Third-order many-body contributions from the general $\mathcal{O}(p^2)$ NN-contact interaction

(Explorations in many-body perturbation theory)

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- Perturbative calculations of nuclear matter with chiral NN-interactions
- New feature at third order: Particle-hole ring diagrams
- Startup: Antisymmetrized $\mathcal{O}(p^2)$ NN-contact interaction
- Euclidean polarization functions to compute four-loop coefficients
- Third-order ring energy from 1π-exchange and chiral NN-potentials
- Third-order ladder contributions from $\mathcal{O}(p^2)$ NN-contact interaction

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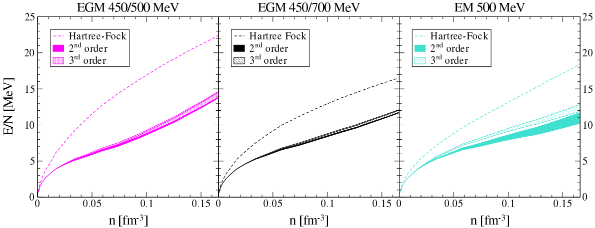
- By using chiral low-momentum interactions, nuclear matter can be calculated reliably in many-body perturbation theory, also at finite T
- Recent work of Darmstadt-group [PRC88, 025802 ('15)]: Neutron matter equation-of-state up to third-order ladder diagrams



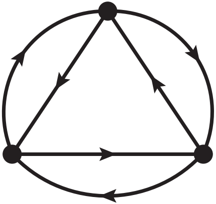
- New feature at third order: particle-hole ring diagrams
- First considered by L. Coraggio, J.W. Holt, et al., PRC91, 054311 ('15): 3rd order ph-diagrams provide at $k_f = 1.3\,\text{fm}^{-1}$ a few MeV repulsion
- No details for 3rd order calc. with NN-potential in partial-wave represent.
- Tensor interaction with mixing $L = J \pm 1$ leads to complicated recouplings
- Present semi-analytical approach allows for tests with model interactions
- Improved calc. of 3rd order ph-diagram [Holt + Kaiser, PRC95, 034326 ('17)]

3rd order ph-ring diagrams with contact-interactions

- Known from low-density expansion: 3rd order ph-contribution from contact-interaction proportional to S-wave scattering length a

$$\bar{E}(k_f)^{3ph} = (g-1)(3-g)\,\frac{a^3 k_f^5}{\pi^4 M}\cdot 2.7950523$$

g is spin-degeneracy factor, density $\rho = g k_f^3/6\pi^2$, $a > 0$ attraction



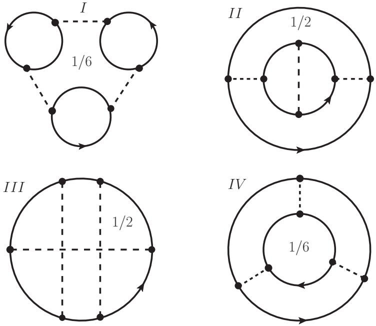
- Nuclear Fermi gas with two different scattering lengths: a_s and a_t

$$\bar{E}(k_f)^{3ph} = 1.0481446\,\frac{(a_s + a_t)k_f^5}{\pi^4 M}\left(5{a_s}^2 + 5{a_t}^2 - 14 a_s a_t\right)$$

- Extend this result to general $\mathcal{O}(p^2)$ NN-contact interaction (9 parameters)
- Derive 3rd order ring energy per particle due to finite-range interactions

3rd order ph-ring diagrams with contact-interactions

- Direct and exchange-type 3-ring diagrams: $I + II + III + IV = (dir - exc)^3/6$



- Antisymmetrized Galilei-invariant contact-interaction (a la Skyrme)

$$V_{\rm Sk} - P_\sigma P_\tau V_{\rm Sk}\Big|_{\vec{q}_{\rm out}\to -\vec{q}_{\rm out}} = (1 - P_\sigma P_\tau)\Big\{ t_0(1 + x_0 P_\sigma) + \frac{t_1}{2}(1 + x_1 P_\sigma)(\vec{q}_{\rm out}^{\,2} + \vec{q}_{\rm in}^{\,2})\Big\}$$
$$+ (1 + P_\sigma P_\tau)\, t_2(1 + x_2 P_\sigma)\, \vec{q}_{\rm out}\cdot\vec{q}_{\rm in} + (1 + P_\tau)\, i W_0 (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{q}_{\rm out}\times\vec{q}_{\rm in})$$

Spin and isospin exchange operators: $P_\sigma = (1+\vec{\sigma}_1\cdot\vec{\sigma}_2)/2$, $P_\tau = (1+\vec{\tau}_1\cdot\vec{\tau}_2)/2$, $\vec{q}_{\rm in} = (\vec{p}_1 - \vec{p}_2)/2$, $\vec{q}_{\rm out} = (\vec{p}_1{}' - \vec{p}_2{}')/2$ momentum differences in initial/final state, completed to general $\mathcal{O}(p^2)$ contact-interaction by adding 2 tensor terms

