

Uncertainties in constraining low-energy constants from ${}^3\text{H}$ β decay



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Philipp Klos

with A. Carbone, K. Hebeler, J. Menéndez, and A. Schwenk

SFB110: Nuclear Dynamics and Threshold Phenomena
Ruhr-Universität Bochum, April 6, 2017



European Research Council
Established by the European Commission

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+ Few-neutron resonances

with S. Gandolfi, H.-W. Hammer, J. E. Lynn, and A. Schwenk

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- Motivation
- Constraining c_D and c_E
- Chiral currents for ${}^3\text{H}$ beta decay

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- Motivation
- Constraining c_D and c_E
- Chiral currents for ${}^3\text{H}$ beta decay

Chiral effective field theory

- ▶ Chiral EFT describes consistently both nuclear forces and currents

Epelbaum, Hammer, and Meißner, RMP **81**, 1773 (2009)

- ▶ Same low-energy constants appear in nuclear forces and currents

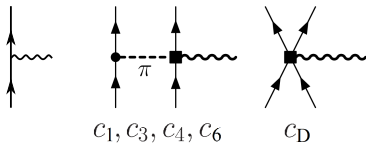
- ▶ Leading axial-vector two-body currents completely determined

Park *et al.*, PRC **67**, 055206 (2003)

Gårdestig and Phillips, PRL **96**, 232301 (2006)

Gazit, Quaglioni, and Navrátil, PRL **103**, 102502 (2009)

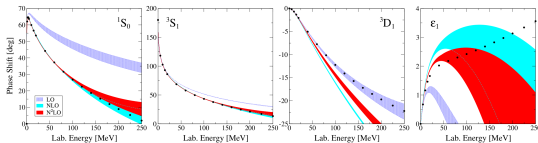
	2N force	3N force	4N force
LO		—	—
NLO		—	—
N ² LO			—
N ³ LO			



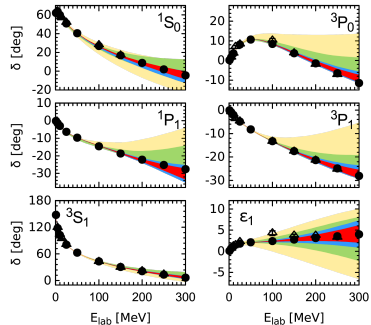
Constraining low-energy constants

- ▶ Low-energy constants in principle defined through underlying theory of QCD
- ▶ In practice fit to experimental data
- ▶ Up to NLO low-energy constants fit to NN and πN scattering data

Patrick Reinert's talk yesterday



Gezerlis et al., PRL 111, 032501 (2013)



E. Epelbaum et al., PRL 115, 122301, (2015)

Chiral forces need to be regularized

Local regulator

$$f_{\Lambda}^{\text{loc}}(\mathbf{p}, \mathbf{p}') = \exp \left[-\frac{(\mathbf{p} - \mathbf{p}')^4}{\Lambda^4} \right]$$

with cutoff Λ

Non-local regulator

$$f_{\Lambda}^{\text{non-loc}}(p^2, p'^2) = \exp \left[-\frac{(p^{2n} + p'^{2n})}{\Lambda^{2n}} \right]$$

Many different regularization schemes and cutoffs currently used!

NN potentials and regulators

	Potentials	Regulators
Nonlocal	e.g. EM, EGM; Carlsson et al., PRX 6, 011019 (2016)	$\exp [-(p^{2n} + p'^{2n})/\Lambda^{2n}]$
Local	Gezerlis et al., PRL 111, 122301 (2013)	$(1 - \exp(-(r/R_0)^4))$
Semilocal	Epelbaum et al., EPJ A 51, 53 (2015), PRL 115, 122301 (2015)	mixture

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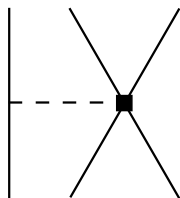


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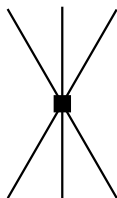
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At N^2 LO two additional LECs enter:

3N forces

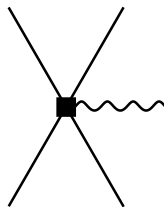


c_D



c_E

2b currents

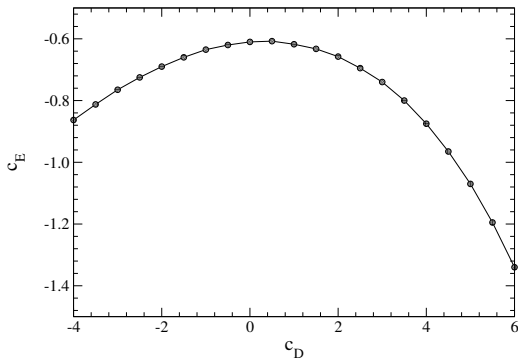
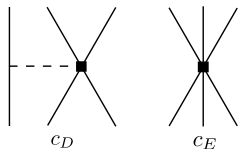


c_D

Fits to three-body observables necessary.

