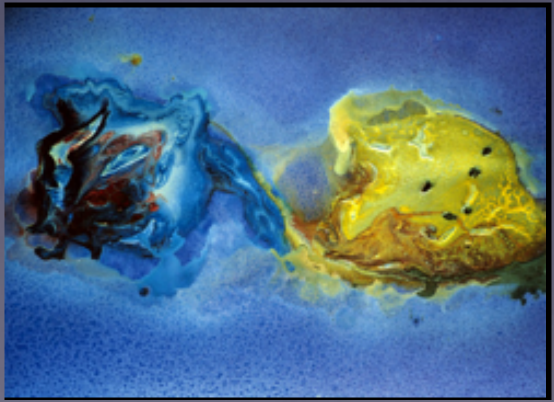
Quarkonia and EXOTICS with Effective Field Theories

Nora Brambilla

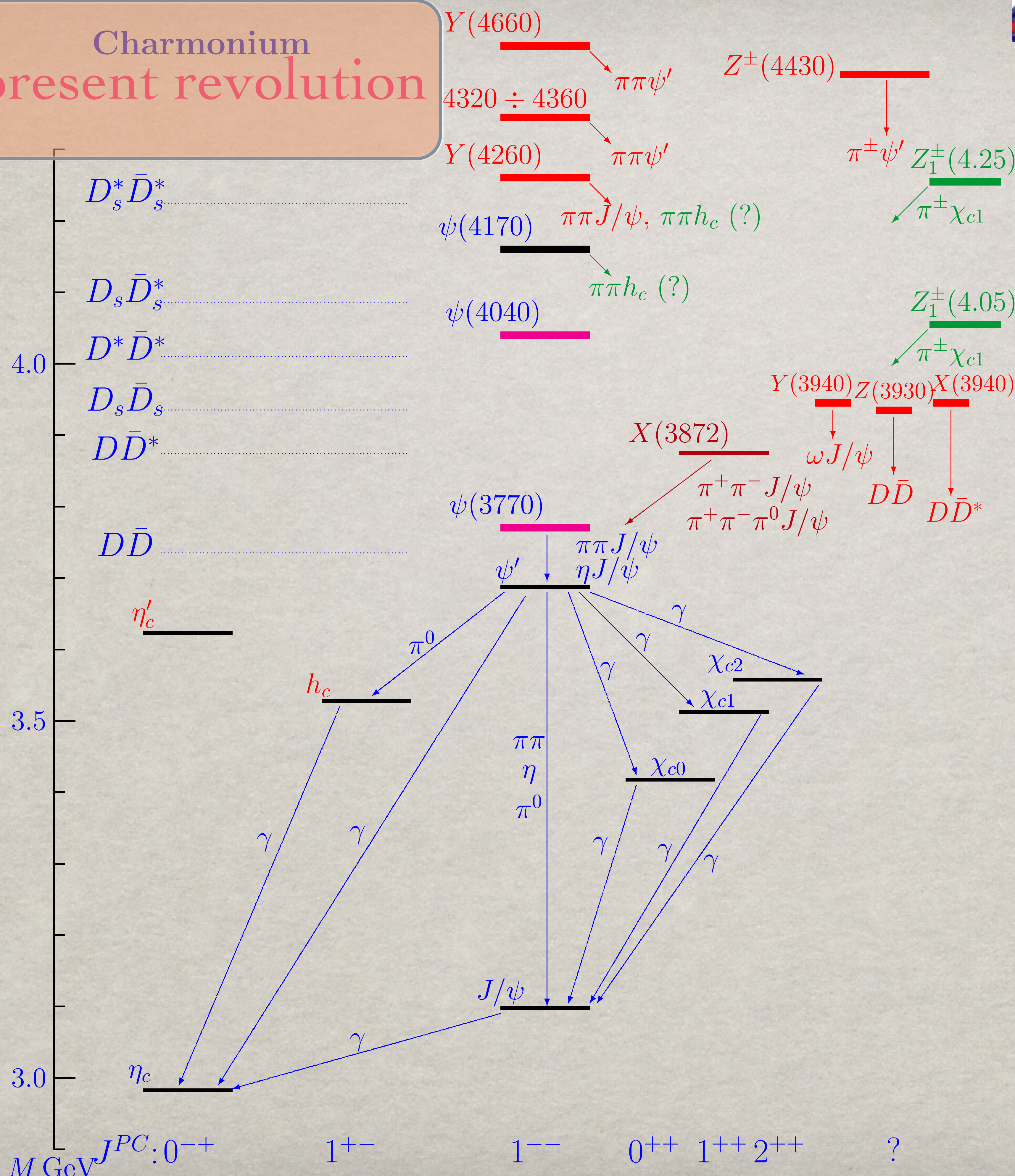
work done in the CRC110 in collaboration with A. Vairo, M. Berwein, G. Krein, J. Tarrus, V. Shabotenko



