Constraints to light-by-light sumrule from two real photon collisions

Lingyun Dai
(IAS, Forschungszentrum Jülich)

with M. R. Pennington (JLab)

Based on: PRD90 (2014) 036004;PRD94 (2016) 116061;
PRD95 (2017) 056007

JÜLICH
FORSCHUNGSZENTRUM

Outlines

1. Introduction
2. Hadronic amplitudes
3. photon amplitudes
4. Constraints to LbL sumrule
5. Summary

1.Introduction

- Muon anomalous magnetic moment is an important indicator of physics beyond SM.
- The difference between Standard Model prediction and experiment is sizable.

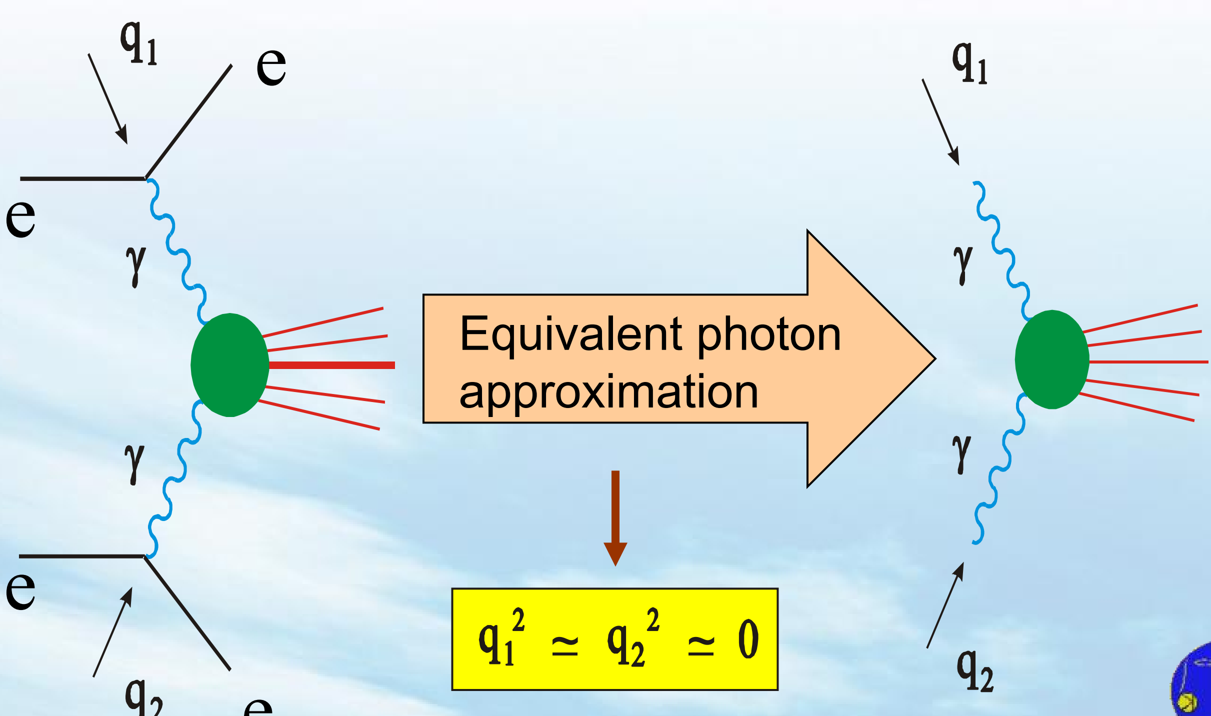
	Value ($\times 10^{-11}$) units
QED ($\gamma + \ell$)	$116\,584\,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_{\alpha}$
HVP(lo) [20]	$6\,923 \pm 42$
HVP(lo) [21]	$6\,949 \pm 43$
HVP(ho) [21]	-98.4 ± 0.7
HLbL	105 ± 26
EW	154 ± 1
Total SM [20]	$116\,591\,802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}}\ (\pm 49_{\text{tot}})$
Total SM [21]	$116\,591\,828 \pm 43_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}}\ (\pm 50_{\text{tot}})$

$$\Delta a_\mu(\text{E821} - \text{SM}) = (287 \pm 80) \times 10^{-11} \ [20]$$
$$= (261 \pm 78) \times 10^{-11} \ [21]$$

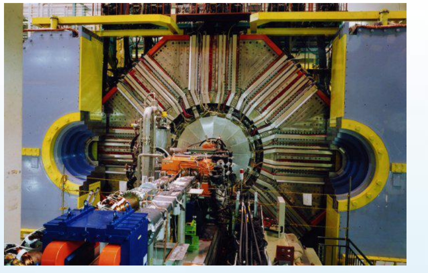
T.Blum arxiv:1311.2198[hep-ph],
R.M.Carey et.al., Fermi Lab proposal 0989

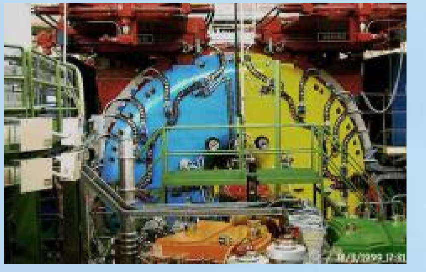
1. Introduction

- $\gamma\gamma \rightarrow$MM has the cleanest background.
- $\gamma\gamma \rightarrow$MM contributes significantly to LbL sumrule



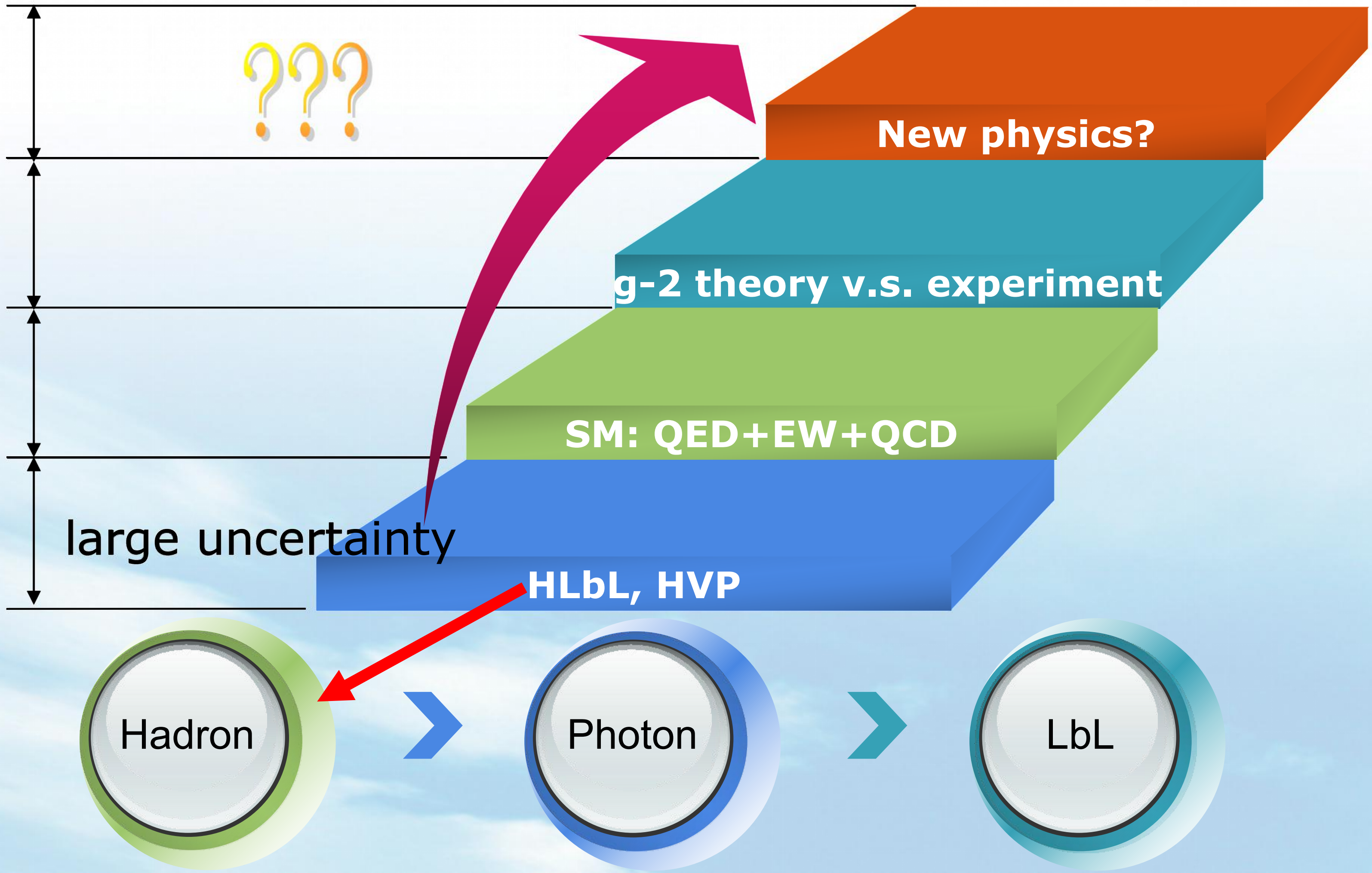
$\pi^+ \pi^-$ | $\pi^0 \pi^0$ | $K^+ K^-$ | $\bar{K}^0 K^0$





Scalars

- EW part is small and reliable. HVP relates to the total cross section of ee-->hadrons, HLbL?



2. Hadronic amplitudes

- **$\pi\pi$-$K\bar{K}$ coupled channel scattering amplitudes**
  - For Isospin 0 waves, We use K-matrix to represent S and D partial waves

$$T = \frac{K}{1 - i\rho K}$$

  - For Isospin 2 partial waves we use the parametrization of Constraint Fits to Data IV (CFDIV).

Pelaez *et al.*
PRD83 (2011) 074004

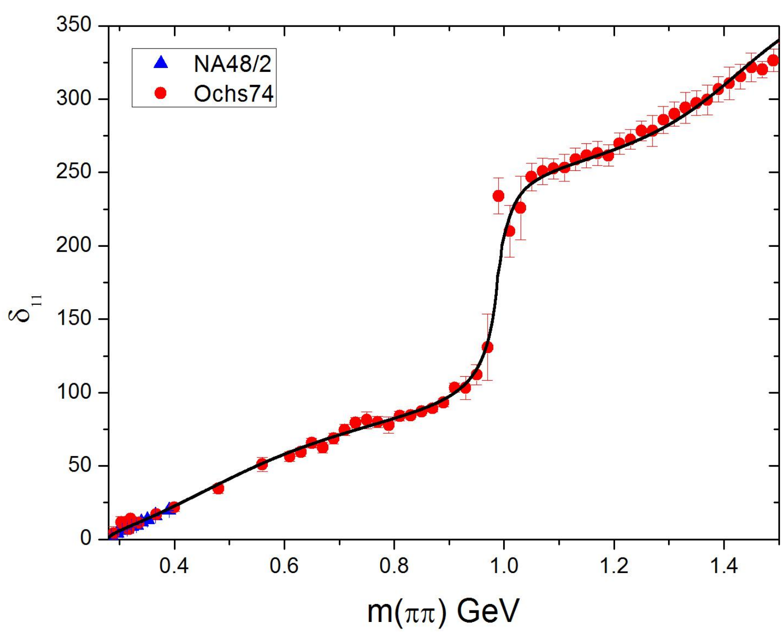
Hadronic scattering

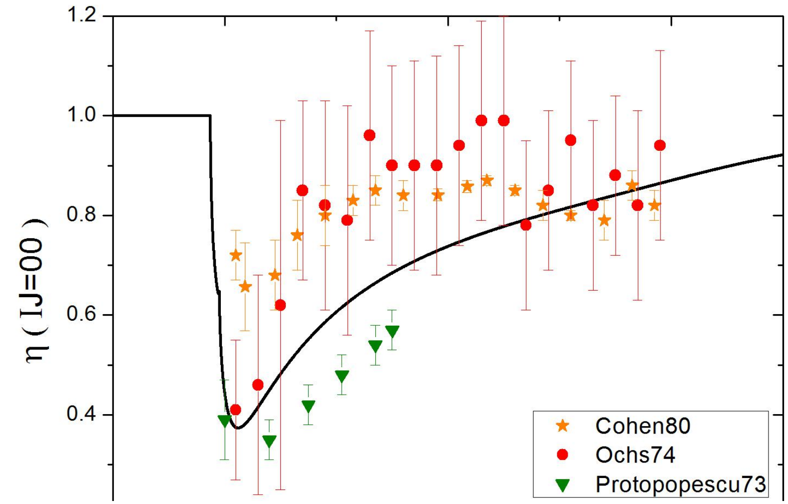
- **$\pi\pi$ - K $\bar{K}$ scattering inputs (I=0)**
  - Data on Phase shifts and inelasticities of $\pi\pi$ - K$\bar{K}$ coupled channel scattering.
  - BABAR's Dalitz plot analysis of $D_s^+ \to (\pi^+\pi^-)\pi^+$ and $D_s^+ \to (K^+K^-)\pi^+$ process.
  - Dispersion analysis based on symmetry and fit to data:
    - T-matrix of $\pi\pi$ scattering by CFDIV .
    - $\pi\pi \to KK$ amplitudes given by Roy-Steiner Equation.