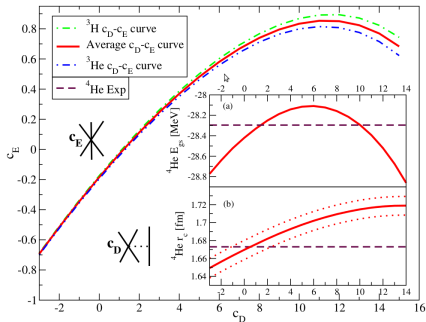
Binding energy of triton

Binding energy of ${}^3\text{H}$ yields relation between c_D and c_E .
Calculation based on non-local EM500 with non-local 3N forces at N^2LO .

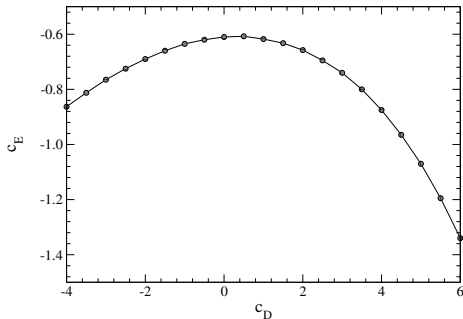


Binding energy of triton

Local 3N regulator



Non-local 3N regulator



Navrátil et al., PRL **99**, 042501 (2007)

Impact of 3N regulator significant

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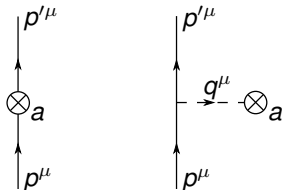
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One-body currents

Axial current at chiral order Q^0

$$A_{1b}^{a\mu} = -g_A \bar{u}(p') \gamma_5 \left(\gamma^\mu - \not{q} \frac{q^\mu}{q^2 - m_\pi^2} \right) \frac{\tau^a}{2} u(p),$$

Pion-decay is momentum dependent

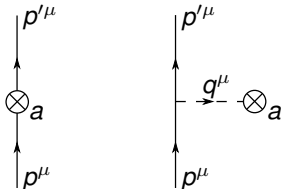


One-body currents

Axial current at chiral order Q^0

$$A_{1b}^{a\mu} = -g_A \bar{u}(p') \gamma_5 \left(\gamma^\mu - \not{q} \frac{q^\mu}{q^2 - m_\pi^2} \right) \frac{\tau^a}{2} u(p),$$

Pion-decay is momentum dependent



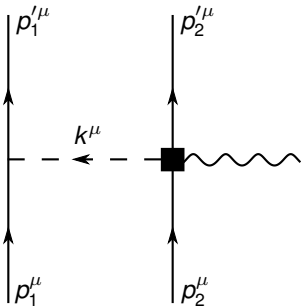
Non-relativistic limit including loop corrections at Q^2 :

$$A_{1b}^{0a} = \frac{\tau^a}{2} g_A(\mathbf{q}^2) \frac{\boldsymbol{\sigma} \cdot \mathbf{P}}{2M} \quad \mathbf{A}_{1b}^a = \frac{\tau^a}{2} \left(g_A(\mathbf{q}^2) \boldsymbol{\sigma} - \frac{g_P(\mathbf{q}^2)}{2M} (\mathbf{q} \cdot \boldsymbol{\sigma}) \mathbf{q} \right)$$

\mathbf{q} momentum transfer, $\mathbf{P} = \mathbf{p} + \mathbf{p}'$ total momentum

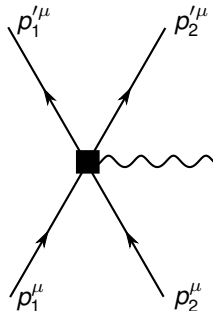
Two-body currents

At order Q^3 , 2b currents enter:



