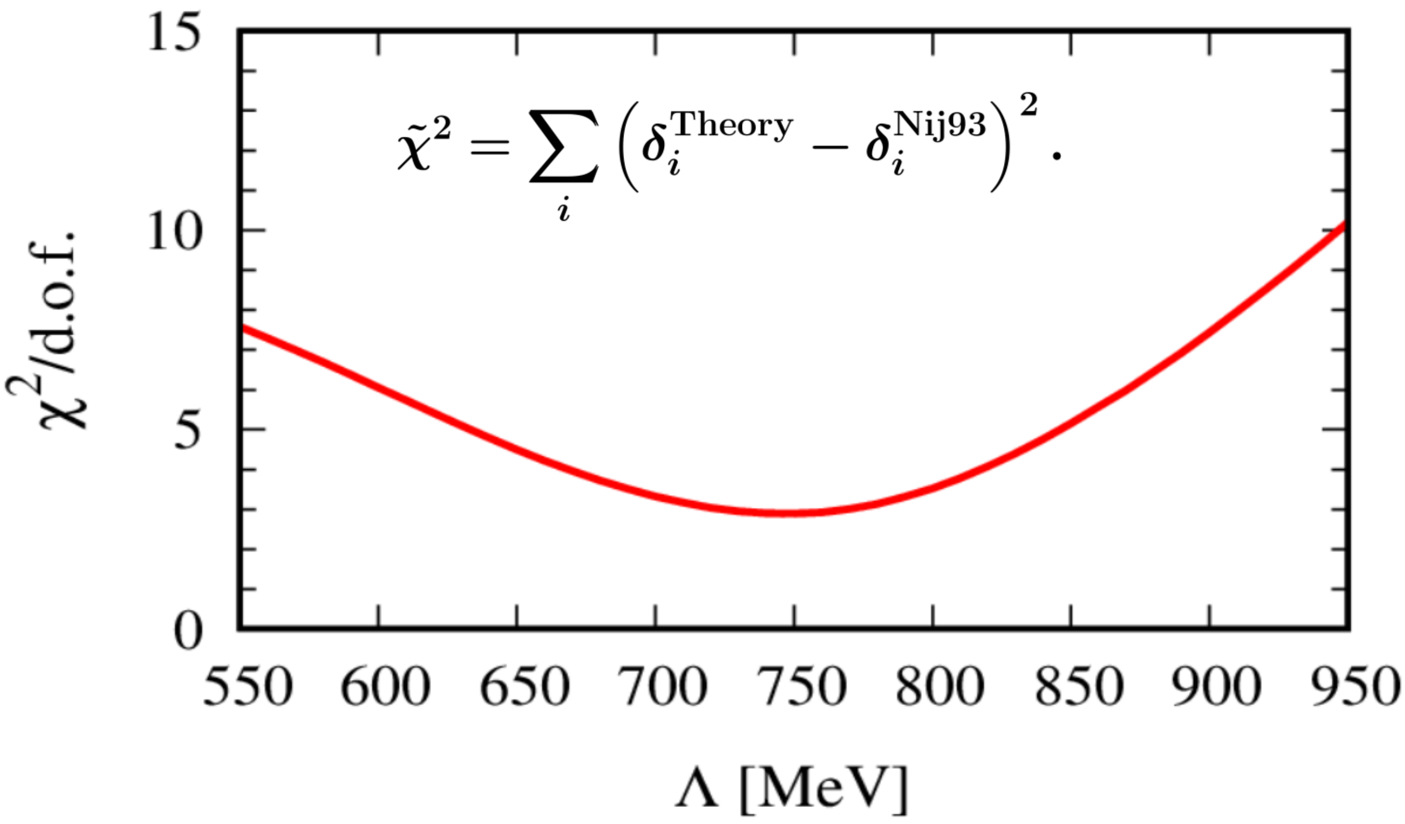
    - **Exponential regulator function:** *U. van Kolck et al., PRL(1994)*