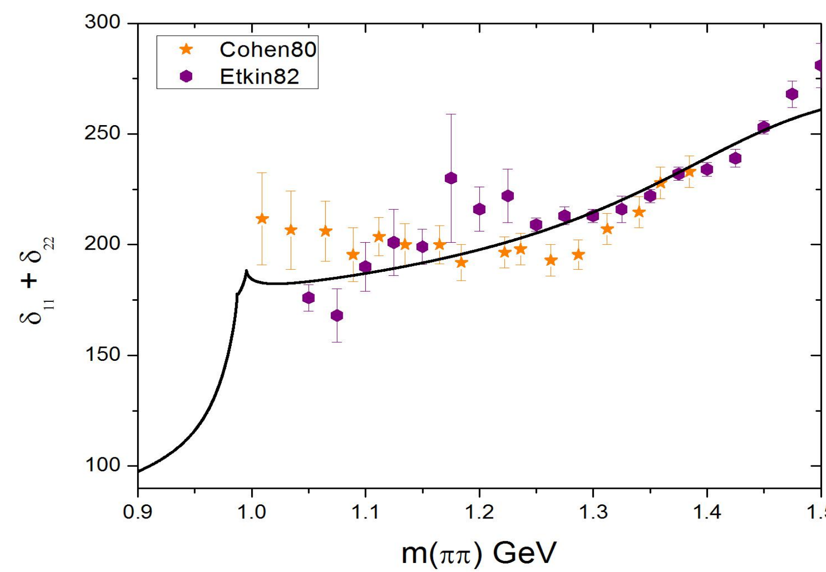
Descotes *et.al*
EPJC33 (2004) 409

Data: phase shift and inelasticity

- $\pi\pi \rightarrow \pi\pi$,KK phase shift and inelasticity

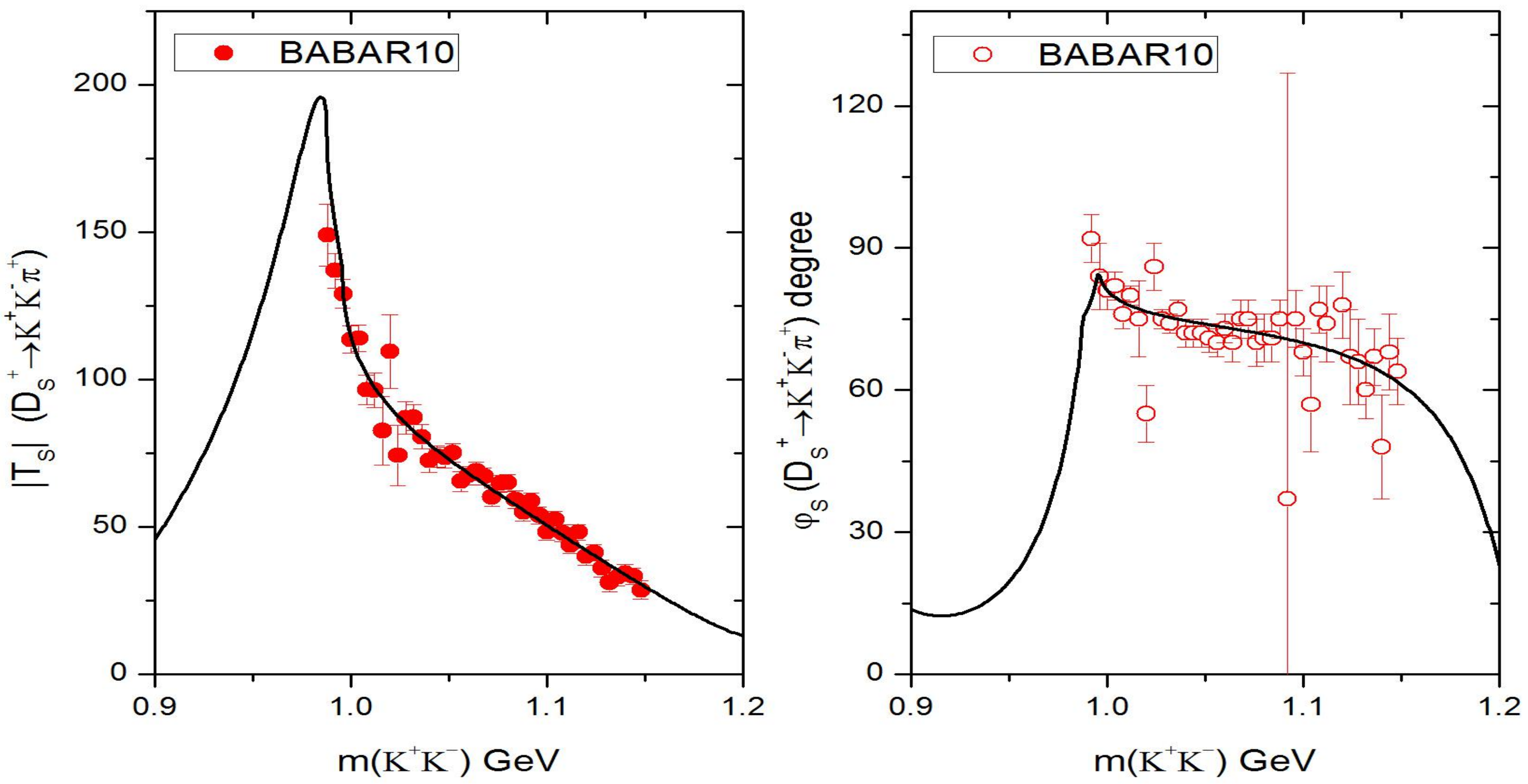






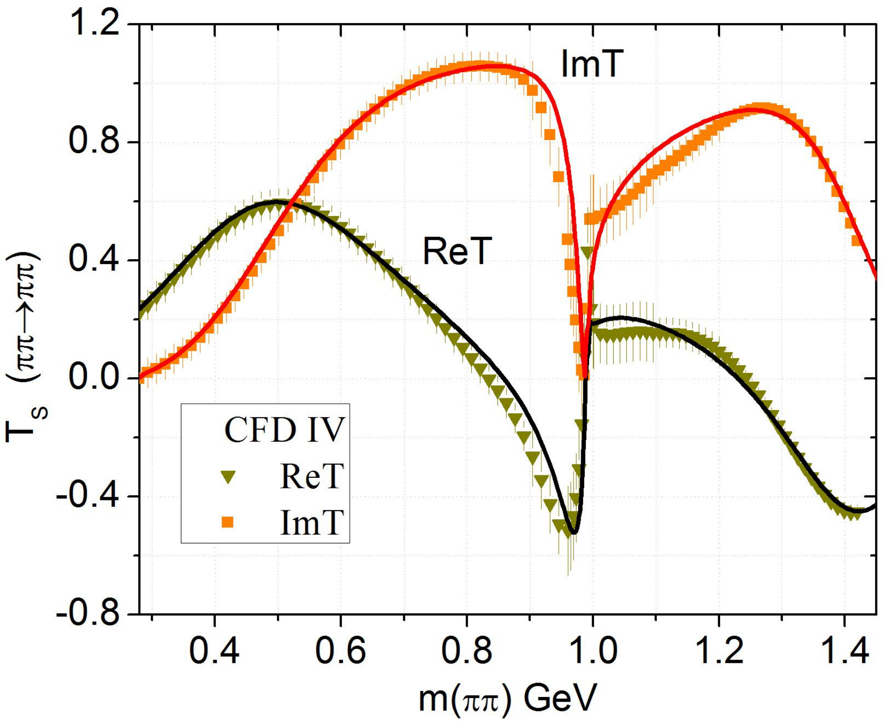
BABAR's Dalitz plot analysis

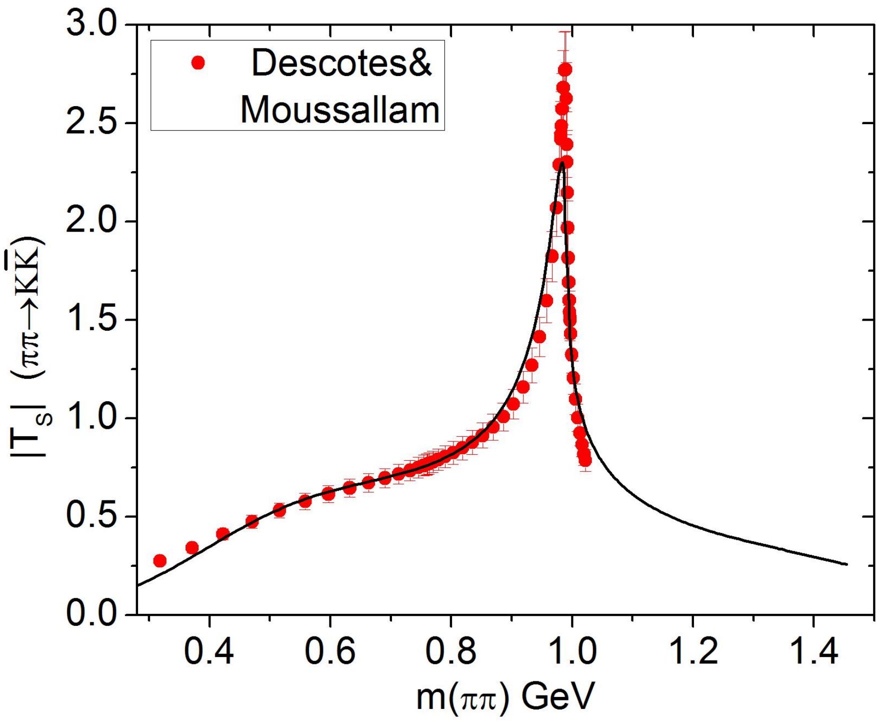
- $\pi\pi$ - KK **scattering inputs**
  - KK threshold region is quite important as it is around $f_0(980)$.



Dispersion analysis constraints

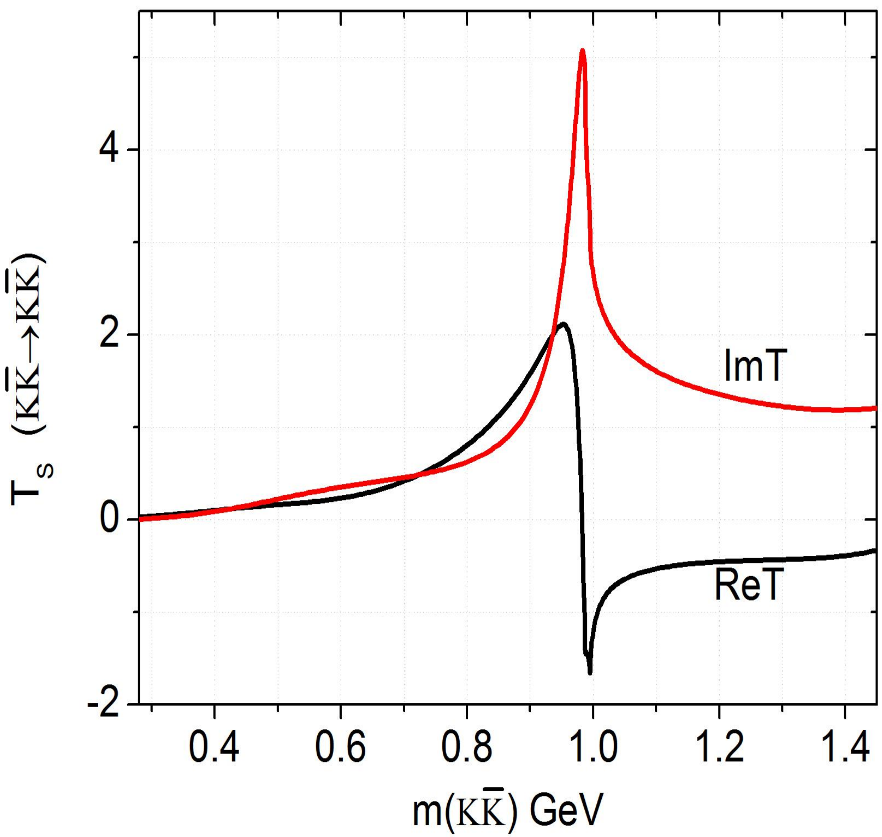
- The constraints from Roy like equation, they have taken crossing symmetry, unitarity into account.

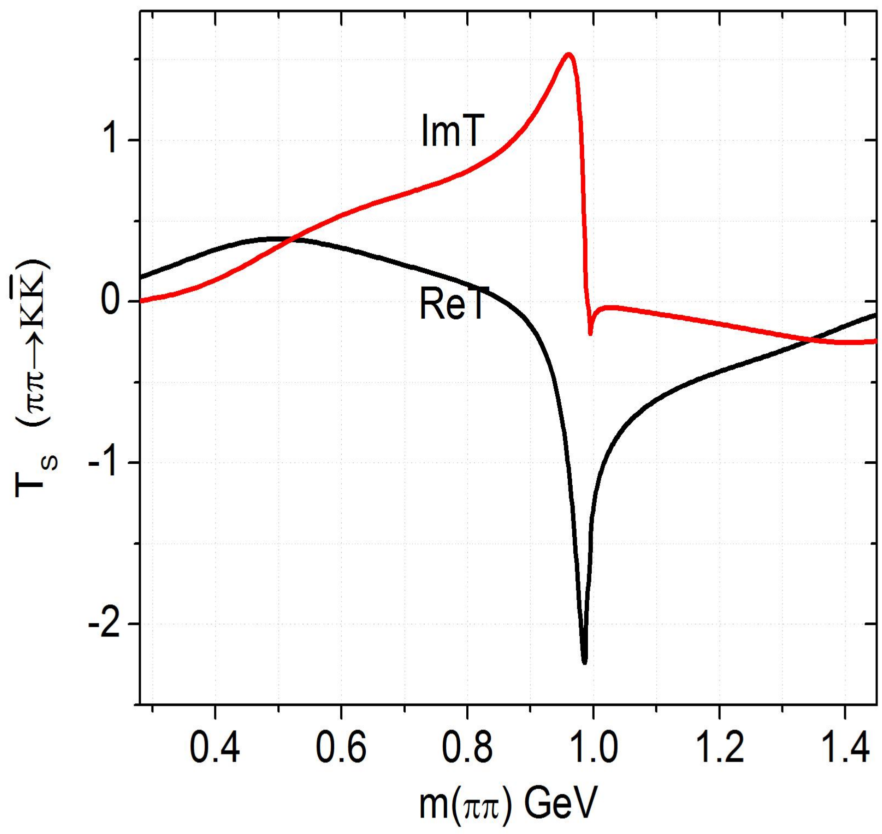




Final T matrix

- We only list ππ-KK coupled channel S-wave here, for Isospin 2 waves we use CFDIV.



