

Generating a resonance-like structure in the reaction $B_c \rightarrow B_s \pi\pi$

Xiao-Hai Liu

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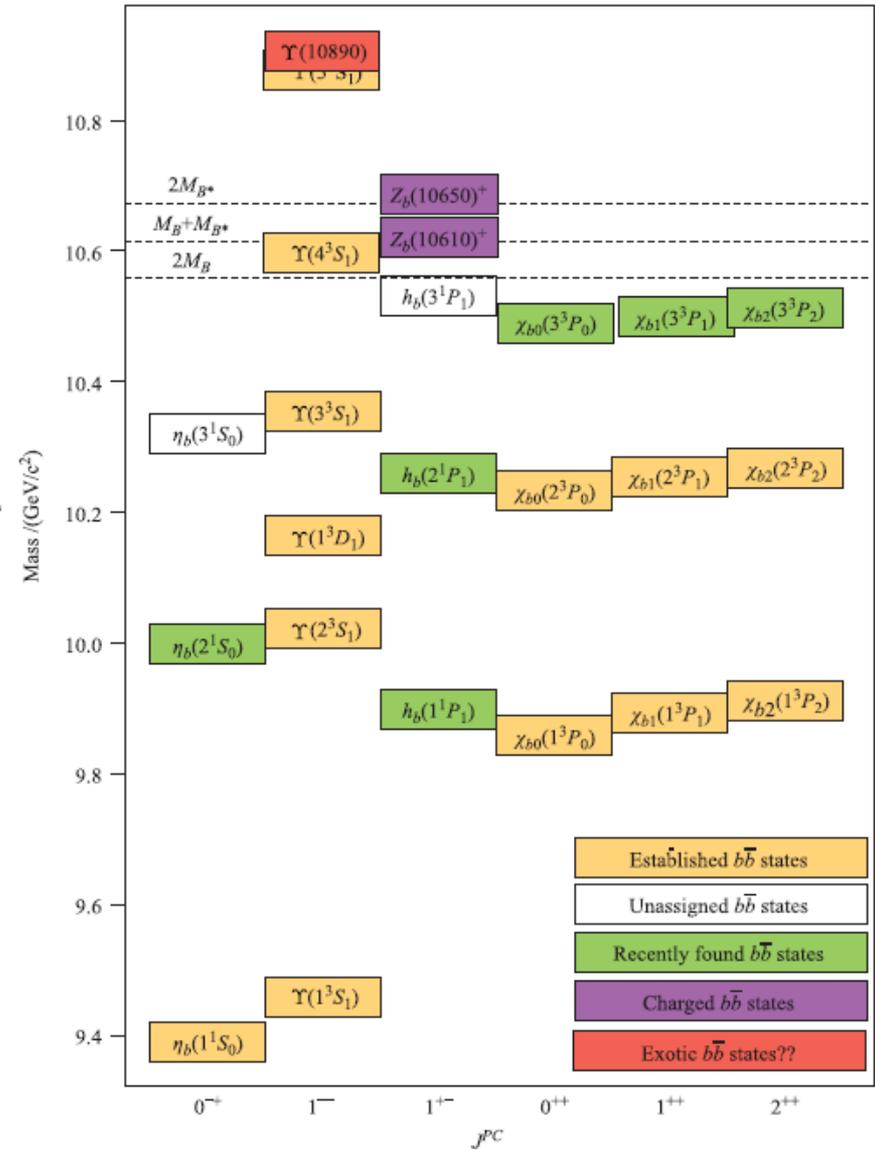
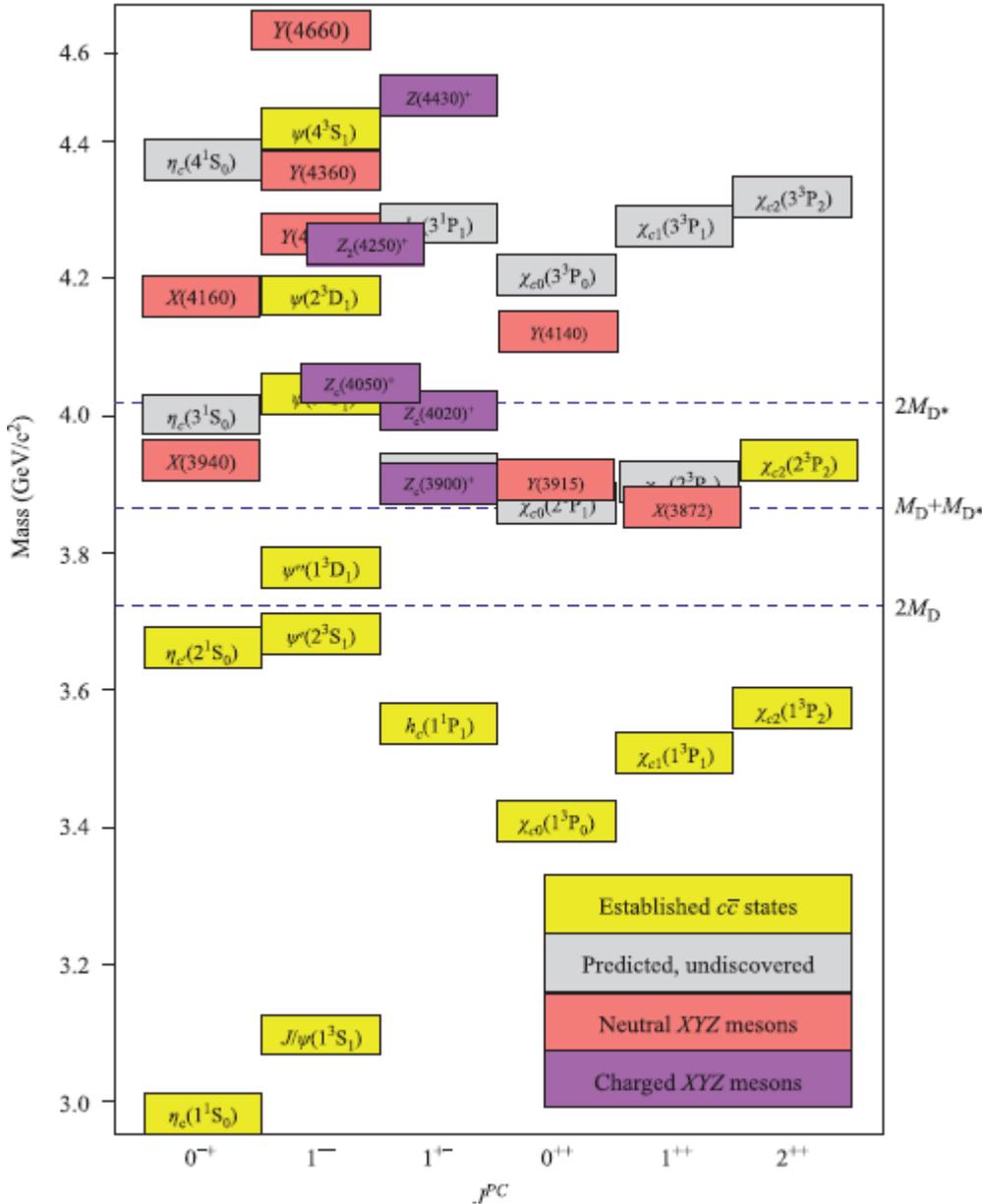
Based on: X.H. Liu & Ulf-G. Meissner, arXiv:1703.09043;

X.H. Liu, M. Oka, Q. Zhao, PLB753,297(2016).

Outline

- Brief review of exotic states
- Triangle singularity (TS) mechanism
(see also Fengkun's talk)
- Resonance-like structure “X(5777)” in $B_c \rightarrow B_s \pi\pi$
- Analysis of the background
- Summary

Unconventional states in heavy quarkonium region



Theoretical Interpretation

✓ Molecular States

✓ Tetraquark, Pentaquark

✓ Hybrid

✓ **Threshold effect (cusp, kinematic singularity)** (*Non-resonance interpretation*)

✓



Genuine resonance interpretations

Theoretical Methods

✓ Phenomenological models (quark model, potential model,

✓ Lattice QCD

✓ Effective Field Theory

✓

(see also Christoph's talk)

Triangle Singularity Mechanism

Early study in 1960s

➤ **Connections between kinematic singularities and resonance-like peaks: e.g. Peierls mechanism**

R.F.Peierls, PRL6,641(1961);

R.C.Hwa, PhysRev130,2580(1963);

C.Goebel,PRL13,143(1964);

P.Landshoff&S.Treiman Phys.Rev.127,649(1962);

.....