3rd order ph ring-diagrams from contact interactions

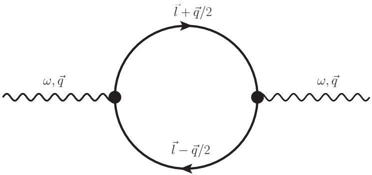
- Triple spin- and isospin-traces:

$$\frac{1}{64}\mathrm{tr}_1\mathrm{tr}_2\mathrm{tr}_3\Big\{(A+B\vec\sigma_1\cdot\vec\sigma_2+C\vec\tau_1\cdot\vec\tau_2+D\vec\sigma_1\cdot\vec\sigma_2\vec\tau_1\cdot\vec\tau_2)(A'+B'\vec\sigma_2\cdot\vec\sigma_3+C'\vec\tau_2\cdot\vec\tau_3+D'\vec\sigma_2\cdot\vec\sigma_3\vec\tau_2\cdot\vec\tau_3)$$
$$\times(A''+B''\vec\sigma_3\cdot\vec\sigma_1+C''\vec\tau_3\cdot\vec\tau_1+D''\vec\sigma_3\cdot\vec\sigma_1\vec\tau_3\cdot\vec\tau_1)\Big\} = AA'A''+3BB'B''+3CC'C''+9DD'D''$$

- Resulting interaction product, exploiting permutational symmetry (123)

$$12t_0^3(1-6x_0^2)+9t_0^2t_1(1-2x_0^2-4x_0x_1)(\vec l_{12}^{\,2}+\vec q^{\,2})+9t_0^2t_2\,[5+4x_2+2x_0^2(1+2x_2)]\,(\vec l_{12}^{\,2}-\vec q^{\,2})$$
$$+\frac{9}{4}t_0t_1^2(1-4x_0x_1-2x_1^2)(\vec l_{12}^{\,2}\vec l_{13}^{\,2}+2\vec l_{12}^{\,2}\vec q^{\,2}+\vec q^{\,4})+\frac{9}{2}t_0t_1t_2\,[5+4x_2+2x_0x_1(1+2x_2)]$$
$$\times(\vec l_{12}^{\,2}\vec l_{13}^{\,2}-\vec q^{\,4})+\frac{9}{4}t_0t_2^2(5+8x_2+2x_2^2)(\vec l_{12}^{\,2}\vec l_{13}^{\,2}-2\vec l_{12}^{\,2}\vec q^{\,2}+\vec q^{\,4})$$
$$+\frac{9}{16}t_1^2t_2\,[5+2x_1^2+4x_2(1+x_1^2)]\,(\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec l_{23}^{\,2}+\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec q^{\,2}-\vec l_{12}^{\,2}\vec q^{\,4}-\vec q^{\,6})$$
$$+\frac{9}{16}t_1t_2^2(5+8x_2+2x_2^2)(\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec l_{23}^{\,2}-\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec q^{\,2}-\vec l_{12}^{\,2}\vec q^{\,4}+\vec q^{\,6})$$
$$+\frac{3}{16}t_1^3(1-6x_1^2)(\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec l_{23}^{\,2}+3\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec q^{\,2}+3\vec l_{12}^{\,2}\vec q^{\,4}+\vec q^{\,6})$$
$$+\frac{t_2^3}{16}(35+84x_2+78x_2^2+28x_2^3)(\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec l_{23}^{\,2}-3\vec l_{12}^{\,2}\vec l_{13}^{\,2}\vec q^{\,2}+3\vec l_{12}^{\,2}\vec q^{\,4}-\vec q^{\,6})$$
$$+9W_0^2(\vec l_{12}\times\vec q)\cdot(\vec l_{13}\times\vec q)\{4t_0(1+x_0)+t_1(1+x_1)(\vec l_{23}^{\,2}+\vec q^{\,2})+5t_2(1+x_2)(\vec l_{23}^{\,2}-\vec q^{\,2})\}$$

$\vec l_{ij}=\vec l_i-\vec l_j$ difference of loop-momenta, $\vec q$ flows through polarization-bubbles



- Finite-temperature formalism in limit $T \to 0$ gives the representation:

$$\Pi(\omega, \vec{q}\,) = \int \frac{d^3 l}{(2\pi)^3} \frac{1}{i\omega + \vec{l}\cdot\vec{q}/M} \Big\{\theta(k_f - |\vec{l} - \vec{q}/2|) - \theta(k_f - |\vec{l} + \vec{q}/2|)\Big\}$$

Fermionic Matsubara-sum yields Fermi-distributions $\to$ step-functions

- Euclidean polarization function: $\Pi[1](\omega, \vec{q}\,) = Mk_f\, Q_0(s, \kappa)/(4\pi^2 s)$, setting $|\vec{q}\,| = 2sk_f$, $\omega = 2s\kappa k_f^2/M$, agrees with alternative derivations

$$Q_0(s, \kappa) = s - s\kappa \arctan\frac{1+s}{\kappa} - s\kappa \arctan\frac{1-s}{\kappa} + \frac{1}{4}(1 - s^2 + \kappa^2)\ln\frac{(1+s)^2 + \kappa^2}{(1-s)^2 + \kappa^2}$$

- For contact-interaction: 3-loops factorize into "cube" of 1-loop

$$\Pi[\vec{l}\,] = -\frac{Mk_f^2}{4\pi^2 s}\, i\kappa\, Q_0(s, \kappa)\,\hat{q}\,, \quad \Pi[l_i l_j] = \frac{Mk_f^3}{4\pi^2 s}\left\{\frac{\delta_{ij}}{3}\, Q_1(s, \kappa) + \left(\hat{q}_i \hat{q}_j - \frac{\delta_{ij}}{3}\right) Q_2(s, \kappa)\right\}$$

