



# Deformation and threshold effects in halo nuclei

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*Supported by:*

NSFC & MOST;

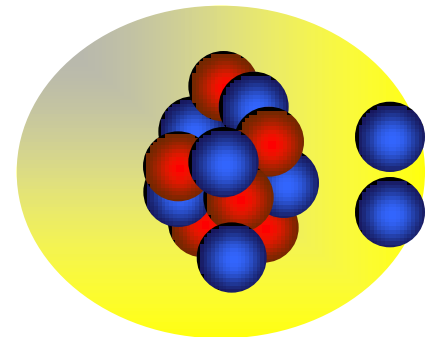
HPC Cluster of SKLTP/ITP-CAS

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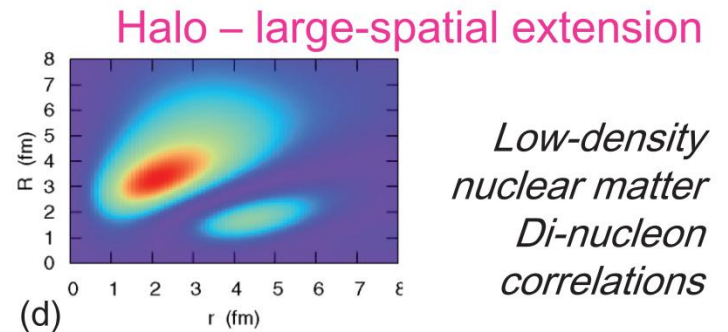
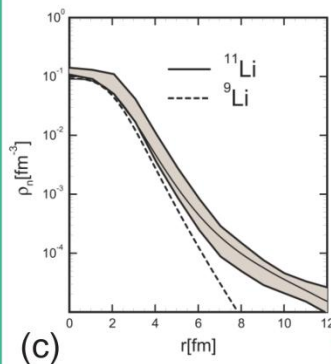
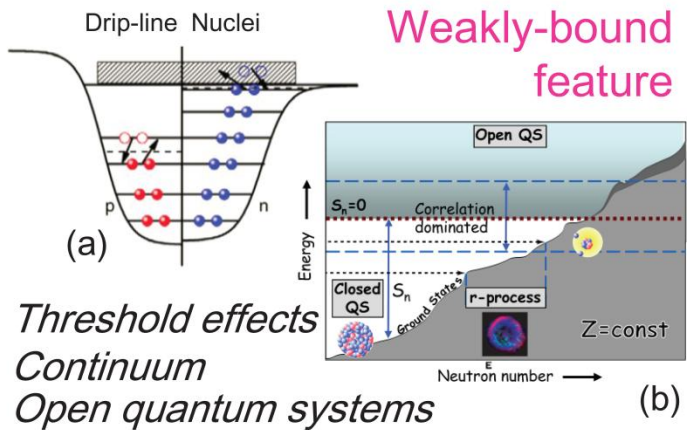
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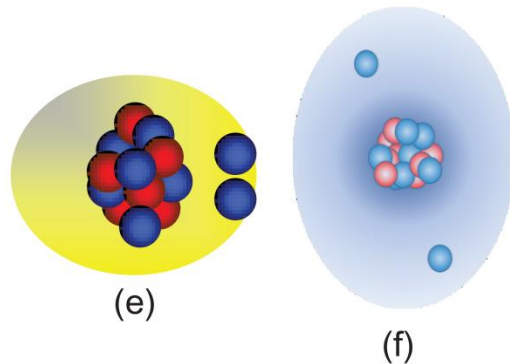


# Physics in exotic nuclear structure

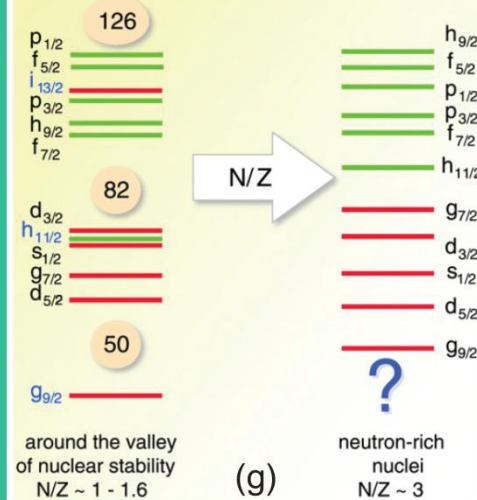


## Shell evolution

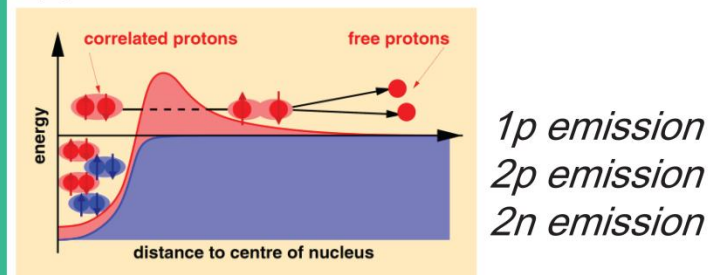
## Halo – deformation effects



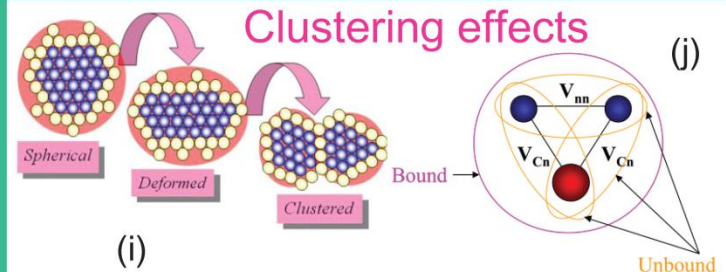
*Shape decoupling*



## New radioactivities

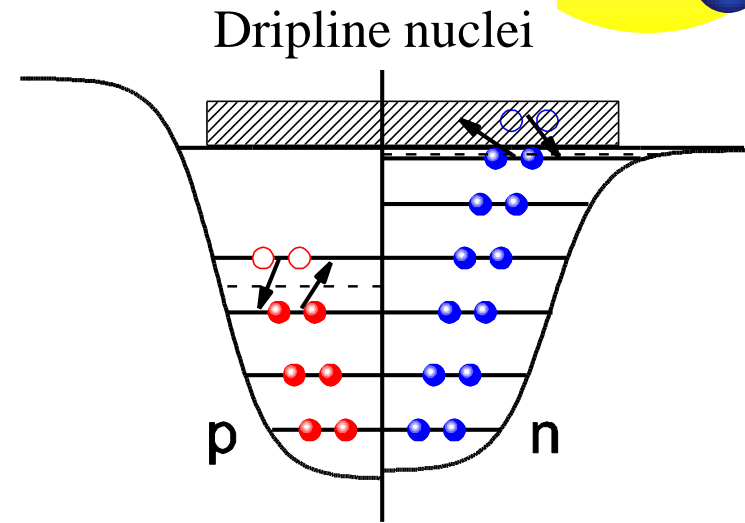
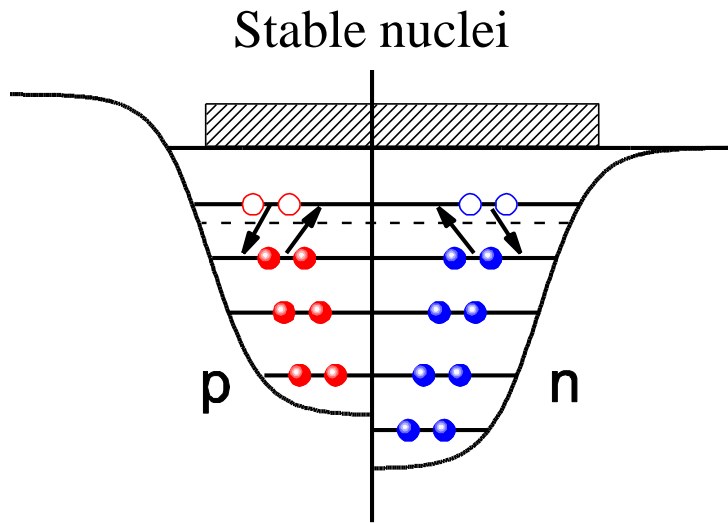
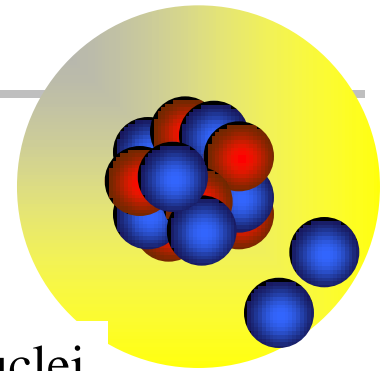


## Clustering effects



# Characteristics of halo nuclei

- Weakly bound; large spatial extension
- Continuum can not be ignored



## *Self-consistent description:*

- Weakly bound, continuum
- Large spatial distribution
- Couplings among ...

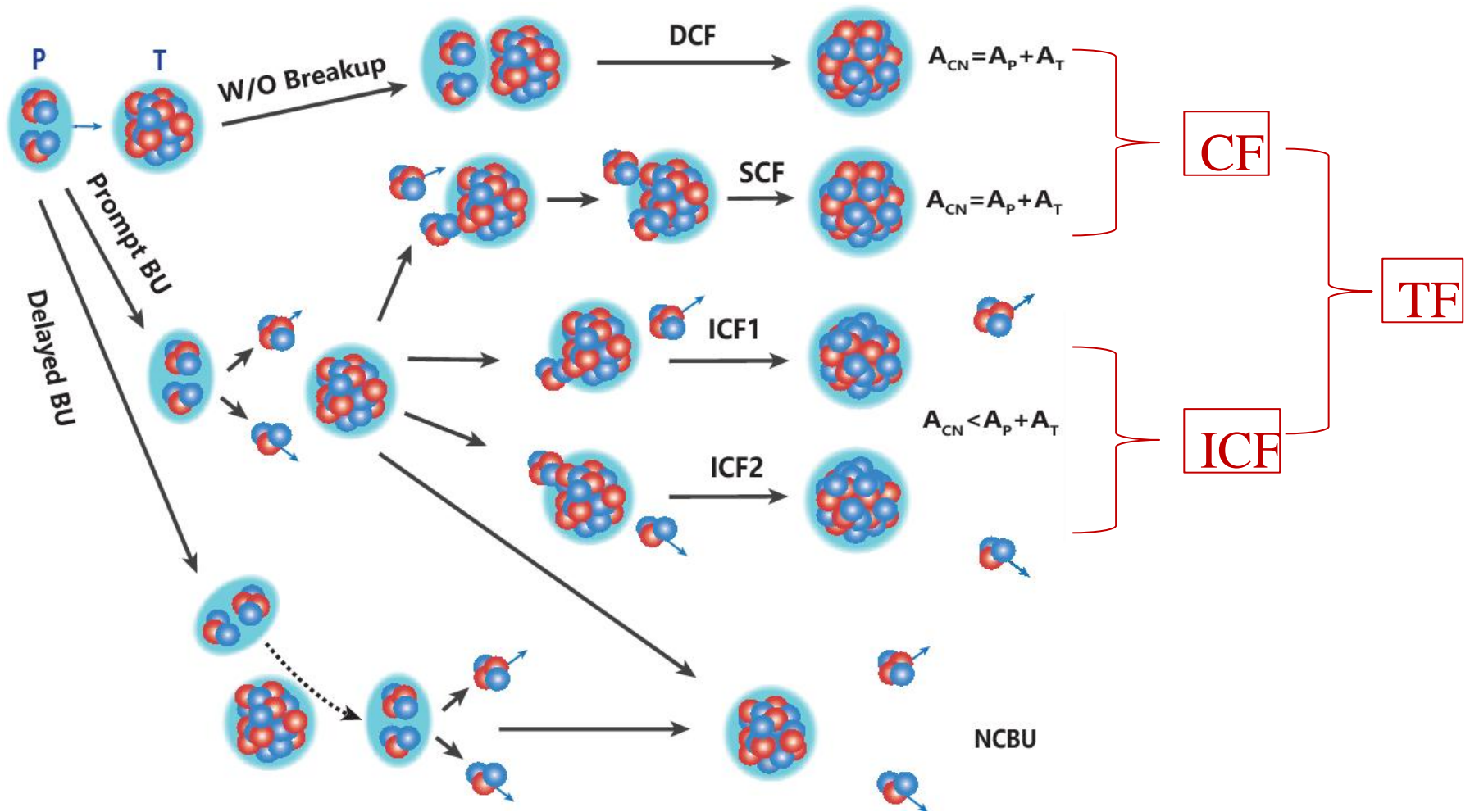
Meng\_Toki\_SGZ\_Zhang\_Long\_Geng2006  
Prog. Part. Nucl. Phys. 57 – 470  
Meng & SGZ 2015, J. Phys. G42-093101