➤ **Some disadvantages:**

✓ **Few experiments to search for the effects;**

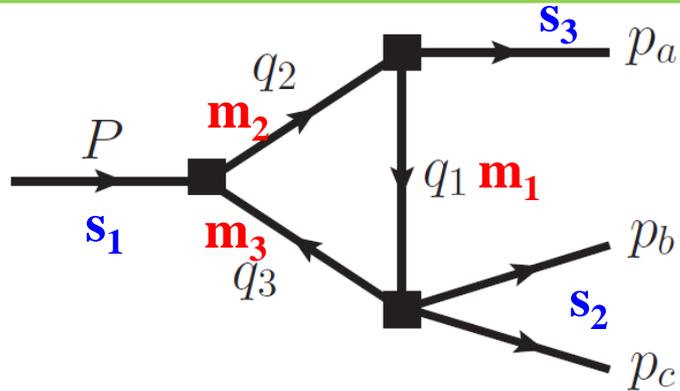
✓ **Low statistics;**

✓ **For the elastic scattering process, the effect of the triangle diagram is nothing more than a multiplication of the singularity from the tree diagram by a phase factor, according to the so-called Schmid theorem**

C.Schmid,Phys.Rev.154,1363(1967);

I. J. R. Aitchison & C. Kacser, Phys.Rev.173,1700(1968); A.V. Anisovich, PLB345,321(1995)

Triangle Singularity Mechanism



$$P^2 = s_1, (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

$$\Gamma_3(s_1, s_2, s_3) = \frac{-1}{16\pi^2} \int_0^1 \int_0^1 \int_0^1 da_1 da_2 da_3 \frac{\delta(1 - a_1 - a_2 - a_3)}{D - i\epsilon}$$

$$D = \sum_{i,j=1}^3 a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2} [m_i^2 + m_j^2 - (q_i - q_j)^2]$$

✓ Singularity in the complex space

Necessary conditions (Landau Equation)

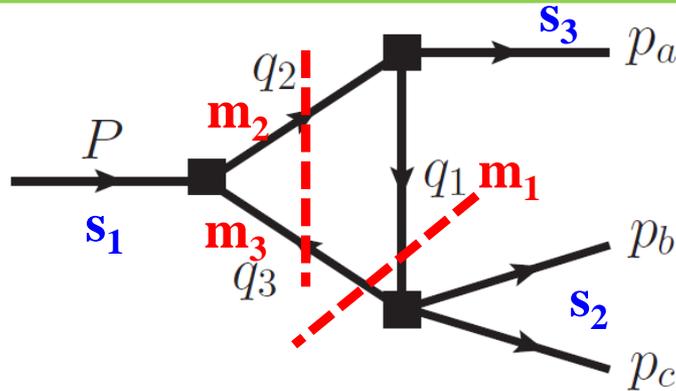
$$D = 0,$$

$$\text{either } a_j = 0 \text{ or } \frac{\partial D}{\partial a_j} = 0.$$

Leading singularity

Landau, Nucl.Phys.13,181(1959)

Triangle Singularity Mechanism



$$P^2 = s_1, (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

✓ Singularity in the complex space

The position of the singularity is obtained by solving

$$\det[Y_{ij}] = 0$$

Normal Threshold

$s_1, s_3, m_{1,2,3}$ fixed

$$s_2^\pm = (m_1 + m_3)^2 + \frac{1}{2m_2^2} [(m_1^2 + m_2^2 - s_3)(s_1 - m_2^2 - m_3^2) - 4m_2^2 m_1 m_3$$

$$\pm \lambda^{1/2}(s_1, m_2^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)], \quad \lambda(x, y, z) \equiv (x - y - z)^2 - 4yz$$

Anomalous Threshold

$$s_1^\pm = (m_2 + m_3)^2 + \frac{1}{2m_1^2} [(m_1^2 + m_2^2 - s_3)(s_2 - m_1^2 - m_3^2) - 4m_1^2 m_2 m_3$$

$$\pm \lambda^{1/2}(s_2, m_1^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)].$$

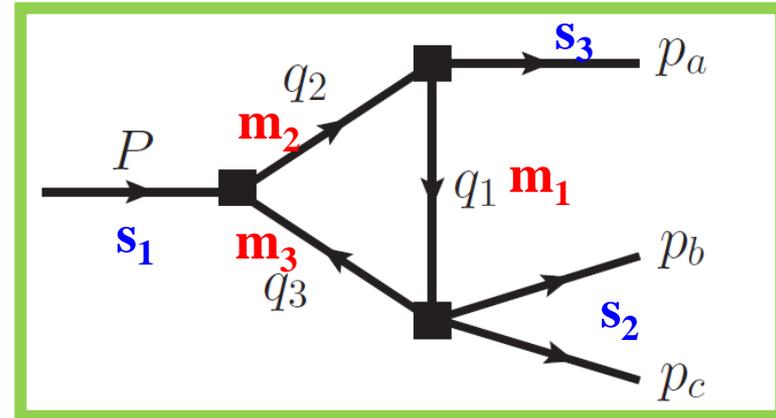
$s_2, s_3, m_{1,2,3}$ fixed

Triangle Singularity Mechanism

Single dispersion representation

$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi} \int_{(m_1+m_3)^2}^{\infty} \frac{ds'_2}{s'_2 - s_2 - i\epsilon} \sigma(s_1, s'_2, s_3)$$

$$\sigma(s_1, s_2, s_3) = \sigma_+ - \sigma_-$$



$$\sigma_{\pm}(s_1, s_2, s_3) = \frac{-1}{16\pi\lambda^{1/2}(s_1, s_2, s_3)} \log[-s_2(s_1 + s_3 - s_2 + m_1^2 + m_3^2 - 2m_2^2) - (s_1 - s_3)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_1, s_2, s_3)\lambda^{1/2}(s_2, m_1^2, m_3^2)].$$

Work in the kinematical region

$$s_1 \leq (m_2 + m_3)^2, \quad s_3 \leq (m_2 - m_1)^2 \quad 0 < s_2 < (m_1 + m_3)^2$$

By analytic continuation, it can be extended into the over threshold region **Fronsdal&Norton, J.Math.Phys.5,100(1964)**

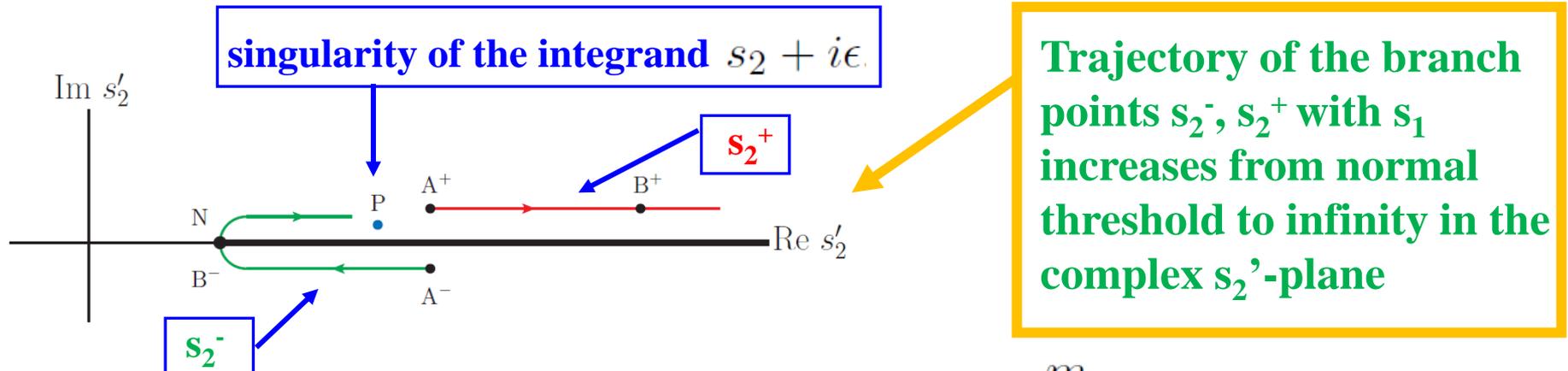
$$s_1 \geq (m_2 + m_3)^2, \quad (m_1 + m_3)^2 \leq s_2 \leq (\sqrt{s_1} - \sqrt{s_3})^2, \quad 0 \leq \sqrt{s_3} \leq m_2 - m_1$$

Branch points of the log function s_2^{\pm}

Triangle Singularity Mechanism

Locations of s_2^\pm in the s_2' -plane can be determined by

$$s_2^\pm(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^\pm}{\partial s_1}$$



Trajectory of the branch points s_2^- , s_2^+ with s_1 increases from normal threshold to infinity in the complex s_2' -plane

$$A^\pm : s_1 = (m_2 + m_3)^2, \quad B^\pm : s_1 = (m_2 + m_3)^2 + \frac{m_3}{m_1} [(m_2 - m_1)^2 - s_3] \quad =s_{1C}$$

$$A^- : s_2^- = (m_1 + m_3)^2 + \frac{m_3}{m_2} [(m_2 - m_1)^2 - s_3] - i\epsilon \quad =s_{2C}$$

$$B^- : s_2^- = (m_1 + m_3)^2 \quad =s_{2N}$$

s_2^- and P will pinch the integral contour

This pinch singularity is the triangle singularity (TS)

Triangle Singularity Mechanism

Locations of s_2^\pm in the s_2 '-plane can be determined by

$$s_2^\pm(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^\pm}{\partial s_1}$$

singularity of the integrand $s_2 + i\epsilon$

Trajectory of the branch

“The kinematic conditions for the existence of singularities on the physical boundary are equivalent to the condition that the relevant Feynman diagram be interpretable as a picture of an energy and momentum-conserving process occurring in space-time, with all internal particles real, on the mass shell and moving forward in time.”