$$f(p', p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}].$$

$$n = 2$$

$$\Lambda = 550 \sim 950\ \text{MeV}$$

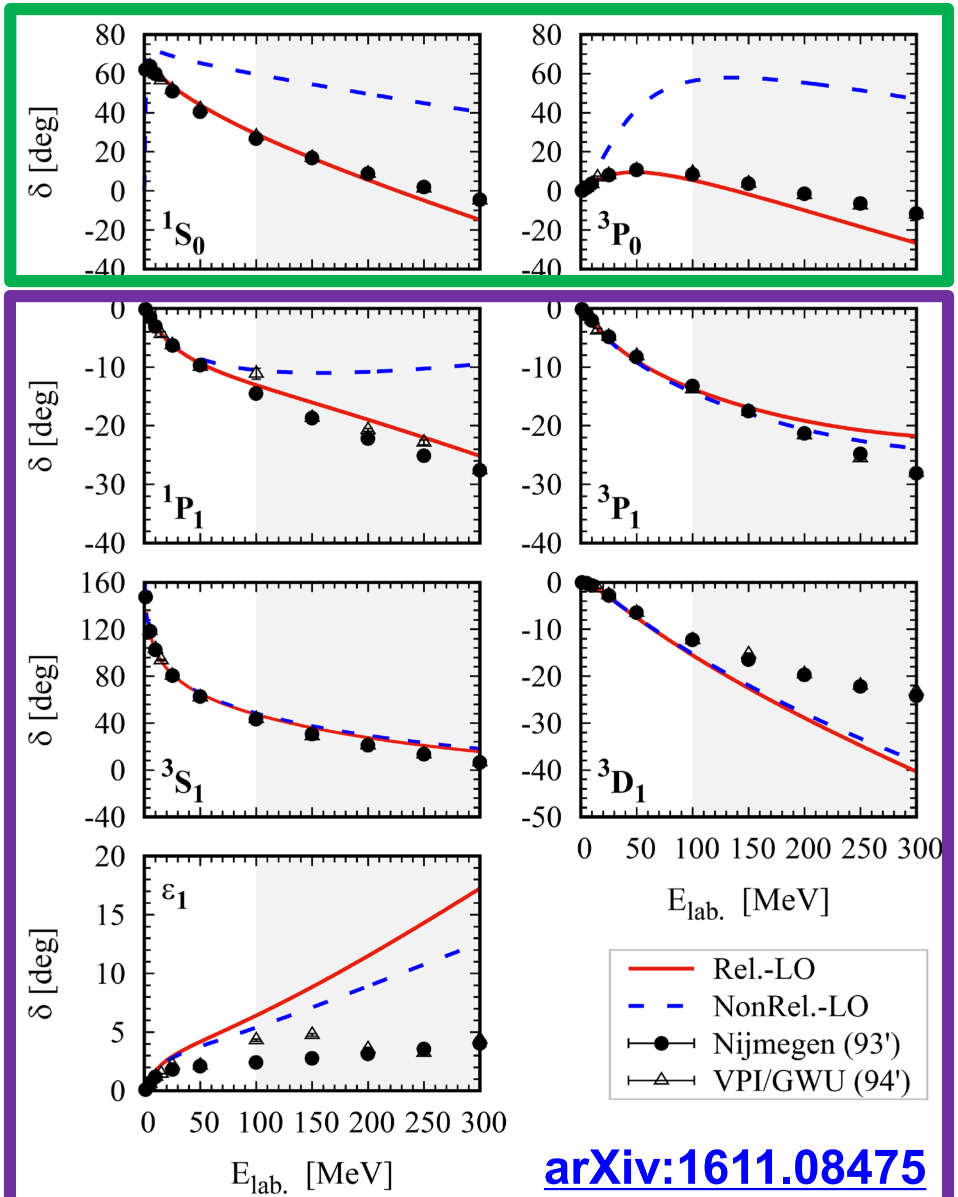
Best fit results



LECs	Values $[10^4\ \mathrm{GeV}^{-2}]$
C_S	-0.135(3)
C_A	0.056(18)
C_V	0.269(12)
C_{AV}	0.244(11)
C_T	0.0625(13)

Λ=747 MeV, the minimum of fit-χ^2/d.o.f. = 2.9

Description of J=0, 1 partial waves



arXiv:1611.08475

- **Improve description** of $^1S_0$, $^3P_0$, $^1P_1$ phase shifts

$$V_{1S0} = 4\pi\left[C_{1S0} + (C_{1S0} + \hat{C}_{1S0})\left(\frac{\vec{p}^2 + \vec{p}'^2}{4M_N^2} + \cdots\right)\right] + \frac{\pi g_A^2}{2f_\pi^2}\int_{-1}^{1}\frac{dz}{\vec{q}^2 + m_\pi^2}\left[\vec{q}^2 - \left(\frac{(\vec{p}^2 - \vec{p}'^2)^2}{4M_N^2} + \cdots\right)\right].$$

- **Quantitatively similar** to the nonrelativistic case for J=1 partial waves

Relativistic corrections are much more suppressed.

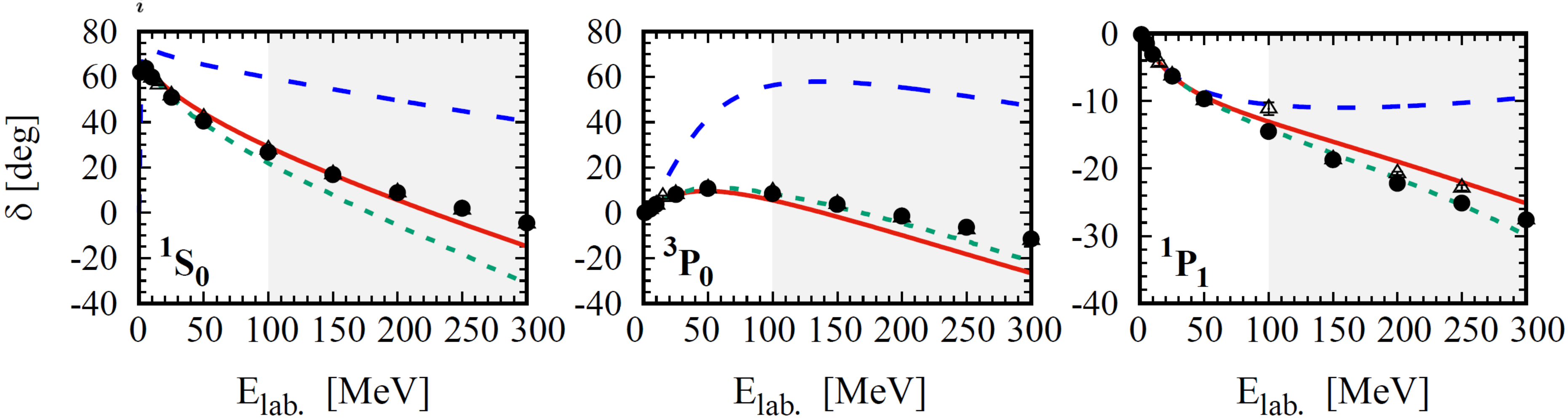
$$V_{3D1} = \frac{8\xi_N}{9}C_{3S1}R_p^2R_{p'}^2 \sim \mathbf{1/M_N^4}.$$