3rd order ph-ring energy per particle

- Translate products of scalar-products into cubic expressions in $Q_j(s,\kappa)$
- Resulting 3-ring energy per particle for isospin-symmetric nuclear matter

$$
\begin{aligned}
\bar{E}(k_f)^{\rm 3ph} = \frac{M^2 k_f^5}{32\pi^7}\Big\{ & t_0^3(1-6x_0^2)\mathcal{N}_1 + k_f^2\, t_0^2 t_1(1-2x_0^2-4x_0x_1)\mathcal{N}_2 \\
& + k_f^2\, t_0^2 t_2\left[5+4x_2+2x_0^2(1+2x_2)\right]\mathcal{N}_3 + k_f^4\, t_0 t_1^2(4x_0x_1+2x_1^2-1)\mathcal{N}_4 \\
& + k_f^4\, t_0 t_1 t_2\left[\frac{5}{2}+x_0x_1(1+2x_2)+2x_2\right]\mathcal{N}_5 + k_f^4\, t_0 t_2^2\left[\frac{5}{2}+4x_2+x_2^2\right]\mathcal{N}_6 \\
& + k_f^6\, t_1^2 t_2\left[\frac{5}{2}+x_1^2+2x_2(1+x_1^2)\right]\mathcal{N}_7 + k_f^6\, t_1 t_2^2\left[\frac{5}{2}+4x_2+x_2^2\right]\mathcal{N}_8 \\
& + k_f^6\, t_1^3(1-6x_1^2)\mathcal{N}_9 + k_f^6\, t_2^3\left[\frac{5}{4}+3x_2+\frac{39}{14}x_2^2+x_2^3\right]\mathcal{N}_{10} \\
& + k_f^4\, W_0^2\left[t_0(1+x_0)\mathcal{N}_{11} + k_f^2\, t_1(1+x_1)\mathcal{N}_{12} + k_f^2\, t_2(1+x_2)\mathcal{N}_{13}\right]\Big\}
\end{aligned}
$$

- For neutron matter isospin-factors change, no $\mathcal{N}_{11,12}$ terms ($^3S_1$-state)

$$
\begin{aligned}
\bar{E}_n(k_n)^{\rm 3ph} = \frac{M^2 k_n^5}{96\pi^7}\Big\{ & t_0^3(x_0-1)^3\mathcal{N}_1 + k_n^2\, t_0^2 t_1(x_0-1)^2(x_1-1)\mathcal{N}_2 \\
& + k_n^2\, t_0^2 t_2(x_0-1)^2(x_2+1)3\mathcal{N}_3 + k_n^4\, t_0 t_1^2(1-x_0)(x_1-1)^2\mathcal{N}_4 \\
& + k_n^4\, t_0 t_1 t_2(x_0-1)(x_1-1)(x_2+1)\frac{3\mathcal{N}_5}{2} + k_n^4\, t_0 t_2^2(1-x_0)(1+x_2)^2\,\frac{3\mathcal{N}_6}{2} \\
& + k_n^6\, t_1^2 t_2(1-x_1)^2(1+x_2)\frac{3\mathcal{N}_7}{2} + k_n^6\, t_1 t_2^2(1-x_1)(1+x_2)^2\,\frac{3\mathcal{N}_8}{2} \\
& + k_n^6\, t_1^3(x_1-1)^3\mathcal{N}_9 + k_n^6\, t_2^3(1+x_2)^3\,\frac{45\mathcal{N}_{10}}{28} + k_n^6\, W_0^2 t_2(1+x_2)\frac{8\mathcal{N}_{13}}{5}\Big\}
\end{aligned}
$$

- For fourth loop-integral $\int d\omega \int d^3q/(2\pi)^4$ introduce polar coordinates in $s\kappa$-plane: $s = r\cos\varphi$, $\kappa = r\sin\varphi$

$$\mathcal{N}_1 = 12\int_0^\infty dr \int_0^{\pi/2} d\varphi\, r\,[Q_0(s,\kappa)]^3 = 4.1925784$$

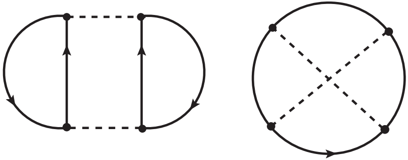
- Dimensional regularization of $\mathcal{N}_j$: subtract power divergences $\int_0^{r_{\max}} dr\, r^{2n}$

$$\mathcal{N}_2 = \int_0^\infty dr \int_0^{\pi/2} d\varphi \Big\{ 18r\,Q_0^2\,[Q_1 + (2s^2+\kappa^2)Q_0] - \frac{16}{3}\cos^3\varphi\,(2+\cos 2\varphi)\Big\} = -0.4633512$$

$$\mathcal{N}_3 = \int_0^\infty dr \int_0^{\pi/2} d\varphi \Big\{ 18r\,Q_0^2\,[Q_1 + (\kappa^2 - 2s^2)Q_0] + \frac{16}{3}\cos^3\varphi\,\cos 2\varphi\Big\} = -2.259163$$