Bulgac1980; nucl-th/9907088

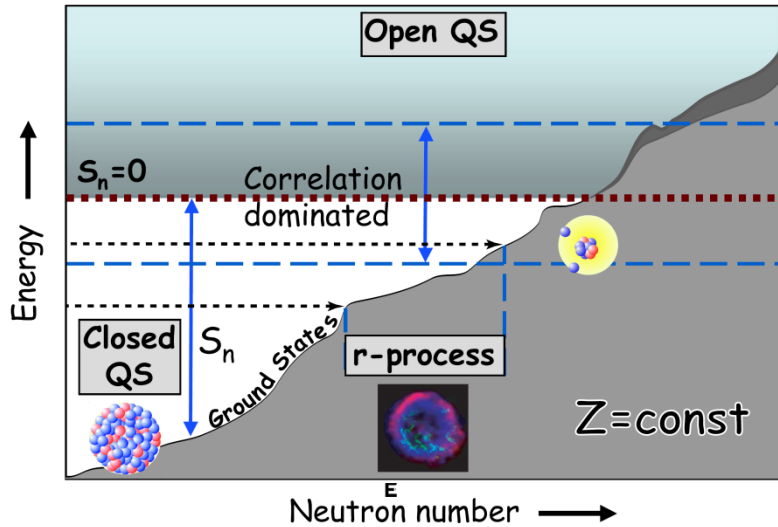
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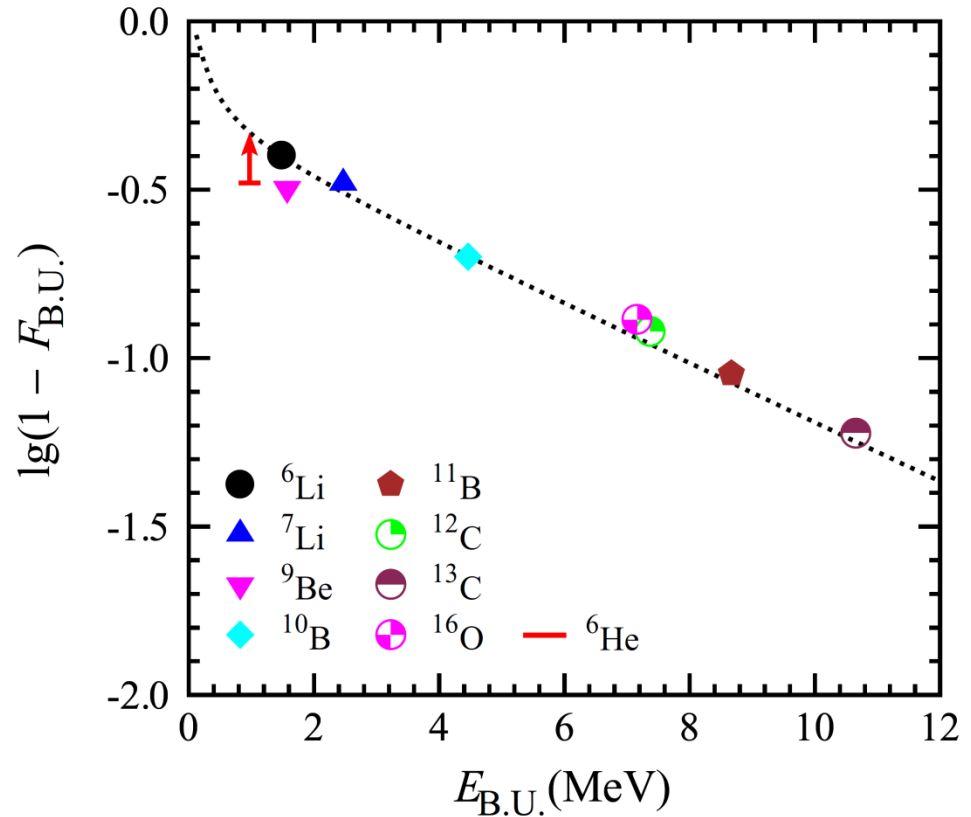
# Breakup effects on fusion of weakly bound projectiles



# Open quantum systems & threshold effects



Dobaczewski+2007\_PPNP59-432  
 Michel+2009\_JPG36-013101

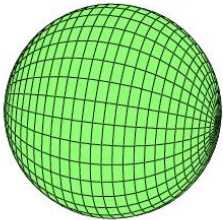
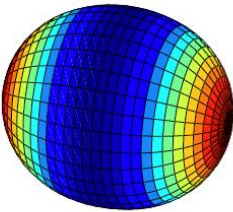
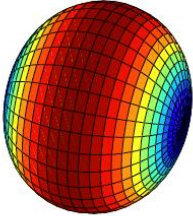
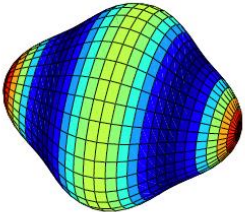
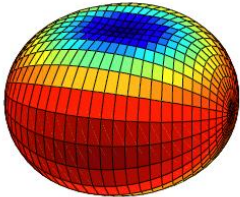
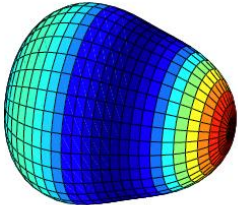
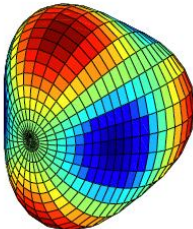
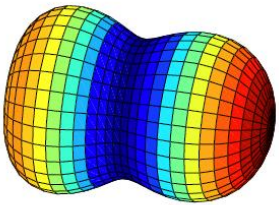


Wang\_Zhao\_Gomes\_Zhao\_SGZ2014\_PRC90-034612  
 Wang\_Zhao\_Diaz-Torres\_Zhao\_SGZ2016\_PRC93-014615

# Various shapes of atomic nuclei

SGZ 2016, Phys. Scr. 91, 063008

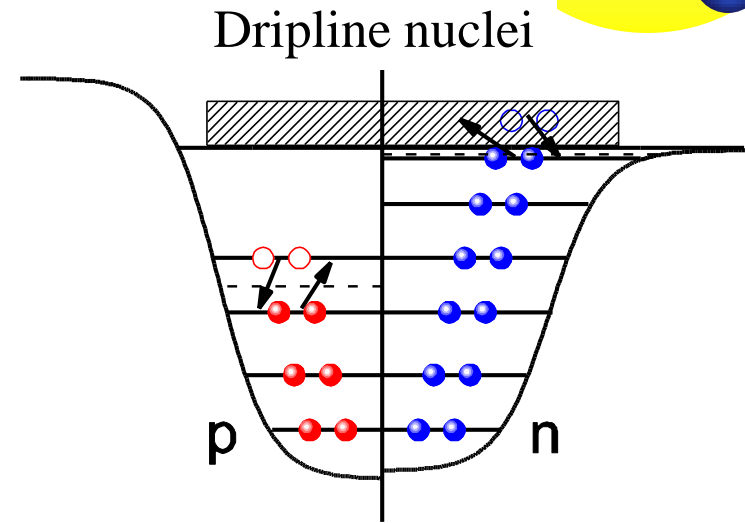
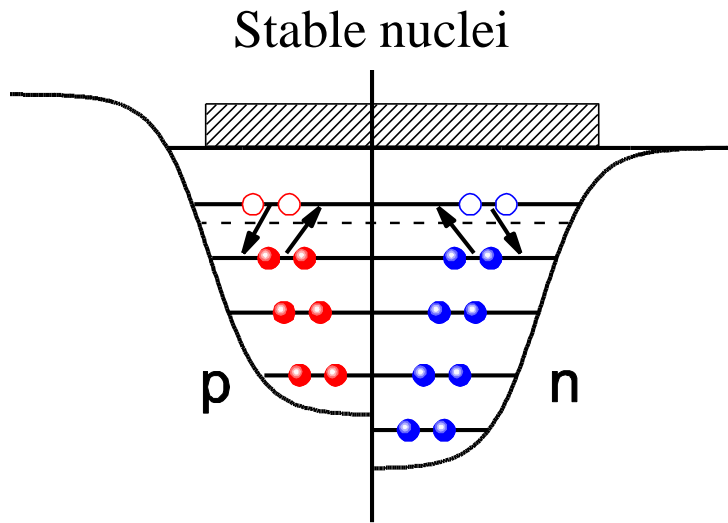
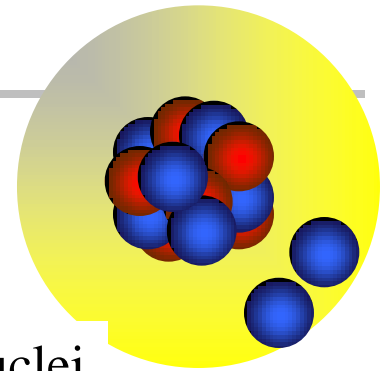
$$R(\theta, \varphi) = R_0 \left[ 1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

(a) $\beta_{\lambda\mu} = 0$	(b) $\beta_{20} > 0$	(c) $\beta_{20} < 0$	(d) $\beta_{40} > 0$
			
(e) $\beta_{22} \neq 0$	(f) $\beta_{30} \neq 0$	(g) $\beta_{32} \neq 0$	(h) $\beta_{20} \gg 0$
			



# Characteristics of halo nuclei

- Weakly bound; large spatial extension
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## *Self-consistent description:*

- Weakly bound, continuum
- Large spatial distribution
- Couplings among ...

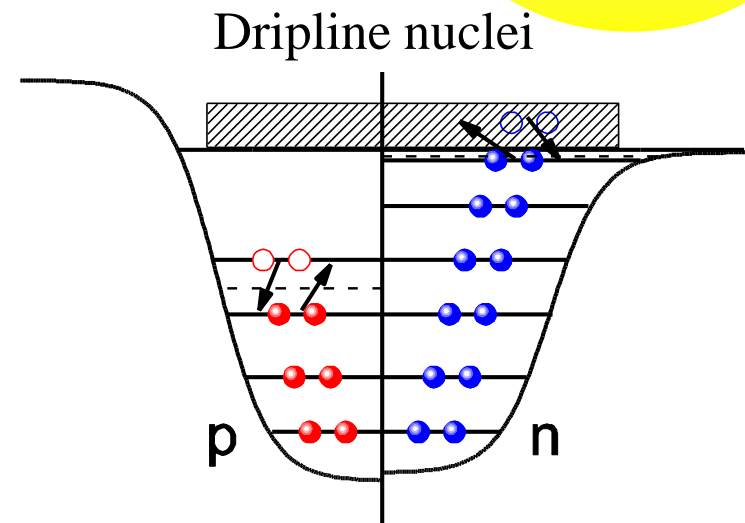
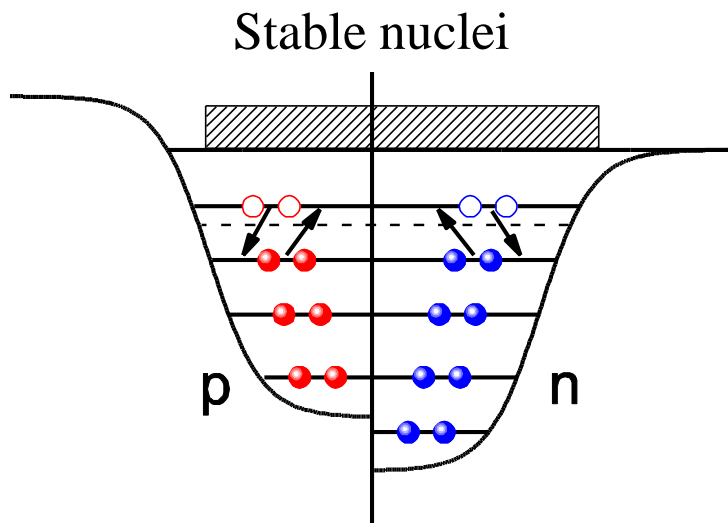
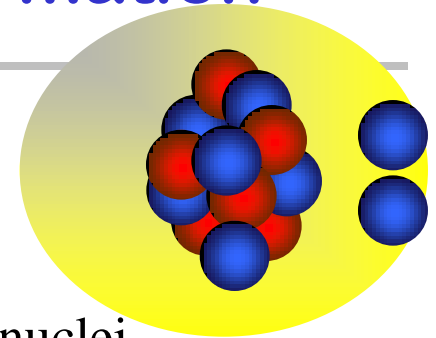
Meng\_Toki\_SGZ\_Zhang\_Long\_Geng2006  
Prog. Part. Nucl. Phys. 57 – 470  
Meng & SGZ 2015, J. Phys. G42-093101

Bulgac1980; nucl-th/9907088

Dobaczewski\_Flocard\_Treiner1984\_NPA422-103

# Characteristics of halo nuclei w/ deformation

- Weakly bound; large spatial extension
- Continuum can not be ignored



## *Self-consistent description:*

- Weakly bound, continuum
- Large spatial distribution
- **Deformation effects**
- Couplings among ...

Meng\_Toki\_SGZ\_Zhang\_Long\_Geng2006  
Prog. Part. Nucl. Phys. 57 – 470  
Meng & SGZ 2015, J. Phys. G42-093101

Bulgac1980; nucl-th/9907088

Dobaczewski\_Flocard\_Treiner1984\_NPA422-103

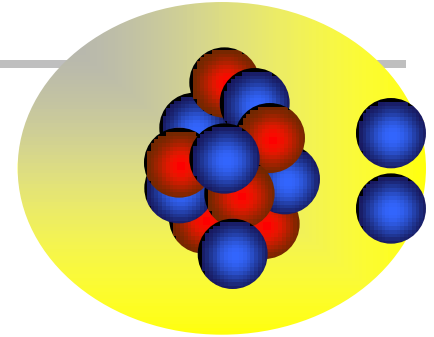
# What we aim at

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*A self-consistent description of*

- ✓ Deformation
- ✓ Continuum contribution
- ✓ Large spatial distribution
- ✓ Interplays among them

by developing a  
relativistic Hartree-Bogoliubov model



# Covariant Density Functional Theory (CDFT)

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$$\mathcal{L} = \bar{\psi}_i (i\partial - M) \psi_i + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - g_\sigma \bar{\psi}_i \sigma \psi_i$$

$$- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - g_\omega \bar{\psi}_i \psi \psi_i$$

$$- \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu - g_\rho \bar{\psi}_i \vec{\rho} \vec{\tau} \psi_i$$

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_i \frac{1 - \tau_3}{2} A \psi_i,$$

Serot\_Walecka1986\_ANP16-1

Reinhard1989\_RPP52-439

Ring1996\_PPNP37-193

Vretenar\_Afanasjev\_Lalazissis\_Ring2005\_PR409-101

Meng\_Toki\_SGZ\_Zhang\_Long\_Geng2006\_PPNP57-470

$$(\alpha \cdot \mathbf{p} + \beta(M + S(\mathbf{r})) + V(\mathbf{r})) \psi_i = \epsilon_i \psi_i$$

Liang\_Meng\_SGZ2015\_PR570-1

$$(-\nabla^2 + m_\sigma^2) \sigma = -g_\sigma \rho_S - g_2 \sigma^2 - g_3 \sigma^3$$