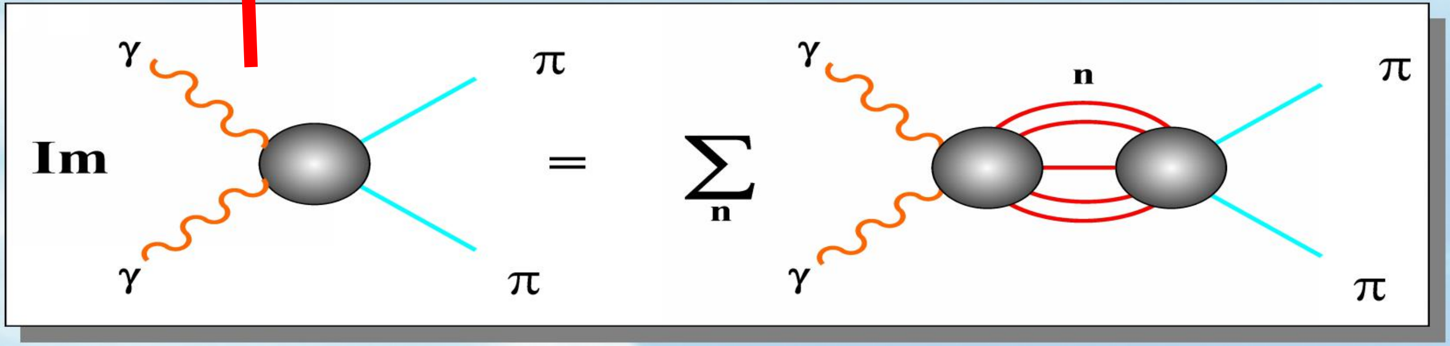


3. Photon amplitudes

- With Final State Interaction Theorem (FSIT), We can construct the amplitudes of photon-photon scattering into meson pairs:

$$\mathcal{F}^I_{J\lambda}(\gamma\gamma \to \pi\pi; s) = {\alpha_1}^I_{J\lambda}(s)\,\hat{T}^I_J(\pi\pi \to \pi\pi; s) + {\alpha_2}^I_{J\lambda}(s)\,\hat{T}^I_J(\pi\pi \to \overline{K}K; s)\,,$$

$$\mathcal{F}^I_{J\lambda}(\gamma\gamma \to \overline{K}K; s) = {\alpha_1}^I_{J\lambda}(s)\,\hat{T}^I_J(\pi\pi \to \overline{K}K; s) + {\alpha_2}^I_{J\lambda}(s)\,\hat{T}^I_J(\overline{K}K \to \overline{K}K; s)\,.$$



Photon-photon collision

- To constraint the di-photon amplitudes, we follow such steps:
  - We use dispersion relation to calculate the amplitudes below 0.6GeV, and give errors.
  - Fit the overall $\gamma\gamma\rightarrow\pi\pi$ and $\gamma\gamma\rightarrow K\bar{K}$ datasets, get a very narrowed patch of solutions.

Low energy di-photon amplitudes

- There are contribution of left hand cuts(LHCs), right hand cuts(RHCs) near real axis in $F^I_{J\lambda}$.
- According to FSIT, its phase should be the same as that of hadronic amplitudes. Thus we define

$$F^I_{J\lambda}=P^I_{J\lambda}\,\Omega^I_{J\lambda},$$

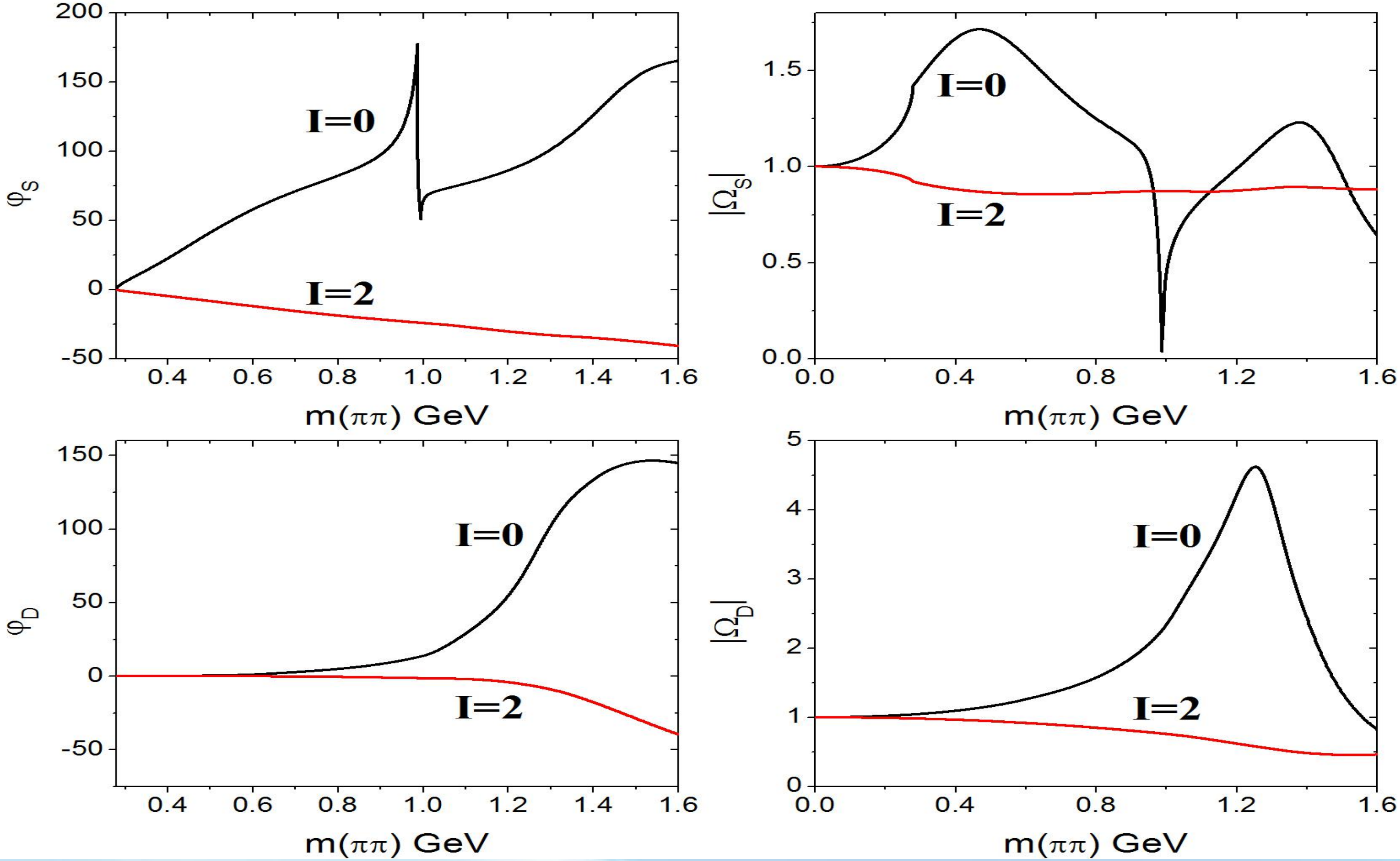
where $P^I_{J\lambda}$ has only LHCs, and $\Omega^I_{J\lambda}$, contains RHCs.

$$\Omega^I_{J\lambda}(s) = \exp\left(\frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\varphi^I_{J\lambda}(s')}{s'(s'-s)}\right)$$

- Now $\varphi^I_{J\lambda}$, should be continued to high energy region, which give error bands for the low energy amplitudes.

Phases and Omnes function

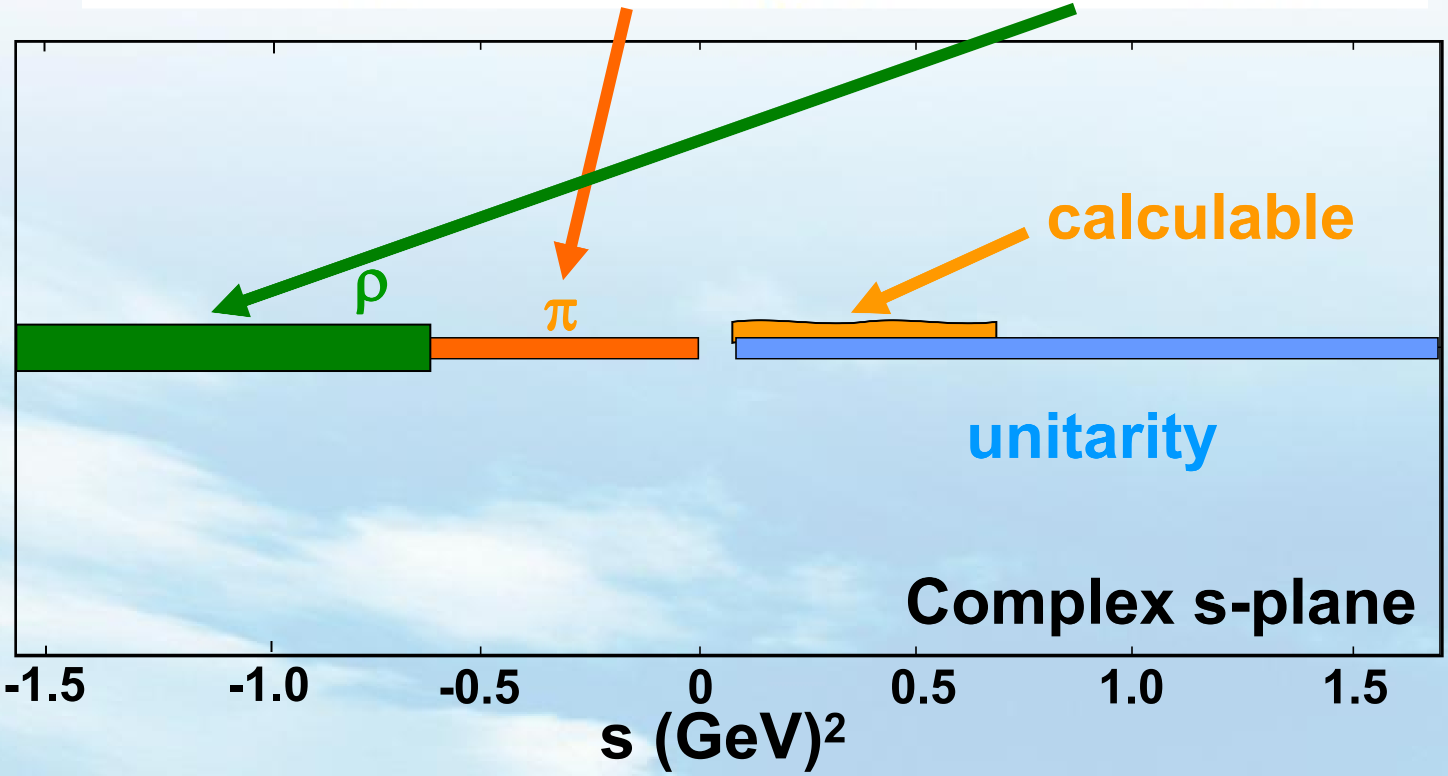
- Below 1.5 GeV it is from our fit, above 2 GeV from Regge behaviour.



Dispersion relations

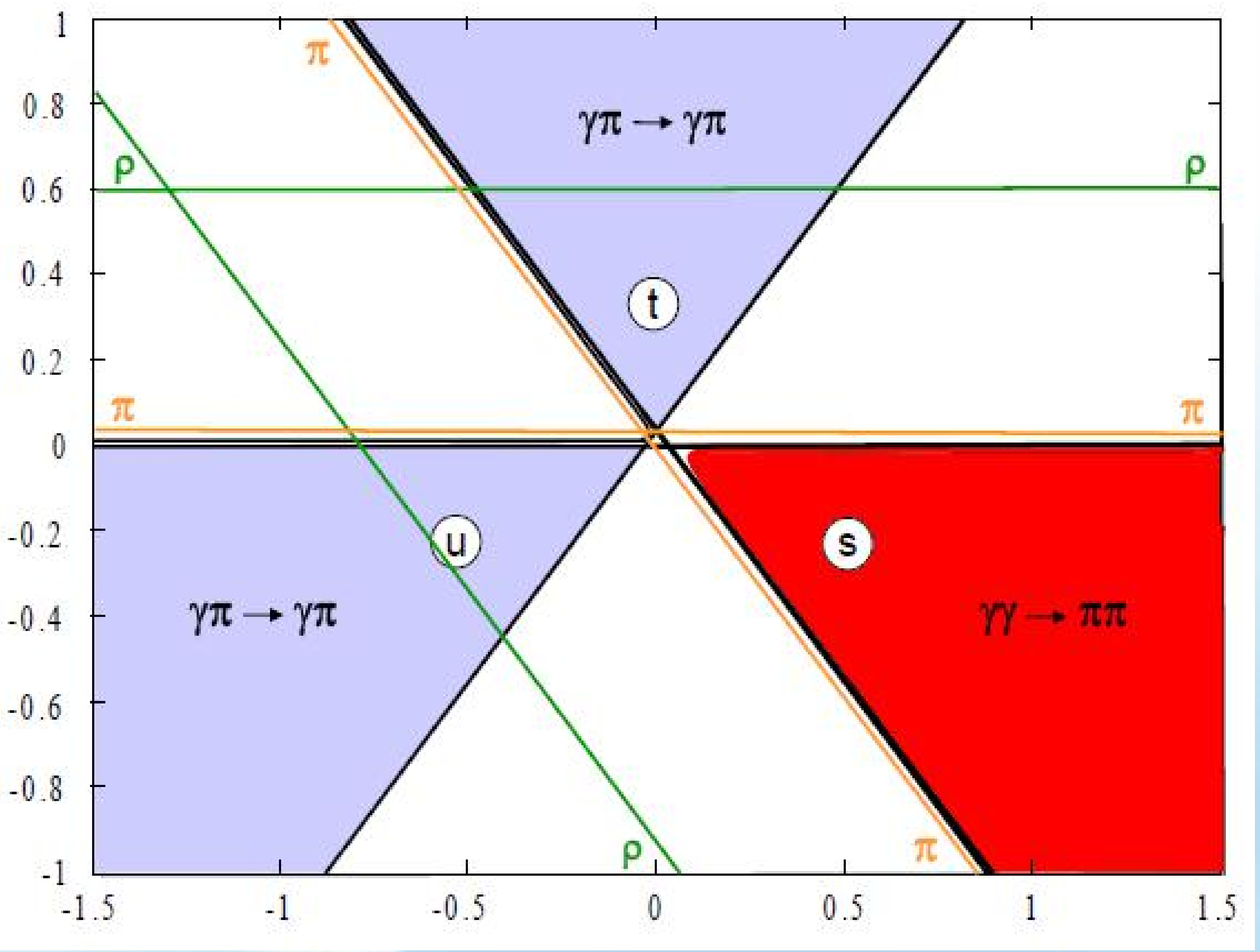
- Now what we need is the LHCs $P^I_{J\lambda}$, It is divided into two parts: Born term and other cross channel exchange terms. When s<0:

$$\mathrm{Im}\,\mathcal{F}^I_{J\lambda} = \mathrm{Im}\,\mathcal{B}^I_{J\lambda}(s) + \mathrm{Im}\,\mathcal{L}^I_{J\lambda}(s)$$



Vector, Axial-Vector, Tensor contributions

- LHCs of ρ, ω, a_1, b_1, h_1 give an error band of low energy amplitudes,
- Remain parts are parametrized as an effective pole 'T'.