Coleman&Norton, Nuovo Cimento 38,5018 (1965)

Fronsdal&Norton, J.Math.Phys.5, 100(1964)

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TS Kinematic Region

Normal threshold and critical point

$$s_{1N} = (m_2 + m_3)^2, \quad s_{1C} = (m_2 + m_3)^2 + \frac{m_3}{m_1} [(m_2 - m_1)^2 - s_3],$$

$$s_{2N} = (m_1 + m_3)^2, \quad s_{2C} = (m_1 + m_3)^2 + \frac{m_3}{m_2} [(m_2 - m_1)^2 - s_3],$$

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

When $s_2 = s_{2N}$

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2 + m_3)} [(m_2 - m_1)^2 - s_3],$$

When $s_1 = s_{1N}$

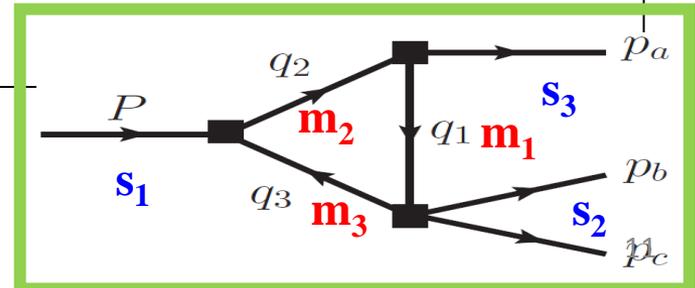
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1 + m_3)} [(m_2 - m_1)^2 - s_3].$$

Discrepancy between anomalous and normal threshold

Largest discrepancy

Enlarge

How to amplify the discrepancy between normal and anomalous threshold?



Liu, Oka, Zhao, PLB753,297 (2016)

arXiv:1507.01674

TS Kinematic Region

Normal threshold and critical point

$$s_{1N} = (m_2 + m_3)^2, \quad s_{1C} = (m_2 + m_3)^2 + \frac{m_3}{m_1} [(m_2 - m_1)^2 - s_3],$$

$$s_{2N} = (m_1 + m_3)^2, \quad s_{2C} = (m_1 + m_3)^2 + \frac{m_3}{m_2} [(m_2 - m_1)^2 - s_3],$$

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Discrepancy between anomalous and normal threshold

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2 + m_3)} [(m_2 - m_1)^2 - s_3],$$

$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1 + m_3)} [(m_2 - m_1)^2 - s_3].$$

Largest discrepancy

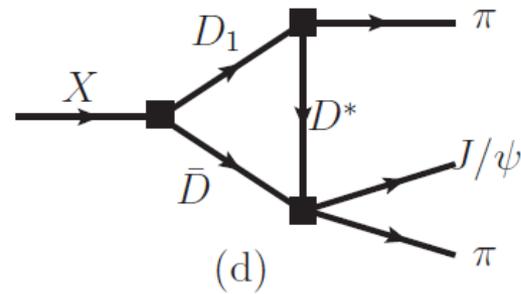
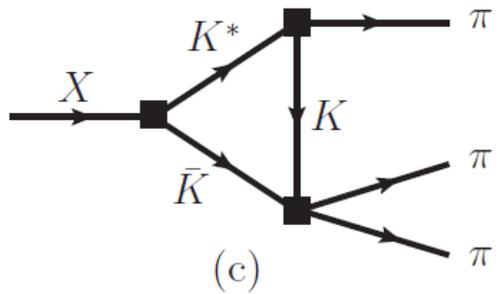
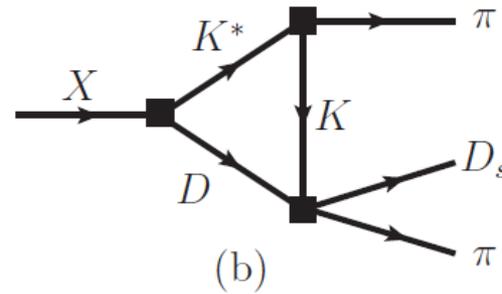
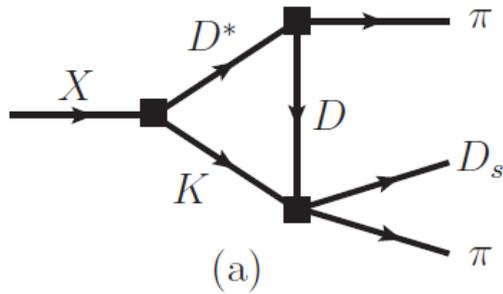
How to amplify the discrepancy between normal and anomalous threshold?



Liu, Oka, Zhao, PLB753,297 (2016)
arXiv:1507.01674

If the discrepancy is larger, it could be used to distinguish the kinematic singularities from genuine particles.

Possible rescatterings to detect TS



Wu, Liu, Zhao & Zou, PRL108,081803(2012)

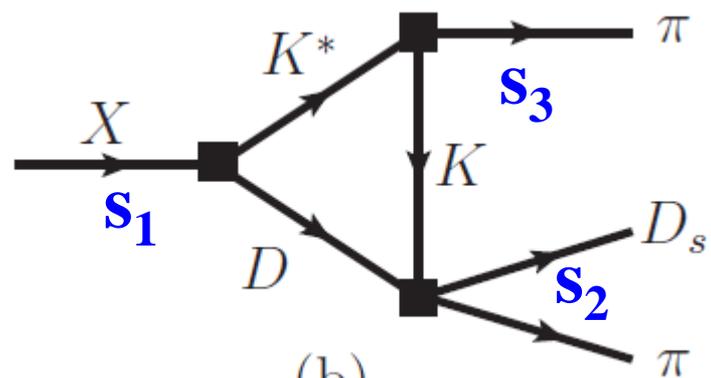
Wang, Hanhart, Zhao, PRL111,132003(2013)

Kinematic region of ATS

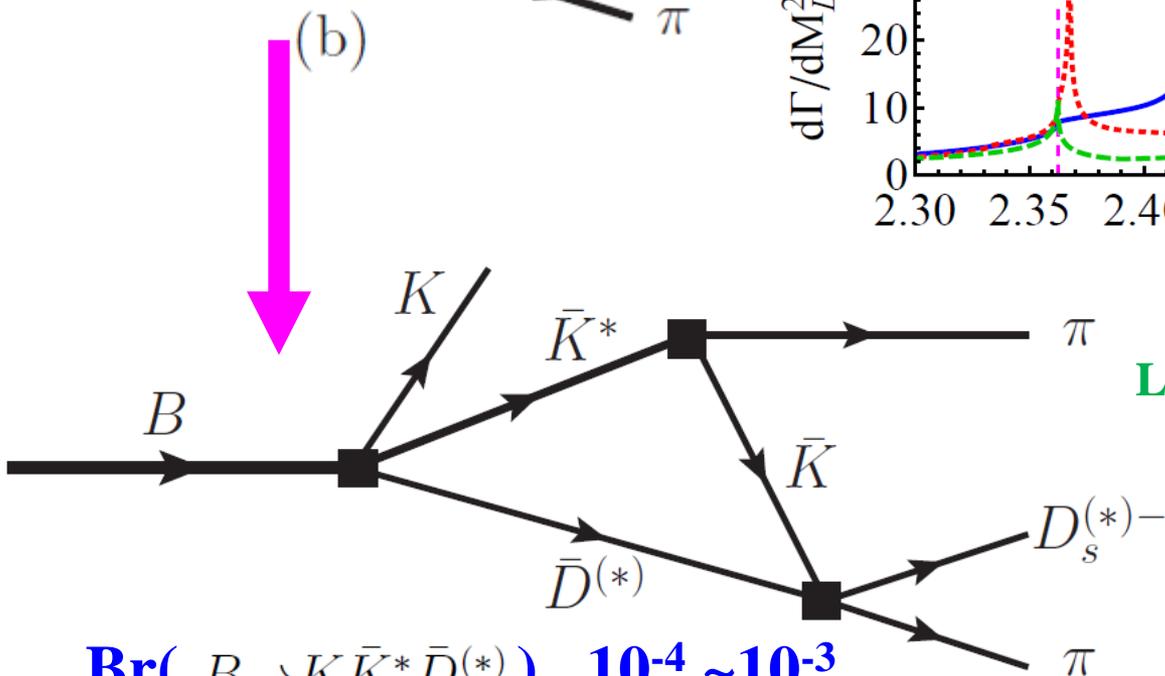
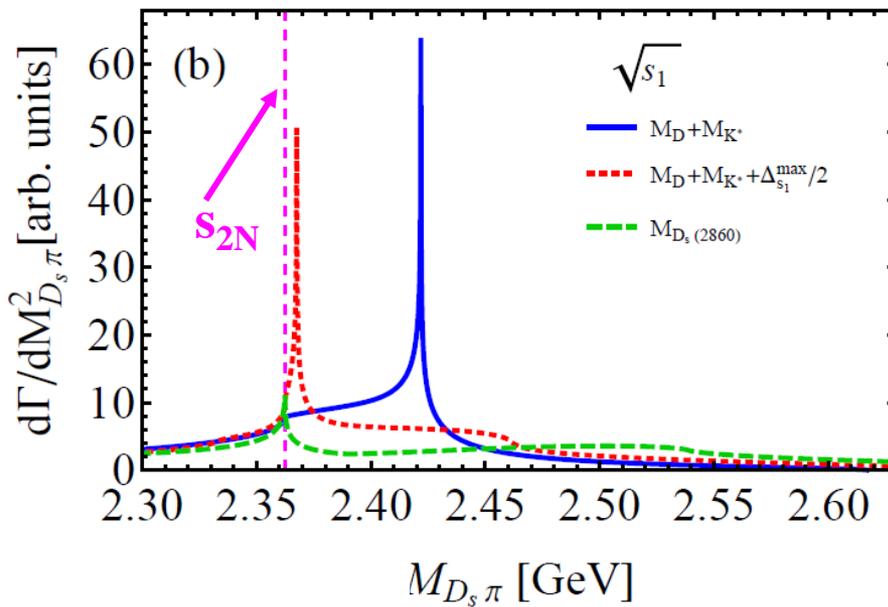
[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s_1}^{\max}$	0.089	96	49	16
$\Delta_{s_2}^{\max}$	0.087	62	38	15