Meng\_SGZ2015\_JPG42-093101

$$(-\nabla^2 + m_\omega^2) \omega = g_\omega \rho_V - c_3 \omega^3$$

$$(-\nabla^2 + m_\rho^2) \rho = g_\rho \rho_3$$

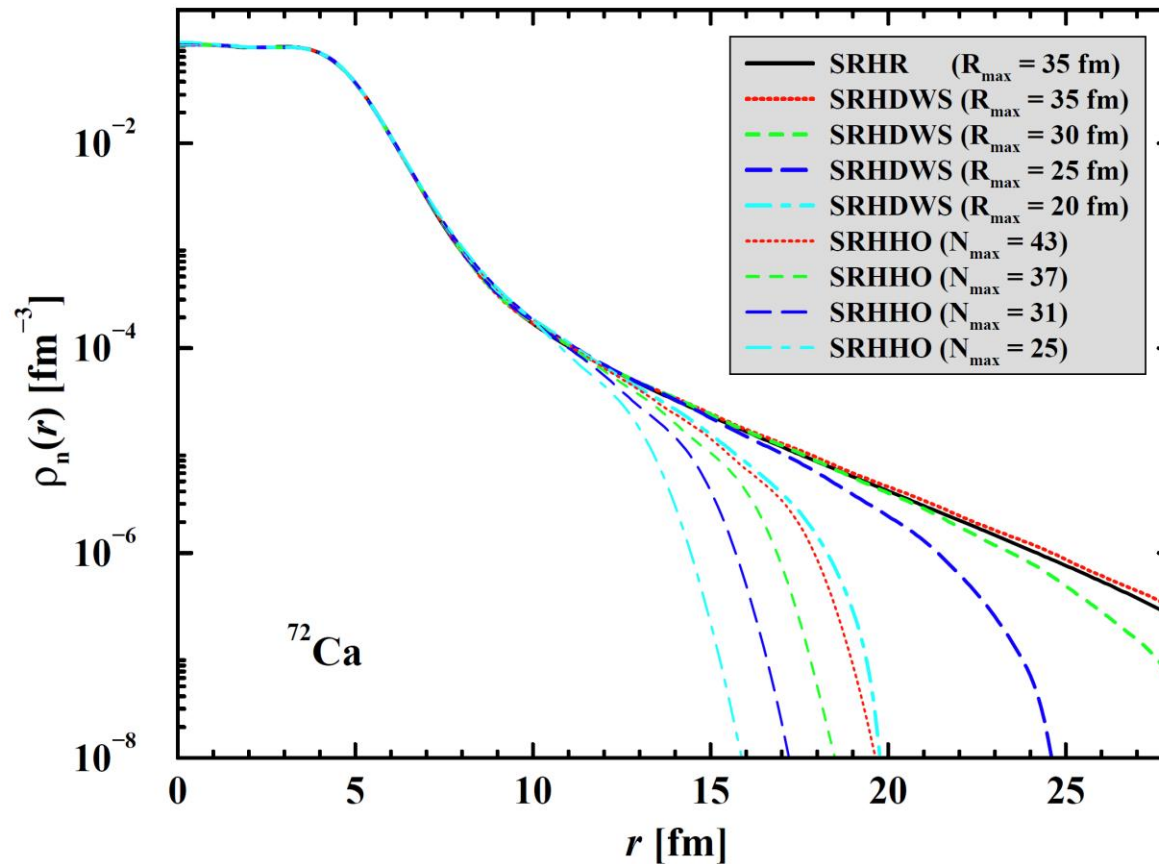
$$-\nabla^2 A = e \rho_C$$

# RMF theories in a Woods-Saxon basis

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Shapes	Model	Schrödinger W-S basis	Dirac W-S basis	
Spherical	Rela. Hartree	SRH SWS <a href="#">SGZ_Meng_Ring2003_PRC91-262501</a>	SRH DWS	✓

# Why Woods-Saxon basis ?



Woods-Saxon basis is a reconciler between the HO basis &  $r$  space

- Reproduces results of  $r$  space
- Matrix diagonalization, numerically less complicated than HO

# RMF theories in a Woods-Saxon basis

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Axially deformed	Rela. Hartree + BCS		DRH DWS	✓

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Axially deformed	Rela. Hartree + BCS  <a href="#">SGZ_Meng_Ring2006_AIP Conf. Proc. 865-90</a>		DRH DWS	✓
Axially deformed	Rela. Hartree-Bogoliubov  <a href="#">SGZ_Meng_Ring 2007_ISPUN Proc.</a> <a href="#">SGZ_Meng_Ring_Zhao 2010_PRC82-011301R</a> <a href="#">SGZ_Meng_Ring_Zhao 2011_JPConfProc312-092067</a> <a href="#">Li_Meng_Ring_Zhao_SGZ 2012_PRC85-024312</a> <a href="#">Li_Meng_Ring_Zhao_SGZ 2012_ChinPhysLett29-042101</a>		DRHB DWS	✓

Woods-Saxon basis is a reconciler between the HO basis &  $r$  space



# RMF theories in a Woods-Saxon basis

Shapes	Model	Schrödinger W-S basis	Dirac W-S basis	
Spherical	Rela. Hartree	SRH SWS SGZ_Meng_Ring2003_PRC91-262501	SRH DWS	✓
Axially deformed	Rela. Hartree + BCS		DRH DWS	✓
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SGZ\_Meng\_Ring 2007\_ISPUN Proc.  
 SGZ\_Meng\_Ring\_Zhao 2010\_PRC82-011301R  
 SGZ\_Meng\_Ring\_Zhao 2011\_JPConfProc312-092067  
 Li\_Meng\_Ring\_Zhao\_SGZ 2012\_PRC85-024312  
 Li\_Meng\_Ring\_Zhao\_SGZ 2012\_ChinPhysLett29-042101

Woods-Saxon basis is a reconciler between the HO basis &  $r$  space

Density dependent DRHB theory in continuum

Chen\_Li\_Liang\_Meng2012\_PRC85-067301

Schunck\_Egido2008\_PRC77-011301R; PRC78-064305

Long\_Ring\_Giai\_Meng2010\_PRC81-024308

# Deformed RHB theory in continuum

Kucharek\_Ring1991\_ZPA339-23

$$\sum_{\sigma'p'} \int d^3\mathbf{r}' \begin{pmatrix} h_D(\mathbf{r}\sigma p, \mathbf{r}'\sigma'p') - \lambda & \Delta(\mathbf{r}\sigma p, \mathbf{r}'\sigma'p') \\ -\Delta^*(\mathbf{r}\sigma p, \mathbf{r}'\sigma'p') & -h_D(\mathbf{r}\sigma p, \mathbf{r}'\sigma'p') + \lambda \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}'\sigma'p') \\ V_k(\mathbf{r}'\sigma'p') \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}\sigma p) \\ V_k(\mathbf{r}\sigma p) \end{pmatrix}$$

Axially deformed nuclei

Woods-Saxon basis

$$\varphi_{i\kappa m}(\mathbf{r}\sigma) = \frac{1}{r} \begin{pmatrix} iG_{i\kappa}(r)Y_{jm}^l(\Omega\sigma) \\ -F_{i\kappa}(r)Y_{jm}^{\tilde{l}}(\Omega\sigma) \end{pmatrix}$$

$$U_k(\mathbf{r}\sigma p) = \sum_{i\kappa} \begin{pmatrix} u_{k,(i\kappa)}^{(m)} \varphi_{i\kappa m}(\mathbf{r}\sigma p) \\ u_{k,(\tilde{i}\kappa)}^{(\bar{m})} \tilde{\varphi}_{i\kappa m}(\mathbf{r}\sigma p) \end{pmatrix}$$

$$V_k(\mathbf{r}\sigma p) = \sum_{i\kappa} \begin{pmatrix} v_{k,(i\kappa)}^{(m)} \varphi_{i\kappa m}(\mathbf{r}\sigma p) \\ v_{k,(\tilde{i}\kappa)}^{(\bar{m})} \tilde{\varphi}_{i\kappa m}(\mathbf{r}\sigma p) \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \mathcal{U} \\ \mathcal{V} \end{pmatrix} = E \begin{pmatrix} \mathcal{U} \\ \mathcal{V} \end{pmatrix}$$

$$\mathcal{U} = \begin{pmatrix} u_{k,(i\kappa)}^{(m)} \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} v_{k,(\tilde{i}\kappa)}^{(\bar{m})} \end{pmatrix}$$

# Parameter set for ph & pp channels

SGZ\_Meng\_Ring\_Zhao 2010\_PRC82-011301R

SGZ\_Meng\_Ring\_Zhao 2011\_JPConfProc312-092067

Li\_Meng\_Ring\_Zhao\_SGZ 2012\_PRC85-024312

Li\_Meng\_Ring\_Zhao\_SGZ 2012\_ChinPhysLett29-042101

NL3, PK1, ...  $R_{\max} = 20 \text{ fm}$ ,  $\Delta r = 0.1 \text{ fm}$

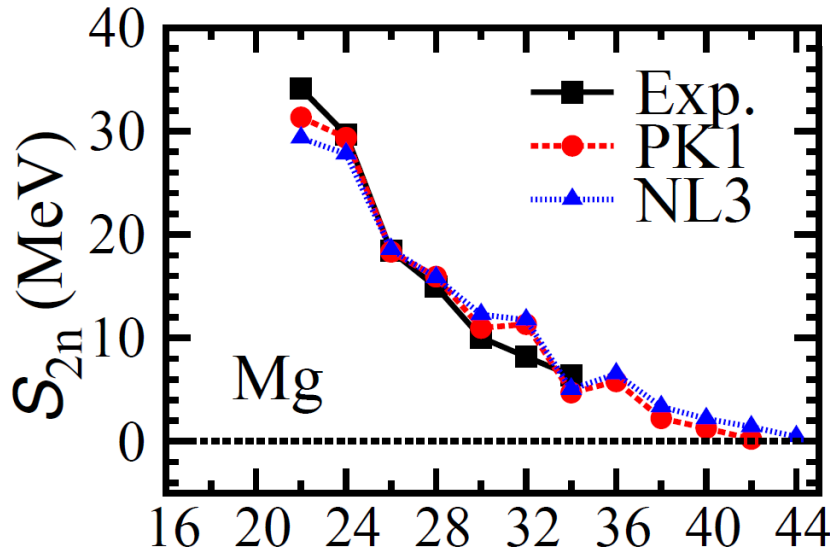
$$V^{\text{pp}}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \frac{1}{2} (1 - P^\sigma) \delta(\mathbf{r}_1 - \mathbf{r}_2) \left( 1 - \frac{\rho(\mathbf{r}_1)}{\rho_{\text{sat}}} \right)$$

$^{20}\text{Mg}$ : spherical from DRHBWS calculation

Model	Pairing force	Parameters	$E_{\text{pair}}^{\text{p}}$ (MeV)
SRHBHO	Gogny	D1S	<b>-9.2382</b>
RCHB	Surface $\delta$	$V_0 = 374 \text{ MeV fm}^3$ $\rho_0 = 0.152 \text{ fm}^3$	<b>-9.2387</b>
	Sharp cutoff	$E_{\text{cut}}^{\text{q.p.}} = 60 \text{ MeV}$	
DRHBWS	Surface $\delta$	$V_0 = 380 \text{ MeV fm}^3$ $\rho_0 = 0.152 \text{ fm}^3$	<b>-9.2383</b>
	Smooth cutoff	$E_{\text{cut}}^{\text{q.p.}} = 60 \text{ MeV}$ $\Gamma = 5.65 \text{ MeV}$	

# Ground states of Mg isotopes

Li\_Meng\_Ring\_Zhao\_SGZ 2012\_PRC85-024312

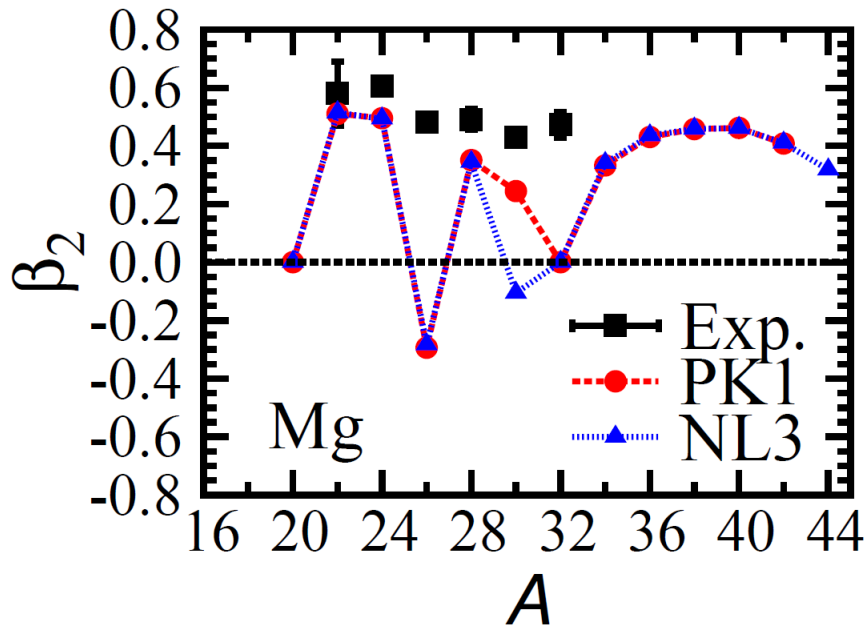


□ The calc. reproduce well the experiment

□  $^{42}\text{Mg}$  ( $^{44}\text{Mg}$ ) is the last bound deformed nucl. from PK1 (NL3)

□ A problem of many mean field models:  $N = 20$  shell quenching can not be obtained

➤  $^{32}\text{Mg}$  is deformed according to the expt., but spherical from many MF calc.