Pion exchange currents

c_1, c_3, c_4, c_6



Contact currents

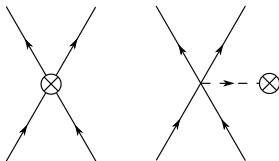
d_1, d_2

Application to many-body nuclear states

Apply two-body antisymmetrizer: $\mathcal{A} = \frac{1}{2}(1 - P_{ij})$ on contact currents:

$$\mathcal{A} \mathbf{A}_{12\text{contact}} = (d_1 + 2d_2) \mathcal{O}_{12}$$

with \mathcal{O}_{12} operator structure of the 2b current

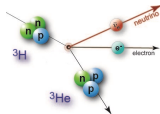


Replace the two constants d_1 and d_2 by one single constant:

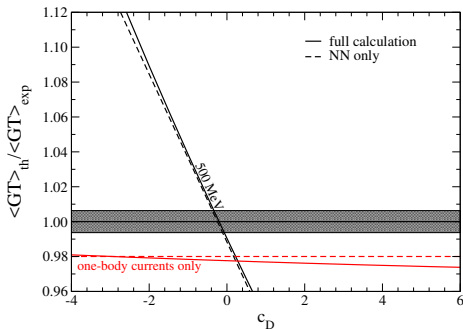
$$c_D = \Lambda_\chi F_\pi^2 (d_1 + 2d_2)$$

NOTE: This is no longer strictly the case for local regulators

Fit to ^3H β decay

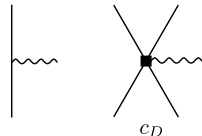


- ▶ Beta-decay of triton to determine c_D
 - ▶ ^3H half-life precisely known
 - ▶ Uncorrelated with ^3H binding energy
- ▶ c_D and c_E fully determined from independent three-body observables



Gamow-Teller matrix element

$$\langle GT \rangle = \frac{1}{g_A} \langle ^3\text{He} \| \sum_{i=1}^3 \mathbf{J}_{i,1b}^+ + \sum_{i < j} \mathbf{J}_{ij,2b}^+ \| ^3\text{H} \rangle$$



2b current regulator

Similar to the nuclear forces currents need to be regulated.

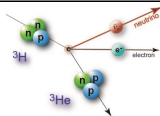
Local regulator

$$f_{\Lambda}^{\text{loc}}(\mathbf{p}, \mathbf{p}') = \exp \left[-(\mathbf{p} - \mathbf{p}')^4 / \Lambda_{2bc}^4 \right]$$

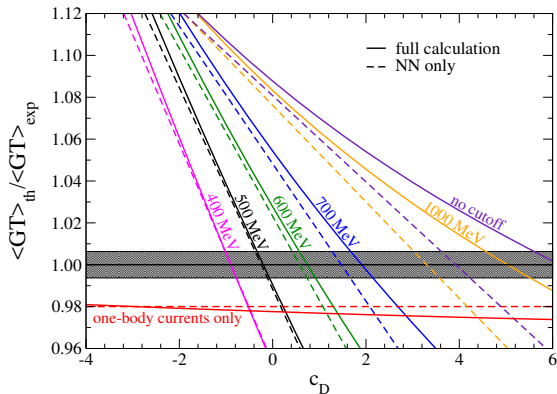
with cutoff Λ_{2bc} [Gazit, Quaglioni, Navrátil, PRL 103, 102502 \(2009\)](#)

- ▶ Unclear how to choose regulator and cutoff consistently in both forces and currents
- ▶ Continuity equation needs to be checked
[Krebs et al., Annals Phys. 378 \(2017\)](#)
- ▶ Here: Study cutoff dependence in 2b currents only

Cutoff dependence of c_D



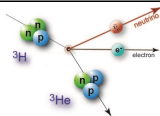
Consider different cutoffs for two-body currents using **local 2bc regulator**:



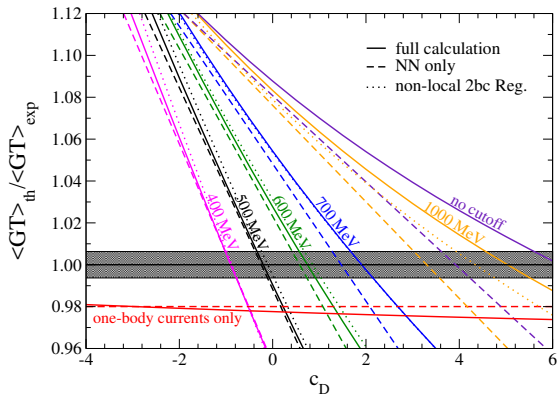
Carbone, Hebeler, Menéndez, Schwenk, PK, arXiv 1612.08010

Significant current-regulator dependence of c_D !