Relativistic vs. Non Relativistic

	Relativistic Chiral NF	Non-relativistic Chiral NF	
Chiral order	LO	LO	NLO*
No. of LECs	**5**	2	**9**
$\tilde{\chi}^2$/d.o.f.	**2.9**	**147.9**	**~2.5**

$$\tilde{\chi}^2 = \sum_i \left(\delta_i^{\text{Theory}} - \delta_i^{\text{Nij93}}\right)^2 .$$

**E. Epelbaum, et. al., NPA(2000)*



- Relativistic chiral NF at LO **can be comparable with** the nonrelativistic case up to NLO
- Relativistic chiral NF provides a **more efficient description** of the phase shifts

Best fit results with BbS equation

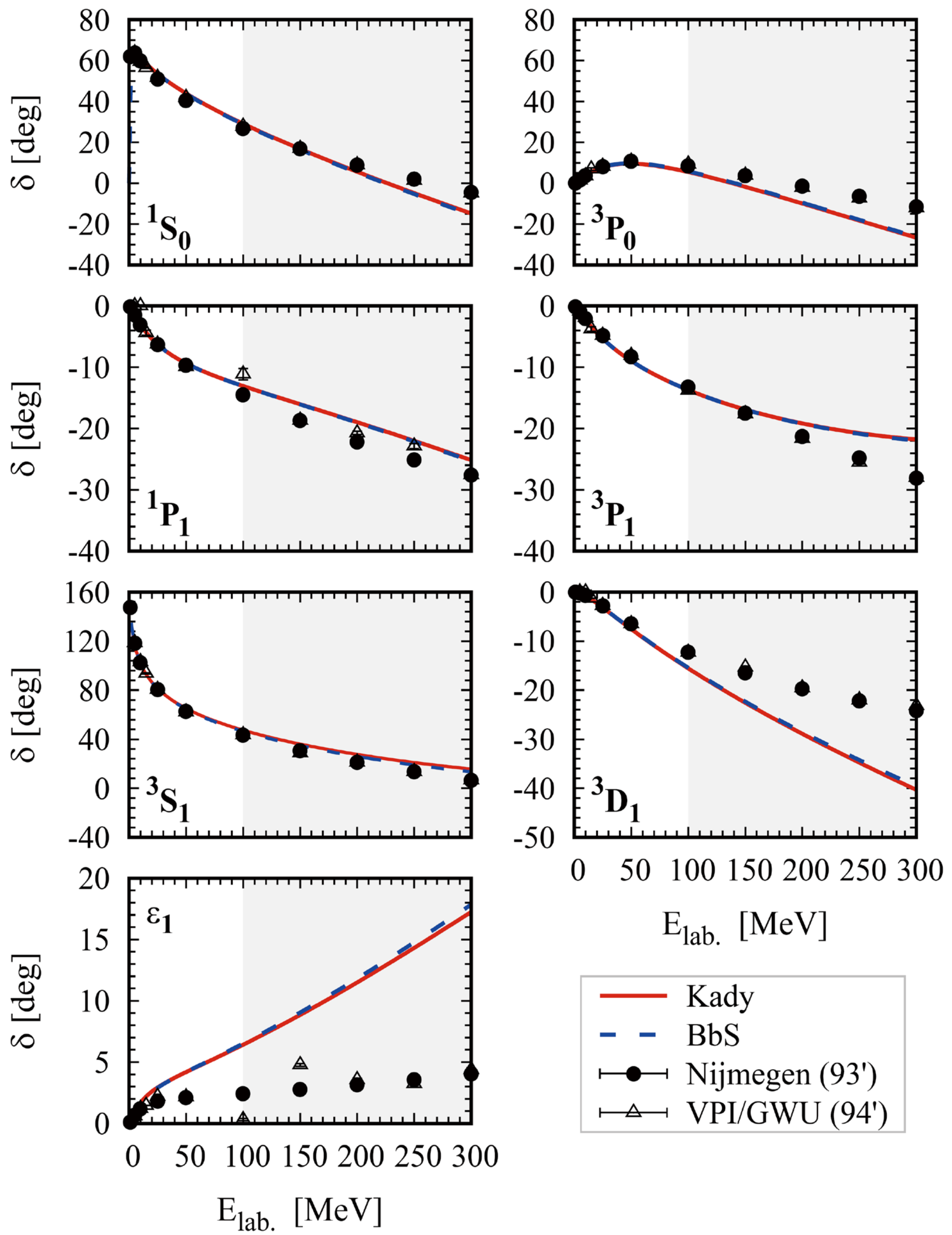
- Replace the scattering equation from the **Kadyshevsky** eq. to the **Blankenbecler-Sugar** eq.

$$T(p',p) = V(p',p) + \int_0^{+\infty} \frac{d\boldsymbol{k}}{(2\pi)^3} V(p',k) \times M_N^2 \frac{1}{\sqrt{k^2+M_N^2}(p^2-k^2)+i\epsilon} T(k,p).$$

R.Blankenbecler & R. Sugar, Phys.Rev.(1966)