# Conditions for occurrence of a halo & its shape

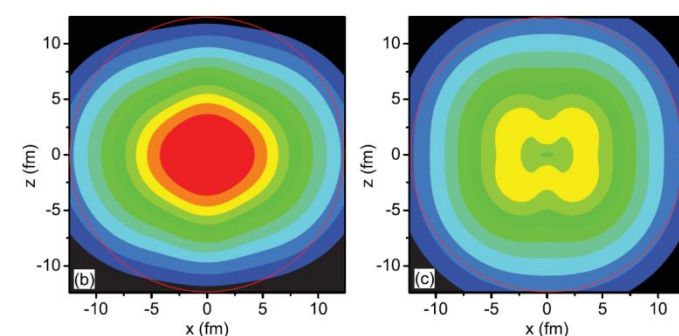
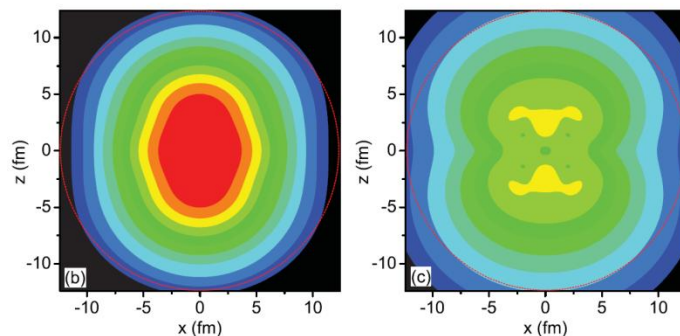
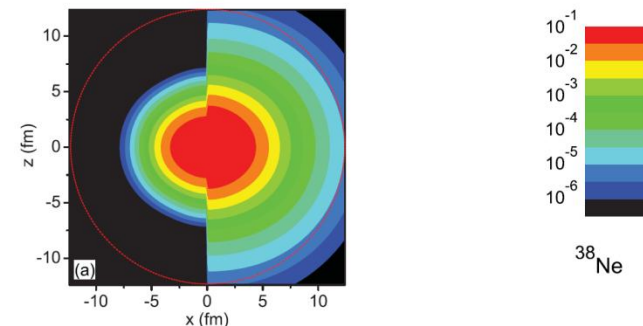
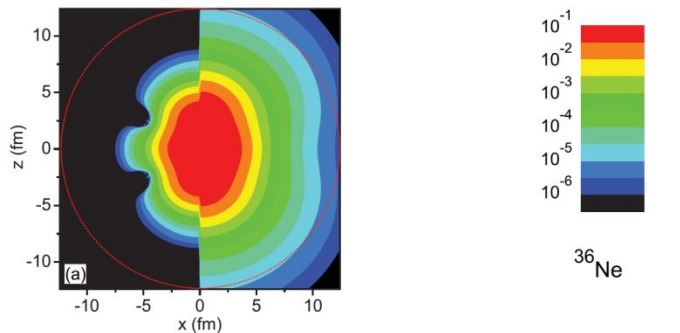
- Existence & deformation of neutron halo depend on quantum numbers of the main components of the s.p. orbits around Fermi surface
  - s levels with  $\Lambda = 0 \Rightarrow$  spherical halos
  - p levels with  $\Lambda = 0 \Rightarrow$  prolate halos
  - p levels with  $\Lambda = 1 \Rightarrow$  oblate halos
  - d, f, ... levels: no halos

SGZ\_Meng\_Ring\_Zhao 2010

PRC82-011301R

Li\_Meng\_Ring\_Zhao\_SGZ 2012

PRC85-024312



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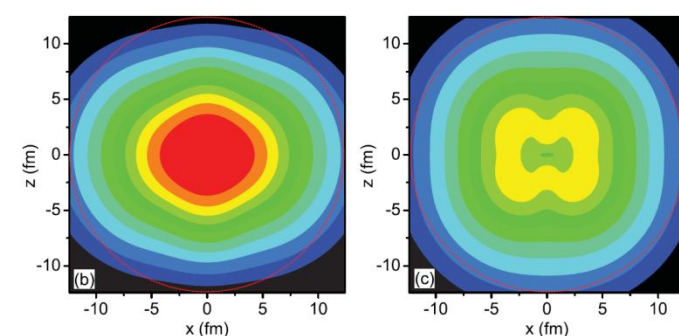
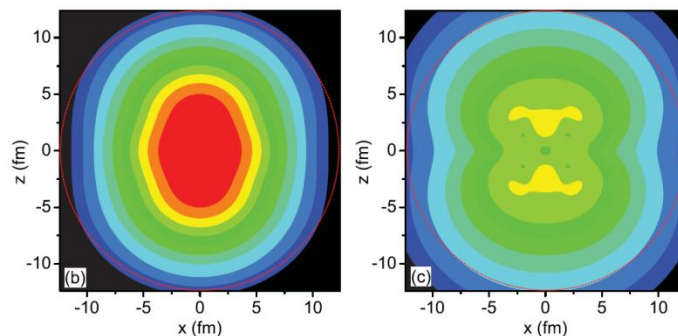
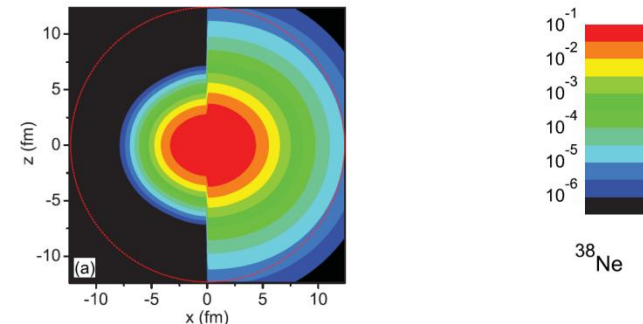
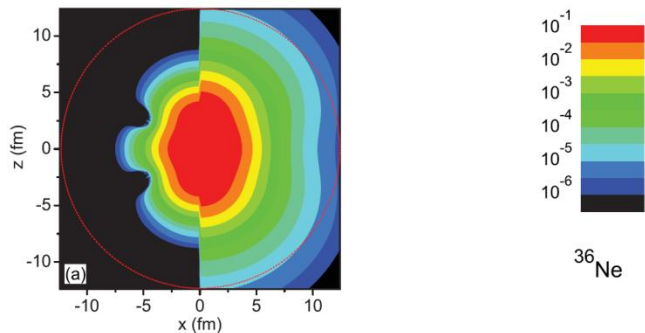
SGZ\_Meng\_Ring\_Zhao 2010

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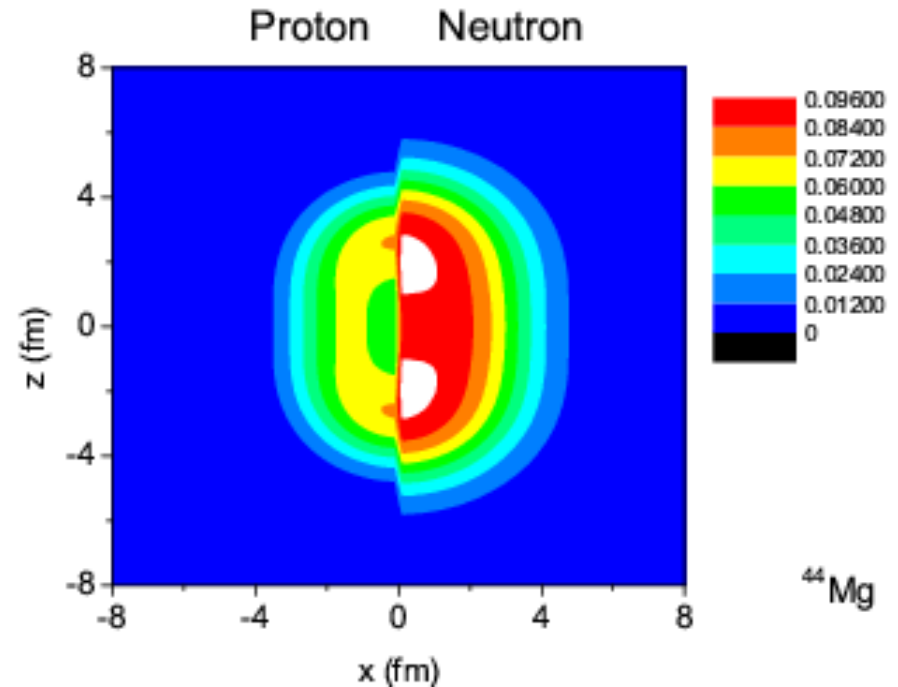
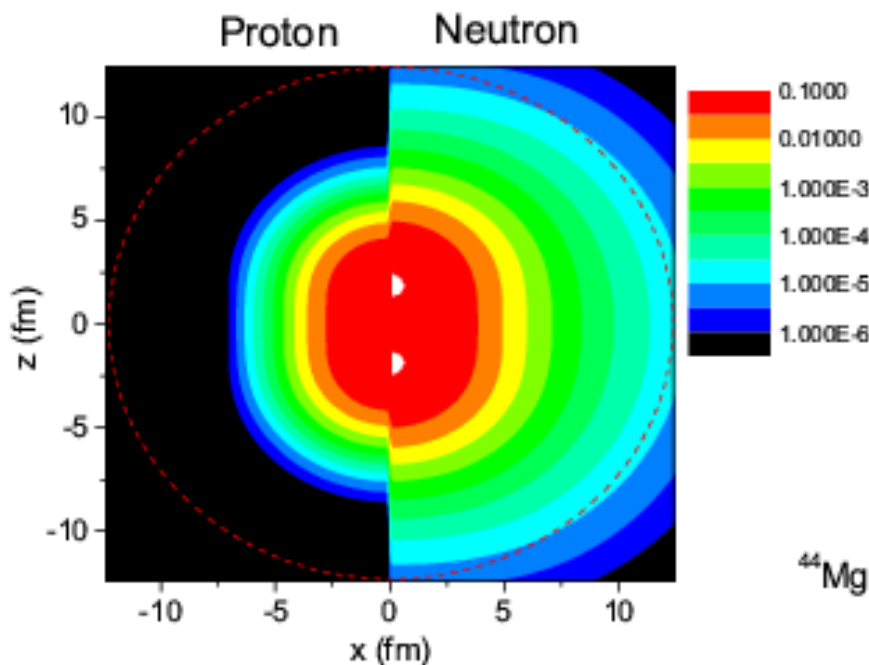
PRC85-024312

Pei\_Zhang\_Xu2013PRC87-051302R



# $^{44}\text{Mg}$ : Density distributions

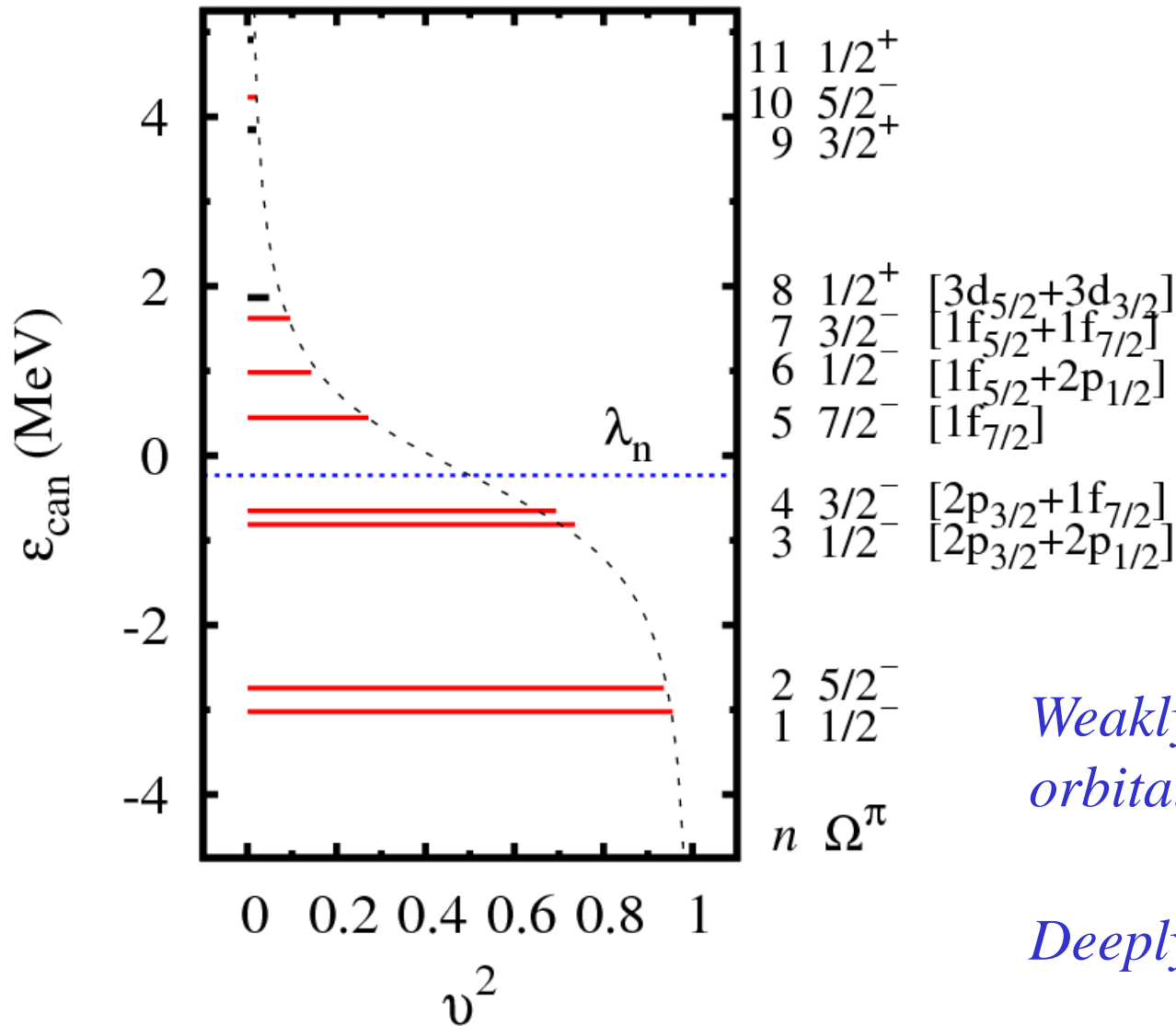
SGZ\_Meng\_Ring\_Zhao 2010 PRC82-011301R  
Li\_Meng\_Ring\_Zhao\_SGZ 2012 PRC85-024312