Dispersion relations

- Low's low energy theorem tells us that:

$$\mathcal{F}^I_{J\lambda}(s) \;\rightarrow\; \mathcal{B}^I_{J\lambda}(s) \quad \text{as } s \rightarrow 0$$

- Thus we can write Dispersion relations of

$$\mathcal{F}^I_{00}(s) \;=\; \mathcal{B}^I_{00}(s) + b^I s\, \Omega^I_{00}(s) + \frac{s^2\,\Omega^I_{00}(s)}{\pi}\int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right]\,\Omega^I_{00}(s')^{-1}}{s'^2(s'-s)}$$

$$-\frac{s^2\,\Omega^I_{00}(s)}{\pi}\int_R ds' \frac{\mathcal{B}^I_{00}(s')\,\mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2(s'-s)}$$

Solved by ChPT

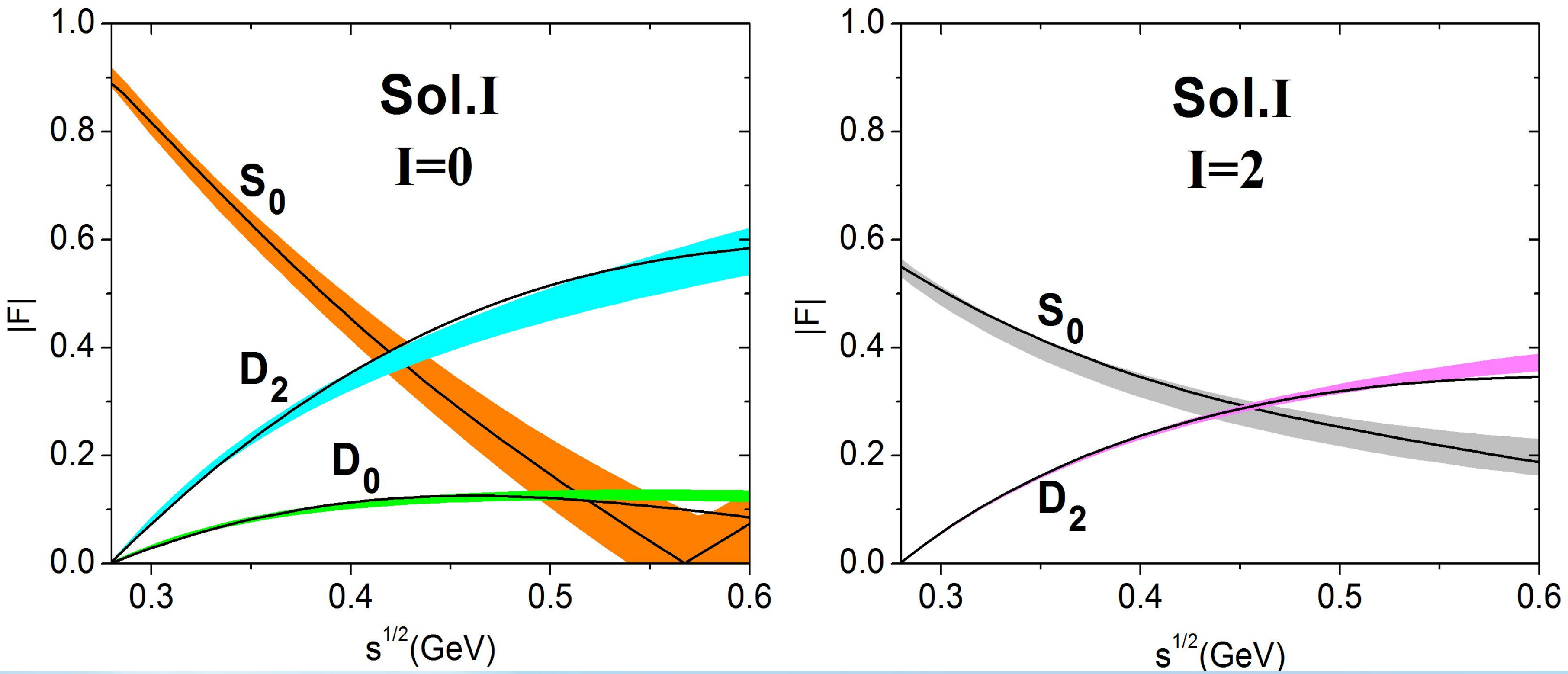
$$\mathcal{F}^I_{J\lambda}(s) \;=\; \mathcal{B}^I_{J\lambda}(s) + \frac{s^n(s-4m_\pi^2)^{J/2}}{\pi}\,\Omega^I_{J\lambda}(s)\int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{J\lambda}(s')\right]\;\Omega^I_{J\lambda}(s')^{-1}}{s'^n(s'-4m_\pi^2)(s'-s)}$$

$$-\frac{s^n(s-4m_\pi^2)^{J/2}}{\pi}\,\Omega^I_{J\lambda}(s)\int_R ds' \frac{B^I_{J\lambda}(s')\,\mathrm{Im}\left[\Omega^I_{J\lambda}(s')^{-1}\right]}{s'^n(s'-4m_\pi^2)(s'-s)}\,.$$

Threshold behaviour

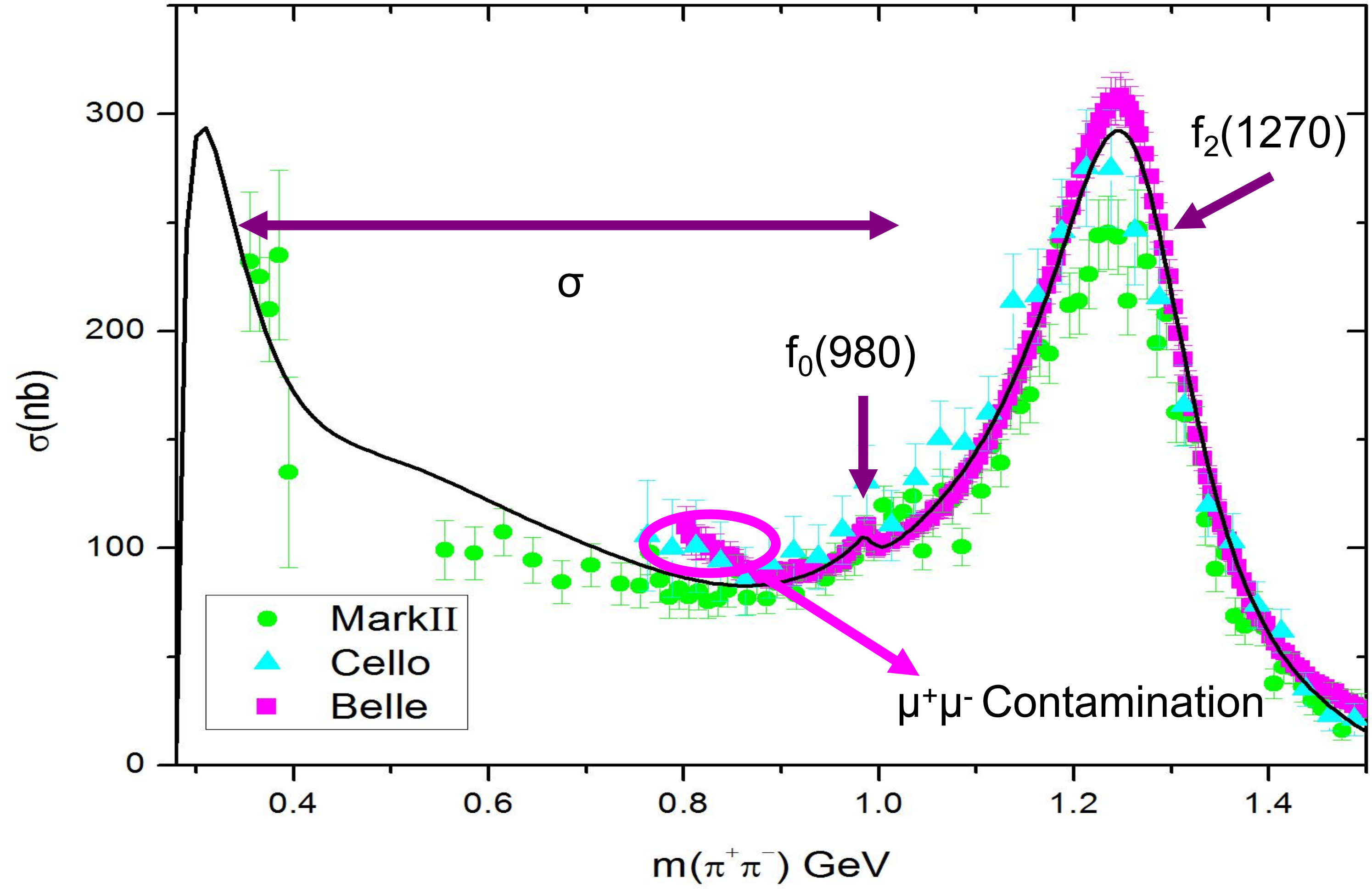
Constraints on low energy amplitudes

- Finally we have the bands given by dispersion relations:

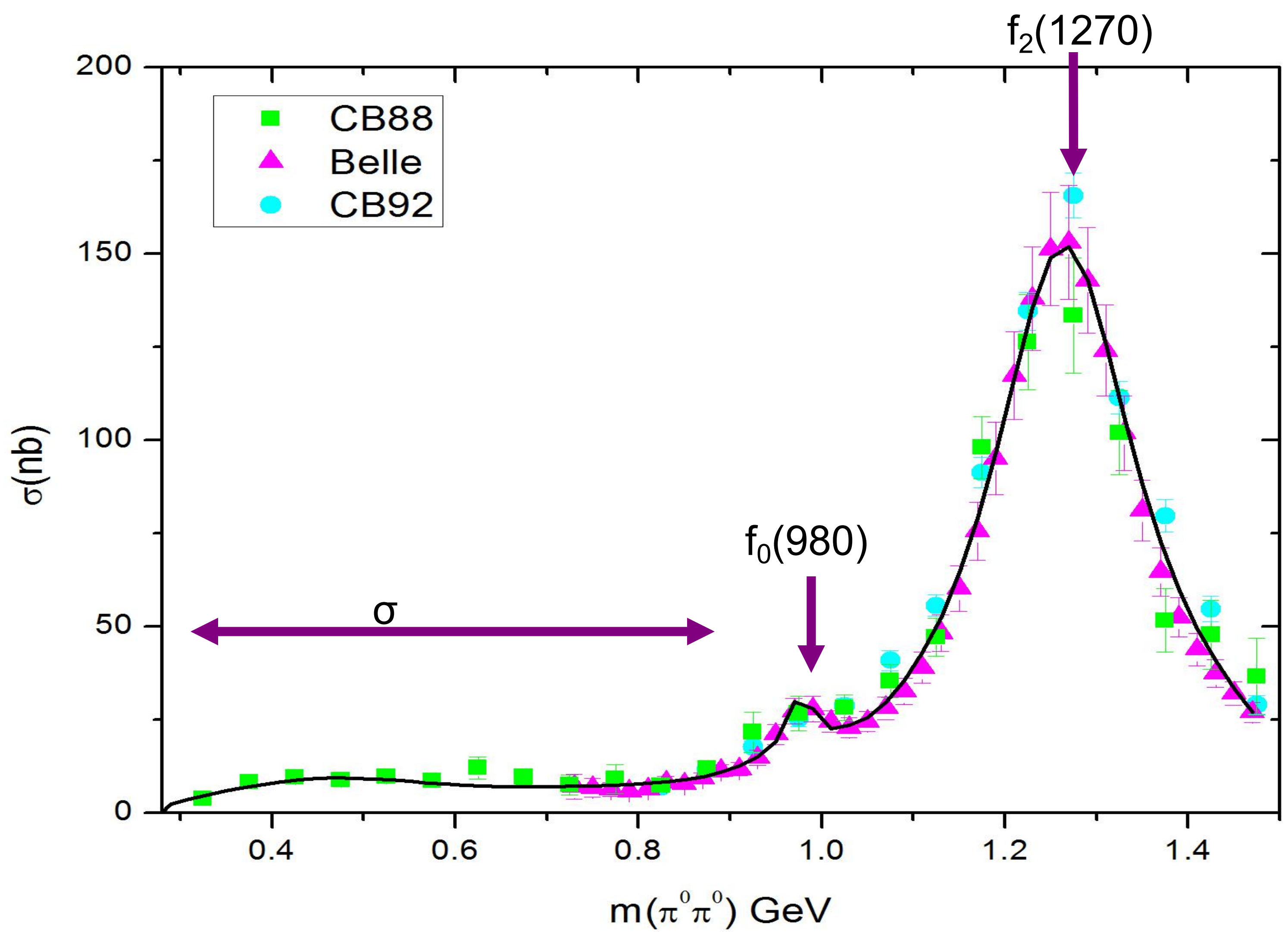


$\gamma\gamma \to \pi^+\pi^-$ integrated cross section

- With these constraints, we fit all datasets. The integrated cross sections with limited angular coverage

