Parametrisation for near-threshold states

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C. Hanhart, Y.S. Kalashnikova, P. Matuschek, R.V. Mizuk, A.V. Nefediev, QW,
PRL115(2015)202001, J.Phys.Conf. Ser. 675(2016)022016

F.K. Guo, C. Hanhart, Y.S. Kalashnikova, P. Matuschek, R.V. Mizuk, A.V. Nefediev, QW,
J. L. Wynen, PRD93(2016)074031

M.L. Du, C. Hanhart, U.-G. Meißner, G.J.Wang, QW, in preparation

Outline

Mini-review of exotic candidates

Cusp alone?

Parametrisation for near-threshold states

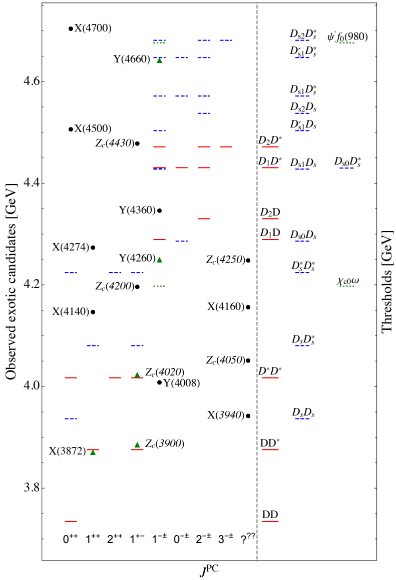
Apply to the two Z_b cases

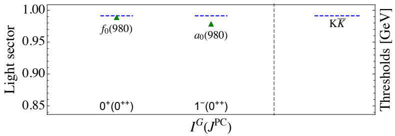
Summary

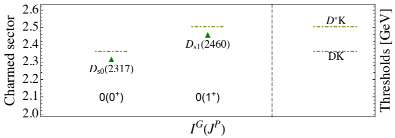
Mini-review of exotic candidates

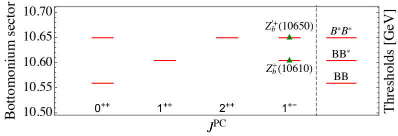
Near-threshold states

PDG2016



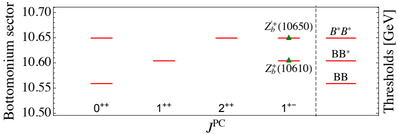






Mini-review of exotic candidates

Properties of near-threshold states



★ Molecular scenario

$E_{Z_b}^{\text{Exp}} \sim 3$ MeV

$\Rightarrow$ Extended object

Size $\frac{\hbar c}{\gamma} \sim 1.56$ fm $\gg R_0$

$R_0 \ll 1$ fm confinement radius

$\gamma = \sqrt{2\mu E}$ binding momentum

μ reduced mass

$\Rightarrow$ Probability to find Z_b

M.Cleven et al., EPJA47(2011)120

$$1 - \left[1 + \frac{\mu^2 g_{\text{bare}}^2}{8\pi\gamma}\right]^{-1} \leq 1|_{g_{\text{bare}} \to \infty}$$

Large coupling $g_{Z_b BB^*}$

$$g_{\text{eff}}^2 = \frac{g_{\text{bare}}^2}{1 + \frac{\mu^2 g_{\text{bare}}^2}{8\pi\gamma}} \leq \frac{8\pi\gamma}{\mu^2}\Big|_{g_{\text{bare}} \to \infty}$$

Exp: Belle, PRL116(2016)212001

$$\mathcal{BR}(Z_b \to B\bar{B}^* + c.c.) \sim 85.6\%$$

★ Cusp alone ?

★ BW does not work,

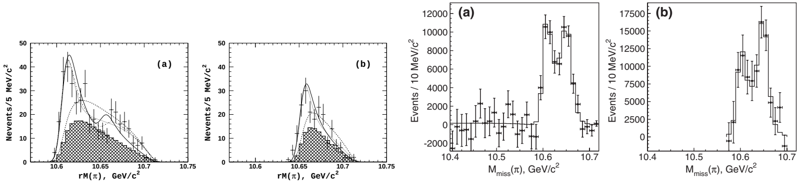
$\Rightarrow E_{Z_b^{(\prime)}}^{\text{Exp}} \ll \Gamma_{Z_b^{(\prime)}}$

$\Rightarrow$ two Z_b states

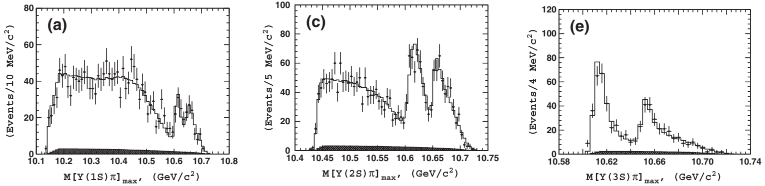
Cusp alone?