Liu, Oka, Zhao, arXiv:1507.01674

Possible rescatterings to detect TS



Movement of TS peak



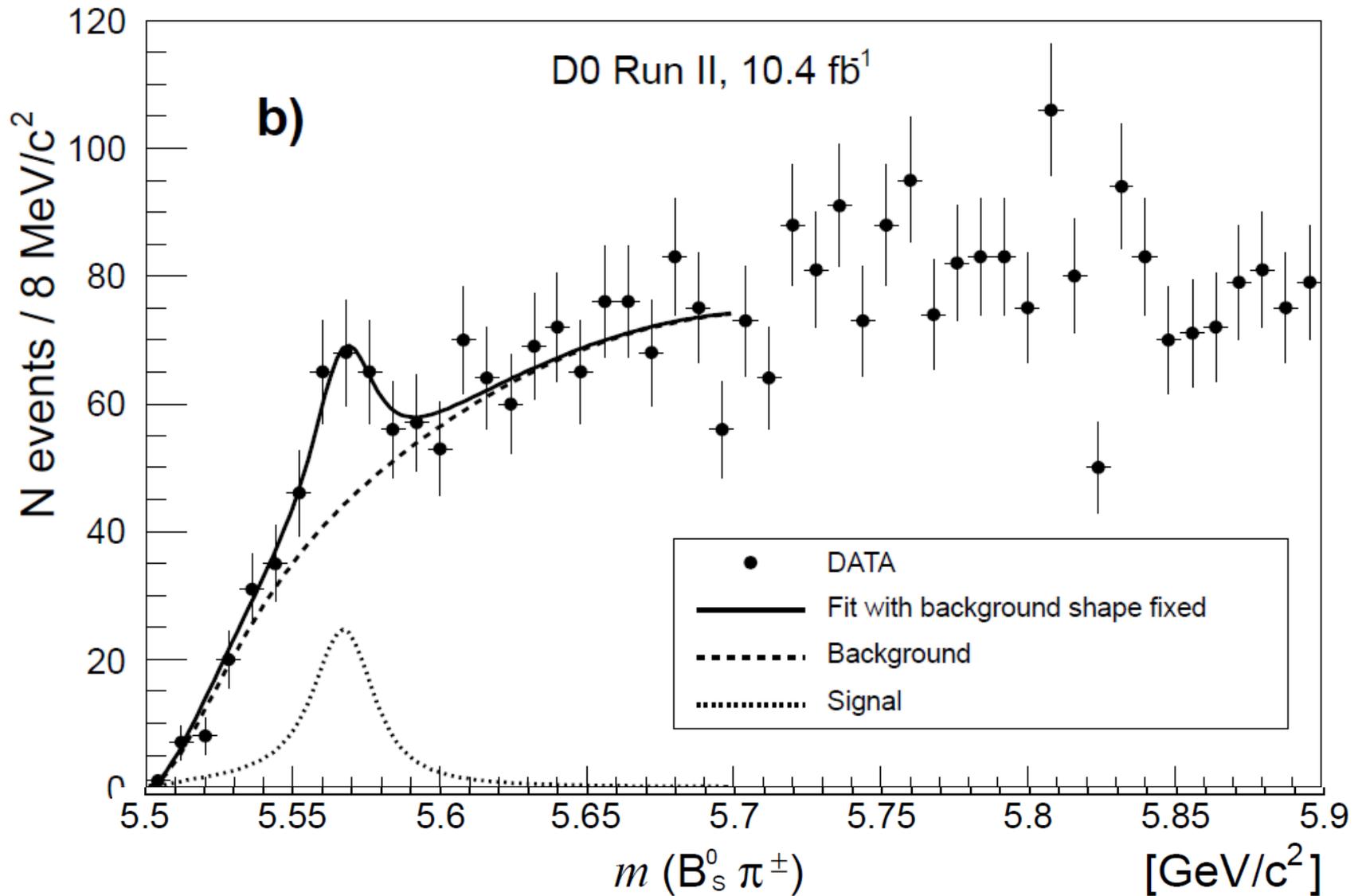
Liu, Oka, Zhao, arXiv:1507.01674

$$\text{Br}(B \rightarrow K \bar{K}^* \bar{D}^{(*)}) \quad 10^{-4} \sim 10^{-3}$$

$$\text{Br}(B \rightarrow K D_s^{(*)-} \pi \pi) \quad \sim 10^{-4}$$

[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s_1}^{\max}$	0.089	96	49	16
$\Delta_{s_2}^{\max}$	0.087	62	38	15

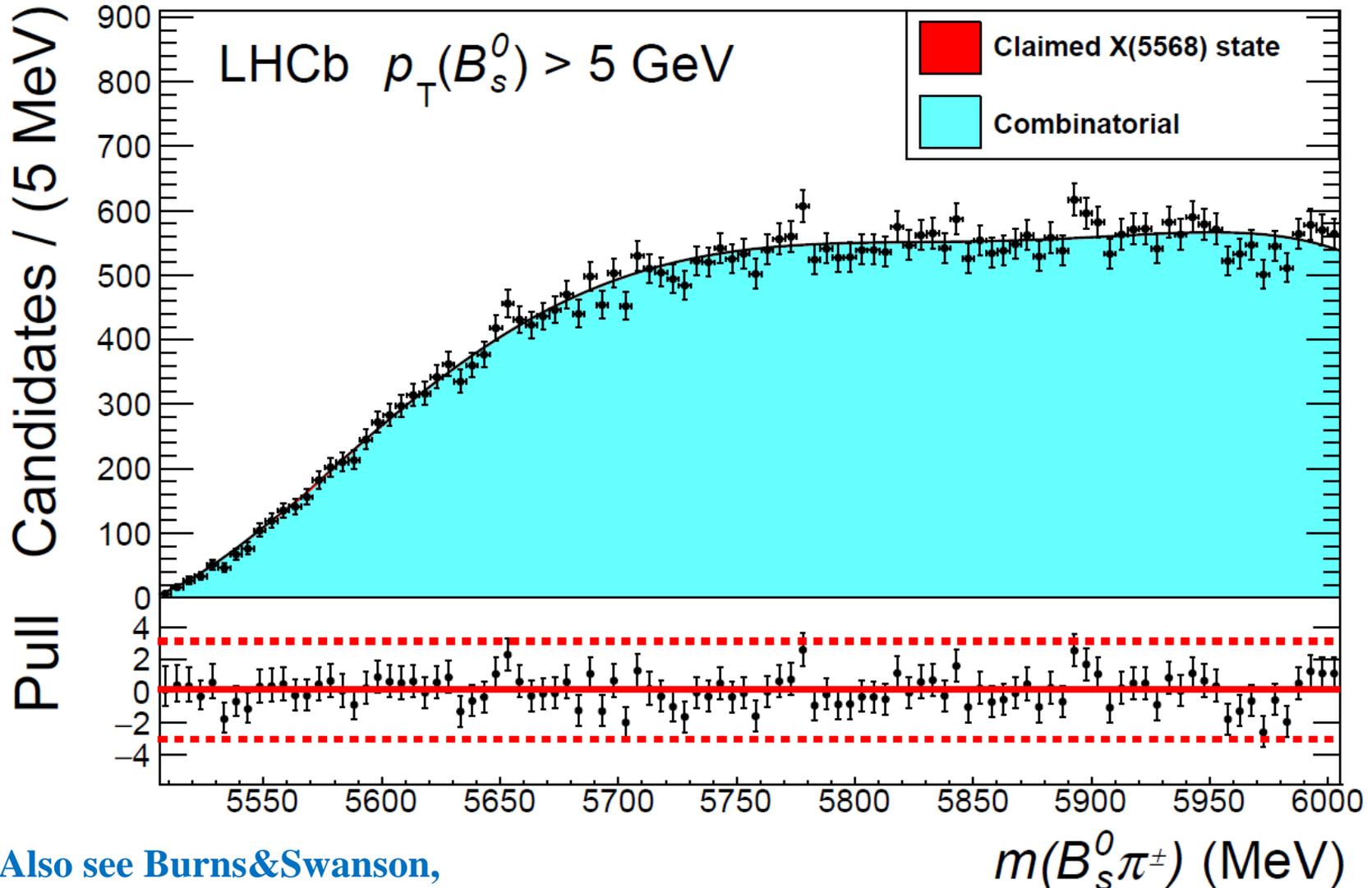
Observation of X(5568) at Fermilab



Significance 5.1σ

**D0 Collaboration, arXiv:1602.07588,
PRL 117, 022003 (2016)**

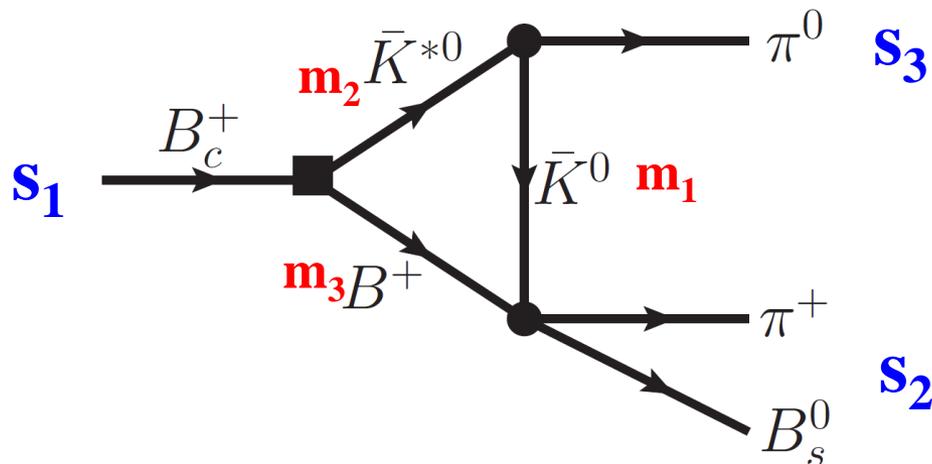
Non-confirmation of X(5568) tetraquark at LHCb