- Expand integrands up to r^{-4} and include $a_2/r_{\max} + a_4/3r_{\max}^3$ for outside region $r > r_{\max}$, <u>accurate</u> and well converged results for $20 < r_{\max} < 40$
- Method verified by rederiving analytical results for 2nd order contribut.
- Remaining four-loop coefficients in dimensional regularization:

$$\mathcal{N}_4 = 2.902123\,,\quad \mathcal{N}_5 = 2.12658\,,\quad \mathcal{N}_6 = 0.438970\,,\quad \mathcal{N}_7 = 0.48756\,,$$
$$\mathcal{N}_8 = -0.27614\,,\quad \mathcal{N}_9 = -1.01924\,,\quad \mathcal{N}_{10} = 0.315484\,,$$
$$\mathcal{N}_{11} = -2.244200\,,\quad \mathcal{N}_{12} = -2.30577\,,\quad \mathcal{N}_{13} = 2.53887$$



- 2nd order particle-particle ladder diagram = 2-ring particle-hole diagram, factor 1/2 not to double-count direct + exchange term via $(dir - exc)^2$

$$\bar{E}(k_f)^{\rm 2nd} = \frac{3Mk_f^4}{32\pi^5}\Big\{t_0^2(1+x_0^2)\mathcal{Z}_1 + k_f^2\, t_0 t_1(1+x_0x_1)\mathcal{Z}_2 + k_f^4\, t_1^2(1+x_1^2)\mathcal{Z}_3 + k_f^4\, t_2^2(5+8x_2+5x_2^2)\mathcal{Z}_4 + k_f^4\, W_0^2\,\mathcal{Z}_5\Big\}$$

- Three-loop coefficients $\mathcal{Z}_j$ in dimensional regularization:

$$\mathcal{Z}_1 = -8\int_0^\infty dr\int_0^{\pi/2} d\varphi\Big\{3rs\,Q_0^2 - \frac{4}{3}\cos^3\varphi\Big\} = 3.451697 = \frac{4\pi}{35}(11 - 2\ln 2)\,,$$

$$\mathcal{Z}_2 = -24\int_0^\infty dr\int_0^{\pi/2} d\varphi\Big\{rs\,Q_0[Q_1 + (2s^2+\kappa^2)Q_0]\Big\}_{\rm reg} = 3.99902 = \frac{8\pi}{945}(167 - 24\ln 2)\,,$$

$$\mathcal{Z}_3 = 1.37573 = \frac{\pi}{10395}(4943 - 564\ln 2)\,, \qquad \mathcal{Z}_4 = 0.0931718 = \frac{\pi}{31185}(1033 - 156\ln 2)\,,$$

$$\mathcal{Z}_5 = 128\int_0^\infty dr\int_0^{\pi/2} d\varphi\Big\{rs^3\,Q_0(Q_2 - Q_1)\Big\}_{\rm reg} = 2.70935 = \frac{16\pi}{10395}(631 - 102\ln 2)$$

- Contact-interaction of $\mathcal{O}(p^2)$ gets completed by adding two tensor terms

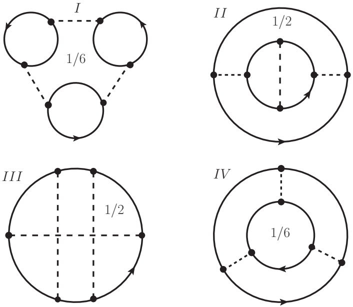
$$V_{\rm ten} - P_\sigma P_\tau V_{\rm ten}\Big|_{\vec q_{\rm out}\to -\vec q_{\rm out}} = (1-P_\tau)\, t_4 \Big\{ \vec\sigma_1\cdot\vec q_{\rm out}\,\vec\sigma_2\cdot\vec q_{\rm out} + \vec\sigma_1\cdot\vec q_{\rm in}\,\vec\sigma_2\cdot\vec q_{\rm in} - \frac{1}{3}\vec\sigma_1\cdot\vec\sigma_2(\vec q_{\rm out}^{\,2}+\vec q_{\rm in}^{\,2})\Big\}$$
$$+(1+P_\tau)\, t_5 \Big\{ \vec\sigma_1\cdot\vec q_{\rm out}\,\vec\sigma_2\cdot\vec q_{\rm in} + \vec\sigma_1\cdot\vec q_{\rm in}\,\vec\sigma_2\cdot\vec q_{\rm out} - \frac{2}{3}\vec\sigma_1\cdot\vec\sigma_2\,\vec q_{\rm out}\cdot\vec q_{\rm in}\Big\}$$

- Tensor contributions to 3-ring energy per particle (t_4 absent in n-matter)