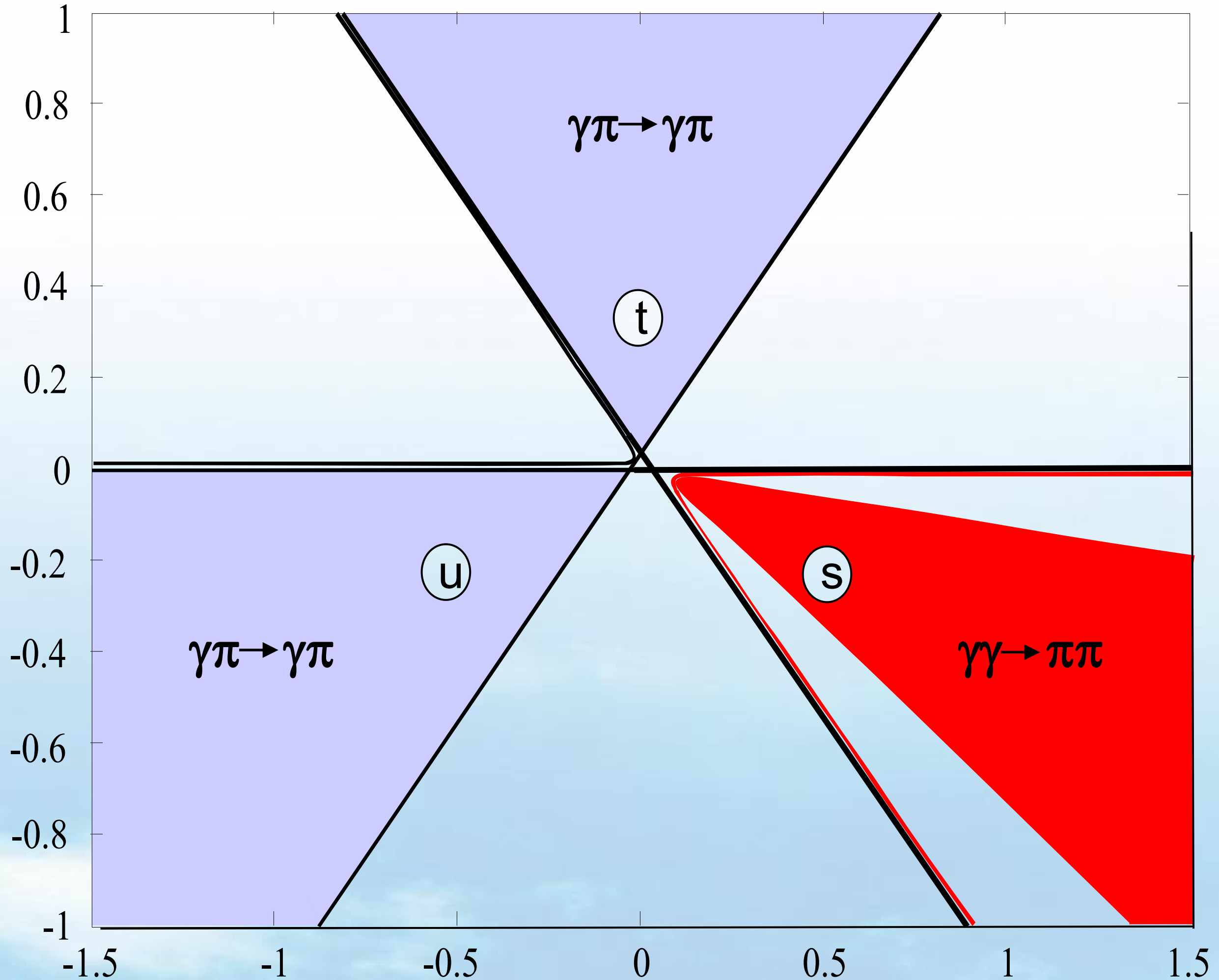


$\gamma\gamma \rightarrow \pi^0\pi^0$ integrated cross section



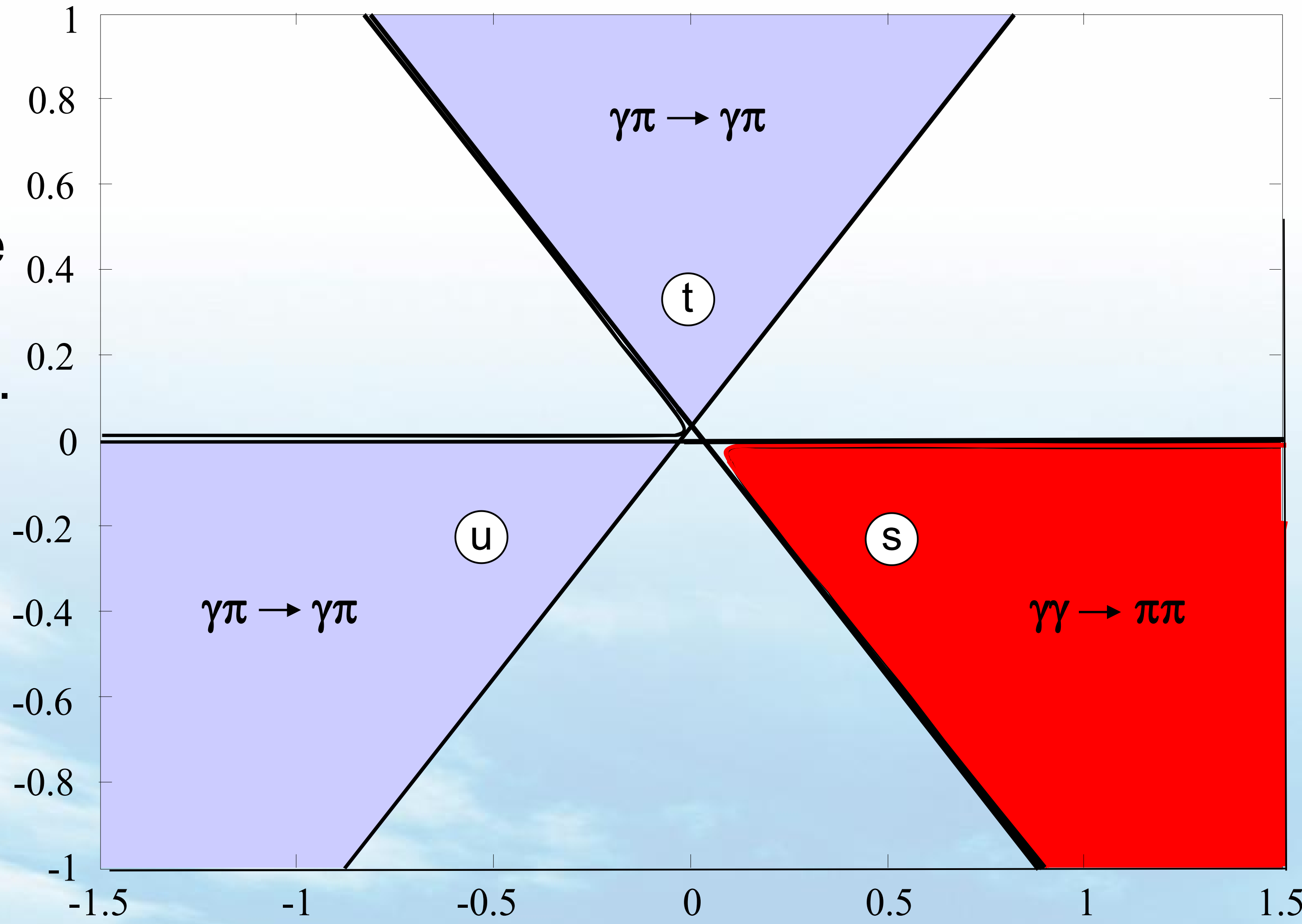
Why angular distribution?

- why angular distribution is so important?
- We only have the limited angular distribution, is it possible to predict the full coveraged one?

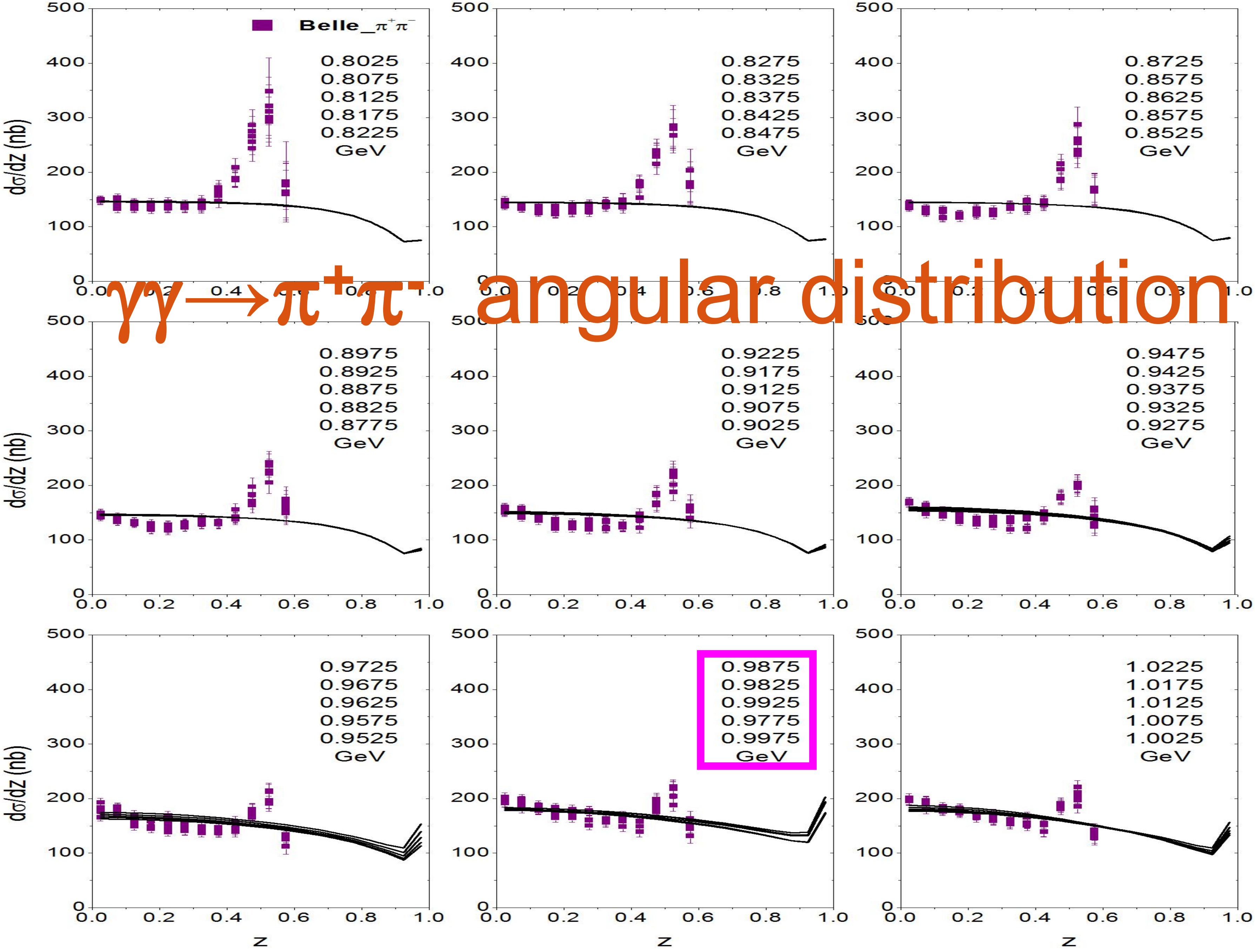


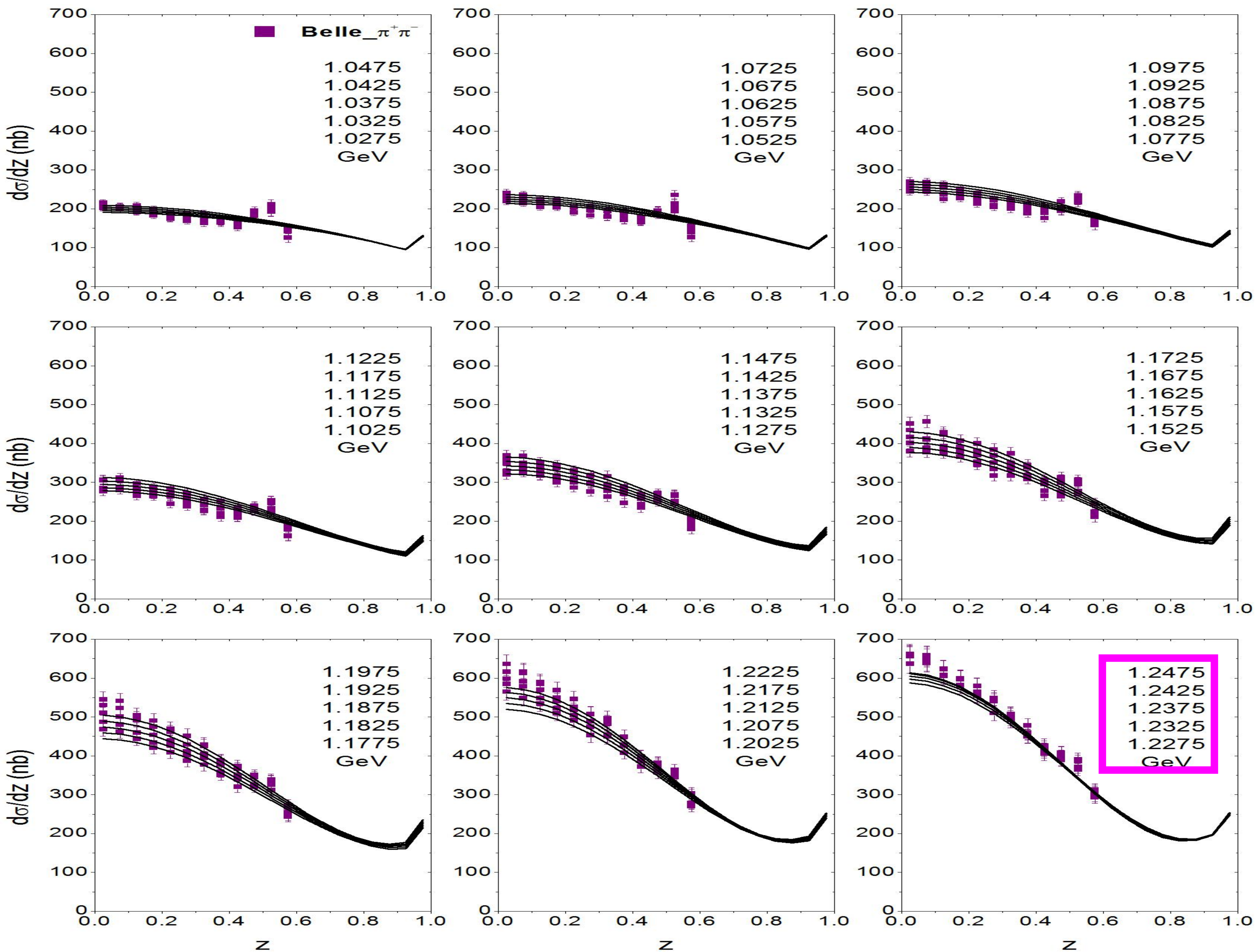
γγ→ππ angular distribution

- we can predict the full cross section if we know each partial wave.
- The angular distribution is helpful to seperate each partial wave.