Cutoff dependence of c_D



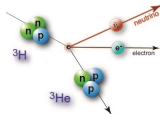
Consider different cutoffs for two-body currents using **non-local 2bc regulator**:



Carbone, Hebeler, Menéndez, Schwenk, PK, arXiv 1612.08010

Significant current-regulator dependence of c_D !

Cutoff dependence of $\langle GT \rangle_{th}$



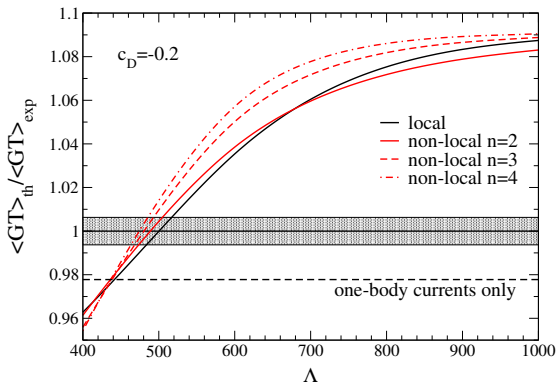
Cutoff-dependence in two-body currents can be larger than two-body-current contribution

Local regulator

$$f_{\Lambda}^{loc}(\mathbf{p}, \mathbf{p}') = \exp[-(\mathbf{p} - \mathbf{p}')^4 / \Lambda_{2bc}^4]$$

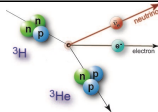
Nonlocal regulator

$$f_{\Lambda}^{non-loc}(\rho^2, \rho'^2) = \exp[-(\rho^{2n} + \rho'^{2n}) / \Lambda^{2n}]$$

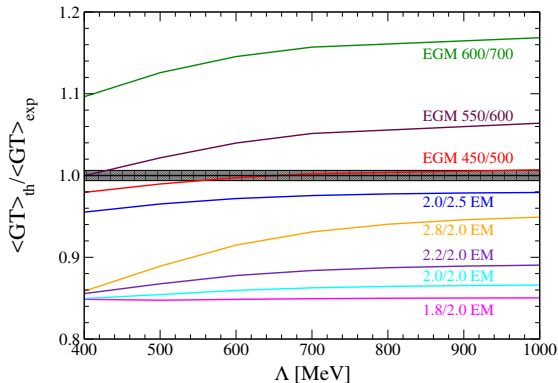


Carbone, Hebeler, Menéndez, Schwenk, PK, arXiv 1612.08010

Cutoff dependence of $\langle GT \rangle_{th}$



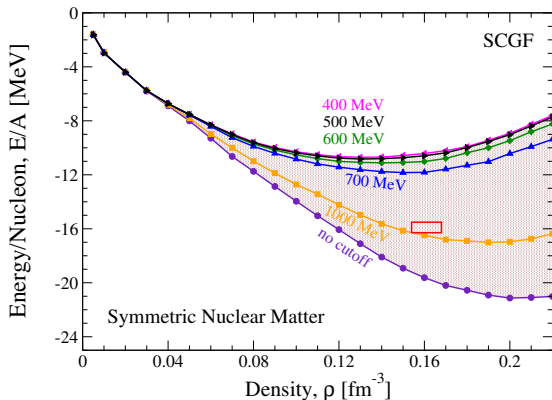
Generally not possible to simultaneously fit all experimental observables



Carbone, Hebeler, Menéndez, Schwenk, PK, arXiv 1612.08010

Impact on nuclear matter

Nuclear matter calculation with c_D , c_E taken from the triton fit

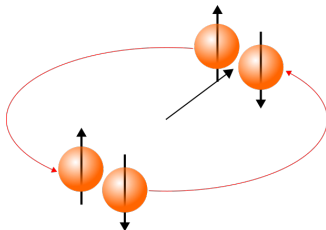


Carbone, Hebeler, Menéndez, Schwenk, PK, arXiv 1612.08010

Few-neutron resonances

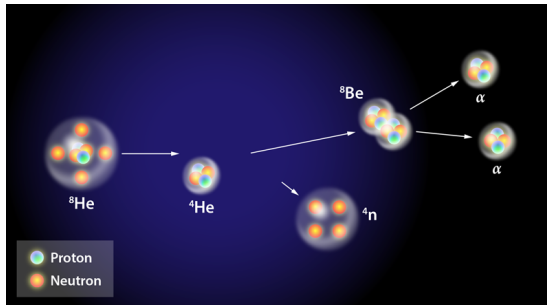


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Four-neutron resonances

Double charge-exchange reaction ${}^8\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be} + 4n$ at RIKEN