□ Prolate deformation

□ Large spatial extension in neutron density distribution

# $^{44}\text{Mg}$ : Single neutron states in canonical basis

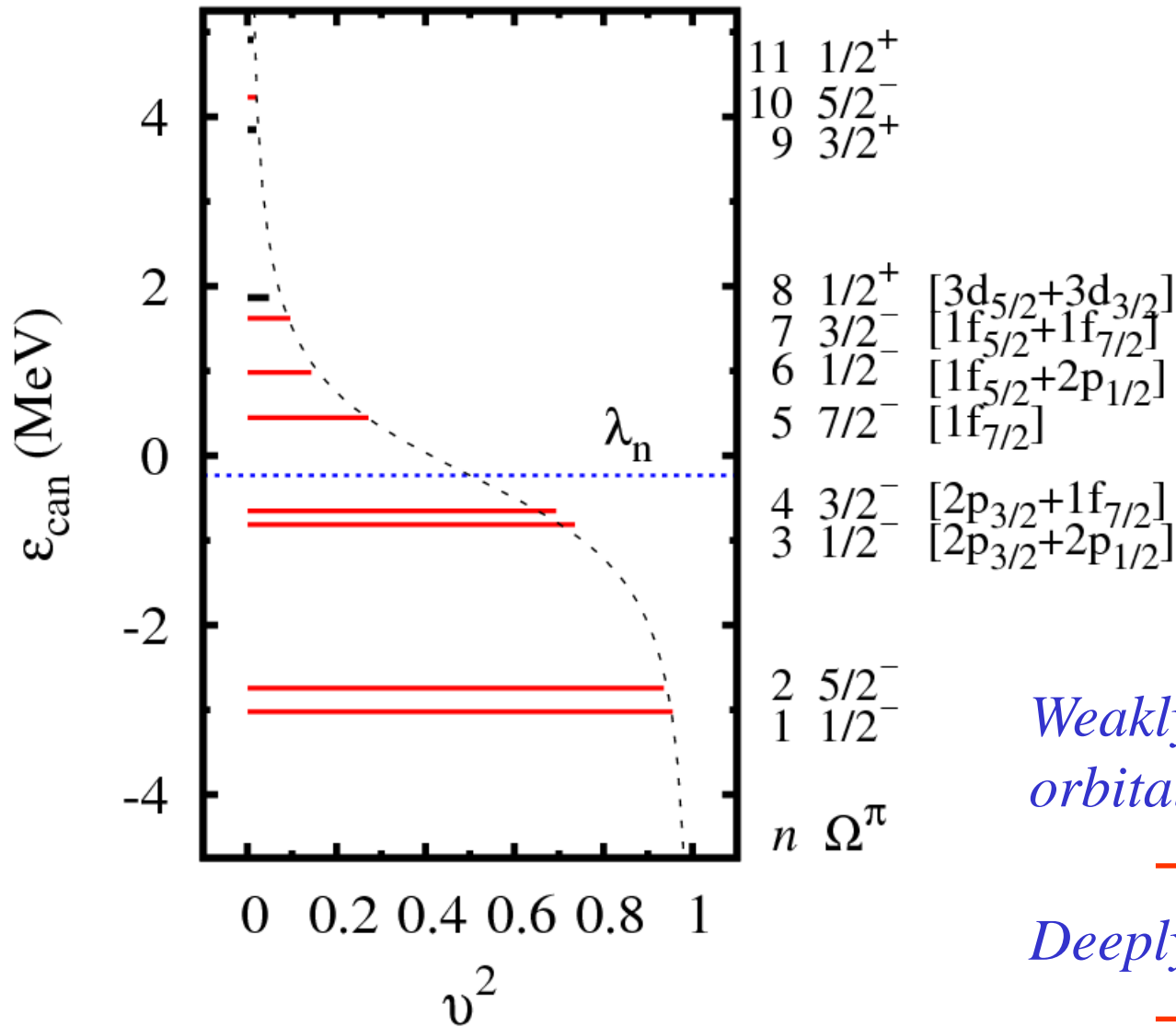


*Weakly bound & continuum orbitals*

*Deeply bound orbitals*



# $^{44}\text{Mg}$ : Single neutron states in canonical basis



*Weakly bound & continuum orbitals*

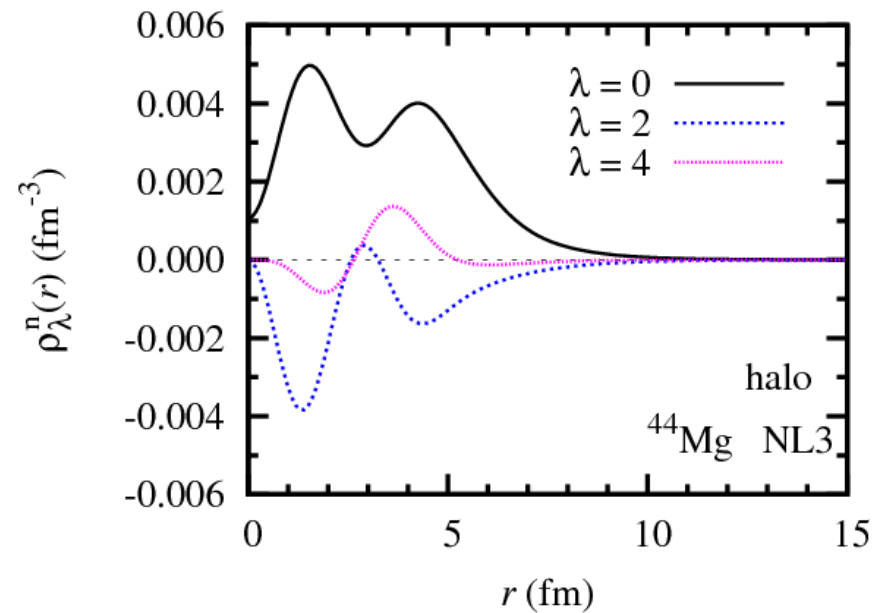
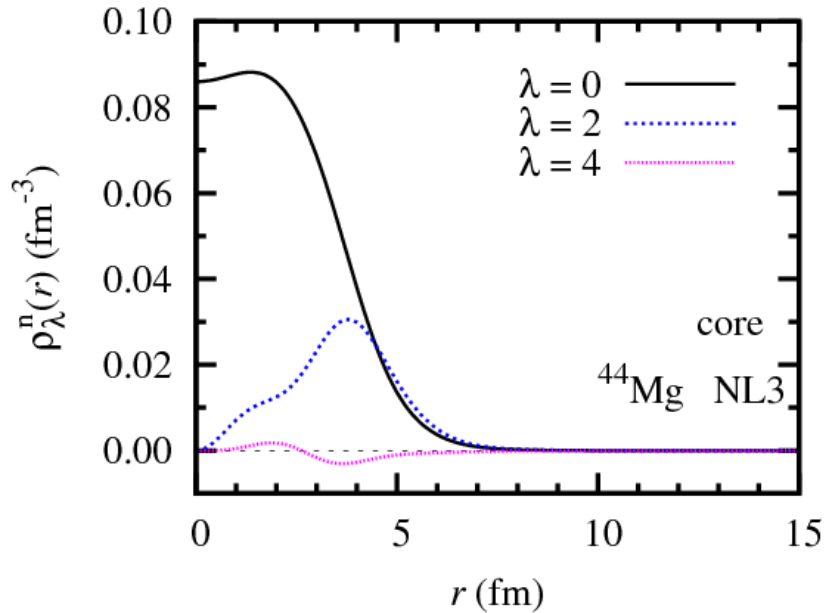
→ halo

*Deeply bound orbitals*

→ core

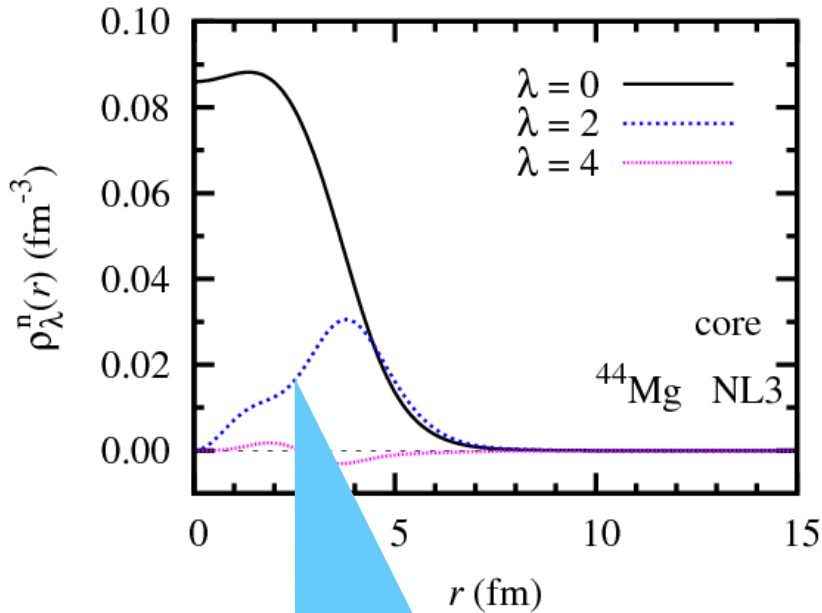
# $^{44}\text{Mg}$ : Density of core & halo---shape decoupling

$$\rho(\mathbf{r}) = \sum_{\lambda} \rho_{\lambda}(r) P_{\lambda}(\cos \theta), \quad \lambda = 0, 2, 4, \dots$$

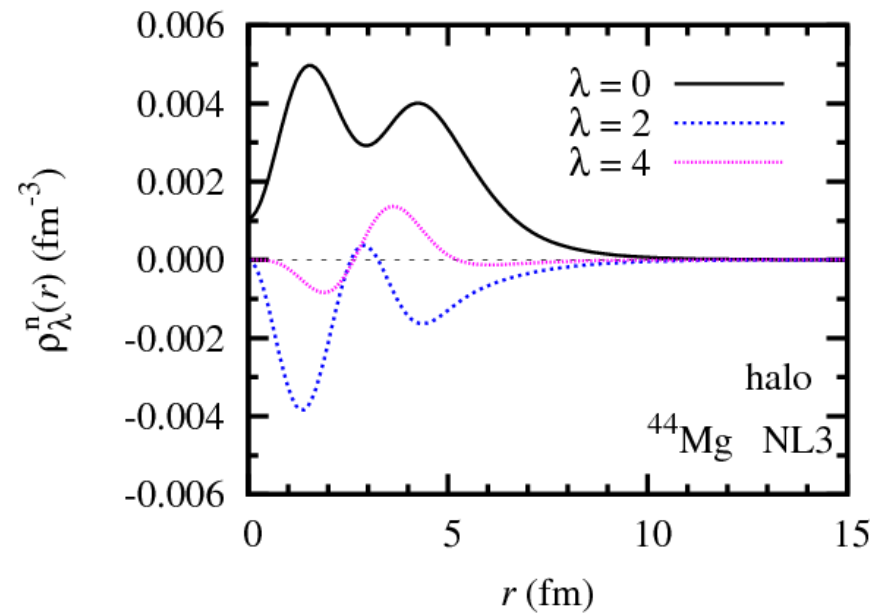


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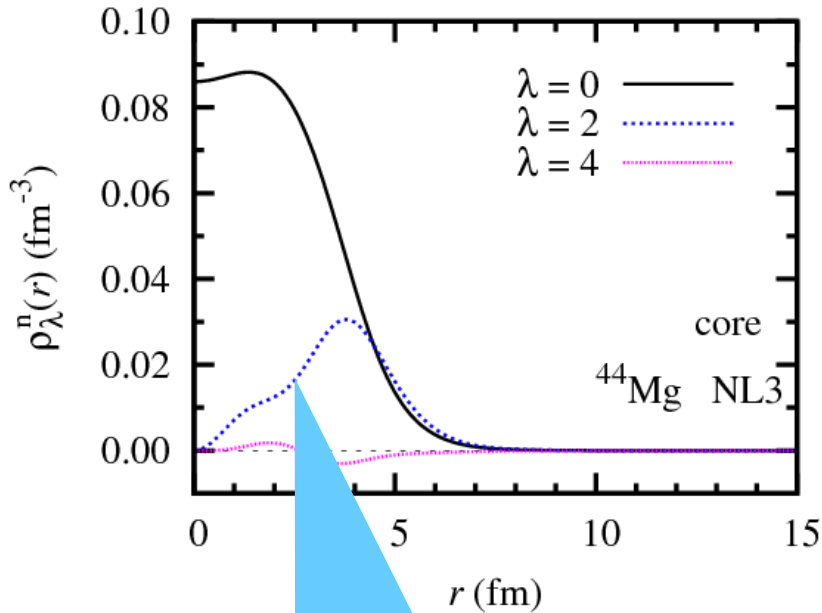