Also see Burns&Swanson,
PLB760,627; Guo et al, 1603.06316;
Yang et al, PLB767,470

LHCb, PRL117,152003
arXiv:1608.00435

Possible exotic structure “X(5777)” around BK -threshold



Particle	B _c	B _s	B	K*	K	π
Mass [GeV]	6.276	5.367	5.279	0.892	0.498	0.135

Kinematic region of triangle singularity (TS)

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2 + m_3)} [(m_2 - m_1)^2 - s_3], \quad \sim 119 \text{ MeV}$$

$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1 + m_3)} [(m_2 - m_1)^2 - s_3]. \quad \sim 71 \text{ MeV}$$

$$\sqrt{s_1}: 6.175 \text{ GeV} \sim 6.294 \text{ GeV}$$

$$\sqrt{s_2}: 5.848 \text{ GeV} \sim 5.777 \text{ GeV}$$

The mass of $B_c \sim 6.276 \text{ GeV}$ perfectly falls into the TS kinematic region!

B_c Decay Modes

Case	Coefficient	CKM factor	Branching ratio	Decay modes
1 a	a_1	$ V_{ud}V_{cs}^* \sim 1$	$\gtrsim 10^{-2}$	$B_s\pi, B_s\rho$
1 b	a_1	$ V_{ud}V_{cd}^* , V_{us}V_{cs}^* \sim \lambda$	$\gtrsim 10^{-3}$	$B_sK, B_d\pi, B_d\rho$
1 c	a_1	$ V_{us}V_{cd}^* \sim \lambda^2$	$\gtrsim 10^{-5}$	B_dK, B_dK^*
2 a	a_2	$ V_{ud}V_{cs}^* \sim 1$	$\gtrsim 10^{-3}$	$B_u^+\bar{K}^0, B_u^+\bar{K}^{*0}$
2 b	a_2	$ V_{ud}V_{cd}^* , V_{us}V_{cs}^* \sim \lambda$	$\gtrsim 10^{-4}$	$B_u\pi, B_u\rho, B_u\omega$
2 c	a_2	$ V_{us}V_{cd}^* \sim \lambda^2$	$\gtrsim 10^{-6}$	$B_u^+K^0, B_u^+K^{*0}$

In the factorization approach

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B^+\bar{K}^{*0}) &= \sqrt{2}G_F F_1^{B_c \rightarrow B_u} f_{K^*} m_{\bar{K}^*} \\ &\times (p_{B_c^+} \cdot \epsilon_{\bar{K}^*}^*) V_{ud} V_{cs}^* a_2, \end{aligned}$$

$$a_2 \sim -0.4$$

$$\begin{aligned} \mathcal{A}(B_c^+ \rightarrow B_s^0 \rho^+) &= \sqrt{2}G_F F_1^{B_c \rightarrow B_s} f_\rho m_\rho \\ &\times (p_{B_c^+} \cdot \epsilon_\rho^*) V_{ud} V_{cs}^* a_1, \end{aligned}$$

$$a_1 \sim 1.22$$

Sun *et al.*, arXiv:1504.01286

Form factor $F_1 \sim 1$

B_c Decay Modes

TABLE III: The CP -averaged branching ratios for the $B_c \rightarrow BP, BV$ decays.

decay mode	case	Ref. [12] ^a	Ref. [18] ^b	Ref. [20] ^c	Ref. [22] ^d	Ref. [23] ^e	Ref. [24] ^f	Ref. [25] ^g	Ref. [26] ^h	Ref. [27] ⁱ	Ref. [29] ^j	QCDF
$B_c \rightarrow B_s^0 \pi^+$	1-a	1.0×10^{-1}	6.8×10^{-2}	1.8×10^{-2}	2.9×10^{-2}	4.0×10^{-2}	4.3×10^{-2} (4.3×10^{-2})	1.3×10^{-1}	4.6×10^{-2}	1.9×10^{-1}	8.8×10^{-2}	$(1.13^{+0.00+0.11+0.01}_{-0.00-0.06-0.01}) \times 10^{-1}$
$B_c \rightarrow B_s^0 K^+$	1-b	7.6×10^{-3}	4.9×10^{-3}	2.0×10^{-3}	2.4×10^{-3}	3.3×10^{-3}	3.3×10^{-3} (3.3×10^{-3})	8.5×10^{-3}	3.4×10^{-3}	1.2×10^{-2}	5.2×10^{-3}	$(7.41^{+0.04+0.70+0.09}_{-0.04-0.39-0.09}) \times 10^{-3}$
$B_c \rightarrow B_s^0 \rho^+$	1-a	6.3×10^{-2}	5.2×10^{-2}	4.6×10^{-2}	1.6×10^{-2}	2.7×10^{-2}	3.0×10^{-2} (2.7×10^{-2})	1.1×10^{-1}	2.7×10^{-2}	8.4×10^{-2}	3.2×10^{-2}	$(4.44^{+0.00+0.41+0.13}_{-0.00-0.23-0.13}) \times 10^{-2}$
$B_c \rightarrow B_s^0 K^{*+}$	1-b	3.2×10^{-4}		1.2×10^{-3}	3.5×10^{-5}	1.5×10^{-4}	8.0×10^{-5} (7.1×10^{-5})	4.0×10^{-4}	1.3×10^{-4}		9.7×10^{-5}	$(1.25^{+0.01+0.12+0.06}_{-0.01-0.07-0.06}) \times 10^{-4}$
$B_c \rightarrow B_d^0 \pi^0$	1-b	7.4×10^{-3}	3.8×10^{-3}	1.2×10^{-3}	1.2×10^{-3}	1.3×10^{-3}	1.8×10^{-3} (1.5×10^{-3})	8.4×10^{-3}	2.4×10^{-3}	1.2×10^{-2}	6.9×10^{-3}	$(7.83^{+0.04+0.73+0.04}_{-0.04-0.41-0.04}) \times 10^{-3}$
$B_c \rightarrow B_d^0 K^+$	1-c		3.0×10^{-4}	1.2×10^{-4}	1.0×10^{-4}	1.1×10^{-4}	1.5×10^{-4} (1.2×10^{-4})	5.9×10^{-4}	1.8×10^{-4}	8.1×10^{-4}	4.4×10^{-4}	$(5.29^{+0.06+0.50+0.07}_{-0.06-0.28-0.07}) \times 10^{-4}$
$B_c \rightarrow B_d^0 \rho^+$	1-b	8.3×10^{-3}	6.9×10^{-3}	3.3×10^{-3}	1.5×10^{-3}	1.6×10^{-3}	2.2×10^{-3} (1.7×10^{-3})	1.4×10^{-2}	2.4×10^{-3}	1.1×10^{-2}	4.3×10^{-3}	$(5.32^{+0.03+0.49+0.15}_{-0.03-0.28-0.15}) \times 10^{-3}$
$B_c \rightarrow B_d^0 K^{*+}$	1-c		2.1×10^{-4}	1.5×10^{-4}	4.6×10^{-5}	4.4×10^{-5}	4.9×10^{-5} (3.7×10^{-5})	3.5×10^{-4}	5.7×10^{-5}	1.7×10^{-4}	8.3×10^{-5}	$(1.06^{+0.01+0.10+0.05}_{-0.01-0.06-0.05}) \times 10^{-4}$
$B_c \rightarrow B_u^+ \bar{K}^0$	2-a	2.1×10^{-2}	1.2×10^{-2}	4.9×10^{-3}	4.2×10^{-3}	4.4×10^{-3}	6.0×10^{-3} (4.9×10^{-3})	2.3×10^{-2}	6.8×10^{-3}	3.6×10^{-2}	2.2×10^{-3}	$(1.97^{+0.00+1.11+0.05}_{-0.00-0.54-0.05}) \times 10^{-2}$
$B_c \rightarrow B_u^+ K^0$	2-c						1.6×10^{-5} (1.3×10^{-5})				6.3×10^{-6}	$(5.71^{+0.06+3.20+0.15}_{-0.06-1.58-0.14}) \times 10^{-5}$
$B_c \rightarrow B_u^+ \bar{K}^{*0}$	2-a	7.8×10^{-3}	8.5×10^{-3}	5.8×10^{-3}	1.6×10^{-3}	1.6×10^{-3}	1.9×10^{-3} (1.4×10^{-3})	1.3×10^{-2}	2.1×10^{-3}	8.0×10^{-3}	2.0×10^{-4}	$(3.72^{+0.00+2.09+0.21}_{-0.00-1.03-0.20}) \times 10^{-3}$
$B_c \rightarrow B_u^+ K^{*0}$	2-c						5.0×10^{-6} (3.7×10^{-6})				5.6×10^{-7}	$(1.07^{+0.01+0.60+0.06}_{-0.01-0.30-0.06}) \times 10^{-5}$
$B_c \rightarrow B_u^+ \pi^0$	2-b	4.0×10^{-4}	2.1×10^{-4}	6.4×10^{-5}	6.2×10^{-5}	6.7×10^{-5}	9.7×10^{-5} (8.1×10^{-5})	4.5×10^{-4}	1.3×10^{-4}	6.6×10^{-4}	5.2×10^{-5}	$(4.23^{+0.02+2.37+0.07}_{-0.02-1.17-0.07}) \times 10^{-4}$
$B_c \rightarrow B_u^+ \rho^0$	2-b	4.4×10^{-4}	3.7×10^{-4}	1.7×10^{-4}	8.7×10^{-5}	8.9×10^{-5}	1.2×10^{-4} (9.4×10^{-5})	7.4×10^{-4}	1.3×10^{-4}	5.5×10^{-4}	1.8×10^{-5}	$(2.86^{+0.01+1.60+0.10}_{-0.01-0.79-0.10}) \times 10^{-4}$
$B_c \rightarrow B_u^+ \omega$	2-b	4.1×10^{-4}					9.0×10^{-5} (7.0×10^{-5})				1.3×10^{-5}	$(2.05^{+0.01+1.15+0.12}_{-0.01-0.57-0.12}) \times 10^{-4}$
$B_c \rightarrow B_u^+ \eta$							5.0×10^{-4} (4.1×10^{-4})				1.4×10^{-4}	$(1.46^{+0.01+0.82+0.13}_{-0.01-0.40-0.12}) \times 10^{-3}$
$B_c \rightarrow B_u^+ \eta'$							6.7×10^{-6} (5.6×10^{-6})				4.2×10^{-6}	$(7.28^{+0.04+4.09+1.66}_{-0.04-2.01-1.47}) \times 10^{-5}$