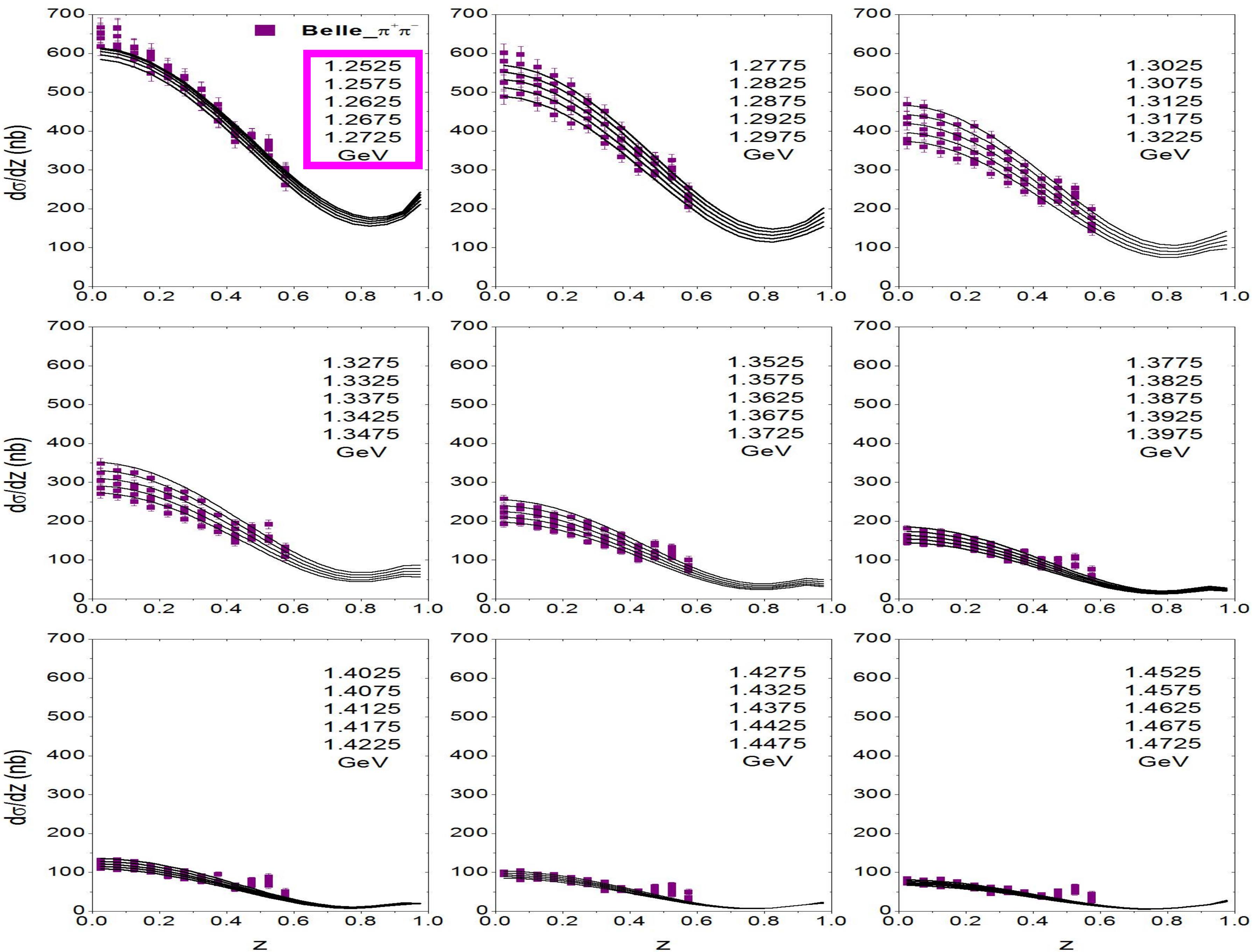


$\gamma\gamma \to \pi^+\pi^-$ angular distribution



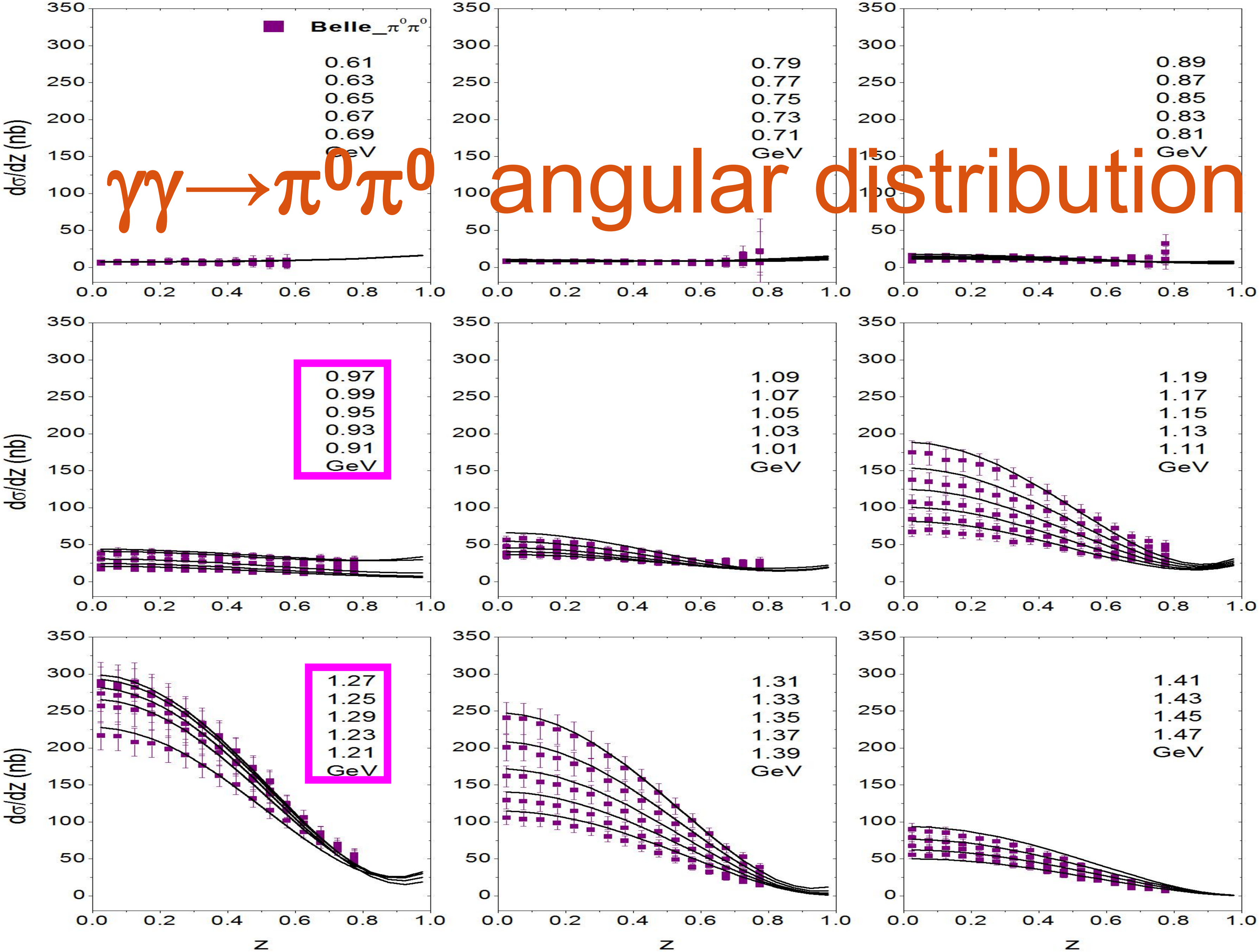


Belle_$\pi^+\pi^-$
1.0475
1.0425
1.0375
1.0325
1.0275
GeV
1.0725
1.0675
1.0625
1.0575
1.0525
GeV
1.0975
1.0925
1.0875
1.0825
1.0775
GeV
1.1225
1.1175
1.1125
1.1075
1.1025
GeV
1.1475
1.1425
1.1375
1.1325
1.1275
GeV
1.1725
1.1675
1.1625
1.1575
1.1525
GeV
1.1975
1.1925
1.1875
1.1825
1.1775
GeV
1.2225
1.2175
1.2125
1.2075
1.2025
GeV
1.2475
1.2425
1.2375
1.2325
1.2275
GeV
dσ/dz (nb)
z



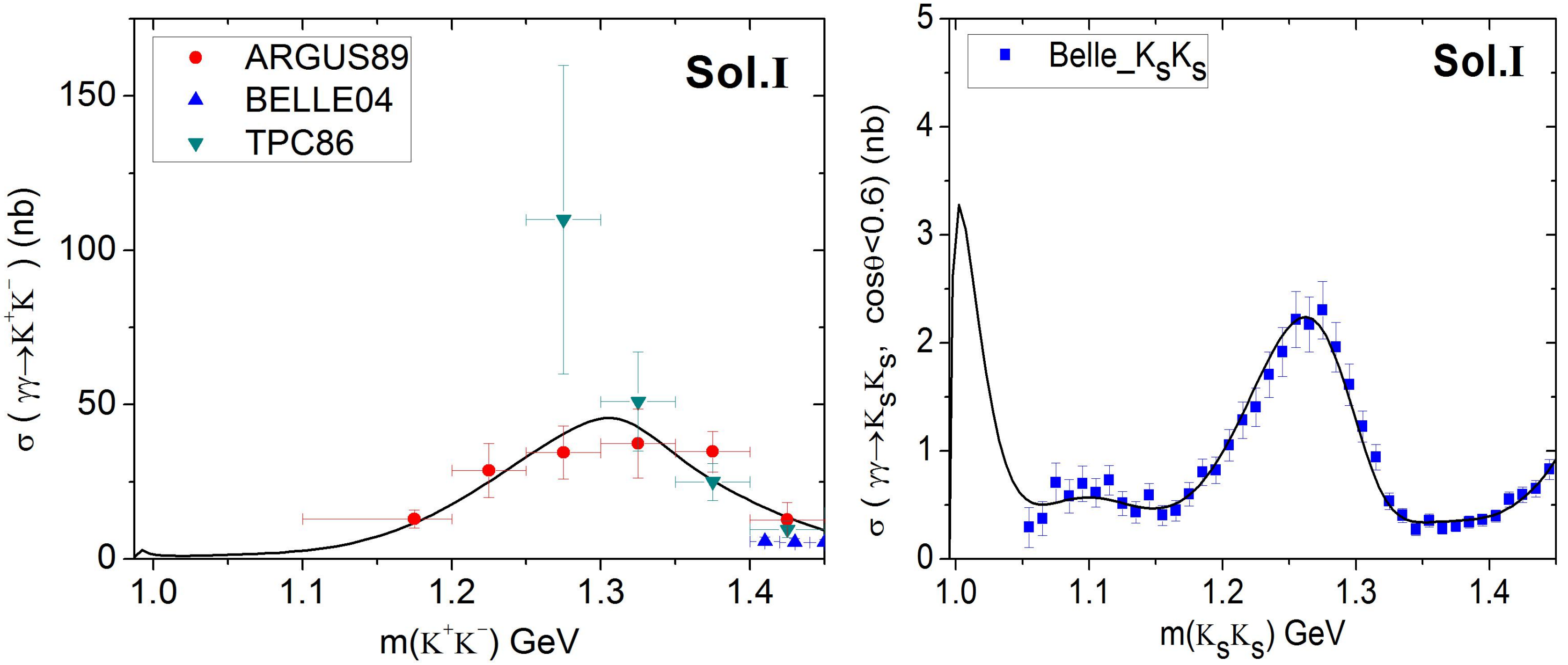
Belle_$\pi^+\pi^-$
1.2525
1.2575
1.2625
1.2675
1.2725
GeV
1.2775
1.2825
1.2875
1.2925
1.2975
GeV
1.3025
1.3075
1.3125
1.3175
1.3225
GeV
1.3275
1.3325
1.3375
1.3425
1.3475
GeV
1.3525
1.3575
1.3625
1.3675
1.3725
GeV
1.3775
1.3825
1.3875
1.3925
1.3975
GeV
1.4025
1.4075
1.4125
1.4175
1.4225
GeV
1.4275
1.4325
1.4375
1.4425
1.4475
GeV
1.4525
1.4575
1.4625
1.4675
1.4725
GeV
dσ/dz (nb)
z