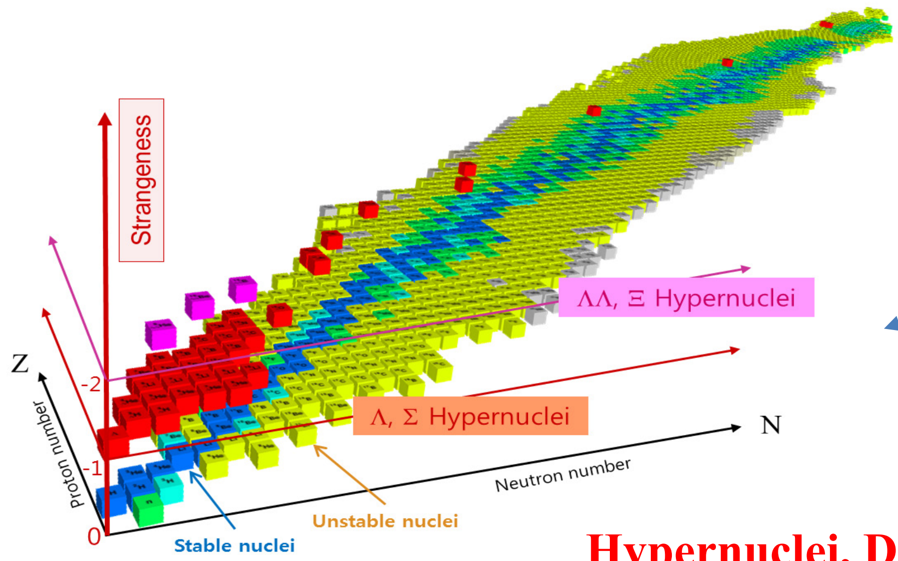
- Best fit results:

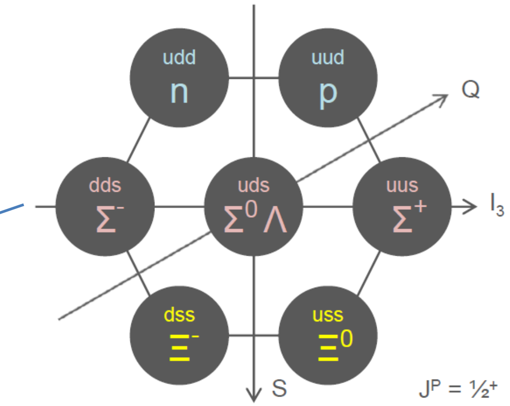
	Kady.	BbS
Cutoff Λ [MeV]	**747**	**743**
Fit-χ^2/d.o.f.	**2.9**	**2.5**



Baryon-Baryon interactions

Key inputs for hypernulcear physics





Hypernuclei, Dibaryon, Neutron stars…

Current status of chiral BB interactions

- Up to NLO from HB approach *J. Haidenbauer, Ulf-G. Meißner, et al., NPA(2006), LNP(2007),PLB(2007),(2010),NPA(2013),(2016)…*
  - **Systematically studied** S = -1, -2, -3, -4 sectors
- Up to NLO from KSW approach *C.L. Korpa, et al., PRC(2001)*
- Up to LO from EG approach *K.-W. Li, et al., PRD(2016)*

Relativistic BB interactions (LO)

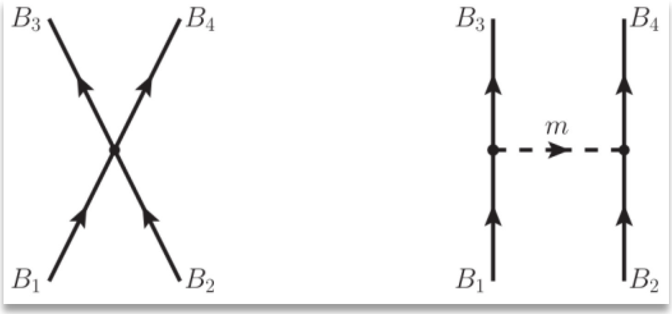
- Covariant effective Lagrangains *H.Polinder, et al.,NPA(2006)*

$$
\begin{aligned}
\mathcal{L}^{\text{eff.}} &= \mathcal{L}^{(0)}_{BB} + \mathcal{L}^{(1)}_{\phi B} \\
&= \frac{C_i^1}{2}\,\mathrm{Tr}\left(\bar{B}_a\bar{B}_b(\Gamma_i B)_b(\Gamma_i B)_a\right) + \frac{C_i^2}{2}\,\mathrm{Tr}\left(\bar{B}_a(\Gamma_i B)_a\bar{B}_b(\Gamma_i B)_b\right) \\
&\quad + \frac{C_i^3}{2}\,\mathrm{Tr}\left(\bar{B}_a(\Gamma_i B)_a\right)\mathrm{Tr}\left(\bar{B}_b(\Gamma_i B)_b\right) \\
&\quad + \mathrm{Tr}\left(\bar{B}\left(i\gamma_\mu D^\mu - M_B\right)B - \frac{D}{2}\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\} - \frac{F}{2}\bar{B}\gamma_\mu\gamma_5[u_\mu, B]\right).
\end{aligned}
$$

Γ_i :Clifford algebra

15 unknown LECs

- BB interactions (momentum space)



$$
V_{\text{CT}}^{B_1B_2\to B_3B_4} = C_i\left(\bar{u}_3\Gamma_i u_1\right)\left(\bar{u}_4\Gamma_i u_2\right),
$$

$$
V_{\text{OME}}^{B_1B_2\to B_3B_4} = N_{B_1B_3\phi}N_{B_2B_4\phi}\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\boldsymbol{q}^2 + m_\phi^2}\mathcal{I}_{B_1B_2\to B_3B_4}.
$$