All the available data for the two Z_b states

$\Upsilon(5S) \to Z_b^{(\prime)\pm}\pi^\mp \to (B^{(*)}\bar{B}^*)\pm\pi^\mp$, $\Upsilon(5S) \to Z_b^{(\prime)\pm}\pi^\mp \to h_b(mP)\pi^\pm\pi^\mp$ with $m = 1, 2$ ☹



$\Upsilon(5S) \to Z_b^{(\prime)\pm}\pi^\mp \to \Upsilon(nS)\pi^\pm\pi^\mp$ with $n = 1, 2, 3$ ☺, with $\frac{\mathcal{B}r(h_b\pi\pi)}{\mathcal{B}r(\Upsilon\pi\pi)} \sim 1$

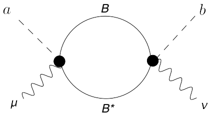


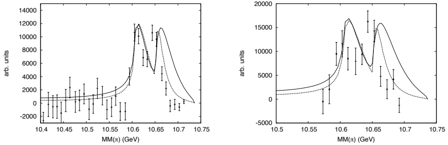
In total: 7 channels, i.e. $B^{(*)}\bar{B}^*$, $h_b(mP)\pi$ and $\Upsilon(nS)\pi$

Belle, PRL108(2012)122001, PRL116(2016)212001

Cusp alone?

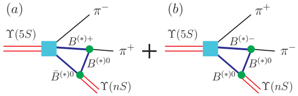
Cusp interpretations of the two Z_b states

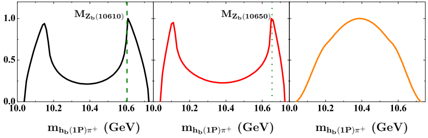




E.S.Swanson,PRD91(2015)034009,Int.J.Mod.Phys.E25(2016)1642010

$B\bar{B}^* + c.c.$ $B^*\bar{B}^*$ $B\bar{B}$





D.Y. Chen, X. Liu, PRD84(2011)094003, D.Y.Chen, X.Liu, T. Matsuki, PRD84(2011)074032

Heavy quark symmetry and light quark symmetry

- $m_Q \gg \Lambda_{QCD} \rightarrow$ physics at the m_Q scale is perturbative
- Heavy quark limit $\rightarrow$ spin symmetry & flavor symmetry

  To the leading order,

$$\mathcal{L}_{QCD} = \bar{h}_v iv \cdot D h_v + \mathcal{O}(\Lambda_{QCD}/m_Q)$$

  No Dirac matrix:

  $\Rightarrow$ spin symmetry (HQSS)

  $\Rightarrow s_Q$ and light degrees of freedom conserved individually

  $\rightarrow \langle HL|\hat{H}_I|H'L'\rangle \equiv V_{HL}\delta_{HH'}\delta_{LL'}$

  $\Rightarrow$spin doublet: $s_l = \frac{1}{2}^-$ (B, B^*) with $m_{B^*} - m_B \sim \Lambda_{QCD}$

  No heavy quark mass: $\rightarrow$ flavor symmetry (HQFS)

★ $V_{H'L} \overset{HQSS}{=} V_{HL} \overset{LQSS}{=} V_{HL'}$

M.B. Voloshin, PRD93(2016)074011

Cusp alone? No!

⇒ Narrow structure in Elastic Channels calls for nearby poles

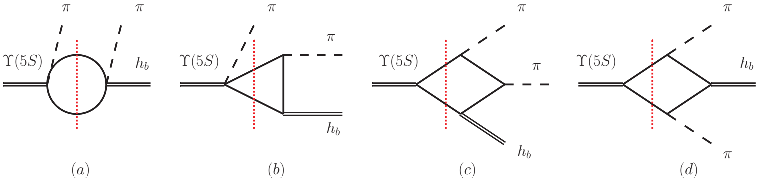
Talk by Christoph Hanhart

⇒ Some to them are observed in HQS flip processes

→ $\Upsilon(5S) \to h_b(mP)\pi\pi$ with $S_{b\bar{b}}^{\Upsilon(5S)} = 1$ and $S_{b\bar{b}}^{h_b(mP)} = 0$

→ no direct two-pion transition

→ all the possible bottomed meson loops are cancelled with each other in the HQ limit, i.e. $m_{B^*} = m_B$.