$$\begin{aligned}
\bar E(k_f)^{\rm 3ph} &= \frac{M^2 k_f^9}{32\pi^7}\Big\{ k_f^2\, W_0^2 \big[t_4\,\mathcal{N}_{14} + t_5\,\mathcal{N}_{15}\big] \\
&\quad + t_4^2\big[t_0(x_0-2)\mathcal{N}_{16} + k_f^2\, t_1(x_1-2)\mathcal{N}_{17} + k_f^2\, t_2(x_2+2)\mathcal{N}_{18}\big] \\
&\quad + t_4 t_5\big[t_0 x_0\,\mathcal{N}_{19} + k_f^2\, t_1 x_1\,\mathcal{N}_{20} + k_f^2\, t_2 x_2\,\mathcal{N}_{21}\big] \\
&\quad + t_5^2\big[t_0(3x_0-2)\mathcal{N}_{22} + k_f^2\, t_1(3x_1-2)\mathcal{N}_{23} + k_f^2\, t_2(3x_2+2)\mathcal{N}_{24}\big] \\
&\quad + k_f^2\big[t_4^3\,\mathcal{N}_{25} + t_4^2 t_5\,\mathcal{N}_{26} + t_4 t_5^2\,\mathcal{N}_{27} + t_5^3\,\mathcal{N}_{28}\big]\Big\}\,, \\
\bar E_n(k_n)^{\rm 3ph} &= \frac{M^2 k_n^9}{24\pi^7}\Big\{ k_n^2\,\frac{2t_5}{5}\big[W_0^2\,\mathcal{N}_{15} + t_5^2\,\mathcal{N}_{28}\big] \\
&\quad + t_5^2\big[t_0(x_0-1)\mathcal{N}_{22} + k_n^2\, t_1(x_1-1)\mathcal{N}_{23} + k_n^2\, t_2(x_2+1)\mathcal{N}_{24}\big]\Big\}
\end{aligned}$$

- Spin-traces $\to$ triple scalar-products $\to$ cubic expressions in $Q_j(s,\kappa)$
- Accurate four-loop coefficients calculated in dimensional regularization:
  $\mathcal{N}_{14} = 0.8722$, $\mathcal{N}_{15} = -5.0175$, $\mathcal{N}_{16} = -2.9160$, $\mathcal{N}_{17} = -2.7834$, $\mathcal{N}_{18} = 0.42202$,
  $\mathcal{N}_{19} = 10.154$, $\mathcal{N}_{20} = 8.0564$, $\mathcal{N}_{21} = -0.7457$, $\mathcal{N}_{22} = -1.0762$, $\dots$, $\mathcal{N}_{28} = 5.4015$

3rd order ring diagrams with 1π-exchange



- Antisymmetrized one-pion exchange interaction:

$$V_{1\pi} = -\frac{g_A^2}{4f_\pi^2}\left\{\vec{\tau}_1\cdot\vec{\tau}_2\,\frac{\vec{\sigma}_1\cdot(\vec{q}_{\rm in}-\vec{q}_{\rm out})\,\vec{\sigma}_2\cdot(\vec{q}_{\rm in}-\vec{q}_{\rm out})}{m_\pi^2+(\vec{q}_{\rm in}-\vec{q}_{\rm out})^2}\right.$$
$$\left.+\frac{1}{4}(\vec{\tau}_1\cdot\vec{\tau}_2-3)\,\frac{2\vec{\sigma}_1\cdot(\vec{q}_{\rm in}+\vec{q}_{\rm out})\,\vec{\sigma}_2\cdot(\vec{q}_{\rm in}+\vec{q}_{\rm out})+(1-\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{q}_{\rm in}+\vec{q}_{\rm out})^2}{m_\pi^2+(\vec{q}_{\rm in}+\vec{q}_{\rm out})^2}\right\}$$

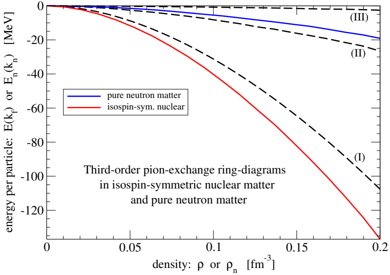
- Direct 3-ring energy per particle in symmetric nuclear matter: $\beta = m_\pi^2/4k_f^2$

$$\bar{E}(k_f)^{(\mathrm{I})} = -\frac{3g_A^6 M^2 k_f^5}{32\pi^7 f_\pi^6}\int_0^\infty ds\int_0^\infty d\kappa\left[\frac{s^2 Q_0(s,\kappa)}{s^2+\beta}\right]^3 < 0\,, \qquad \text{sizeable!}$$

- Contribution of exchange-type diagram $II = -dir^2 \cdot exc/2$

$$\bar{E}(k_f)^{(\mathrm{II})} = \frac{9g_A^6 M^2 k_f^5}{(2\pi)^7 f_\pi^6} \int_0^\infty ds \int_0^\infty d\kappa \int_0^1 dl_1 \int_0^1 dl_2 \int_{-l_1}^{l_1} dx \int_{-l_2}^{l_2} dy \, \frac{l_1 l_2 s^4 \, Q_0(s,\kappa)\,(s^2+\beta)^{-2}}{[(s+x)^2+\kappa^2][(s+y)^2+\kappa^2]} \Big\{(s+x)(s+y)$$
$$+[2\beta+(x-y)^2]\,[\kappa^2-(s+x)(s+y)]\,W_a^{-1/2} - [2\beta+(2s+x+y)^2]\,[\kappa^2+(s+x)(s+y)]\,W_b^{-1/2}\Big\}$$

- Outcome for third-order pion ring energy per particle

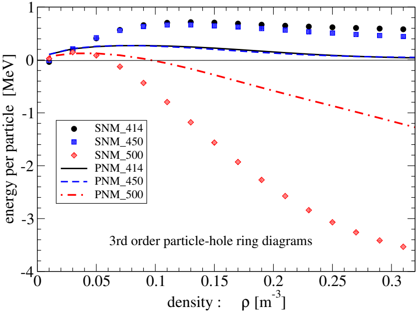


- Exchange-type contributions suppressed but attractive: $|I| \gg |II| \gg |III|$
- Net result $\bar{E}(k_{f0}) \simeq -92\,\mathrm{MeV}$ at $\rho_0 = 0.16\,\mathrm{fm}^{-3}$ very large, $\bar{E}_n \simeq -12.7\,\mathrm{MeV}$
- Semi-analytical approach allows to benchmark numerical computations of 3ph ring diagram based on partial-wave decomp. for test-interactions