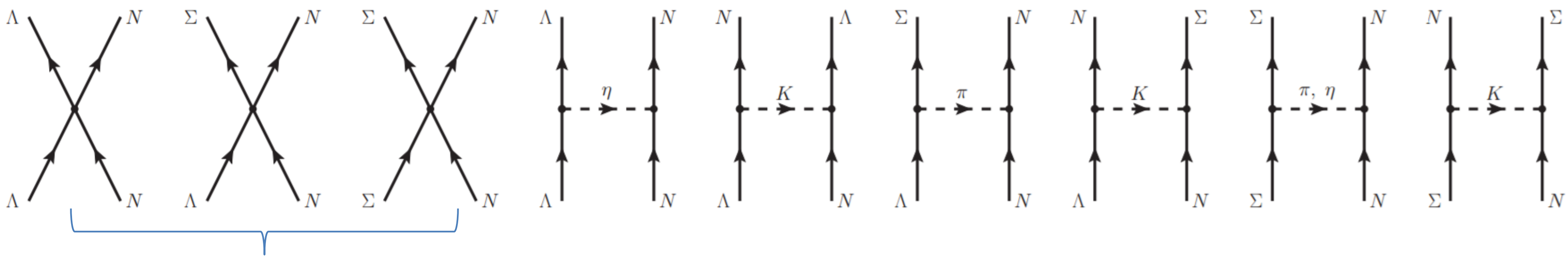
Strangeness = -1 sector

K.-W. Li, XLR, L.-S.Geng, B. Long, 1612.08482

- S = -1; I = 3/2, 1/2

$\Sigma^+ p$ (+3/2) $\quad \Lambda p, \Sigma^+ n, \Sigma^0 p$ (+1/2) $\quad \Lambda n, \Sigma^0 n, \Sigma^- p$ (-1/2) $\quad \Sigma^- n$ (-3/2) $\quad I_3$

- Contact diagrams and OME diagrams



12 unknown LECs

- Coulomb force in charged channels: Vincent-Phatak method

C.Vincent&S.Phatak, PRC(1974)

- Kadyshevsky equation

$$T_{\rho\rho'}^{\nu\nu',J}(p',p;\sqrt{s}) = V_{\rho\rho'}^{\nu\nu',J}(p',p) + \sum_{\rho'',\nu''} \int_0^\infty \frac{dp'' p''^2}{(2\pi)^3} \frac{2\mu_{\nu''}^2\, V_{\rho\rho''}^{\nu\nu'',J}(p',p'')\, T_{\rho''\rho'}^{\nu''\nu',J}(p'',p;\sqrt{s})}{(p''^2+4\mu_{\nu''}^2)\left(\sqrt{q_{\nu''}^2+4\mu_{\nu''}^2} - \sqrt{p''^2+4\mu_{\nu''}^2}+i\varepsilon\right)}$$

Fitting procedure

- 36 YN scattering data: 35 cross section + 1 $\Sigma^- p$ capture ratio

$$\Lambda p \to \Lambda p: \ (12) \qquad \Sigma^+ p \to \Sigma^+ p: \ (4) \qquad \Sigma^- p \to \Sigma^- p: \ (7)$$

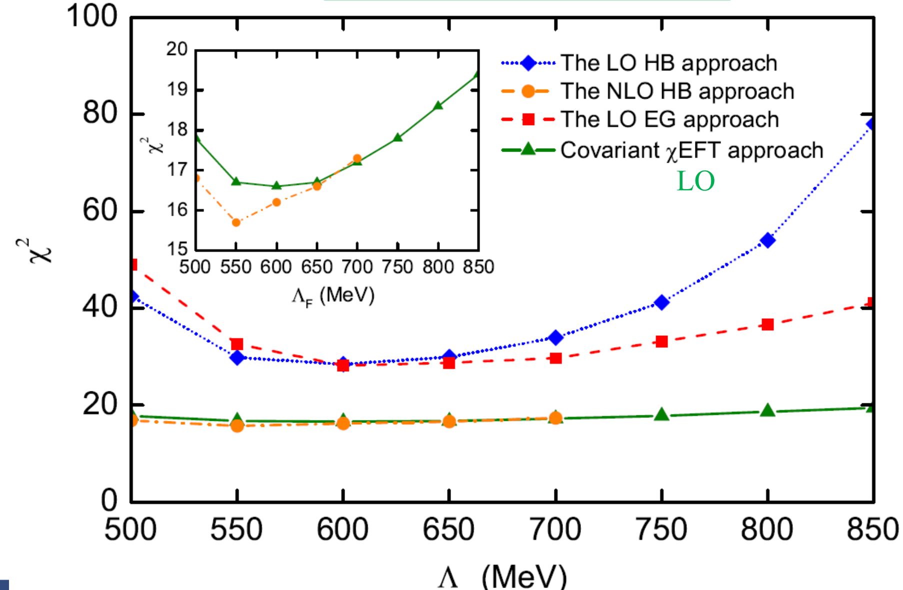
$$\Sigma^- p \to \Lambda n: \ (6) \qquad \Sigma^- p \to \Sigma^0 n: \ (6)$$

- Hypertriton ${}^3_\Lambda H$ binding energy (we are unable to calculate)
  - Λp S-wave scattering lengths
  - $\Sigma^+ p$ S-wave scattering length
- Regulator function

$$f(p', p) = \exp\left[-\left(\frac{p'}{\Lambda}\right)^{2n} - \left(\frac{p}{\Lambda}\right)^{2n}\right].$$