Core: prolate

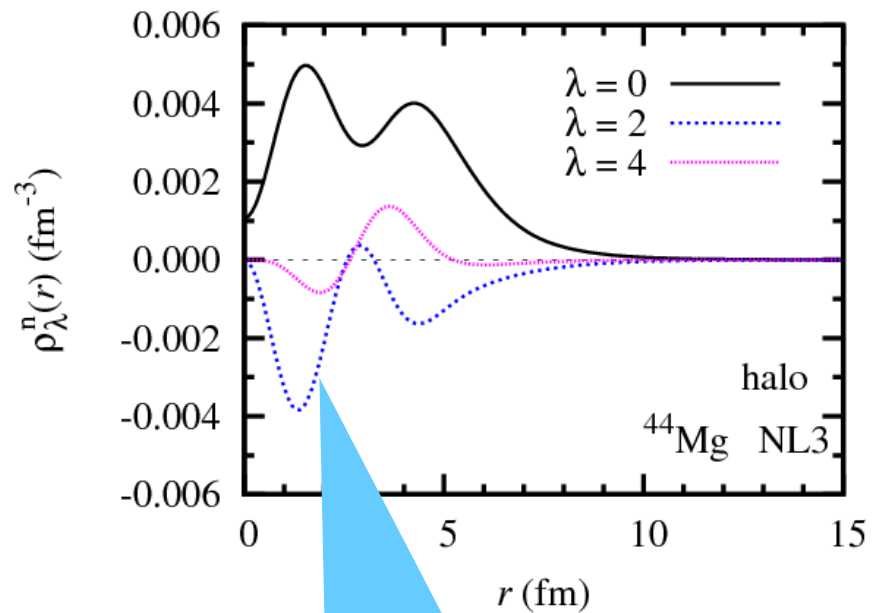


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$$\rho(\mathbf{r}) = \sum_{\lambda} \rho_{\lambda}(r) P_{\lambda}(\cos \theta), \quad \lambda = 0, 2, 4, \dots$$

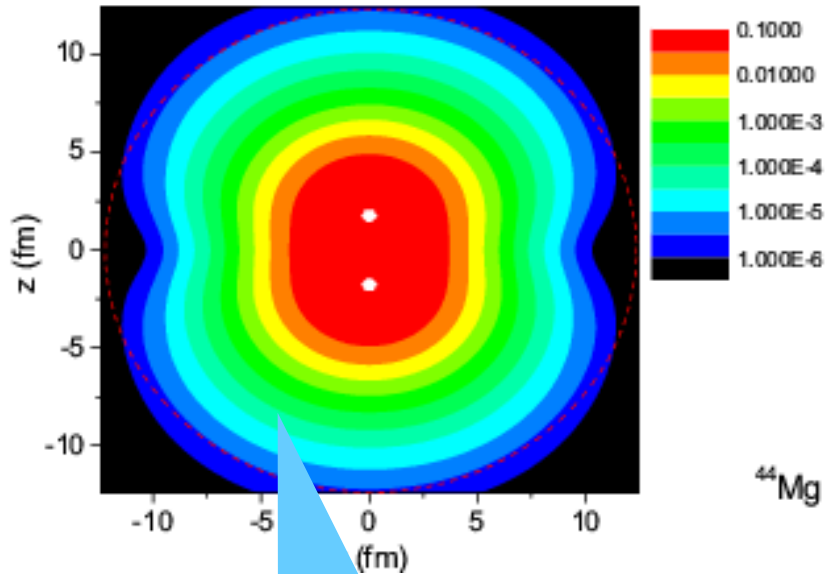


Core: prolate

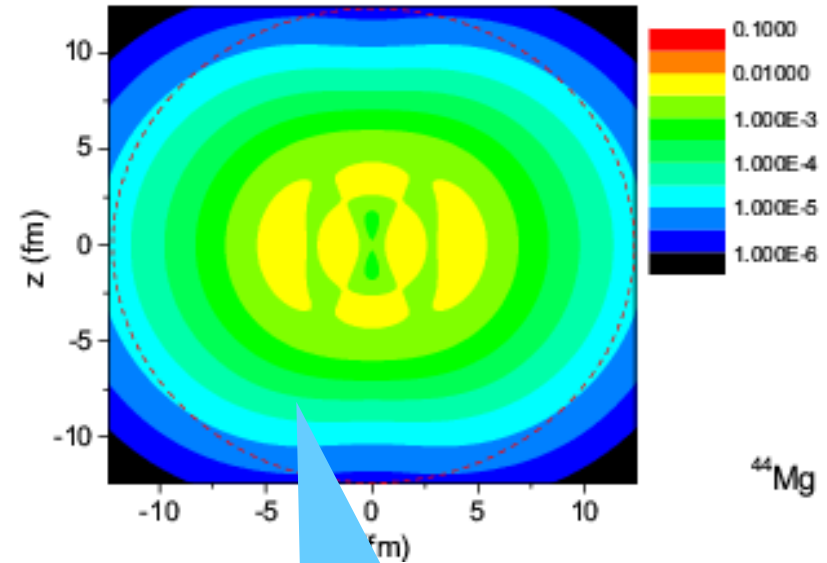


halo: oblate

# $^{44}\text{Mg}$ : Density of core & halo---shape decoupling



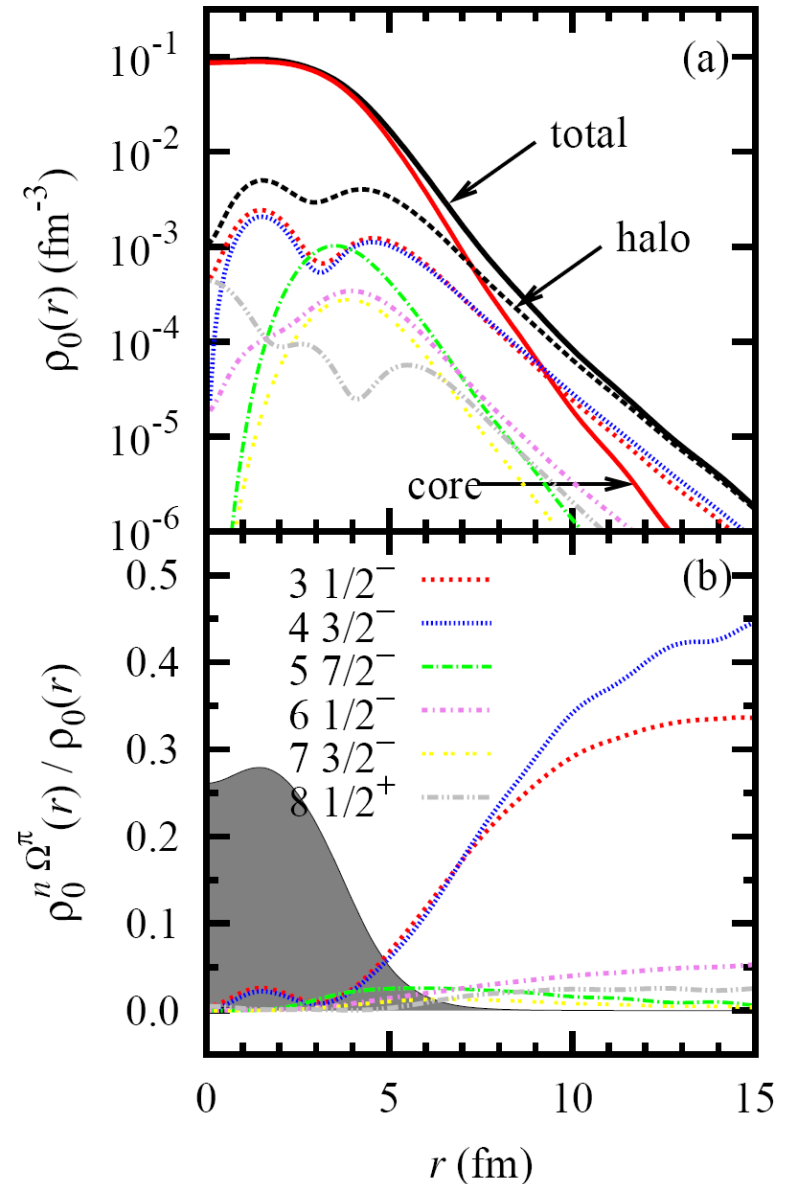
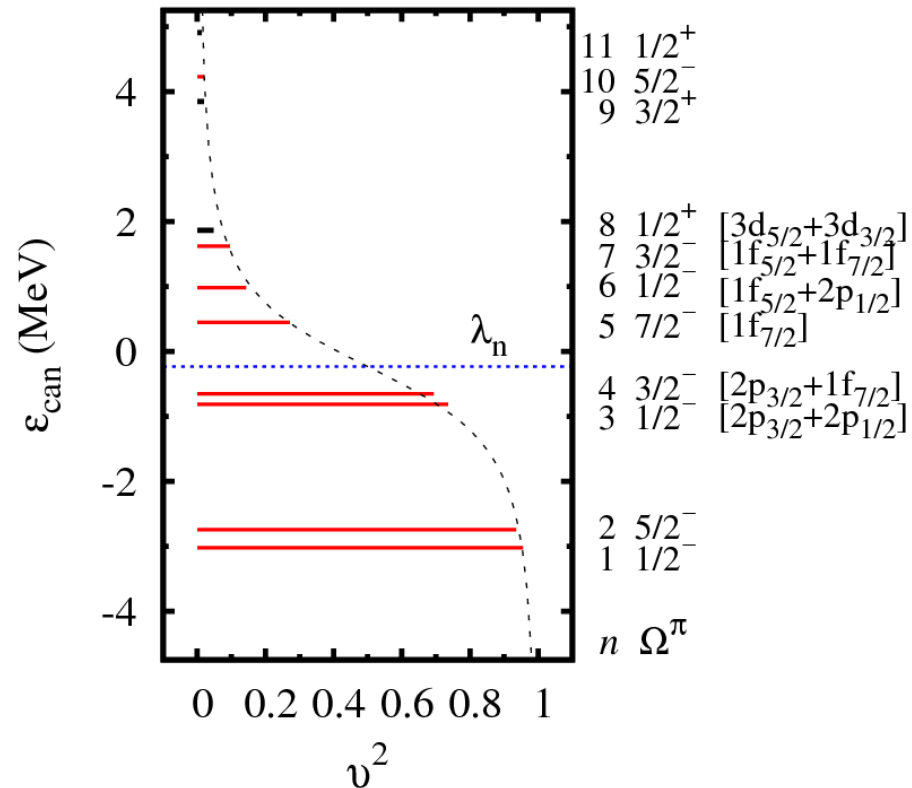
Core: prolate



halo: oblate

# $^{44}\text{Mg}$ : Decomposition of neutron density distribution

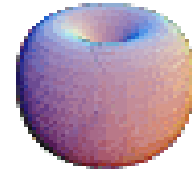
- The 3rd & 4th states contribute to tail part of neutron density distribution
- Main component:  $2p_{3/2}$
- $R_{\text{core}} = 3.72$  fm,  $R_{\text{halo}} = 5.86$  fm



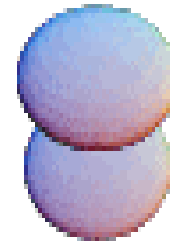
# Shape of low- $\Lambda$ single particle orbital

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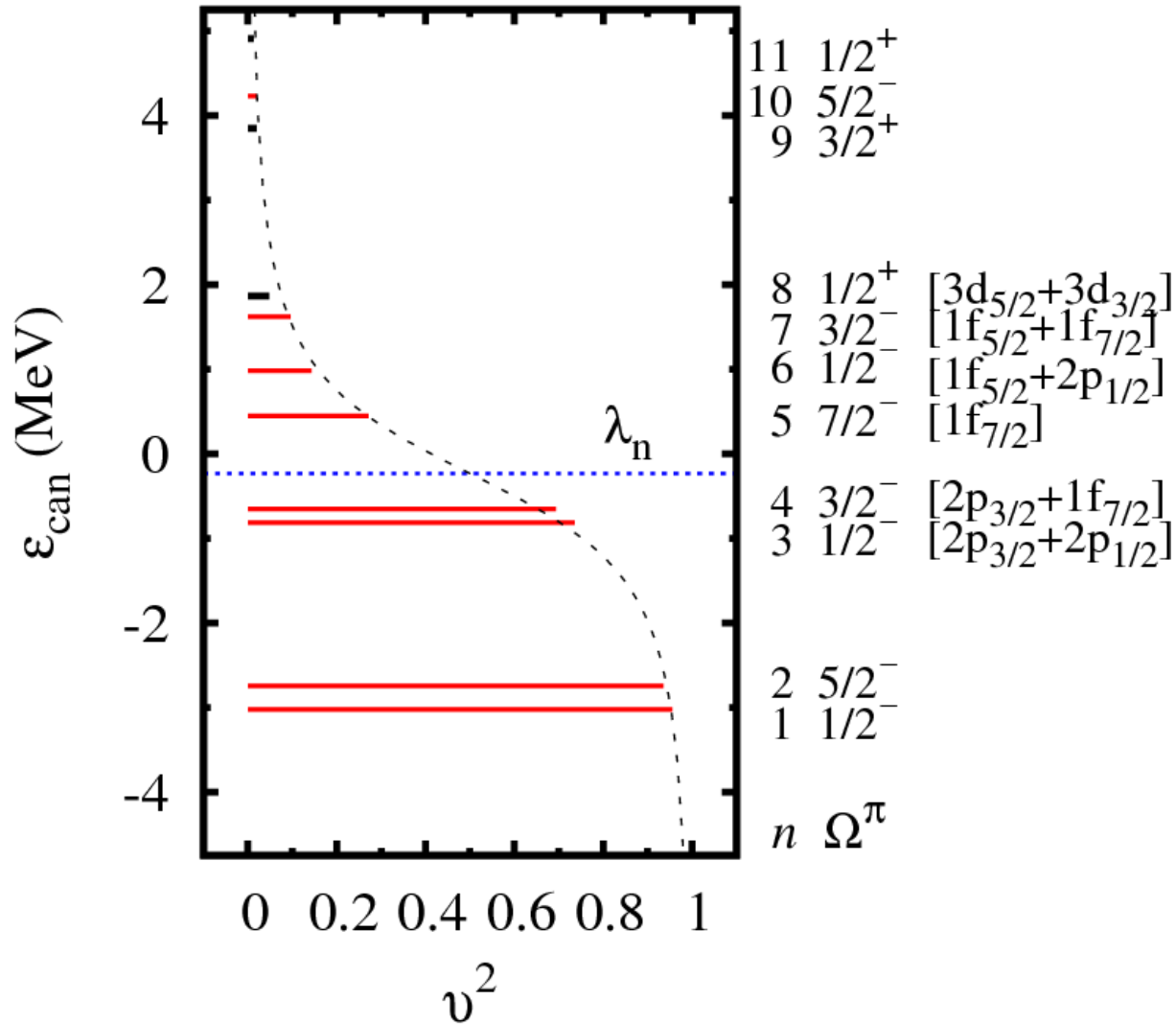
$$l = 1, \Lambda = \pm 1 \quad |Y_{1\pm 1}(\theta, \phi)|^2 \propto \sin^2(\theta)$$