$\gamma\gamma \rightarrow \pi^0\pi^0$ angular distribution

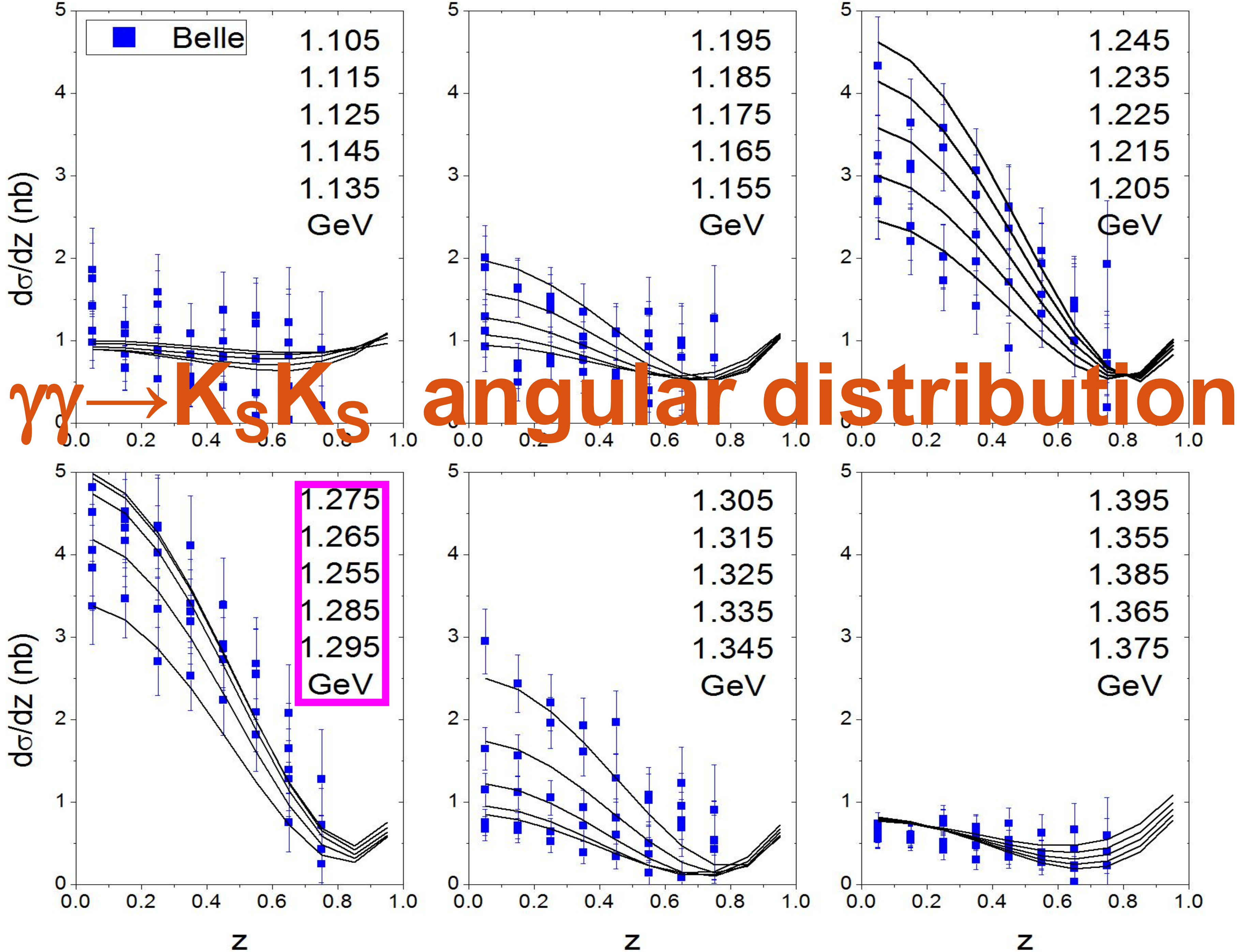


$\gamma\gamma \rightarrow KK$ integrated cross section

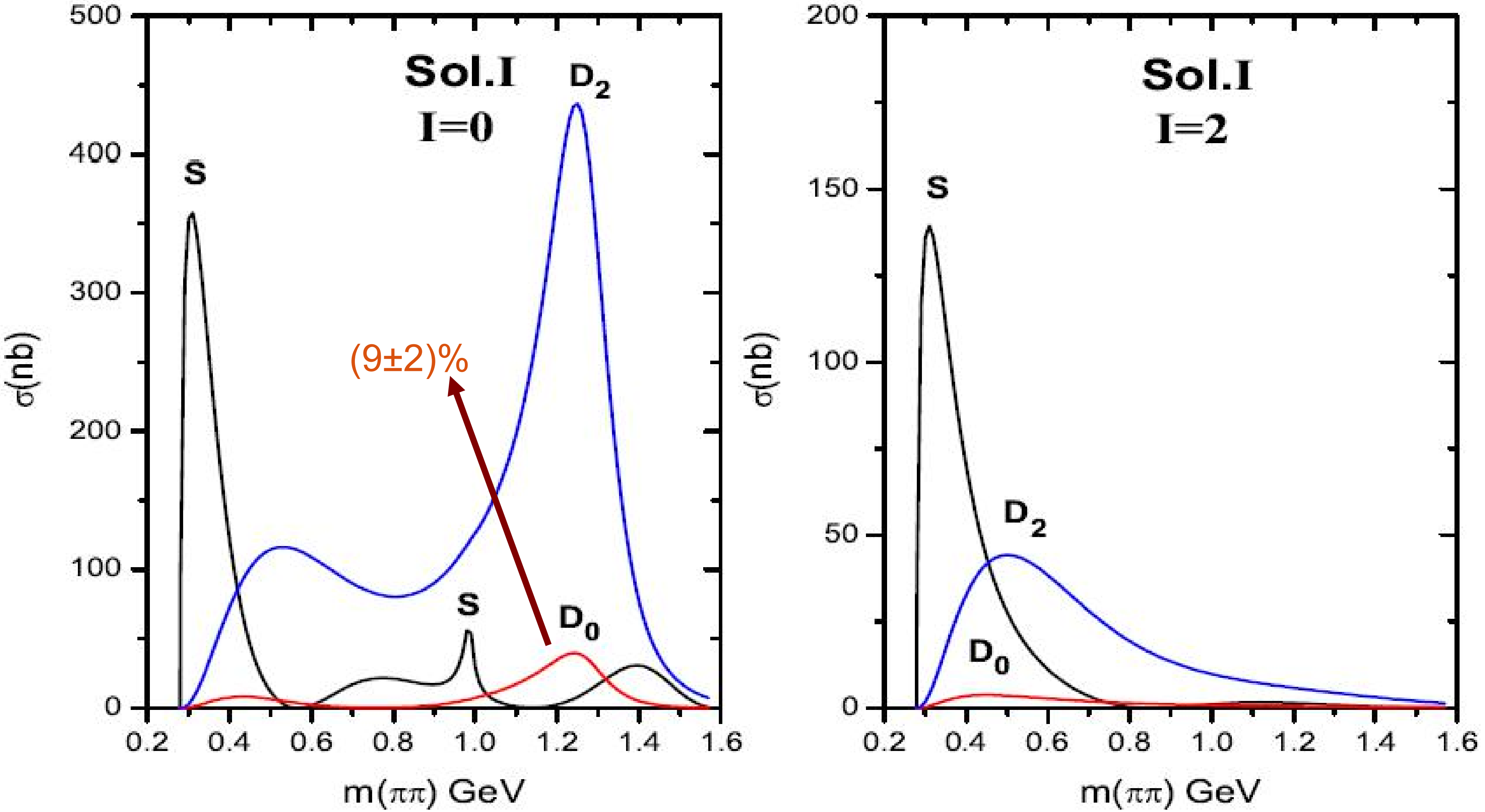
- If only fit to $\gamma\gamma \rightarrow \pi\pi$, we will get a region of solutions. $\gamma\gamma \rightarrow K\bar{K}$ data is helpful to select solutions.
- The latest $K_S K_S$ data of Belle make the accurate coupled channel analysis possible. Especially the angular distribution.



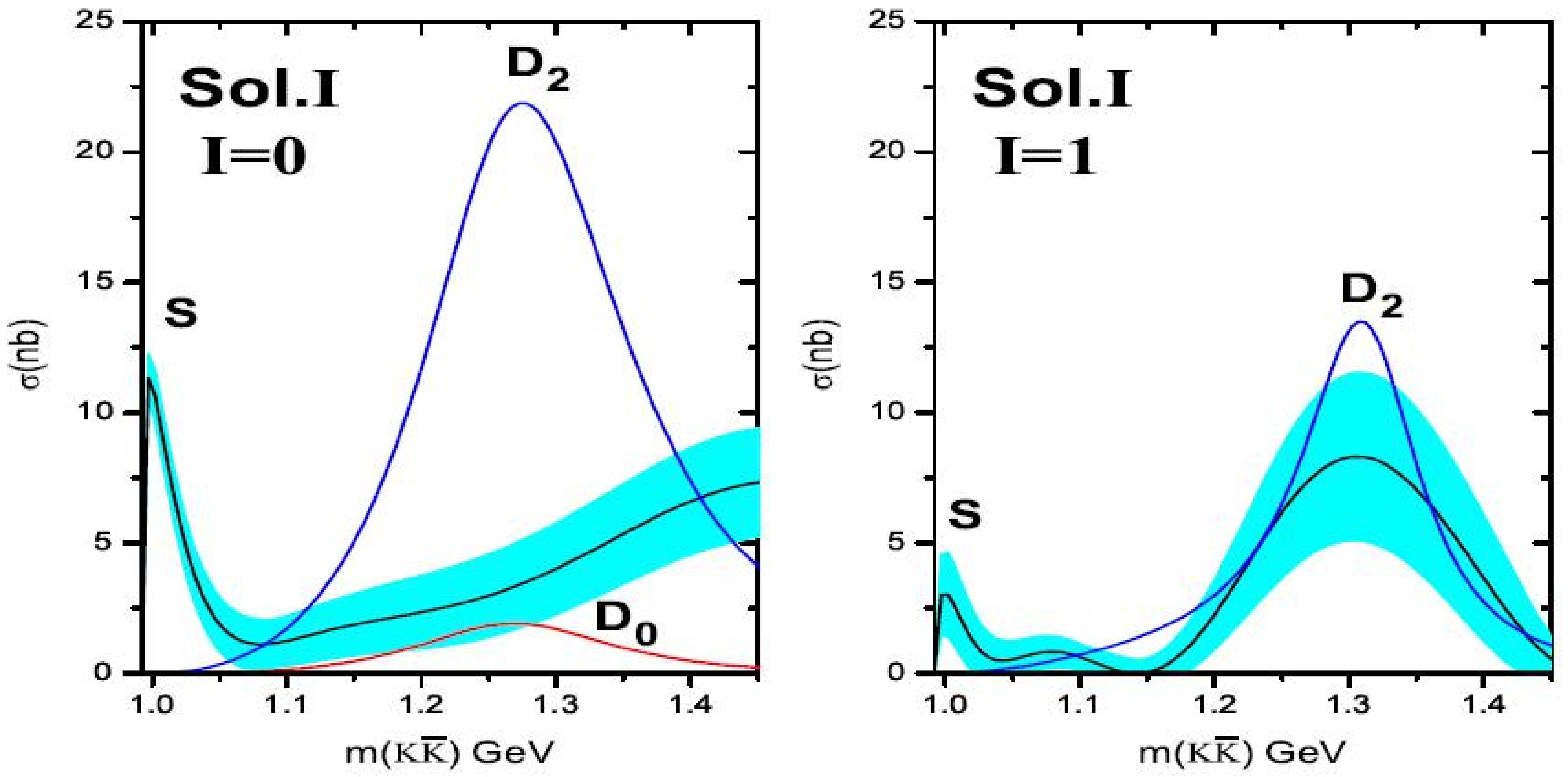
γγ→$K_S K_S$ angular distribution



γγ→ππ individual partial waves



γγ→KK individual partial waves, I=1



4.Constraints to light-by-light sumrule

- For LbL one needs photons with virtualities from threshold up to around 2 GeV^2. Our massless photon amplitudes are boundary values when $Q^2 = 0$.
- Narrow resonance estimates from the tensor mesons are not a good approximation.
- With our amplitudes we can test the simplest Pascalutsa-Vanderhaeghen sumrule.

$$0 = \int_0^\infty \mathrm{d}s \, \frac{\Delta\sigma(s)}{s},$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_0^\infty \mathrm{d}s \, \frac{\sigma_{||}(s) \pm \sigma_\perp(s)}{s^2}.$$

$\sigma_2(s) - \sigma_0(s)$

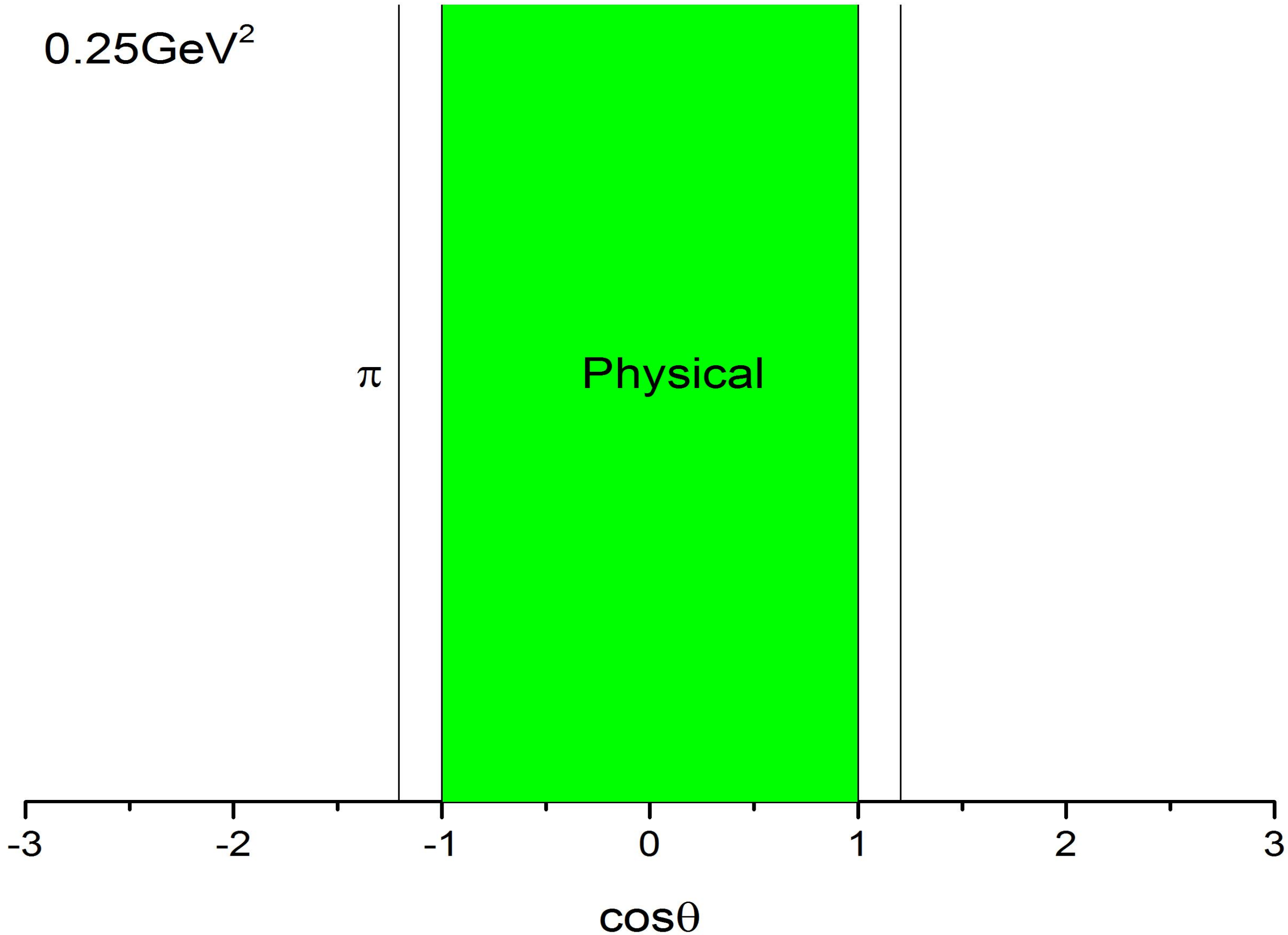
V.Pascalutsa & M.Vanderhaeghen, PRL105 (2010) 201603.