- Now, the real thing: third-order ring diagrams with chiral NN-interaction
- Partial-wave based num. computation tested for model-type interactions:

$$V_{\text{central}} = -\frac{g^2}{m^2+q^2}\,, \qquad V_{\text{tensor}} = -g^2\,\vec{\tau}_1\cdot\vec{\tau}_2\,\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{(m^2+q^2)^2}$$

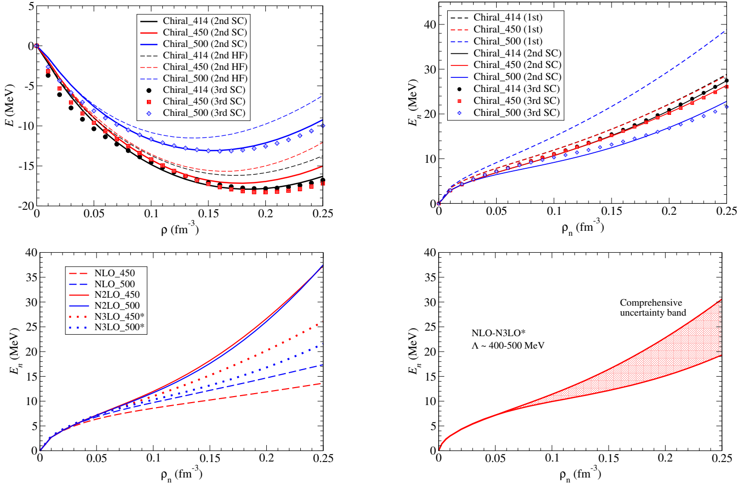
- For pure spin-orbit interaction: 3rd order ring diagrams vanish identically, spin-trace zero $\leftrightarrow$ delicate cancelations in multiple partial-wave sums



- For N^{3}LO chiral NN-potential with low resolution scale $\Lambda = (410-500)$ MeV, 3rd order particle-hole contribution bounded by a few MeV up to $\rho = 2\rho_0$
- Extensive numerical computations performed by J.W. Holt (TAMU)

Nuclear and neutron matter at third order from chiral NN-interactions

- Third-order calculations with selfconsistent single-particle energies, chiral N^3LO potential + 3N interaction with cutoffs $\Lambda = (414, 450, 500)$ MeV



- Convergence of many-body perturbation theory well under control
- Largest uncertainty from higher-order chiral forces [PRC95, 034326 ('17)]

Third-order ladder diagrams with contact-interactions

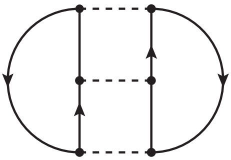
- Known from low-density expansion: contributions up to 3rd order ladder diagrams from contact-interaction prop. to S-wave scattering length a

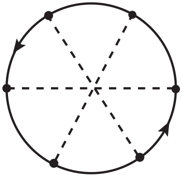
$$\bar{E}(k_f)^{\rm lad} = (g-1)\frac{k_f^2}{M}\left\{-\frac{ak_f}{3\pi} + \frac{2}{35}(11-2\ln 2)\left(\frac{ak_f}{\pi}\right)^2 - 1.1716223\left(\frac{ak_f}{\pi}\right)^3\right\}$$

- Extend this result to general $\mathcal{O}(p^2)$ contact-interaction (7+2 parameters)

$$V_{\rm cont} = t_0(1+x_0P_\sigma) + \frac{t_1}{2}(1+x_1P_\sigma)(\vec{q}_{\rm out}^{\,2} + \vec{q}_{\rm in}^{\,2}) + t_2(1+x_2P_\sigma)\,\vec{q}_{\rm out}\cdot\vec{q}_{\rm in}$$
$$+iW_0(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_{\rm out}\times\vec{q}_{\rm in}) + V_{\rm ten}(t_4,t_5)$$

- Third-order Hartree (direct) and Fock (exchange) ladder diagrams



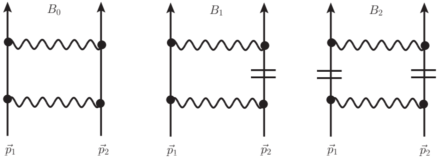


- Twice-iterated interaction in medium integrated over two Fermi spheres

In-medium loop functions

- Inner two loop integrals factorize via complex in-medium loop function

$$i\left(\frac{\theta(|\vec{p}\,|-k_f)}{p_0-\vec{p}^{\,2}/2M+i\epsilon}+\frac{\theta(k_f-|\vec{p}\,|)}{p_0-\vec{p}^{\,2}/2M-i\epsilon}\right)=\frac{i}{p_0-\vec{p}^{\,2}/2M+i\epsilon}-2\pi\,\delta(p_0-\vec{p}^{\,2}/2M)\,\theta(k_f-|\vec{p}\,|)$$