$$l = 1, \Lambda = 0 \quad |Y_{10}(\theta, \phi)|^2 \propto \cos^2(\theta)$$

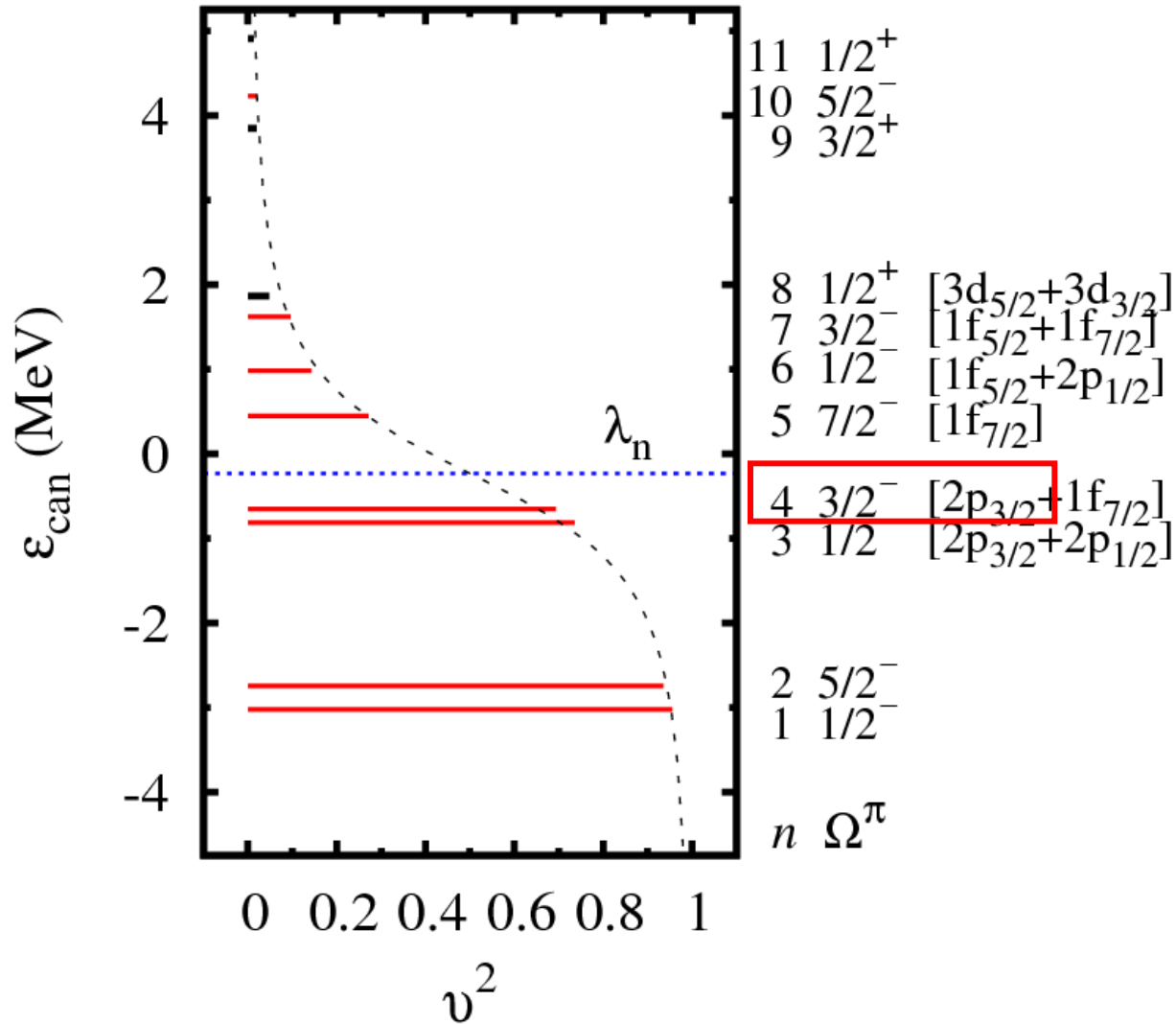


# Mechanism of shape decoupling

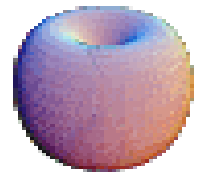




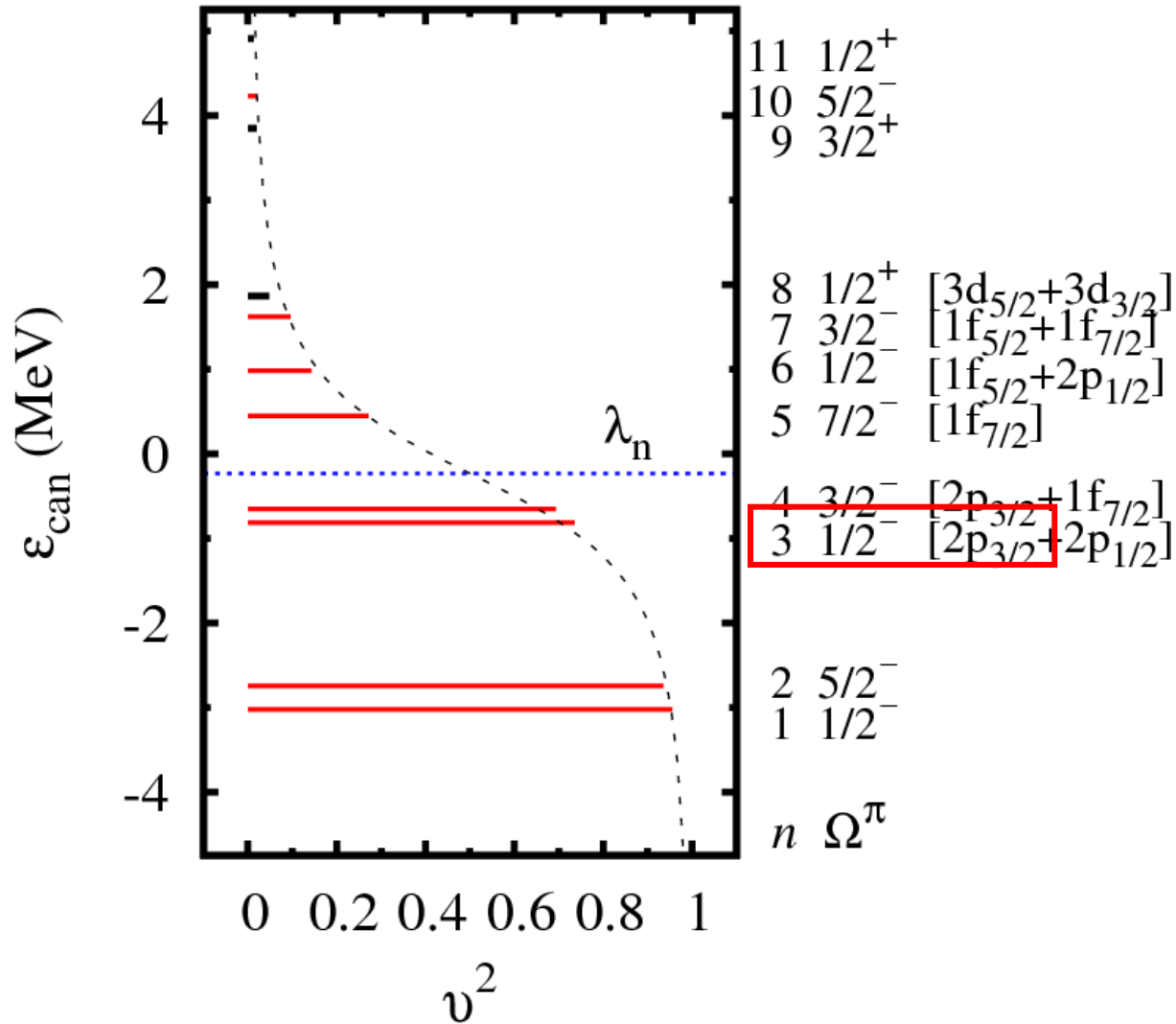
# Mechanism of shape decoupling



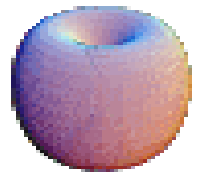
$\Lambda = \pm 1$



# Mechanism of shape decoupling

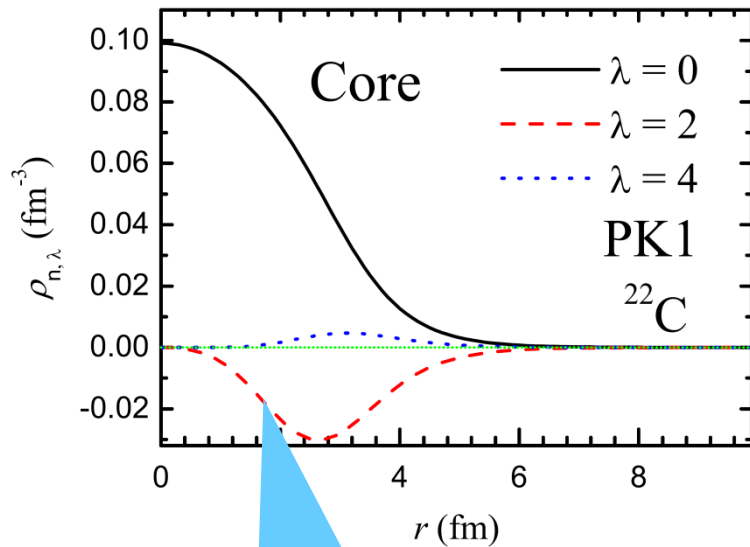


$\Lambda = \pm 1$   
 $\Lambda = 0$

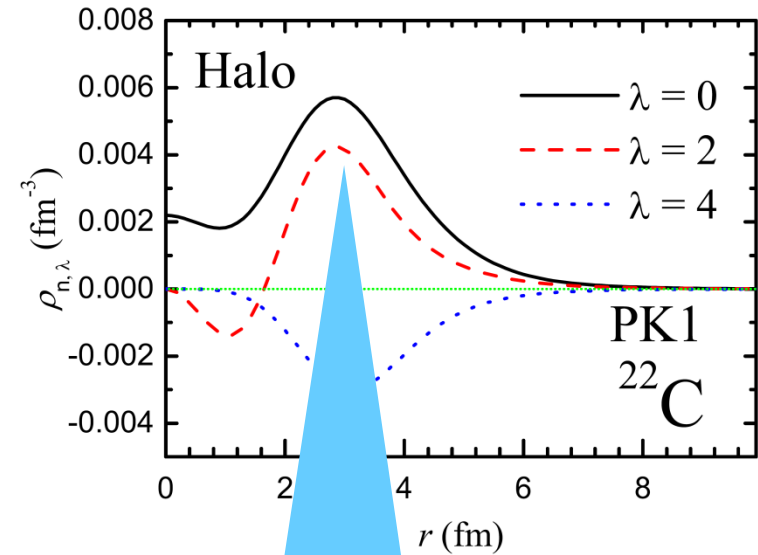


# $^{22}\text{C}$ : Density of core & halo---shape decoupling

$$\rho(\mathbf{r}) = \sum_{\lambda} \rho_{\lambda}(r) P_{\lambda}(\cos \theta), \quad \lambda = 0, 2, 4, \dots$$



Core: oblate

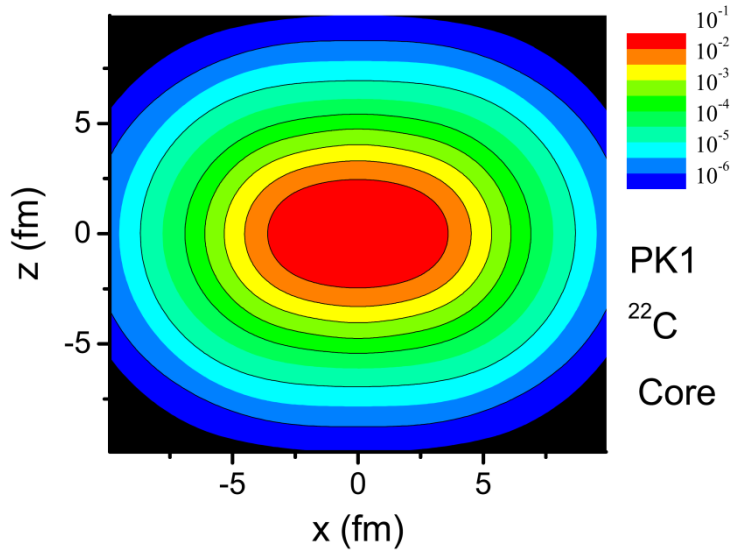


halo: prolate

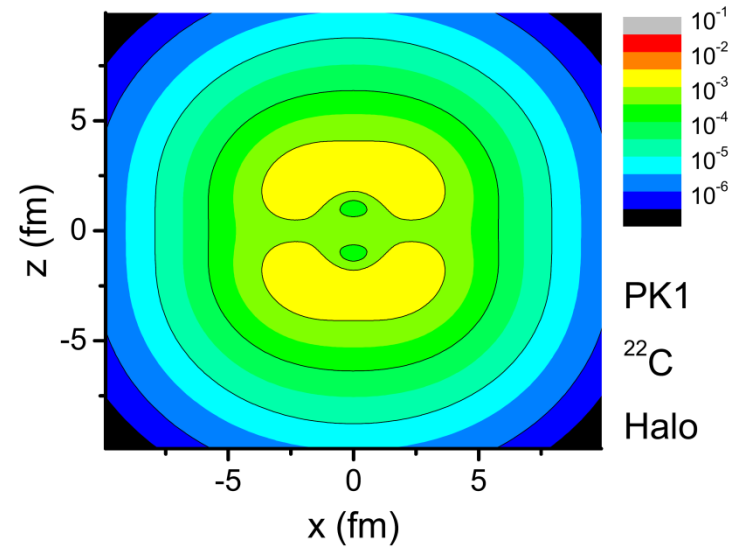
Xiang-Xiang Sun et al., in preparation

# $^{22}\text{C}$ : Density of core & halo---shape decoupling

$$\rho(\mathbf{r}) = \sum_{\lambda} \rho_{\lambda}(r) P_{\lambda}(\cos \theta), \quad \lambda = 0, 2, 4, \dots$$