DK interaction

Scattering length a_0 [fm]	LQCD [1]	LQCD [2]	
$l=0$	-0.86 ± 0.03	$-1.33(20)$	strong
$l=1$	$0.07 \pm 0.03 + 0.17 i$		weak

[1] L. Liu et al., PRD87,014508 (2013)

[2] D. Mohler et al., PRL111,222001(2013)

See also Chen et al., 1609.08928 for a review

$D_{s0}(2317)/D_{s1}(2460)$, isoscalar hadronic molecule of DK/D*K

Heavy quark symmetry

$B\bar{K}$ interaction

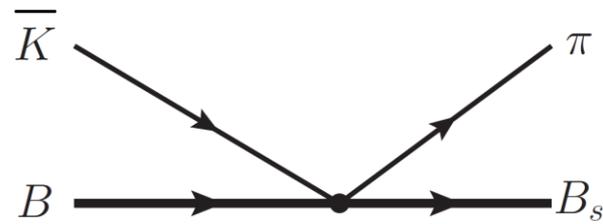
Scattering length a_0 ($l=1$) $\sim 0.02 + 0.23 i$ [fm]

Geng et al.,
PRD89,014026(2014); EPJC77,94(2017)

B_{s0}/B_{s1} , isoscalar hadronic molecule of BK/B*K, not observed yet

The relative weak interaction ($I=1$) does not support the existence of an isovector hadronic molecule

$B\bar{K} - B_s\pi$ interaction



Unitarized amplitude

Oller&UGM, PLB500,263(2001);Oller,Oset&Ramos, PPNP45,157(2000)

$$T = (1 - VG)^{-1}V,$$

Leading order interaction

$$\mathcal{L}_{D\phi}^{(1)} = \mathcal{D}_\mu D D^\mu D^\dagger - M_0^2 D D^\dagger$$

Existence of a BK ($I=1$)molecular state is rather questionable

Non-zero S-wave potential

$$V_{12}(s) = \frac{1}{8f^2} \left(3s - (M_1^2 + M_2^2 + m_1^2 + m_2^2) - \frac{\Delta_1 \Delta_2}{s} \right)$$

Whether there is dynamically generated resonance or not is largely depend on the cutoff

Oset *et al.*, arXiv:1603.09230

cutoff~ 2.8 GeV, much larger than the normal cutoff~1 GeV

L.S. Geng *et al.*, arXiv:1607.06326

$B\bar{K} - B_s\pi$ interaction

NLO potential adopted in the covariant formalism of UChPT

Geng et al.,

PRD89,014026(2014);EPJC77,94(2017)

$$\begin{aligned} & \mathcal{V}_{\text{NLO}}(P(p_1)\phi(p_2) \rightarrow P(p_3)\phi(p_4)) \\ &= -\frac{8}{f_0^2}C_{24} \left(c_2 p_2 \cdot p_4 - \frac{c_4}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 + p_1 \cdot p_2 p_3 \cdot p_4) \right) \\ & \quad - \frac{4}{f_0^2}C_{35} \left(c_3 p_2 \cdot p_4 - \frac{c_5}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 + p_1 \cdot p_2 p_3 \cdot p_4) \right) \\ & \quad - \frac{4}{f_0^2}C_6 \frac{c_6}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 - p_1 \cdot p_2 p_3 \cdot p_4) \\ & \quad - \frac{8}{f_0^2}C_0 c_0 + \frac{4}{f_0^2}C_1 c_1, \end{aligned}$$

No dynamically generated poles are found on the Riemann sheets

LECs are determined by fitting the Lattice data of

L. Liu et al., PRD87,014508 (2013)

$$V^{S\text{-wave}} = \frac{1}{2} \int_{-1}^1 V(\cos \theta) d \cos \theta$$

Contributions of higher partial waves will be highly suppressed for the near-threshold scattering.

Invariant mass distribution of $B_s\pi$

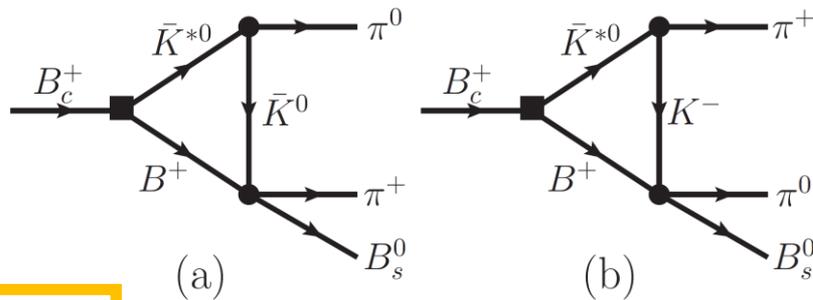
Rescattering amplitude via the triangle diagram

$$\begin{aligned} \mathcal{A}_{B_c^+ \rightarrow B_s^0 \pi^+ \pi^0}^{[\bar{K}^{*0} B^+ \bar{K}^0]} &= \frac{1}{i} \int \frac{d^4 q_3}{(2\pi)^4} \frac{\mathcal{A}(B_c^+ \rightarrow B^+ \bar{K}^{*0})}{(q_1^2 - m_{\bar{K}^*}^2 + im_{\bar{K}^*} \Gamma_{\bar{K}^*})} \\ &\times \frac{\mathcal{A}(\bar{K}^{*0} \rightarrow \bar{K}^0 \pi^0) \mathcal{A}(B^+ \bar{K}^0 \rightarrow B_s^0 \pi^+)}{(q_2^2 - m_{B^+}^2)(q_3^2 - m_{\bar{K}^0}^2)} \mathbb{F}(q_3^2), \quad (5) \end{aligned}$$

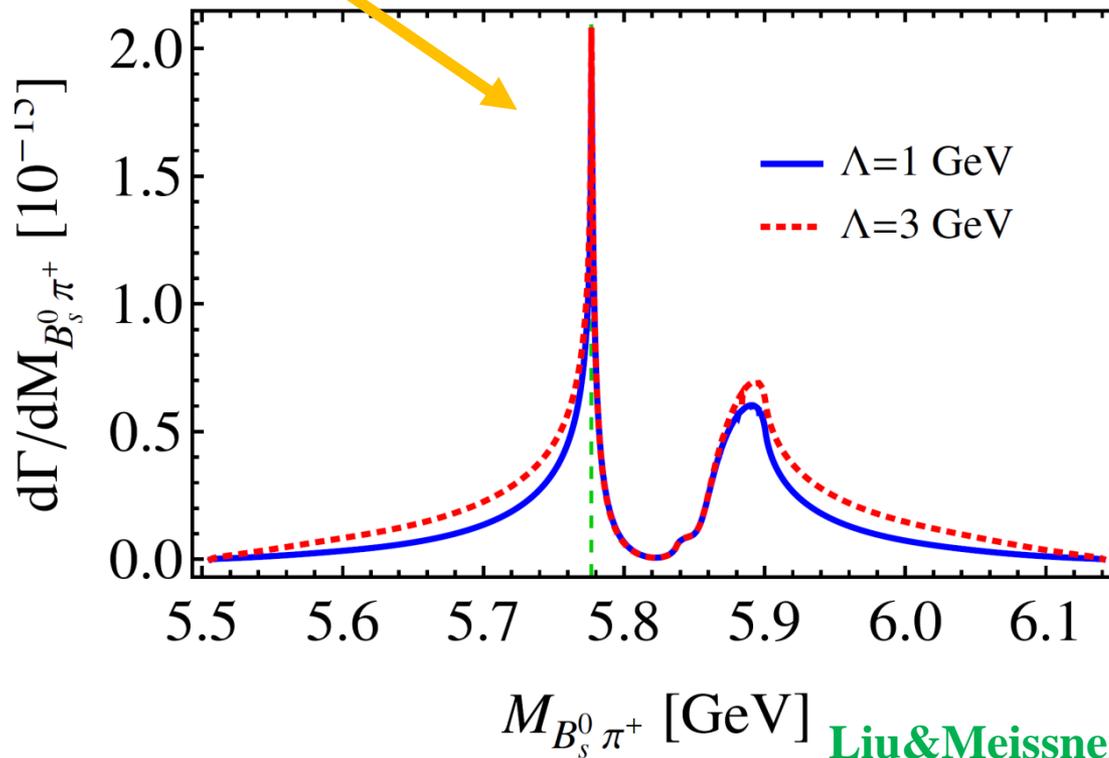
$$\mathbb{F}(q_3^2) = (m_{\bar{K}^0}^2 - \Lambda^2)/(q_3^2 - \Lambda^2)$$