Does NOT depend on (1) topology (2) multiplets

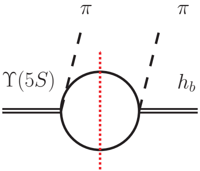
Cusp alone? No! Why?

From hadron basis to $|HL\rangle$ basis ($S_{b\bar{b}}^{\Upsilon} = 1$ and $S_{b\bar{b}}^{h_b} = 0$)

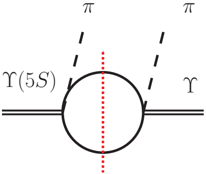
$$|B\bar{B}^*\rangle_{1^{+-}} = -\frac{1}{\sqrt{2}}|1_H \otimes 0_L\rangle - \frac{1}{\sqrt{2}}|0_H \otimes 1_L\rangle$$

$$|B^*\bar{B}^*\rangle_{1^{+-}} = \frac{1}{\sqrt{2}}|1_H \otimes 0_L\rangle - \frac{1}{\sqrt{2}}|0_H \otimes 1_L\rangle$$

A.E. Bondar et al., PRD84(2011)054010



$$\begin{cases} \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{2} & \text{for} \quad B\bar{B}^* + c.c. \\ \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2} & \text{for} \quad B^*\bar{B}^* \end{cases}$$



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\end{aligned}
$$

A.E. Bondar et al., PRD84(2011)054010

⇒ HQSS breaking effect $\frac{\Lambda_{QCD}}{m_b}$

⇒ Experimental fact:

$$
\frac{Br(\Upsilon(5S) \to h_b\pi\pi)}{Br(\Upsilon(5S) \to \Upsilon\pi\pi)} \propto \frac{|\mathcal{M}_{B\bar{B}^*+c.c.} - \mathcal{M}_{B^*\bar{B}^*}|^2}{|\mathcal{M}_{B\bar{B}^*+c.c.} + \mathcal{M}_{B^*\bar{B}^*}|^2} \sim 1 \, \textbf{?}
$$

Cusp alone? No! Why?

The effective lagrangians to the leading order

$$\mathcal{L}_{\Upsilon BB\pi} = g_{\Upsilon BB\pi} \left\langle \Upsilon^i \sigma^i \bar{H}_a^\dagger H_b^\dagger \right\rangle A_{ab}^0 + \text{H.c},$$
$$\mathcal{L}_{hBB\pi} = g_{hBB\pi} \left\langle h_b^{i\dagger} H_a \sigma^j \bar{H}_b \right\rangle \epsilon_{ijk} A_{ab}^k + \text{H.c.},$$

with $H_a = H_a^\mu \sigma^\mu = B_a + \sigma^i B_a^{*i}$, $\quad \bar{H}_a^\dagger = \bar{\sigma}^\nu \bar{H}_b^{\dagger\nu} = \bar{B}_a - \sigma^i \bar{B}^{*i}$

and $\sigma^\mu = (\sigma^i, \mathbf{1})$, $\quad \bar{\sigma}^\nu = (-\sigma^i, \mathbf{1})$. The two-point functional

$$Z_2[\Upsilon, A_\mu, h_b^\dagger] = i g_{\Upsilon BB\pi} g_{hBB\pi} \left\langle \sigma^l \bar{\sigma}^\mu \sigma^\nu \right\rangle \left\langle \sigma^\nu \sigma^j \bar{\sigma}^\mu \right\rangle$$
$$\times \int d^4x d^4y \Upsilon^l(x) A_{ab}^0(x) \epsilon^{ijk} h_b^{i\dagger}(y) A_{ba}^k(y) \Delta(x-y) \Delta(y-x)$$

$\left\langle \sigma^l \bar{\sigma}^\mu \sigma^\nu \right\rangle \left\langle \sigma^\nu \sigma^j \bar{\sigma}^\mu \right\rangle \Rightarrow B\bar{B}(0)$, $B^*\bar{B} + c.c.(8\delta^{jl})$, $B^*\bar{B}^*(-8\delta^{jl})$

$\frac{\mathcal{B}r(h_b\pi\pi)}{\mathcal{B}r(\Upsilon\pi\pi)} \sim 1 \Rightarrow$ the intermediate genuine states