APS/Alan Stonebraker

Tetraneutron resonance at $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst.})$ MeV

Kisamori et al., PRL **116**, 052501 (2016)

QMC in three lines:

Ground state: $H|\Psi_0\rangle = E_0|\Psi_0\rangle$

Trial state: $|\Psi_T\rangle = \sum_i \alpha_i |\Psi_i\rangle$

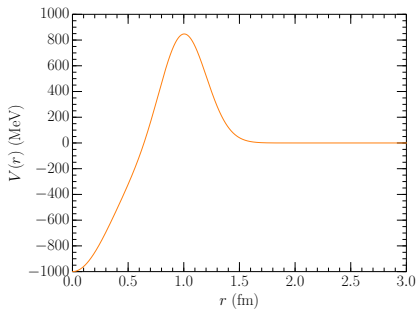
Propagate: $\lim_{\tau \rightarrow \infty} e^{-H\tau} |\Psi_T\rangle \rightarrow |\Psi_0\rangle$

QMC in more than three lines:

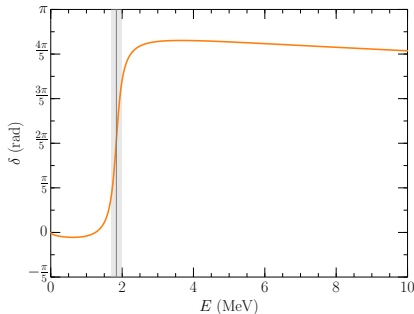
J. Carlson *et al.*, RMP **87**, 1067 (2015).

- ▶ Exact method to solve Schrödinger equation
- ▶ Very successful for light nuclei
- ▶ However, simulates bound states only

2n test potential



Phaseshift



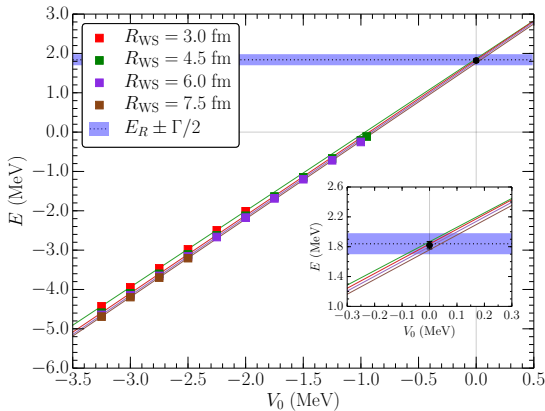
$$V(r) = V_1 e^{-\left(\frac{r}{R_1}\right)^2} + V_2 e^{-\left(\frac{r-r_2}{R_2}\right)^2}$$

adjusted to have a clear resonance
 $E_R = 1.84$ MeV, $\Gamma = 0.282$ MeV

Two-body test potential

- ▶ Add external Woods-Saxon to make the system bound

$$V_{WS} = \frac{V_0}{1 + e^{(r-R_{WS})/a}}$$



Two-body test potential

- ▶ Add external Woods-Saxon to make the system bound

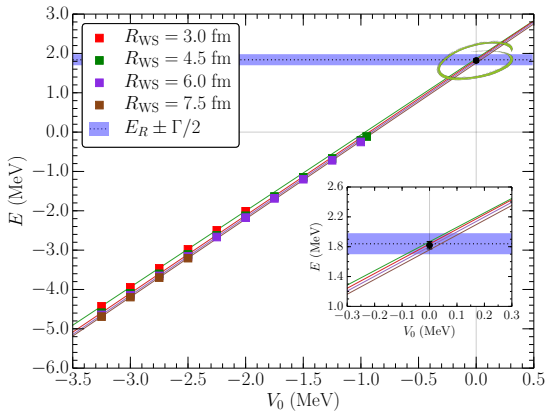
$$V_{WS} = \frac{V_0}{1 + e^{(r-R_{WS})/a}}$$