Breit-Wigner type propagator will remove the TS from the physical boundary by a small distance, but the physical amplitude can still feel the influence of TS. I.J.R. Aitchison & C. Kacser, PR133, B1239 (1964)

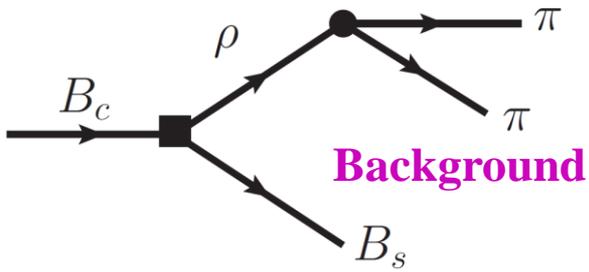
Invariant mass distribution of $B_s\pi$



X(5777)



Background Analysis



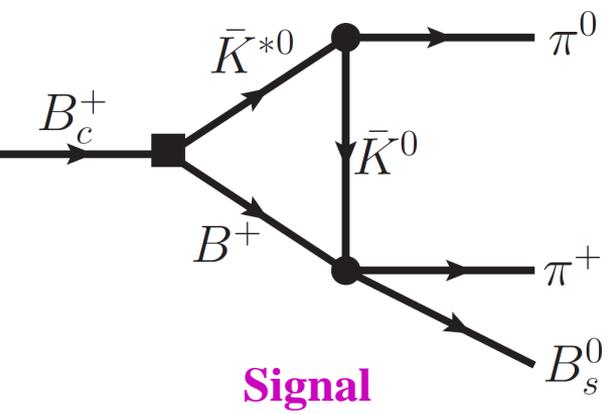
$$\frac{Br(B_c \rightarrow B\bar{K}^*)}{Br(B_c \rightarrow B_s\rho)} \sim \frac{1}{10}$$

$$\mathcal{A}(B_c^+ \rightarrow B_s^0 \pi^+ \pi^0) = e^{i\theta} \mathcal{A}_\rho^{\text{tree}} + \mathcal{A}^{\text{loop}} \mathcal{F}(s_{\pi\pi})$$

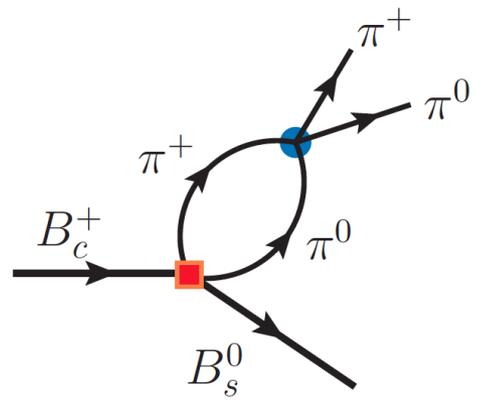
Relative phase

Account for $\pi\pi$ FSI

+

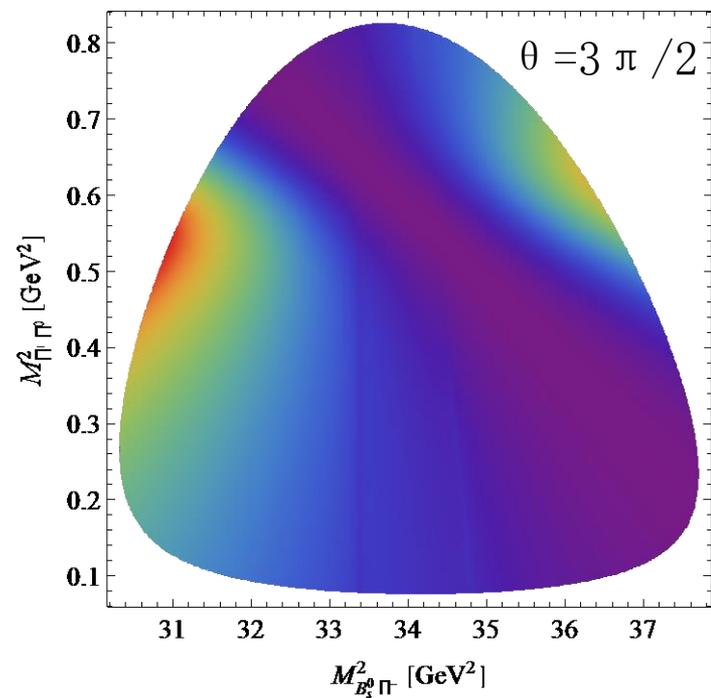
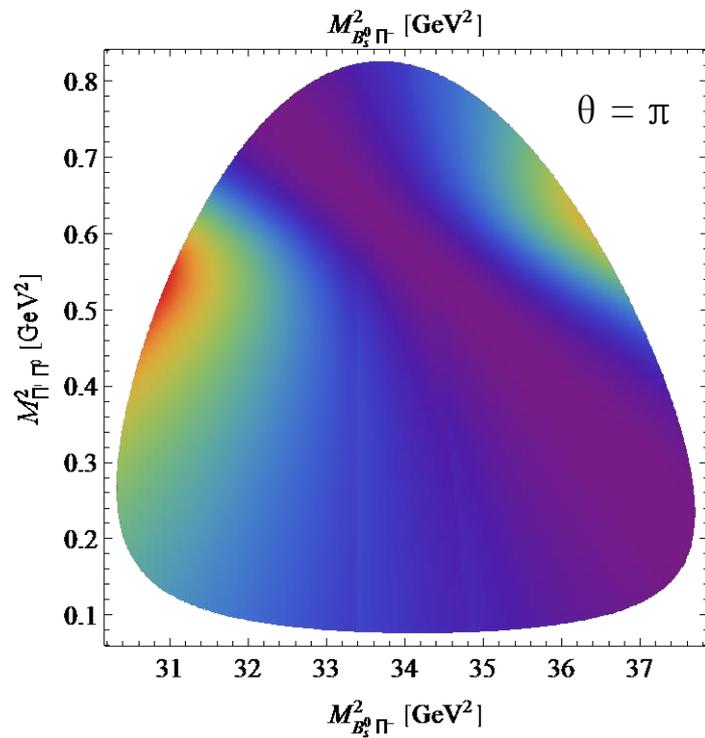
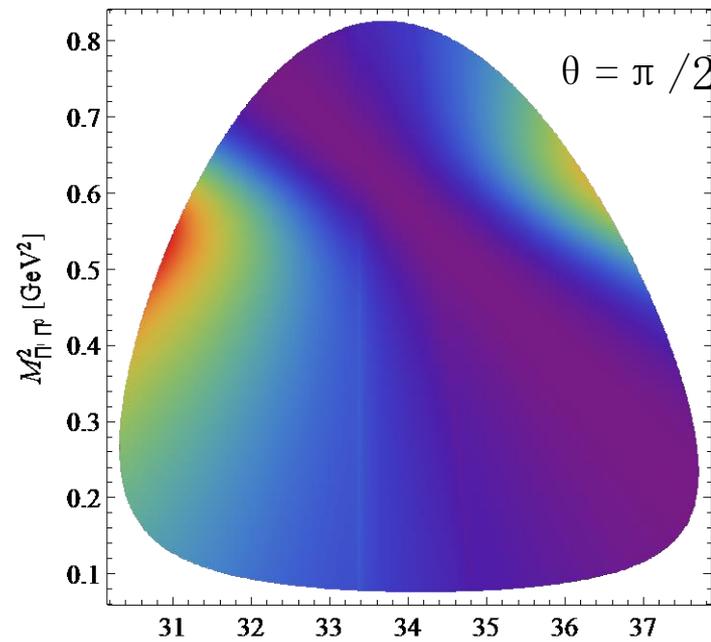
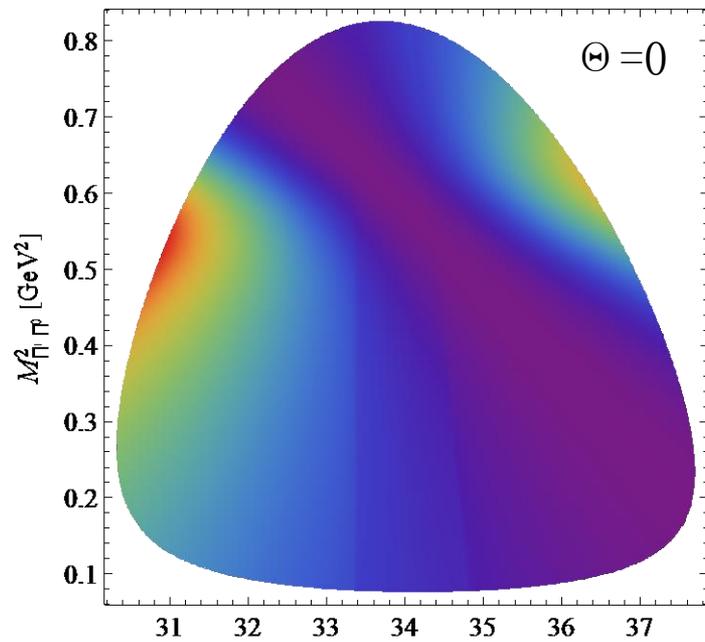