Parametrisation for near-threshold states

Multichannel LSE

Potential $\hat{V}$

$$\begin{array}{cccc} & b=\overline{1,N_\mathrm{p}} & \beta=\overline{1,N_\mathrm{e}} & i=\overline{1,N_\mathrm{in}} \end{array}$$

$$\hat{V}=\begin{pmatrix} v_{ab} & v_{a\beta}(\boldsymbol{p}') & v_{ai}(\boldsymbol{k}) \\ v_{\alpha b}(\boldsymbol{p}) & v_{\alpha\beta}(\boldsymbol{p},\boldsymbol{p}') & v_{\alpha i}(\boldsymbol{p},\boldsymbol{k}) \\ v_{ja}(\boldsymbol{k}') & v_{j\beta}(\boldsymbol{k}',\boldsymbol{p}') & v_{ji}(\boldsymbol{k}',\boldsymbol{k}) \end{pmatrix} \qquad \begin{array}{l} a=\overline{1,N_\mathrm{p}} \\ \alpha=\overline{1,N_\mathrm{e}} \\ j=\overline{1,N_\mathrm{in}}. \end{array}$$

Bare pole terms: with indices a, b, ...

Inelastic channels: hidden-flavor channels with indices i, j, ...

Elastic channels: open-flavor channels with indices α, β, ...

C. Hanhart et al., PRL115(2015)202001, F.K. Guo et al., PRD93(2016)074031

Note: $v_{ij}(\boldsymbol{k},\boldsymbol{k}')\equiv 0$, $\pi-Q\bar{Q}$ scattering length ≤ 0.02 fm

L. Liu et al., Proc. Sci., LATTICE2008(2008)112, W. Detmold, et al., PRD87(2013)094504

Parametrisation for near-threshold states

Multichannel LSE

Potential $\hat{V}$

$$\hat{V} = \begin{pmatrix} v_{AB}(\boldsymbol{p}, \boldsymbol{p}') & v_{Ai}(\boldsymbol{p}, \boldsymbol{k}) \\ v_{jB}(\boldsymbol{k}', \boldsymbol{p}') & 0 \end{pmatrix} \qquad B = \overline{1,\, N_{\rm e} + N_{\rm p}} \quad i = \overline{1, N_{\rm in}} \quad A = \overline{1\, N_{\rm e} + N_{\rm p}} \quad j = \overline{1, N_{\rm in}},$$

Capital greek letters A, B for elastic channels and bare poles.

Bare pole terms: with indices a, b, …

Inelastic channels: hidden-flavor channels with indices i, j, …

Elastic channels: open-flavor channels with indices α, β, …

C. Hanhart et al., PRL115(2015)202001, F.K. Guo et al., PRD93(2016)074031

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L. Liu et al., Proc. Sci., LATTICE2008(2008)112, W. Detmold, et al., PRD87(2013)094504

Parametrisation for near-threshold states

LSE $t = v - vSt$ can be decomposed into two sub sets

$$\begin{cases} t_{iB} = v_{iB} - \sum_A v_{iA} S_A t_{AB} \\ t_{AB} = v_{AB} - \sum_C v_{AC} S_C t_{CB} - \sum_i v_{Ai} S_i t_{iB} \end{cases}$$

$$\begin{cases} t_{Aj} = v_{Aj} - \sum_B t_{AB} S_B v_{Bj} \\ t_{ij} = - \sum_A v_{iA} S_A v_{Aj} + \sum_{AB} v_{iA} S_A t_{AB} S_B v_{Bj} \end{cases}$$

Define effective potential V

$$\begin{aligned} t_{AB} &= v_{AB} - \sum_C v_{AC} S_C t_{CB} - \sum_i v_{Ai} S_i \left(v_{iB} - \sum_C v_{iC} S_C t_{CB} \right) \\ &= \underbrace{v_{AB} - \sum_i v_{Ai} S_i v_{iB}}_{V_{AB}} - \sum_C \underbrace{\left(v_{AC} - \sum_i v_{Ai} S_i v_{iC} \right)}_{V_{AC}} S_C t_{CB} \end{aligned}$$

Parametrisation for near-threshold states

LSE $t = v - vSt$ can be decomposed into two sub sets

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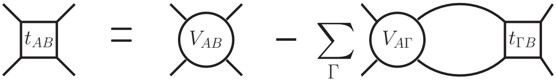
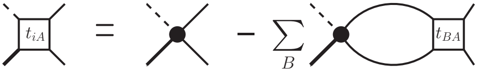
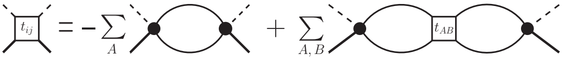


$$t_{AB} = \underbrace{v_{AB} - \sum_i v_{Ai} S_i v_{iB}}_{V_{AB}} - \sum_C \underbrace{\left(v_{AC} - \sum_i v_{Ai} S_i v_{iC} \right)}_{V_{AC}} S_C t_{CB}$$