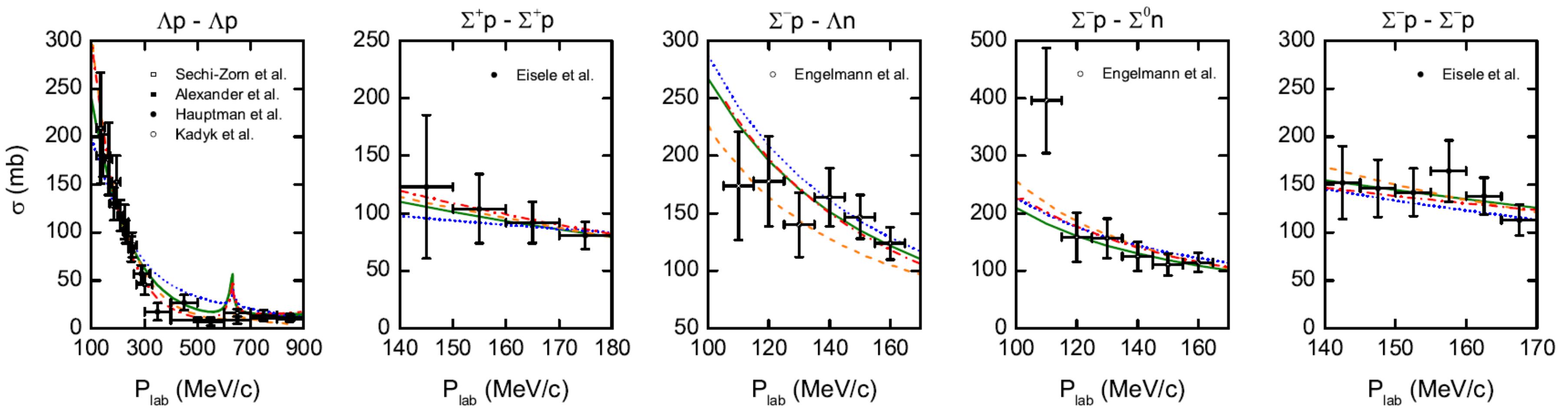
$n = 2 \quad \Lambda = 500 \sim 850$ MeV

Cutoff dependence



Best fitting results

- Description of cross sections (Λ = 600 MeV)



arXiv:1612.08482

Green solid lines: LO covariant ChEFT approach ; Blue dotted lines: LO HB approach

Red dash-dotted lines: NSC97f; Orange dashed lines: Julich 04

36 YN data	HB approach		Covariant ChEFT	NSC97f$
No. of LECs (or parameters)	5 (LO*)	23 (NLO#)	12 (LO)	29
χ^2	28.3	16.2	16.6	16.7

*Polinder NPA 799 (2006) 244 #Haidenbauer NPA 915 (2013) 24 $Rijken PRC 59 (1999) 21

Relativistic effects: better description of experimental data

Summary

- **We performed an exploratory study to construct the relativistic nuclear force up to leading order in covariant ChEFT**
  - Relativistic chiral NF can **self-consistently** include the **spin-orbit interaction**, etc.
  - **Relativistic effects can improve** the description of $^1S_0$, $^3P_0$ and $^1P_1$ phase shifts
  - Relativistic framework presents **a more efficient formulation** of the chiral nuclear force
- **LO relativistic hyperon-nucleon interactions are also studied.**

Perspectives

- **Relativistic chiral nuclear force up to NLO**
  - Calculate the contact potential with two derivatives and two-pion exchange potentials
  - Expect to achieve a better description of phase shifts
- **Our final goal: construct a high precision chiral nuclear force**
  - Study **chiral extrapolation** of nuclear force from LQCD
  - Study few-body systems by using the **Gaussian Expansion Method**
  - Study nuclear structure by using **Relativistic Brueckner–Hartree–Fock theory**