Constraints to light-by-light sumrule

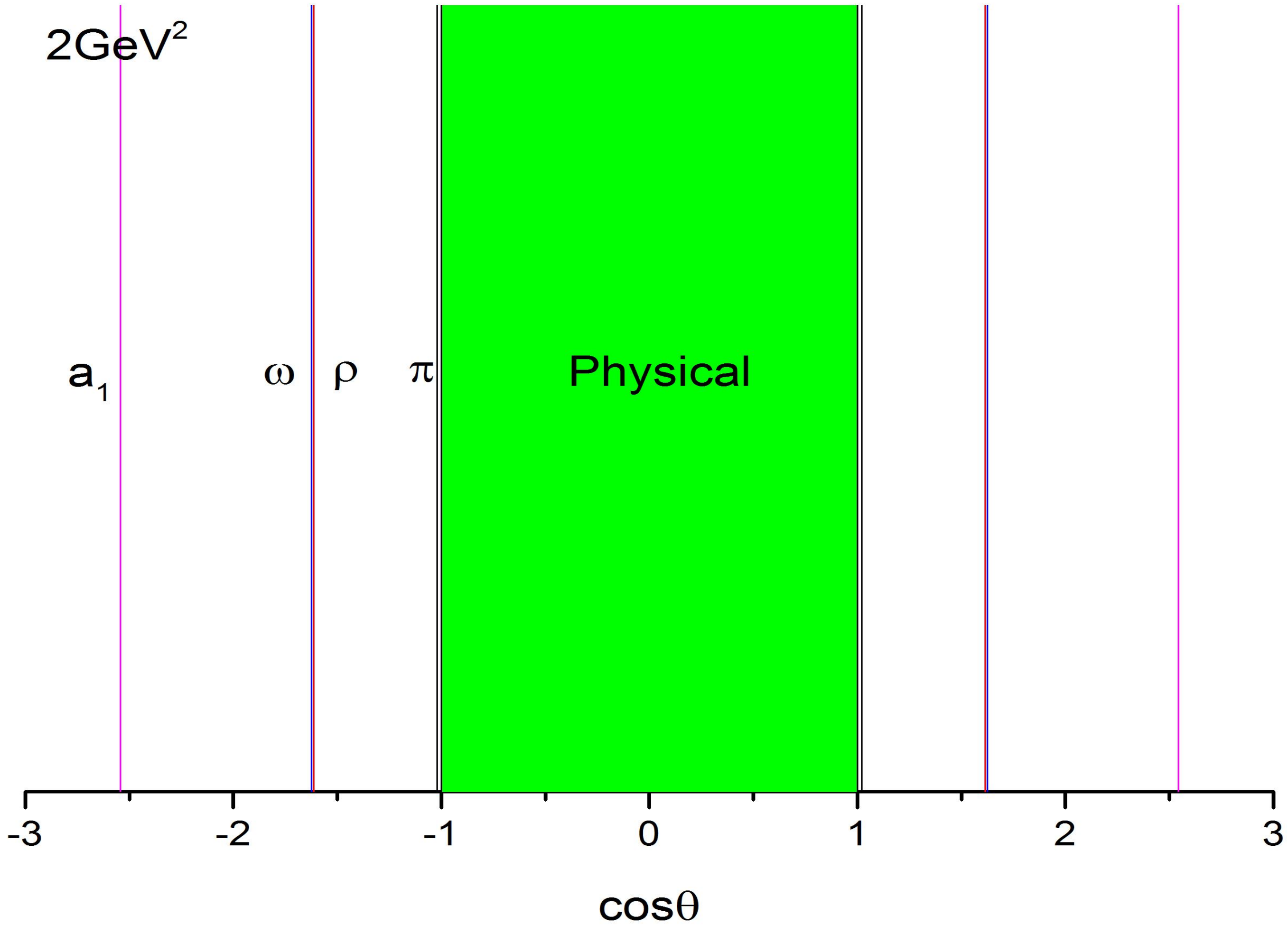
- For $\gamma\gamma\rightarrow\pi\pi$ amplitudes above 2 GeV^2 we use Born terms to estimate, the uncertainty is within 10%.
- For $\gamma\gamma\rightarrow KK$ amplitudes the uncertainty is within 25%. But KK cross section has much less contribution to the PV sumrule.
- The Born term itself satisfies PV sumrule, so higher partial waves do not contribute. Finally one has:

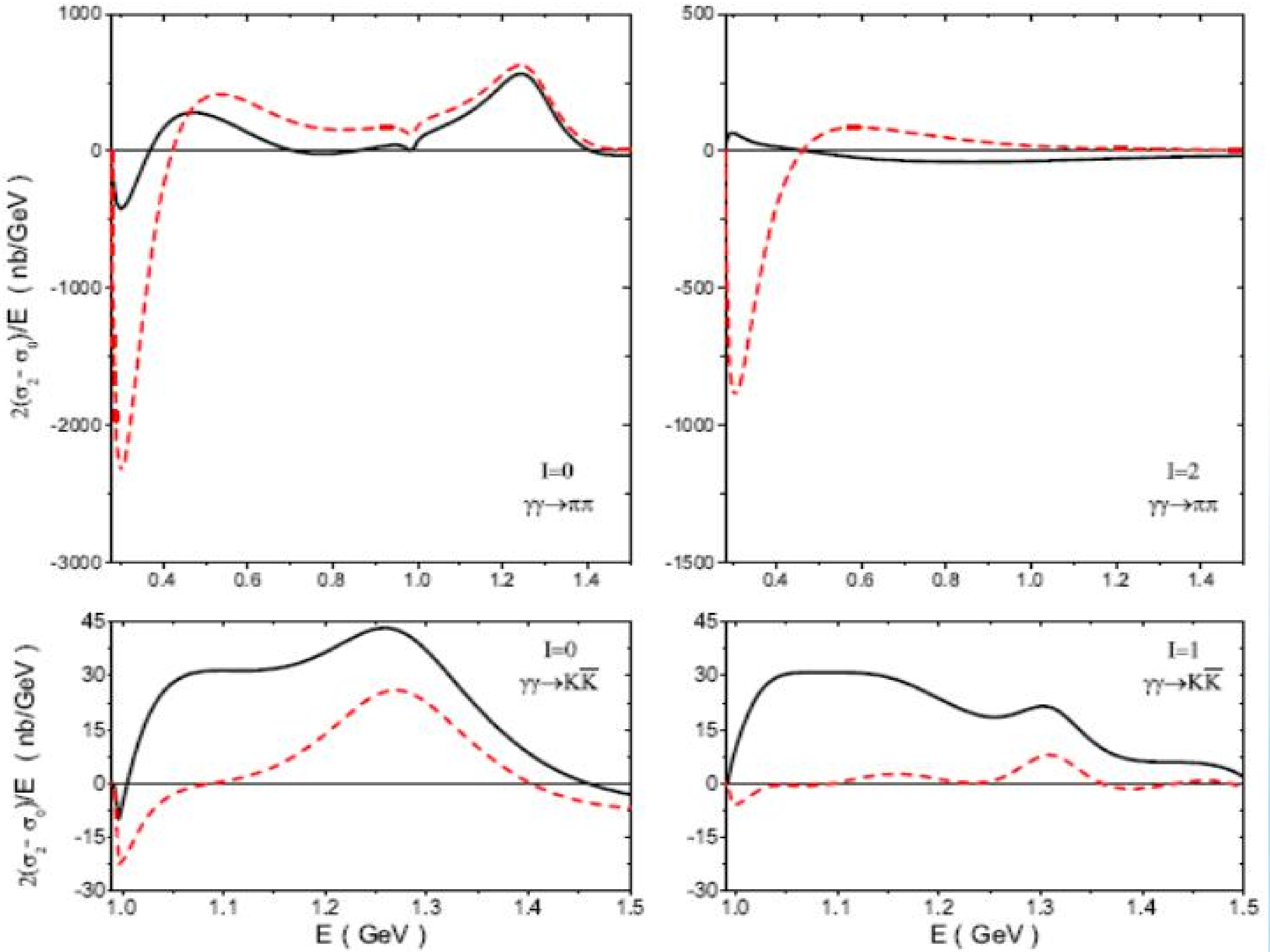
$$\overline{\Delta}\sigma^I(s) = \sigma^I_{D2}(s) - \sigma^I_S(s) - \sigma^I_{D0}(s) - \left[\sigma^I_{D2}(s) - \sigma^I_S(s) - \sigma^I_{D0}(s)\right]_{Born}$$

Constraints to light-by-light sumrule



Constraints to light-by-light sumrule





I=0
γγ→ππ
I=2
γγ→ππ
I=0
γγ→KK̄
I=1
γγ→KK̄
$2(\sigma_2 - \sigma_0)/E$ (nb/GeV)
E (GeV)

Constraints to light-by-light sumrule

- The contribution to PV sumrule is certainly not zero.

evaluation of $\Delta^I(4m_\pi^2, \infty, Z=1)$	$I=0$	$I=1$	$I=2$
$\gamma\gamma \to \pi^0$ [6] (nb)	-	-190.9±4.0	-
$\gamma\gamma \to \eta, \eta'$ [6] (nb)	-497.7±19.3	-	-
$\gamma\gamma \to a_2(1320)$ [6] (nb)	-	*135.0±12±25* †	-
$\gamma\gamma \to \pi\pi$ (nb)	308.0±41.5	-	-44.2±6.1
$\gamma\gamma \to \overline{K}K$ (nb)	23.7±7.5	18.1±4.9	-
SUM (nb)	-166.0±46.4	-37.8±28.4	-44.2±6.1

Multi-particles' contribution

- We have no decomposition information about the amplitudes of multi-particles' channel.

$$\mathcal{R}(s_1, s_2; \text{channel}) = \frac{\Delta(s_1, s_2, Z = 1; \text{channel})}{\Sigma(s_1, s_2, Z_{\text{exp}}; \text{channel})}$$

cosθ

total cross section

- Our amplitude analysis typically gives R=0.65 from s = 1 to 2 GeV^2.
- ππ Born amplitude gives R>1.

Constraints to light-by-light sumrule

- 4π channel's contribution is roughly of 150–200 nb in the I = 0 mode and 50 nb in the I = 2 mode.

Channel	Publication	E_1 (GeV)	E_2 (GeV)	Σ (nb)	$\mathcal{R}(Born)$
$\pi^+\pi^-$ $(Z = 0.6)$	[16]	2.4	4.1	0.44 ± 0.01	1.61
K^+K^- $(Z = 0.6)$	[16]	2.4	4.1	0.39 ± 0.01	1.29
$\pi^0\pi^0$ $(Z = 0.8)$	[17]	1.44	3.3	8.8 ± 0.2	1.18
$\pi^0\pi^0\pi^0$	[18]	1.525	2.425	5.8 ± 0.8	1.55
$\pi^+\pi^-\pi^0$ (non-res.)	[19]	0.8	2.1	23.0 ± 1.3	1.39
$K_sK^\pm\pi^\mp$	[20]	1.4	4.2	9.7 ± 1.6	
$\pi^+\pi^-\pi^+\pi^-$	[21]	1.1	2.5	$215 \pm 11 \pm 21$	1.49
$\pi^+\pi^-\pi^+\pi^-$	[22]	1.0	3.2	$153 \pm 5 \pm 39$	1.48
$\pi^+\pi^-\pi^0\pi^0$	[23]	0.8	3.4	$103 \pm 4 \pm 14$	1.42

Pascalutsa-Vanderhaeghen light-by-light sumrule

- 4π is likely the largest contribution to be added below 2.5 GeV to make the PV sumrule for both I=0,2 zero.
- Experiments on 4π production would be rather helpful, for example $\rho^+\rho^-$, $\rho^0\rho^0$ production from two untagged photon.

BESIII(BEPCII)? Belle(KEKB)?

Polarizabilities

Polarizabilities may play important role on LbL sumrule

K.T.Engel et.al. PRD86 (2012) 037502

fixed by Adler zero and $(\alpha_1 - \beta_1)_{\pi^+} = 4.0$

easiest one to be measured by experiment

Polarizabilities $\lambda = 0$	Model I	Model II	Model III	Model IV	Model V	ChPT + Resonance Model
$(\alpha_1 - \beta_1)_{\pi^+}$	**4.0 ± 1.2 ± 1.4**	**0.0**	**11.6**	**4.0**	**4.0**	5.7±1.0
$(\alpha_2 - \beta_2)_{\pi^+}$	15.7±1.1	13.0±1.1	20.9±1.1	13.2±3.4	18.1±2.5	16.2[21.6]
$(\alpha_1 - \beta_1)_{\pi^0}$	-0.9±0.2	-0.8±0.1	-1.1±0.2	-0.8±0.2	-1.0±0.2	-1.9±0.2
$(\alpha_2 - \beta_2)_{\pi^0}$	20.6±0.8	17.8±0.8	26.0±0.8	18.6±2.4	22.4±1.8	37.6±3.3
$\lambda = 2$						
$(\alpha_1 + \beta_1)_{\pi^+}$	0.26±0.07	0.26±0.07	0.26±0.07	0.17±0.51	0.42±0.22	0.16[0.16]
$(\alpha_2 + \beta_2)_{\pi^+}$	-1.4±0.5	-1.4±0.5	-1.4±0.5	-0.9±3.5	-2.4±1.5	-0.001
$(\alpha_1 + \beta_1)_{\pi^0}$	0.60±0.06	0.60±0.06	0.60±0.06	-0.04±0.52	0.90±0.17	1.1±3.3
$(\alpha_2 + \beta_2)_{\pi^0}$	-3.7±0.4	-3.7±0.4	-3.7±0.4	0.4±3.4	-5.5±1.1	0.04

6. Summary

Amplitudes

Including all new datasets and analyticity, unitarity, crossing symmetry, we perfom an amplitude analysis on photon photon collision.

LbL constraint

Our individual amplitudes are the boundary of LbL amplitudes when virtual photons are changed into real photons.

PV sumrule

We test PV sum rules for real photon case. 4π is likely the largest contribution to be added below 2.5 GeV.

polarizability

We predict pion polarizabilities. They may also play an important role in LbL scattering.

Thank You!