Parametrisation for near-threshold states

- $\Rightarrow S_i$ the ith $(Q\bar{Q})(q\bar{q})$ channel, enters V additively
- $\Rightarrow$ Arbitrary number of inelastic channels non-perturbatively
- $\Rightarrow$ The dimension of LSE is from $N_e + N_p + N_{\text{in}}$ to $N_e + N_p$
- $\Rightarrow$ Other components of t matrix can be obtained algebraically





- $\Rightarrow$ Satisfy unitarity and analyticity
- $\Rightarrow$ Separable interaction: the further analysis in experiment

Apply to the two Z_b cases

The wave functions of $B\bar{B}^* + c.c.$ and $B^*\bar{B}^*$ with $J^{PC} = 1^{+-}$

$$
\begin{aligned}
|B\bar{B}^*\rangle_{1^{+-}} &= -\frac{1}{\sqrt{2}}|1_H \otimes 0_L\rangle - \frac{1}{\sqrt{2}}|0_H \otimes 1_L\rangle \\
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\end{aligned}
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A.E. Bondar et al., PRD84(2011)054010

Direct potential between elastic channels

$$V_0 \equiv \langle 1_H \otimes 0_L|\hat{H}_\mathrm{I}|1_H \otimes 0_L\rangle, \quad V_1 \equiv \langle 0_H \otimes 1_L|\hat{H}_\mathrm{I}|0_H \otimes 1_L\rangle$$

with redefined parameters

$$\gamma_t{}^{-1} \equiv (2\pi)^2\mu V_0, \quad \gamma_s{}^{-1} \equiv (2\pi)^2\mu V_1$$

$$v = \frac{(2\pi)^2\mu}{2}\begin{pmatrix} \gamma_s{}^{-1} + \gamma_t{}^{-1} & \gamma_s{}^{-1} - \gamma_t{}^{-1} \\ \gamma_s{}^{-1} - \gamma_t{}^{-1} & \gamma_s{}^{-1} + \gamma_t{}^{-1} \end{pmatrix}$$

$\Rightarrow t^v \Rightarrow \Delta = \gamma_s\gamma_t - k_1k_2 + \frac{i}{2}(\gamma_s + \gamma_t)(k_1 + k_2)$

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\end{aligned}
$$

A.E. Bondar et al., PRD84(2011)054010

$\Rightarrow$ Total spin of $b\bar{b}$, $S_{b\bar{b}}^{\Upsilon} = 1$, $S_{b\bar{b}}^{h_b} = 0$

$\Rightarrow$Potential between elastic channels and inelastic channels

$$
v^{ei} = \begin{pmatrix} g_{1P} & g_{2P} & g_{1S} & g_{2S} & g_{3S} \\ g_{1P}\xi_{1P} & g_{2P}\xi_{2P} & g_{1S}\xi_{1S} & g_{2S}\xi_{2S} & g_{3S}\xi_{3S} \end{pmatrix}
$$

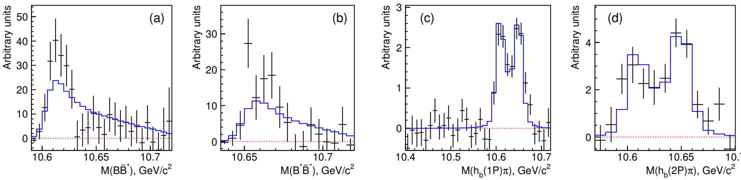
with $\xi_i \equiv g_{iB^*\bar{B}^*}/g_{iB\bar{B}^*}$. In HQSS, $\xi_{nS} = -1$ and $\xi_{mP} = 1$.