- Its real part: $\vec{P}=(\vec{p}_1+\vec{p}_2)/2$, $\vec{q}=(\vec{p}_1-\vec{p}_2)/2$ where $|\vec{p}_{1,2}|<k_f$

$$\int\frac{d^3l}{(2\pi)^3}\,\frac{M}{\vec{l}^{\,2}-\vec{q}^{\,2}}\Big\{\theta(k_f-|\vec{P}-\vec{l}\,|)+\theta(k_f-|\vec{P}+\vec{l}\,|)-1\Big\}=\frac{Mk_f}{4\pi^2}R(s,\kappa)+0,$$

$$R(s,\kappa)=2+\frac{1}{2s}[1-(s+\kappa)^2]\ln\frac{1+s+\kappa}{|1-s-\kappa|}+\frac{1}{2s}[1-(s-\kappa)^2]\ln\frac{1+s-\kappa}{1-s+\kappa}$$

dimensionless variables $s=|\vec{P}|/k_f$ and $\kappa=|\vec{q}|/k_f$ satisfy $s^2+\kappa^2<1$

- Its imaginary part: dropping $[1-\theta(..)][1-\theta(..)]\to 0$ by Pauli-blocking

$$\int\frac{d^3l}{(2\pi)^3}\,M\,\delta(\vec{l}^{\,2}-\vec{q}^{\,2})\,\theta(k_f-|\vec{P}-\vec{l}\,|)\,\theta(k_f-|\vec{P}+\vec{l}\,|)=\frac{Mk_f}{4\pi^2}I(s,\kappa),$$

$$I(s,\kappa)=\kappa\,\theta(1-s-\kappa)+\frac{1}{2s}(1-s^2-\kappa^2)\,\theta(s+\kappa-1)$$

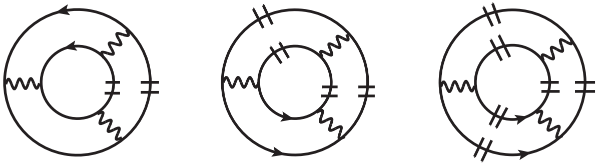
- P-wave interactions introduce factor $l_i l_j$ in loop integrals, decompose this tensor into a transversal and a longitudinal part

$$\text{real part:} \quad \frac{Mk_f^3}{12\pi^2}\Big\{R_\perp(s,\kappa)[\delta_{ij}-\hat{P}_i\hat{P}_j]+R_\|(s,\kappa)\hat{P}_i\hat{P}_j\Big\},$$

$$\text{imaginary part:} \quad \frac{Mk_f^3}{12\pi^2}\Big\{I_\perp(s,\kappa)[\delta_{ij}-\hat{P}_i\hat{P}_j]+I_\|(s,\kappa)\hat{P}_i\hat{P}_j\Big\},$$

$$\text{relations:} \quad 2R_\perp+R_\| = 4+3\kappa^2 R\,, \quad 2I_\perp+I_\| = 3\kappa^2 I$$

- Mixing terms of S- and P-wave interactions vanish in a medium with one single Fermi momentum k_f, i.e. without isospin- or spin-asymmetries, weight functions $\theta(k_f-|\vec{P}\pm\vec{l}|)$ in loop integral are even under $\vec{l}\to-\vec{l}$
- Proper real-valued integrand for energy density $\rho\bar{E}$ at third order



$$(R-i\pi I)^2+(R-i\pi I)(2i\pi I)+\frac{1}{3}(2i\pi I)^2 = R^2-\frac{\pi^2}{3}I^2$$

Evaluation of 3rd order ladder diagrams: S-wave interactions

- Results in pure neutron matter (only $^1S_0, P_\sigma \to -1$), density $\rho_n = k_n^3/3\pi^2$

$$\bar{E}_n(k_n)^{\rm 3lad} = \frac{M^2 k_n^5}{64\pi^6}\Big\{ t_0^3(1-x_0)^3 B_1 + k_n^2\, t_0^2 t_1 (1-x_0)^2(1-x_1) B_2 + k_n^4\, t_0 t_1^2 (1-x_0)(1-x_1)^2 B_3 + k_n^6\, t_1^3 (1-x_1)^3 B_4 \Big\}$$

- Four-loop coefficients with high numerical accuracy, $t_0(x_0-1) = 4\pi a/M$

$$B_1 = 8\int_0^1 ds\, s^2 \int_0^{\sqrt{1-s^2}} d\kappa\, \kappa\, I(s,\kappa)\big[3R^2 - \pi^2 I^2\big] = 1.1716223\,,$$

$$B_2 = 8\int_0^1 ds\, s^2 \int_0^{\sqrt{1-s^2}} d\kappa\, \kappa\, I\big[3\kappa^2(3R^2-\pi^2 I^2)+8R\big] = 1.9893144\,,$$

$$B_3 = 1.360736\,, \qquad B_4 = 0.3344923$$

- Result in isospin-symmetric nuclear matter, density $\rho = 2k_f^3/3\pi^2$

$$\bar{E}(k_f)^{\rm 3lad} = \frac{3M^2 k_f^5}{64\pi^6}\Big\{ t_0^3(1+3x_0^2) B_1 + k_f^2\, t_0^2 t_1 (1+x_0^2+2x_0 x_1) B_2 + k_f^4\, t_0 t_1^2 (1+2x_0 x_1 + x_1^2) B_3 + k_f^6\, t_1^3 (1+3x_1^2) B_4 \Big\}$$