- ▶ Extrapolation to vanishing external potential yields

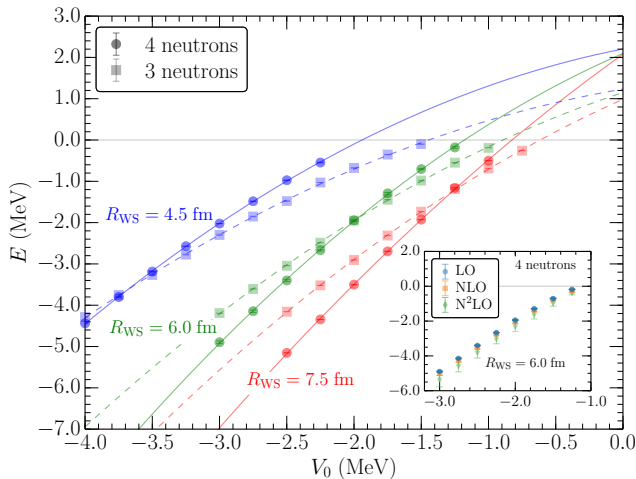
$$E_R = 1.83(5) \text{ MeV}$$

(Compare to 1.84 MeV)

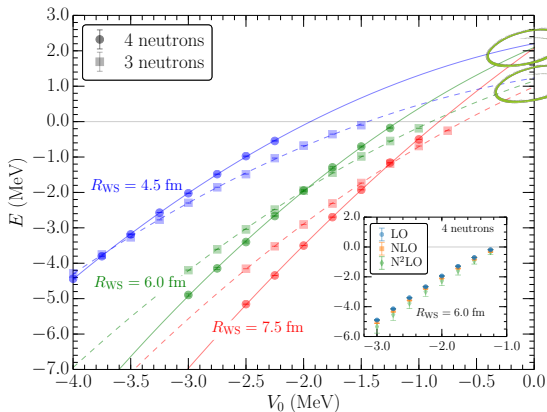
- ▶ Independent of trap geometry



Three- and four-neutron resonances



Three- and four-neutron resonances

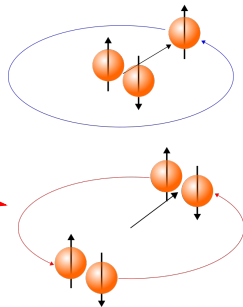
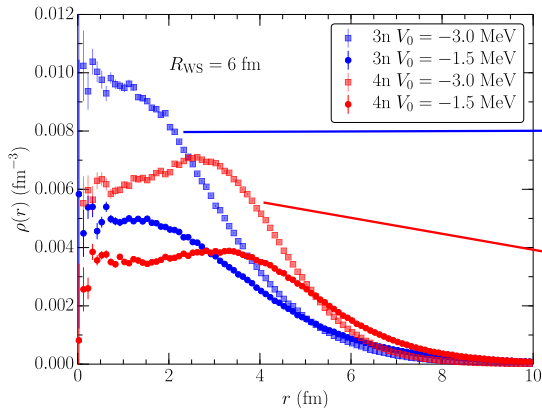


▶ $E_{3n} = 1.1(2)$ MeV
 $E_{4n} = 2.1(2)$ MeV

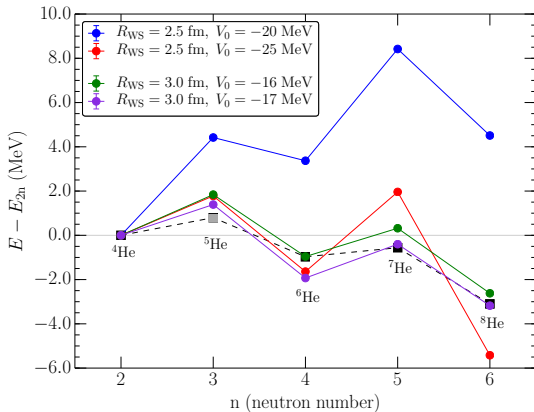
▶ 3n resonance lower than 4n resonance

▶ identical results without 3N forces/different cutoffs

One-body densities



- ▶ ^3n and ^4n are very dilute



- ▶ Odd isotopes always higher in energy than even
- ▶ Ordering of 3n and 4n resonances not an artifact of the trap

Summary

^3H β decay: [arXiv 1612.08010](#)

- ▶ Significant current-cutoff dependence when fitting c_D
- ▶ Additional uncertainties from cutoff variation need to be taken into account
- ▶ Generally not possible to simultaneously fit all experimental observables
- ▶ **How do we choose regulators consistently in forces and currents?**

Few-neutron resonances: [arXiv 1612.01502](#)

- ▶ Chiral EFT interaction supports tetra-neutron resonance
- ▶ Three neutron resonance potentially lower than tetra-neutron resonance (experimentally observable?)
- ▶ Dependence on three-body forces and regulator choice insignificant due to diluteness of the system