Apply to the two Z_b cases

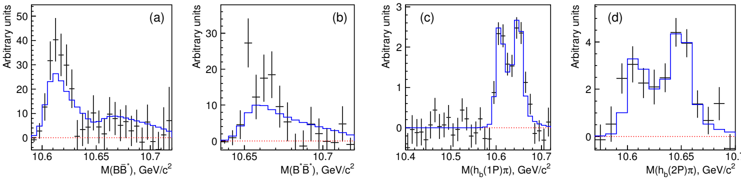
Hadronic molecular picture

HQSS limit $\Rightarrow \gamma_s \approx \gamma_t \Rightarrow$ light-quark spin symmetry (LQSS)

M.B. Voloshin, PRD93(2016)074011



HQSS breaking $\Rightarrow \gamma_s \neq \gamma_t \Rightarrow$ sizable LQSS breaking



Belle, PRD91(2015)072003, PRL108(2012)122001, PRL116(2016)212001

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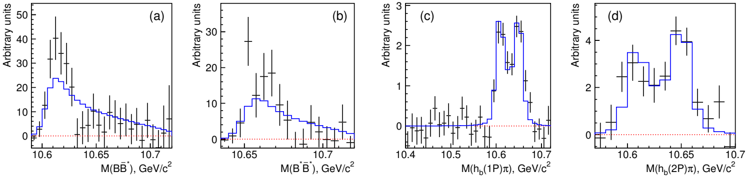
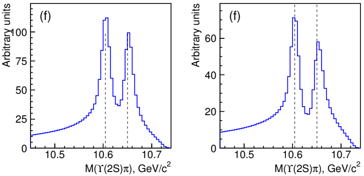
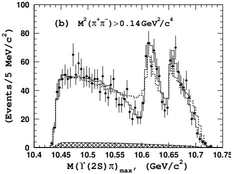


Can accommodate $\Upsilon(nS)\pi$ channels

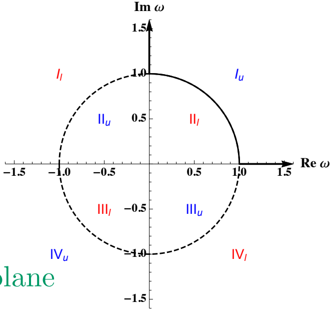




Belle, PRD91(2015)072003, PRL108(2012)122001, PRL116(2016)212001

Apply to the two Z_b cases

Nature of Z_b and Z_b' (2^7 RS to 2^2 RS)



RS-I:	Im $k_1 > 0$,	Im $k_2 > 0$,
RS-II:	Im $k_1 < 0$,	Im $k_2 > 0$,
RS-III:	Im $k_1 > 0$,	Im $k_2 < 0$,
RS-IV:	Im $k_1 < 0$,	Im $k_2 < 0$,

Conformal mapping from k-plane to ω-plane

$$k_1 = \sqrt{\frac{\mu\delta}{2}}\left(\omega + \frac{1}{\omega}\right), \quad k_2 = \sqrt{\frac{\mu\delta}{2}}\left(\omega - \frac{1}{\omega}\right).$$

Energy relative to the $B\bar{B}^*$ threshold

$$E = \frac{k_1^2}{2\mu} = \frac{k_2^2}{2\mu} + \delta = \frac{\delta}{4}\left(\omega^2 + \frac{1}{\omega^2} + 2\right)$$

with $\delta = m_{B^*} - m_B$.

Apply to the two Z_b cases

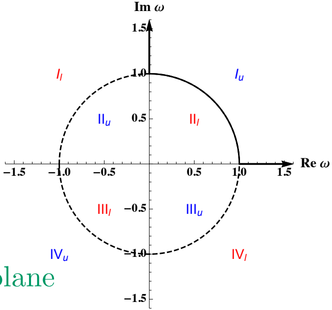
Nature of Z_b and Z_b' (2^7 RS to 2^2 RS)

RS-I: Im $k_1 > 0$, Im $k_2 > 0$,

RS-II: Im $k_1 < 0$, Im $k_2 > 0$,

RS-III: Im $k_1 > 0$, Im $k_2 < 0$,

RS-IV: Im $k_1 < 0$, Im $k_2 < 0$,



Conformal mapping from k-plane to ω-plane

$$k_1 = \sqrt{\frac{\mu\delta}{2}}\left(\omega + \frac{1}{\omega}\right), \quad k_2 = \sqrt{\frac{\mu\delta}{2}}\left(\omega - \frac{1}{\omega}\right).$$

Energy relative to the $B\bar{B}^*$ threshold

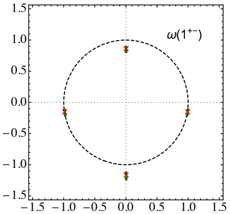
$$E = \frac{k_1^2}{2\mu} = \frac{k_2^2}{2\mu} + \delta = \frac{\delta}{4}\left(\omega^2 + \frac{1}{\omega^2} + 2\right)$$

with $\delta = m_{B^*} - m_B$.