VMD model

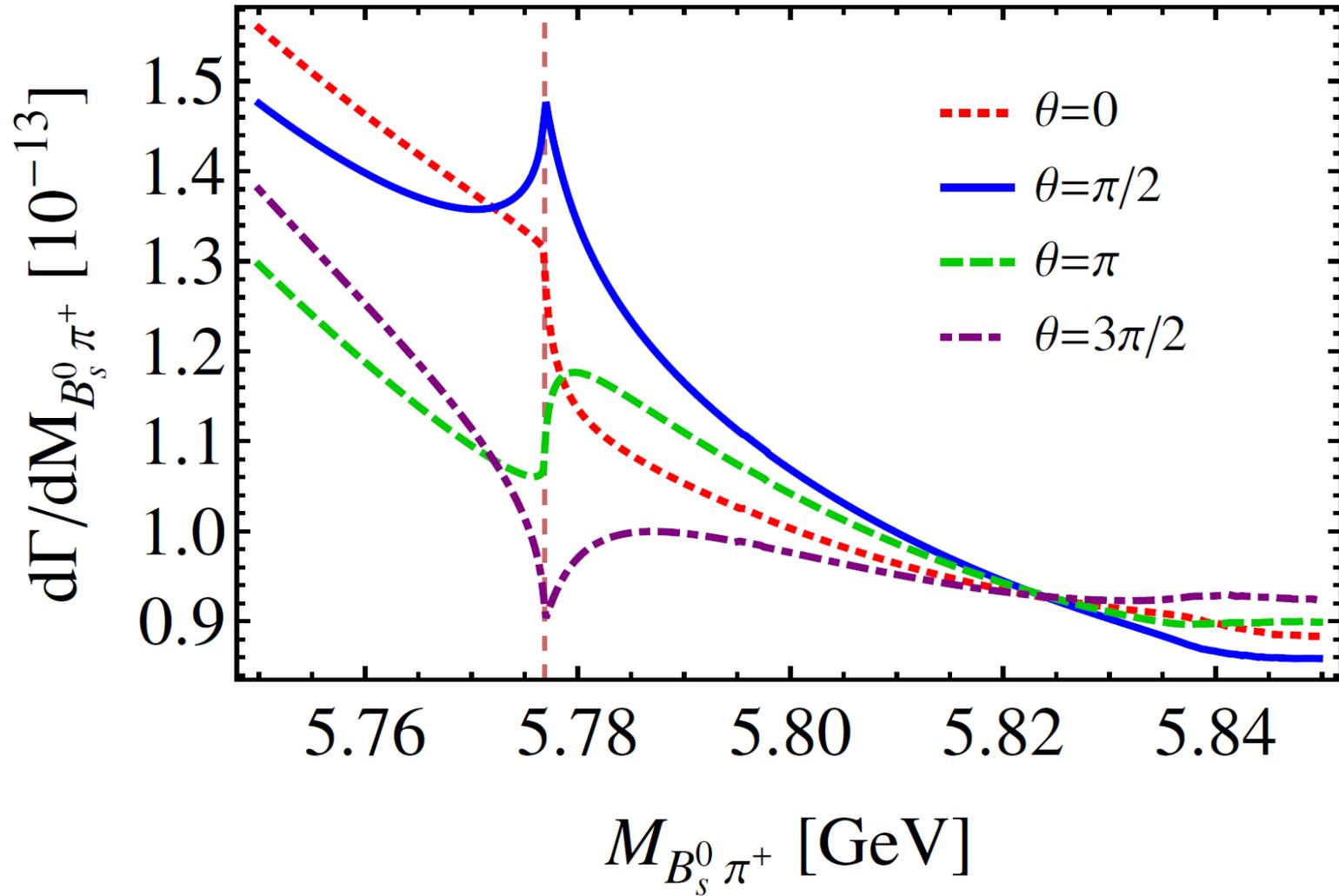


$$\mathcal{F}(s_{\pi\pi}) \simeq \frac{s_{\pi\pi} - \overset{\circ}{m}_\rho^2}{s_{\pi\pi} - m_\rho^2 + im_\rho\Gamma_\rho} \quad \overset{\circ}{m}_\rho \simeq 0.81\text{GeV}$$

Klingl et al., ZPA356,193



Background Analysis



Summary

We investigated the possibility of searching for a resonance-like structure $X(5777)$ in $Bc \rightarrow Bs \pi\pi$, which may help us to establish a non-resonance interpretation for some XYZ particles, i.e., the TS mechanism.

➤ Advantages:

- ✓ Kinematic conditions of TS are perfectly fulfilled;
- ✓ The relative weak $B\bar{c}(I=1)$ interaction does not support the existence of an isovector hadronic molecule;
- ✓ The relevant couplings are under good theoretical control;
- ✓ The branching ratio of $Bc \rightarrow B K^*$ is relatively larger;
- ✓ The background is expected to be simple.

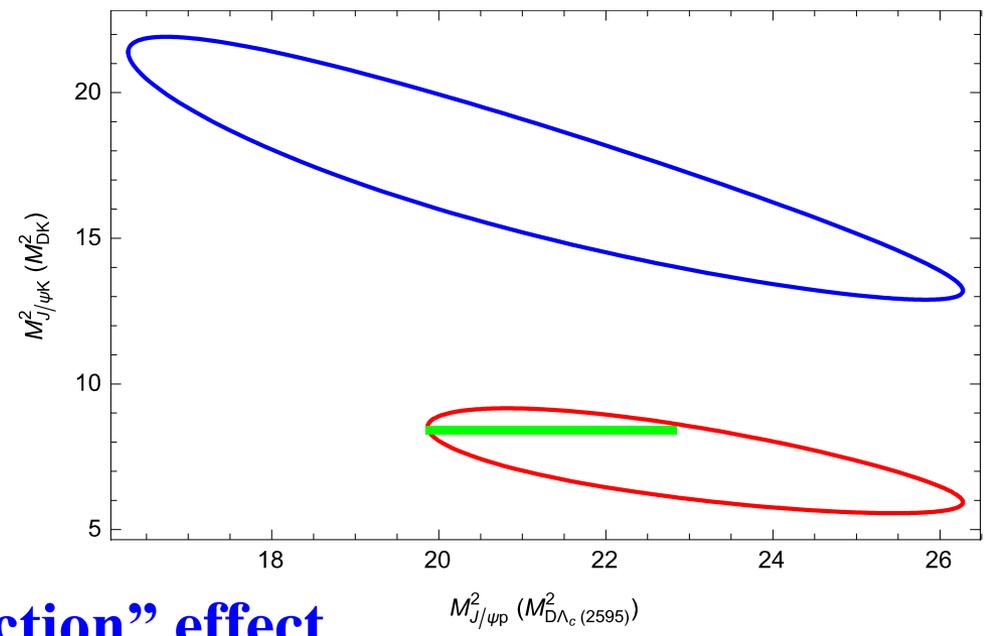
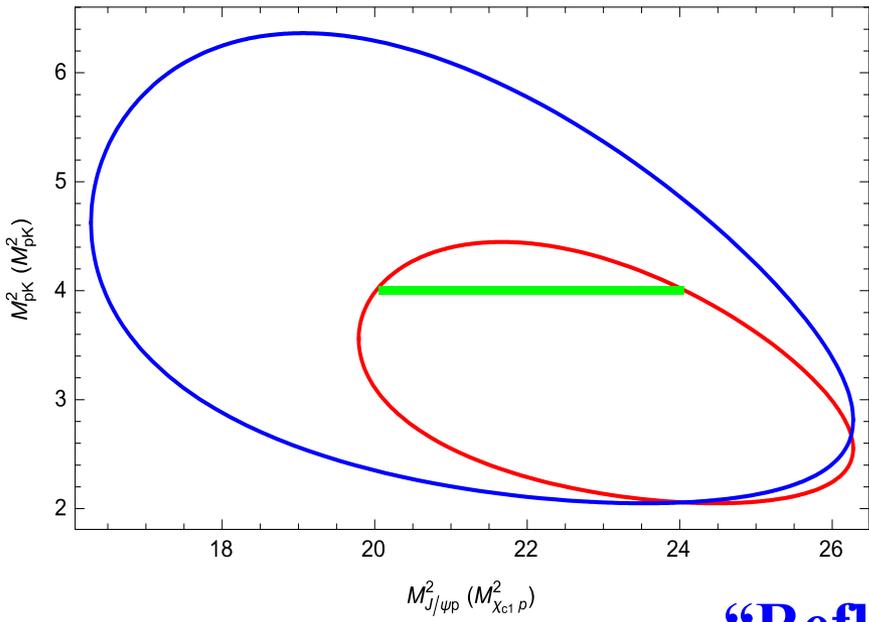
➤ Disadvantages:

- ✓ For LHCb, it is difficult to detect the neutral pion in the final states

Thanks!

Backup

TS mechanism: Reflection in Dalitz plot



“Reflection” effect