- Sum contributions from isotriplet $^1S_0$- and isosinglet $^3S_1$-states

$$\big[3t_0^3(1-x_0)^3 + 3t_0^3(1+x_0)^3\big]/2 = 3t_0^3(1+3x_0^2)\,,$$

$$\big[3t_0^2 t_1(1-x_0)^2(1-x_1) + 3t_0^2 t_1(1+x_0)^2(1+x_1)\big]/2 = 3t_0^2 t_1(1+x_0^2+2x_0 x_1)$$

- Results for neutron matter (only $^3P_{0,1,2}$) and symmetric nuclear matter

$$\bar{E}_n(k_n)^{\rm 3lad} = \frac{M^2 k_n^{11}}{64\pi^6}\Big\{ t_2^3(1+x_2)^3 B_5 + t_2(1+x_2)W_0^2 B_6 + W_0^3 B_7 \Big\},$$

$$\bar{E}(k_f)^{\rm 3lad} = \frac{M^2 k_f^{11}}{64\pi^6}\Big\{ \frac{t_2^3}{3}(5+12x_2+15x_2^2+4x_2^3)B_5 + \frac{3t_2}{2}(1+x_2)W_0^2 B_6 + \frac{3}{2}W_0^3 B_7 \Big\}$$

- Spin-traces: Hartree $\mathrm{tr}_{\sigma_1}\mathrm{tr}_{\sigma_2}$, Fock $\mathrm{tr}_{\sigma_1=\sigma_2}$ taking care of ordering
- t_2^3-term: $9(1+x_2)^3 \to \big[3(1-x_2)^3 + 27(1+x_2)^3\big]/2 = 3(5+12x_2+15x_2^2+4x_2^3)$
- Isotriplet ^3P_J-interactions: 3(Hart+Fock)=4Hart+2Fock, thus: Hart=Fock
- Pertinent four-loop coeffcients computed from double-integrals

$$B_5 = \frac{8}{9}\int_0^1 ds\, s^2 \int_0^{\sqrt{1-s^2}} d\kappa\, \kappa\, \Big[2I_\perp(3R_\perp^2 - \pi^2 I_\perp^2) + I_\parallel(3R_\parallel^2 - \pi^2 I_\parallel^2)\Big] = 0.06699116\,,$$

$$B_6 = \frac{128}{9}\int_0^1 ds\, s^2 \int_0^{\sqrt{1-s^2}} d\kappa\, \kappa \Big\{ I_\perp\Big[3R_\perp^2 + 2R_\perp R_\parallel + R_\parallel^2 - \frac{\pi^2}{3}(3I_\perp^2 + 2I_\perp I_\parallel + I_\parallel^2)\Big] + I_\parallel\Big[R_\perp(R_\perp + 2R_\parallel) - \frac{\pi^2}{3}I_\perp(I_\perp + 2I_\parallel)\Big]\Big\} = 1.327456\,,$$

$$B_7 = \frac{128}{9}\int_0^1 ds\, s^2 \int_0^{\sqrt{1-s^2}} d\kappa\, \kappa \Big\{ 2I_\perp\Big(R_\perp R_\parallel - \frac{\pi^2}{3}I_\perp I_\parallel\Big) + I_\parallel\Big(R_\perp^2 - \frac{\pi^2}{3}I_\perp^2\Big)\Big\} = 0.4527642$$

- Tensor contributions in neutron matter (only ^3P_J) and symmetric N-matter

$$\bar{E}_n(k_n)^{\rm 3lad} = \frac{M^2 k_n^{11}}{64\pi^6}\left\{ t_5 W_0^2 B_8 + t_5^2 t_2(1+x_2) B_9 + t_5^2 W_0 B_{10} + t_5^3 B_{11} \right\},$$

$$\bar{E}(k_f)^{\rm 3lad} = \frac{M^2 k_f^{11}}{64\pi^6}\left\{ \frac{3}{2} t_5 W_0^2 B_8 + \frac{3}{2} t_5^2 t_2(1+x_2) B_9 + \frac{3}{2} t_5^2 W_0 B_{10} + \frac{3}{2} t_5^3 B_{11} + k_f^{-2}\, t_4^2 t_0(1+x_0) B_{12} + t_4^2 t_1(1+x_1) B_{13} \right\}$$

- t_5-term acts in ^3P_J-states, $t_5 t_2^2$ and $t_5 t_2 W_0$ interferences give spin-trace =0
  $B_8 = -2.243263\,,\quad B_9 = 2.042028\,,\quad B_{10} = 6.66812\,,\quad B_{11} = -1.655323$
- Tensorial t_4-term responsible for $^3S_1\,^3D_1$-mixing: $P_\sigma \to 1$, param. $t_{0,1}(1+x_{0,1})$
- Spin-traces give: Hart = Fock, relevant in nuclear matter: 4 Hart - 2 Fock
- Pertinent four-loop coefficients: $B_{12} = 2.421103\,,\quad B_{13} = 1.559127$

Summary: Semi-analytical many-body calculations

- Third-order particle-hole ring diagrams from $\mathcal{O}(p^2)$ contact-interaction
- 3-ring energy per particle from 1π-exchange (very large) and low-momentum chiral NN-potentials (moderately small)
- Third-order ladder diagrams from general $\mathcal{O}(p^2)$ NN-contact interaction
- Four-loop coefficients computed accurately from double-integrals