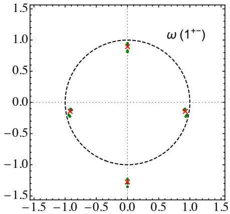
Apply to the two Z_b cases

Pole positions of Z_b and Z_b'

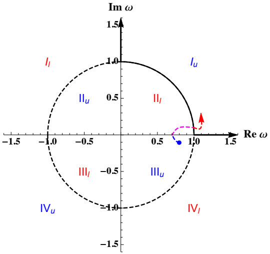
HQSS limit



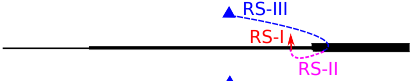
HQSS breaking



Path of the RS-III pole to RS-I

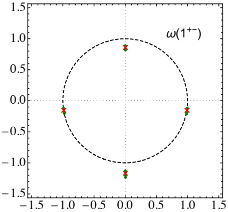
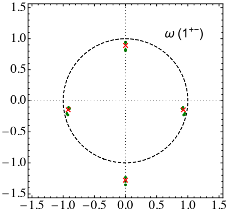


Energy plane



Apply to the two Z_b cases

Pole positions of Z_b and Z_b'

HQSS limit	HQSS breaking
	

Energies of Z_b and Z_b' (below the respective thresholds)

MeV	HQSS limit	HQSS breaking
$\varepsilon_B(Z_b)$	$1.10^{+0.79}_{-0.54} \pm i0.06^{+0.02}_{-0.02}$	$0.60^{+1.40}_{-0.49} \pm i0.02^{+0.02}_{-0.01}$
$\varepsilon_B(Z_b')$	$1.10^{+0.79}_{-0.53} \pm i0.08^{+0.03}_{-0.05}$	$0.97^{+1.42}_{-0.68} \pm i0.84^{+0.22}_{-0.34}$

Apply to the two Z_b cases

Z_b as a virtual state

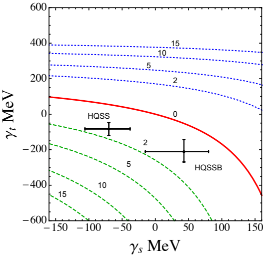
Determinant of t^v

$$\Delta = \gamma_s \gamma_t - k_1 k_2 + \frac{i}{2}(\gamma_s + \gamma_t)(k_1 + k_2)$$

Bound state vs. Virtual state

$$\xrightarrow[k_2=\sqrt{-2\mu\delta}]{k_1=0} \quad \gamma_t = \left(\gamma_s^{-1} - \sqrt{2/(\mu\delta)}\right)^{-1}$$

Parameter space γ_s-γ_t



⇒ Inelastic channels do not change the virtual state Z_b

⇒ Indicate the hadronic molecular nature of Z_b

⇒ A similar conclusion holds for Z_b'

Summary

- Relative large branching ratio in HQSF process indicates the intermediate genuine states
- Practical parametrization for the line shape of near-threshold states compatible with all requirements of unitarity and analyticity
- Can include bare poles and an arbitrary number of elastic and inelastic channels nonperturbatively
- A good description of Z_b and Z_b' as virtual state and resonance, respectively

Thank you very much for your attention!

BackUp

Apply to the two Z_b cases

LSE for a given momentum-independent direct interaction

$$t^v_{\alpha\beta} = v_{\alpha\beta} - \sum_\gamma v_{\alpha\gamma} J_\gamma t^v_{\gamma\beta},$$

with loop integral $J_\gamma = R_\gamma + iI_\gamma$. The real part R can be absorbed into the renormalization of the direct potential

$$(t^v)^{-1} = v^{-1} + (R + iI) = v^{-1}_{\text{ren}} + iI,$$

with $v_{\text{ren}} = Z^{-1}v$ and $Z = 1 + vR$. The t matrix is

$$t^v = \frac{1}{(2\pi)^2\mu}\frac{1}{\Delta}\begin{pmatrix} \frac{1}{2}(\gamma_s + \gamma_t) + ik_2 & \frac{1}{2}(\gamma_t - \gamma_s) \\ \frac{1}{2}(\gamma_t - \gamma_s) & \frac{1}{2}(\gamma_s + \gamma_t) + ik_1 \end{pmatrix},$$

with

$$\Delta = \gamma_s\gamma_t - k_1k_2 + \frac{i}{2}(\gamma_s + \gamma_t)(k_1 + k_2).$$