Perspectives

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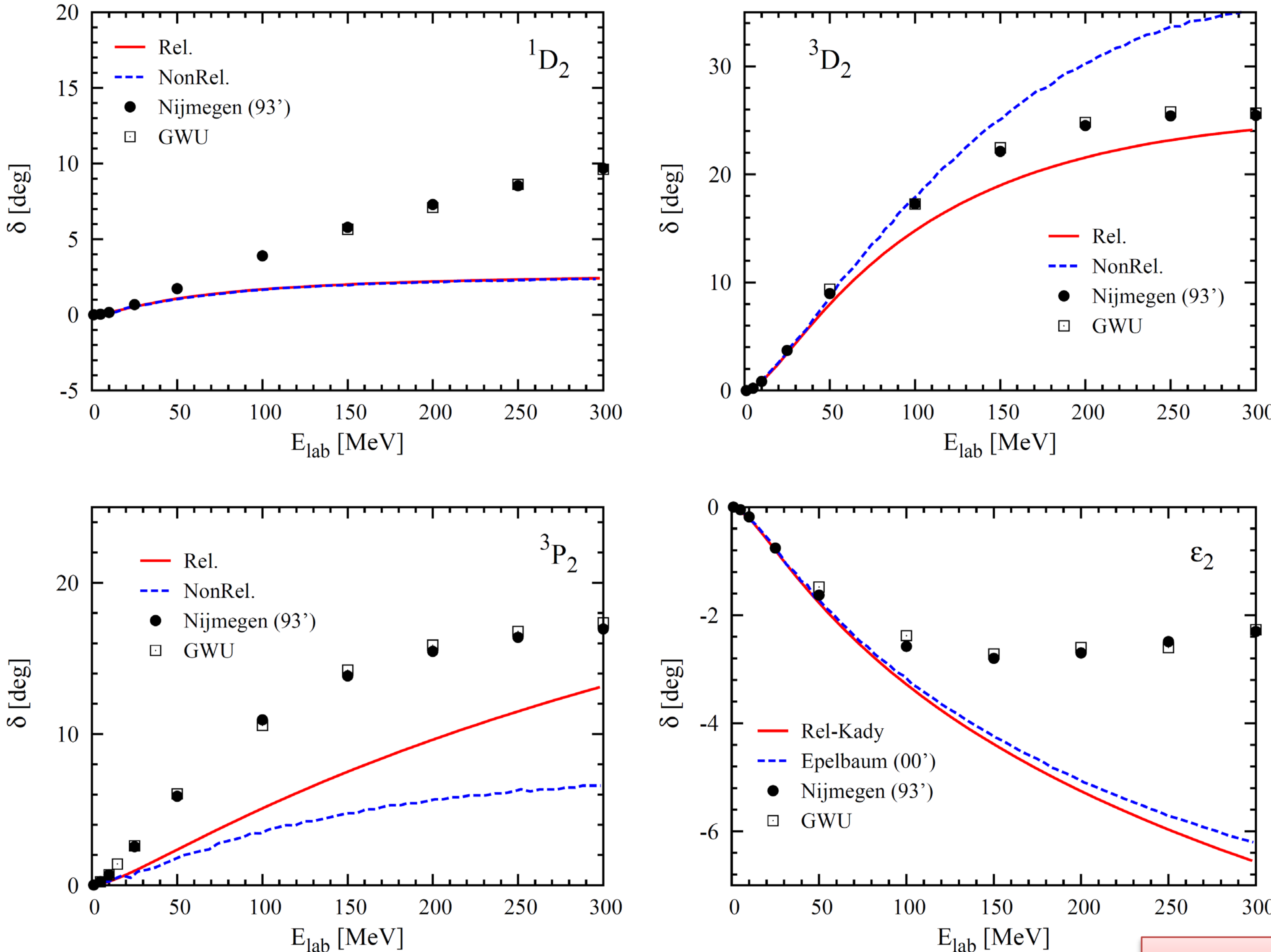
Thank you for your attention!

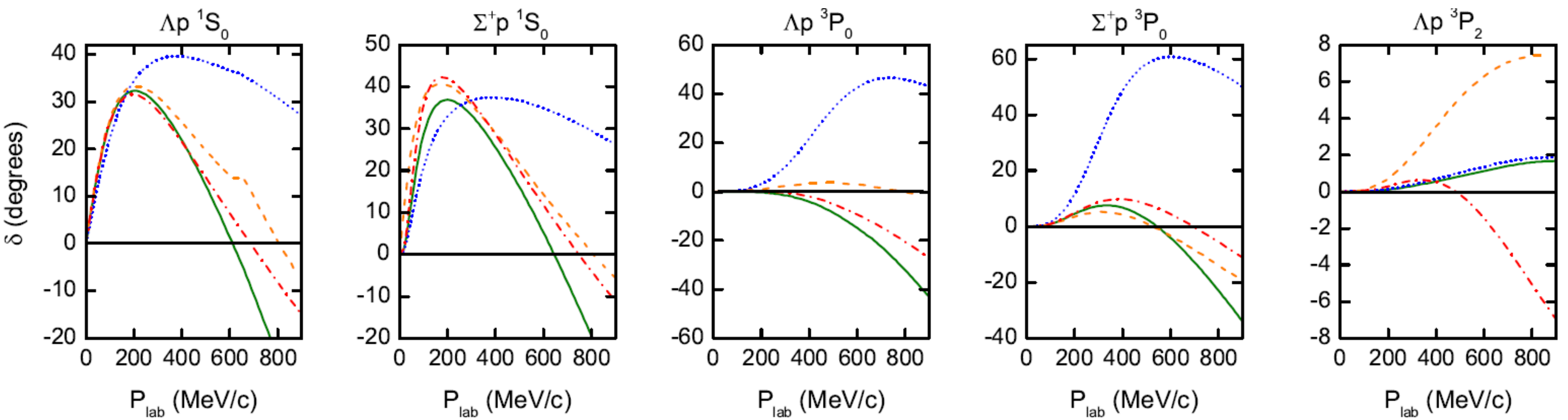
Back up slides

Description of J=2 PWs phase shift



OPE prediction

Phase shifts (S=-1)