Core: oblate

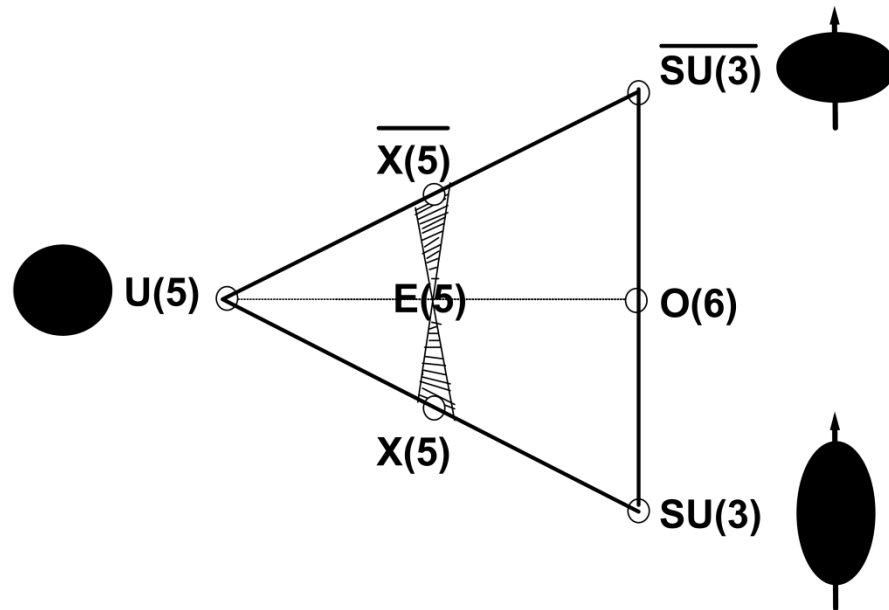


halo: prolate

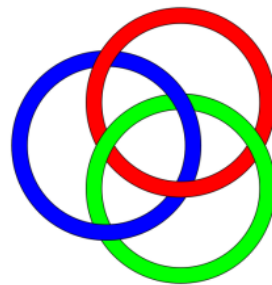
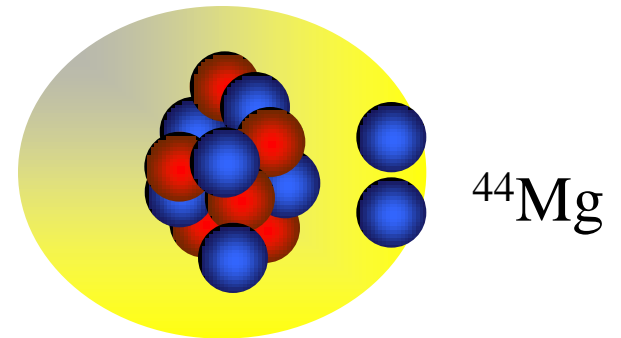
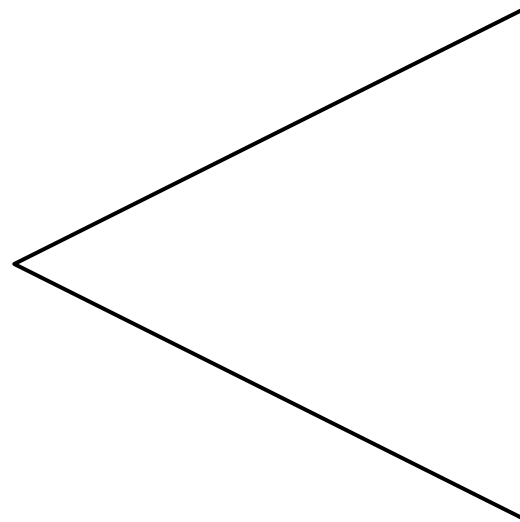
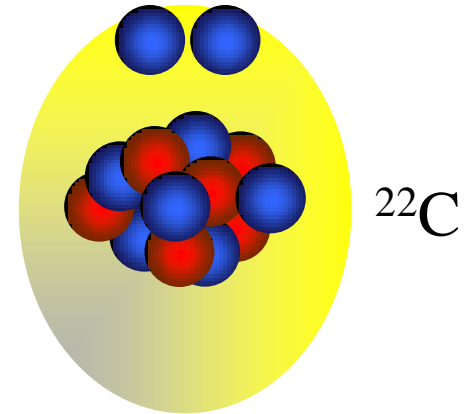
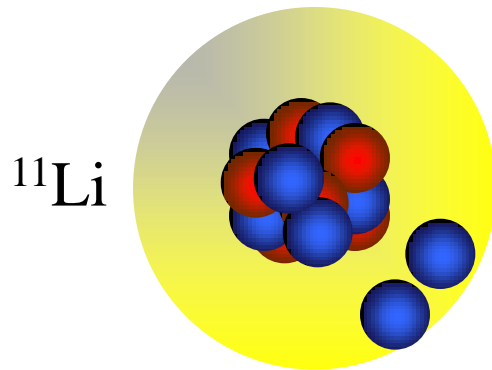
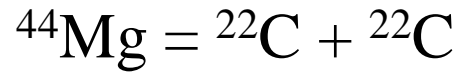
Xiang-Xiang Sun et al., in preparation

# Extended Casten triangle

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# Triangle of Borromean nuclei: $^{11}\text{Li}$ , $^{22}\text{C}$ & $^{44}\text{Mg}$

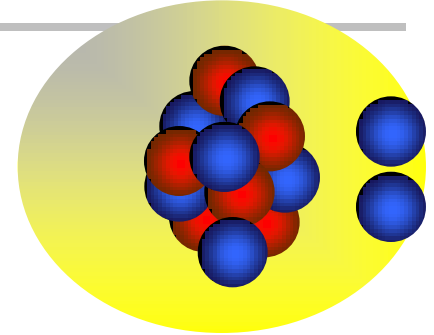


Borromean Ring

# How to probe the shape decoupling?

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- ❑ Larger cross section
- ❑ Narrower momentum distribution
  - Double-hump ?



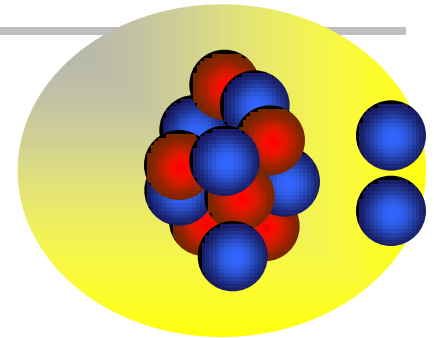
# How to probe the shape decoupling?

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- ❑ Larger cross section
- ❑ Narrower momentum distribution
  - Double-hump ?

Navin...1997\_PRL81-5089

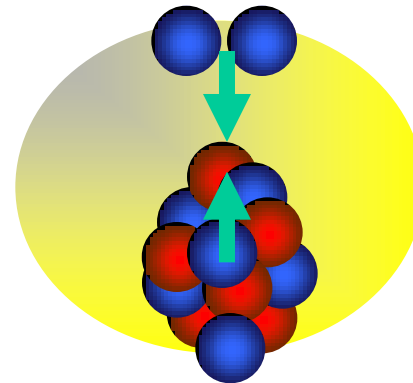
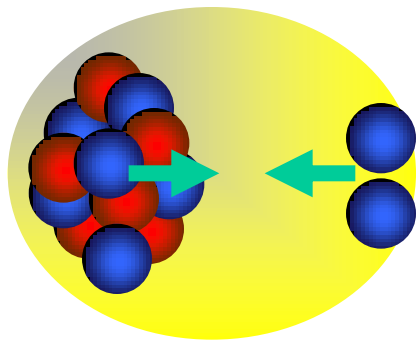
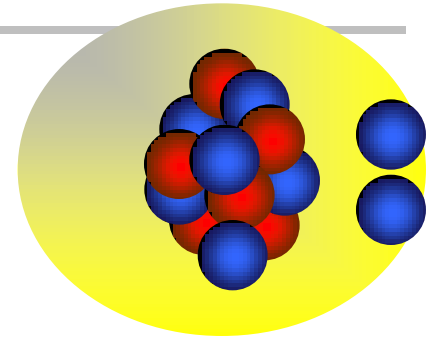
Sakharuk\_Zelevinsky1998\_PRC61-014609





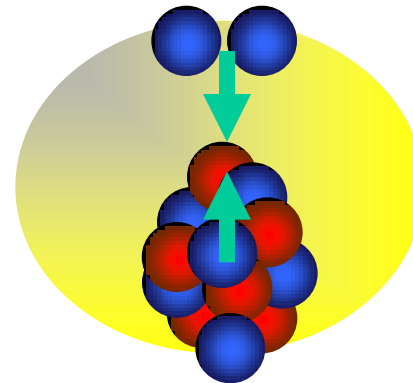
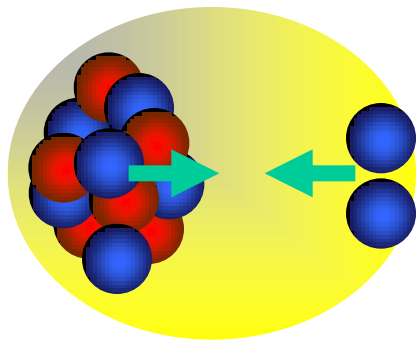
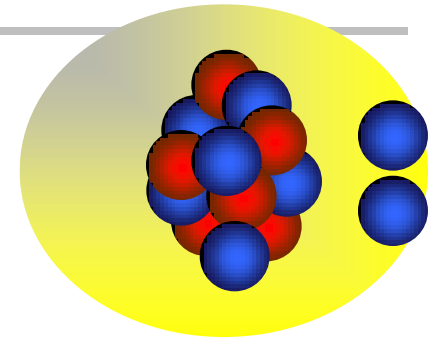
# How to probe the shape decoupling?

- ❑ Larger cross section
- ❑ Narrower momentum distribution
  - Double-hump ?
- ❑ New dipole modes ?



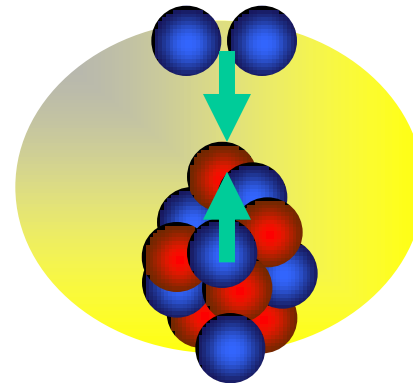
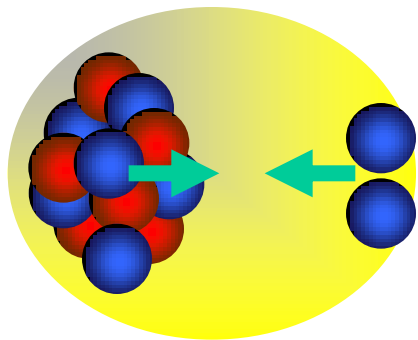
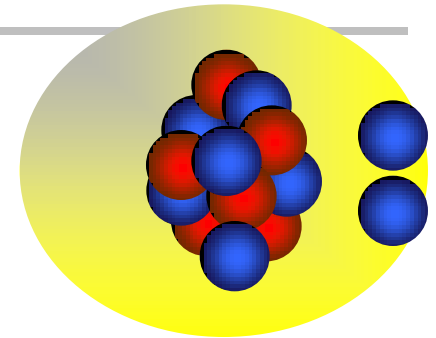
# How to probe the shape decoupling?

- ❑ Larger cross section
- ❑ Narrower momentum distribution
  - Double-hump ?
- ❑ New dipole modes ?
- ❑ Rotation ?



# How to probe the shape decoupling?

- ❑ Larger cross section
- ❑ Narrower momentum distribution
  - Double-hump ?
- ❑ New dipole modes ?
- ❑ Rotation ?
- ❑ Fusion ?



# Summary & perspectives

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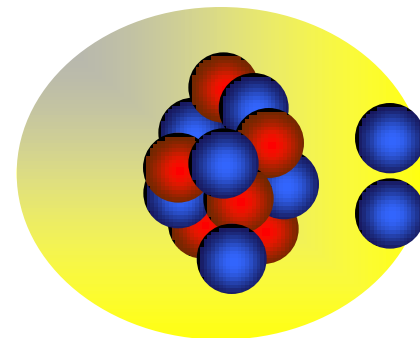
- Deformed relativistic HB theory in a Woods-Saxon basis
  - Occurrence of a halo in deformed nuclei depending on intrinsic structure of valence orbitals
  - Prolate deformed core w/ oblate halo:  $^{44}\text{Mg}$
  - Oblate deformed core w/ prolate halo:  $^{22}\text{C}$
  - Triangle of Borromean nuclei:  $^{11}\text{Li}$ ,  $^{22}\text{C}$  &  $^{44}\text{Mg}$
  
- How to probe shape decoupling ?

Collaborators :

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P. Ring (TU Munich & PKU), Xiang-Xiang Sun (ITP)

Jie Zhao (ITP), En-Guang Zhao (ITP)



Zhou, Shan-Gui

ITP/CAS

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Thanks

谢谢

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URL: [www.itp.ac.cn/~sgzhou](http://www.itp.ac.cn/~sgzhou)