Apply to the two Z_b cases

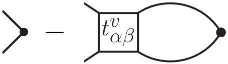
Switch on the $h_b(mP)\pi$ and $\Upsilon(nS)\pi$ channels

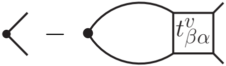
The t matrix is (separable interaction)

$$t = t^v + \psi[\mathcal{G} - G^{-1}]^{-1}\bar{\psi},$$

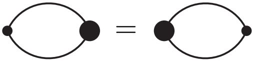
$\Rightarrow$ dressed incoming form factor $\psi_{\alpha\beta} = \delta_{\alpha\beta} - t^v_{\alpha\beta}J_\beta$

$\Rightarrow$ dressed outgoing form factor $\bar{\psi}_{\alpha\beta} = \delta_{\alpha\beta} - J_\alpha t^v_{\alpha\beta}$

$\psi_{\alpha\beta} =$ 

, $\bar{\psi}_{\alpha\beta} =$ 

$$\Rightarrow \mathcal{G}_{\alpha\beta} = J_\alpha\underbrace{(\delta_{\alpha\beta} - t^v_{\alpha\beta}J_\beta)}_{\psi_{\alpha\beta}} = \underbrace{(\delta_{\alpha\beta} - J_\alpha t^v_{\alpha\beta})}_{\bar{\psi}_{\alpha\beta}}J_\beta$$

$\mathcal{G}_{\alpha\beta} =$ 

Apply to the two Z_b cases

⇒ inelastic bubble loop reads as

$$
\begin{aligned}
G_{\alpha\beta} &= \sum_i \int \varphi_{i\alpha}(\boldsymbol{q}) S_i(\boldsymbol{q}) \varphi_{i\beta}(\boldsymbol{q}) d^3 q \\
&\to \frac{i(2\pi)^2}{\sqrt{s}} \sum_i m_{\mathrm{th}_i^{\mathrm{in}}} \mu_i^{\mathrm{in}} g_{i\alpha} g_{i\beta} (k_i^{\mathrm{in}})^{2l_i+1}, \\
&= \sum_i
\end{aligned}
$$

The production amplitudes

$$
\begin{aligned}
\mathcal{M}_\alpha^{\mathrm{e}}(\boldsymbol{p}) &= \mathcal{F}_\alpha(\boldsymbol{p}) - \sum_\beta \int \mathcal{F}_\beta(\boldsymbol{q}) S_\beta(\boldsymbol{q}) t_{\beta\alpha}(\boldsymbol{q}, \boldsymbol{p}) d^3 q, \\
\mathcal{M}_i^{\mathrm{in}}(\boldsymbol{k}) &= -\sum_\alpha \int \mathcal{F}_\alpha(\boldsymbol{q}) S_\alpha(\boldsymbol{q}) t_{\alpha i}(\boldsymbol{q}, \boldsymbol{k}) d^3 q
\end{aligned}
$$

⇒ Elastic bare production amplitude

⇒ Interaction between spectator and other particles is neglected

Decomposition of the P-wave charmonium

$$
\begin{aligned}
|B\bar{B}\rangle_{1^{--}} &= \frac{1}{2}|0_H \otimes 1_L\rangle + \frac{1}{2\sqrt{3}}|1_H \otimes 0_L\rangle - \frac{1}{2}|1_H \otimes 1_L\rangle + \frac{1}{2}\sqrt{\frac{5}{3}}|1_H \otimes 2_L\rangle \ , \\
|B\bar{B}^* + c.c.\rangle_{1^{--}} &= -\frac{1}{\sqrt{3}}|1_H \otimes 0_L\rangle + \frac{1}{2}|1_H \otimes 1_L\rangle + \frac{1}{2}\sqrt{\frac{5}{3}}|1_H \otimes 2_L\rangle \ , \\
|B^*\bar{B}^*\rangle^{s=0}_{1^{--}} &= \frac{1}{2}\sqrt{3}|0_H \otimes 1_L\rangle - \frac{1}{6}|1_H \otimes 0_L\rangle + \frac{1}{2\sqrt{3}}|1_H \otimes 1_L\rangle - \frac{\sqrt{5}}{6}|1_H \otimes 2_L\rangle \ , \\
|B^*\bar{B}^*\rangle^{s=2}_{1^{--}} &= \frac{\sqrt{5}}{3}|1_H \otimes 0_L\rangle + \frac{1}{2}\sqrt{\frac{5}{3}}|1_H \otimes 1_L\rangle + \frac{1}{6}|1_H \otimes 2_L\rangle \ .
\end{aligned}
$$