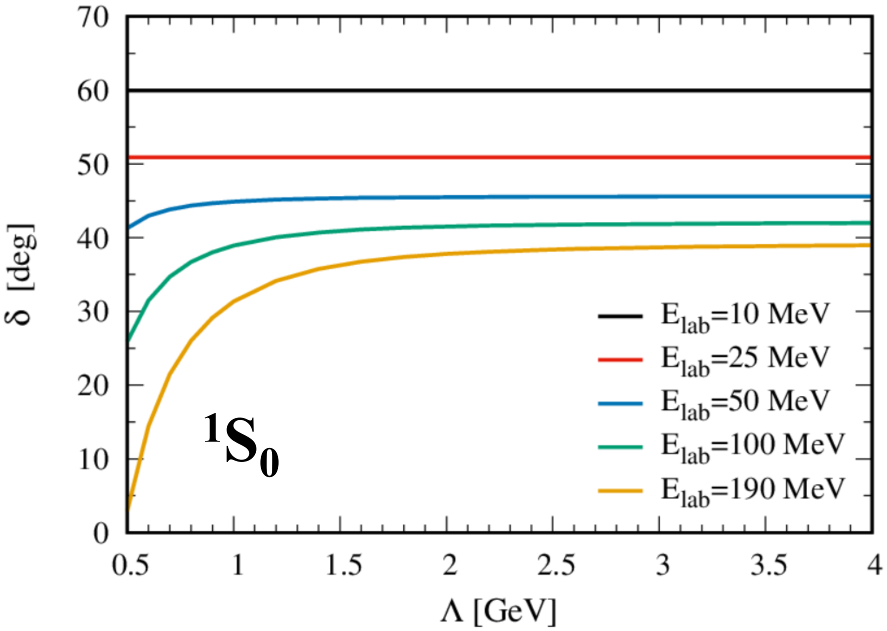


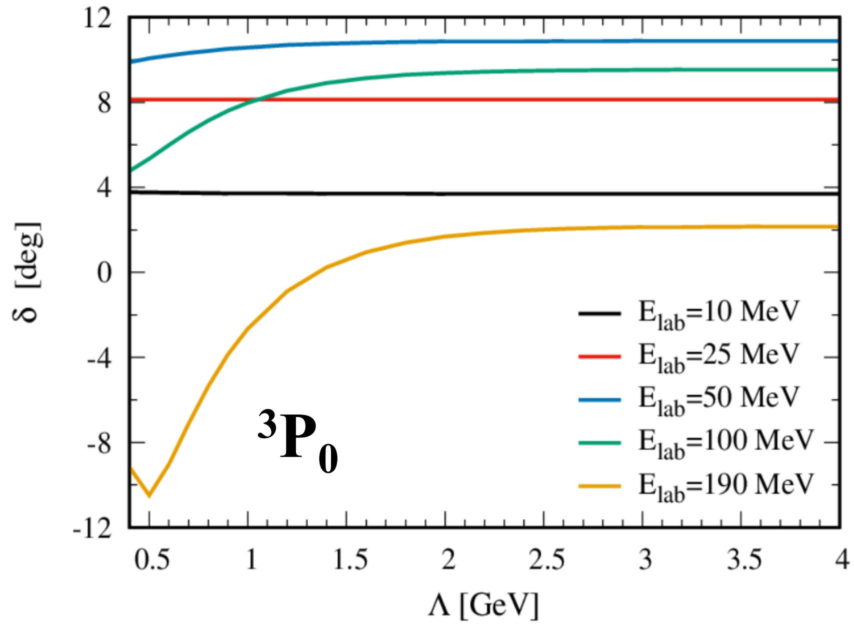
Green solid lines: LO covariant ChEFT approach ; Blue dotted lines: LO HB approach

Red dash-dotted lines: NSC97f; Orange dashed lines: Julich 04

Renormalization Group Invariance

- Rel. Chiral NF up to LO
  - J=0 partial waves:





- We self-consistently **achieved the RGI for 3P0 partial wave**
- One LEC **naturally appeared** in relativistic chiral NF (covariant form)

$$V_{3P0} = -8\pi N_p^2 N_{p'}^2 \boldsymbol{C_{3P0}} \frac{pp'}{\epsilon_p \epsilon_{p'}} + V_{\text{OPEP}}.$$

Errors and correlation matrix

TABLE I: The best fit results of five LECs appearing in the contact terms (in unit of $10^4 \mathrm{GeV}^{-2}$) with the momentum cutoff $\Lambda = 747$ MeV.

LECs	C_S	C_A	C_V	C_{AV}	C_T
Best fit	0.13515 ± 0.00307	-0.055963 ± 0.018217	-0.26857 ± 0.01151	-0.24427 ± 0.01141	-0.062538 ± 0.001319

	C_S	C_A	C_V	C_{AV}	C_T
C_S	**1.00**	0.21	-0.93	-0.58	-0.39
C_A	0.23	1.00	-0.15	0.45	0.21
C_V	**-0.93**	-0.15	1.00	0.77	0.69
C_{AV}	-0.57	0.45	**0.77**	1.00	0.89
C_T	-0.39	0.21	0.69	**0.89**	1.00

Kadyshevsky equation for unequal masses

$$T_{\rho\rho'}^{\nu\nu',J}(\boldsymbol{p}',\boldsymbol{p};\sqrt{s}) = V_{\rho\rho'}^{\nu\nu',J}(\boldsymbol{p}',\boldsymbol{p}) + \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}$$

$$\times \frac{m_{1,\nu''}m_{2,\nu''}\, V_{\rho\rho''}^{\nu\nu'',J}(\boldsymbol{p}',\boldsymbol{p}'')\, T_{\rho''\rho'}^{\nu''\nu',J}(\boldsymbol{p}'',\boldsymbol{p};\sqrt{s})}{\sqrt{\boldsymbol{p}''^2+m_{1,\nu''}^2}\sqrt{\boldsymbol{p}''^2+m_{2,\nu''}^2}\left(\sqrt{\boldsymbol{q}_{\nu''}^2+m_{1,\nu''}^2}+\sqrt{\boldsymbol{q}_{\nu''}^2+m_{2,\nu''}^2}-\sqrt{\boldsymbol{p}''^2+m_{1,\nu''}^2}-\sqrt{\boldsymbol{p}''^2+m_{2,\nu''}^2}+i\epsilon\right)}. \quad (2)$$

Since we are only performing a LO calculation, consistent with the derivation of the kernel potential and the chiral power counting, one can treat the mass difference as a higher order correction. As a result, the common mass is chosen to be twice of the