Physik Department TUM
T30f

- Quarkonium physics and scales
  - Quarkonium below the strong decay threshold: theory known (EFT and lattice)
    - Quarkonium X Y Z at and above threshold:models and degrees of freedom
      - BO versus van der Waals regime: EFTs for QED
        - EFT for hybrids: results and comparison to the data, outlook for tetraquarks
          - van der Waals bottomonia interaction : bound states?

Charmonium
the present revolution



THE COLLIDER DETECTOR AT FERMILAB

DØ

BELLE

BABAR
Collaboration Home Page

BES

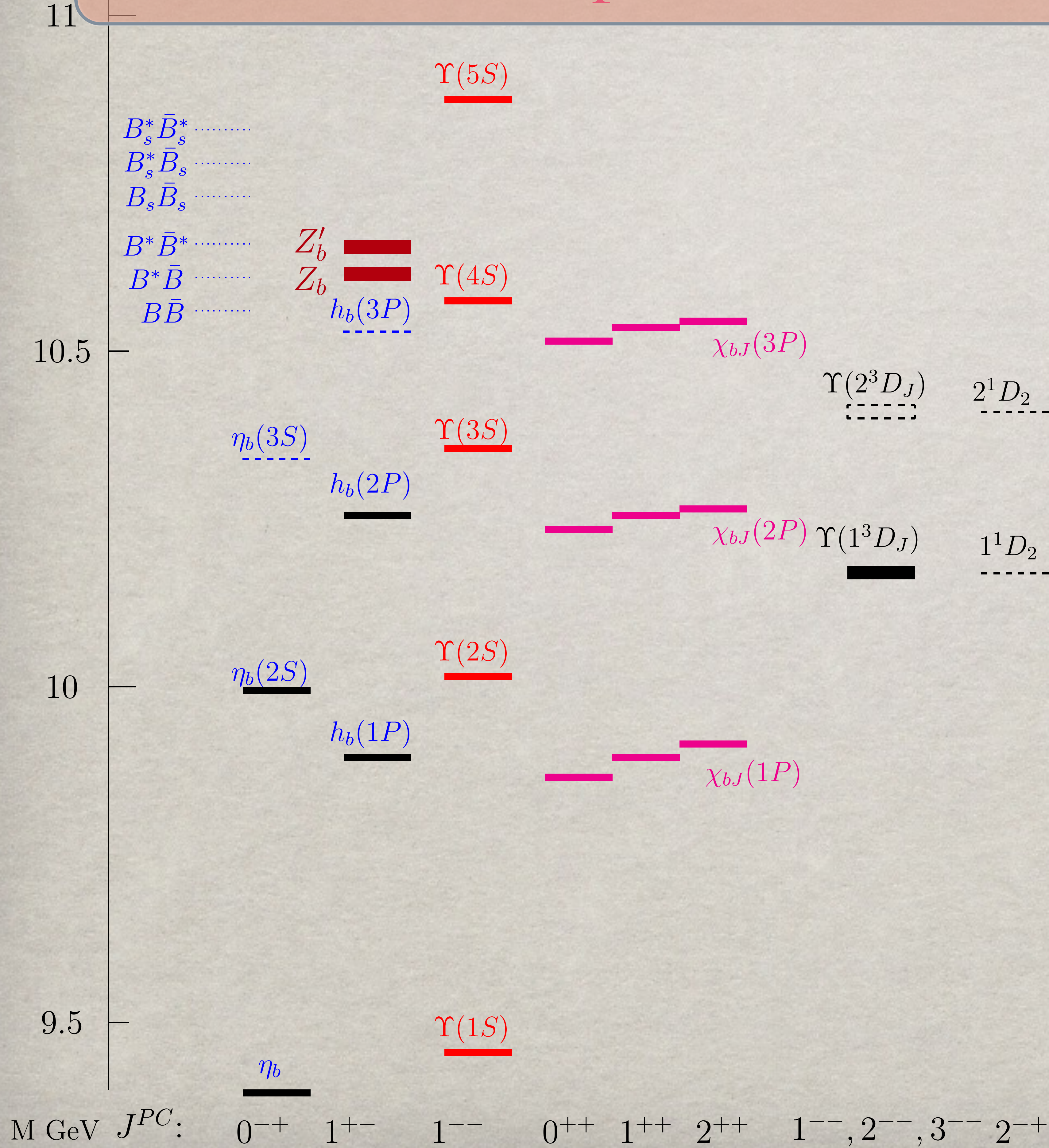
CLEO

ATLAS

CMS

LHCb

bottomonium: the present revolution



THE COLLIDER DETECTOR AT FERMILAB

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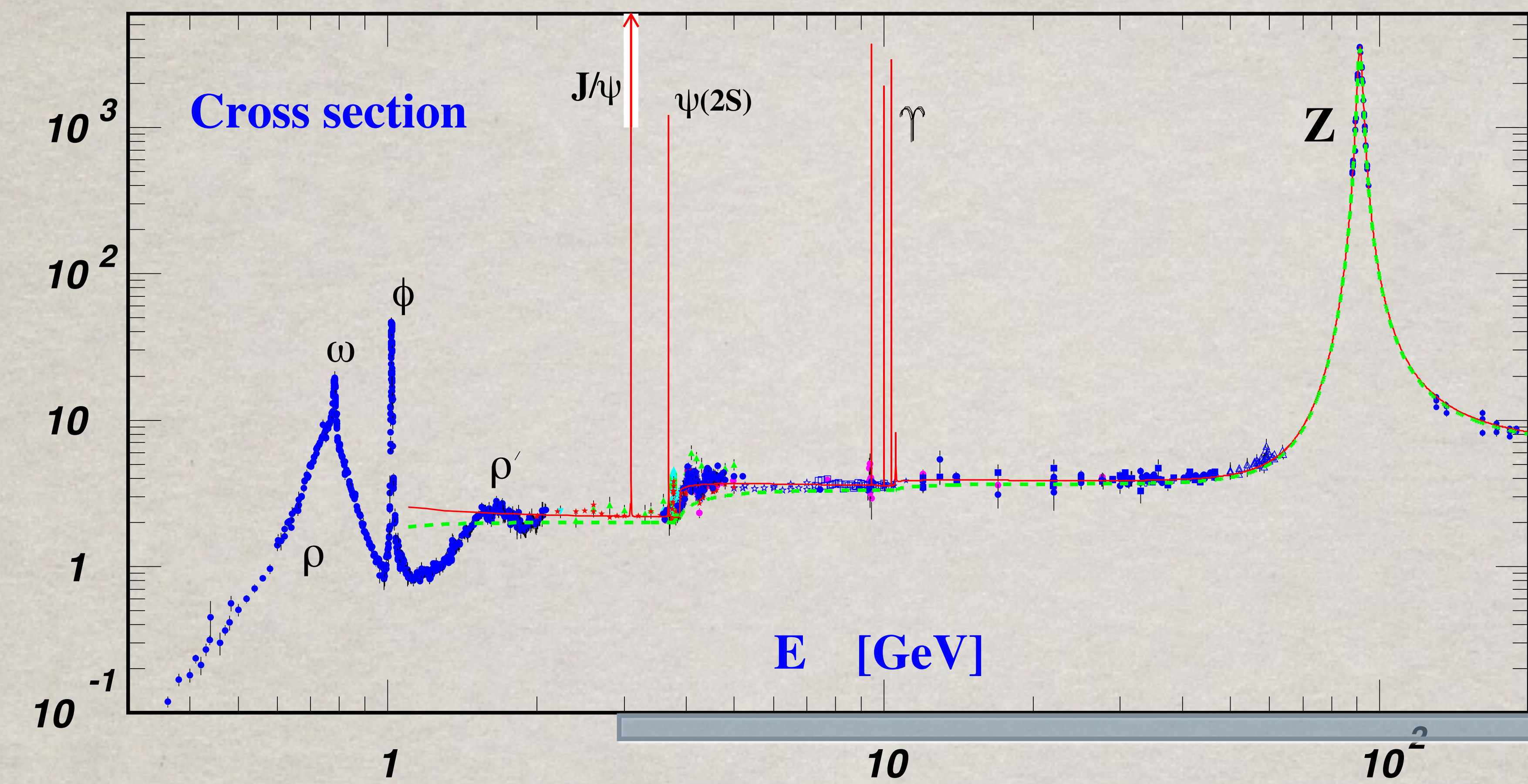
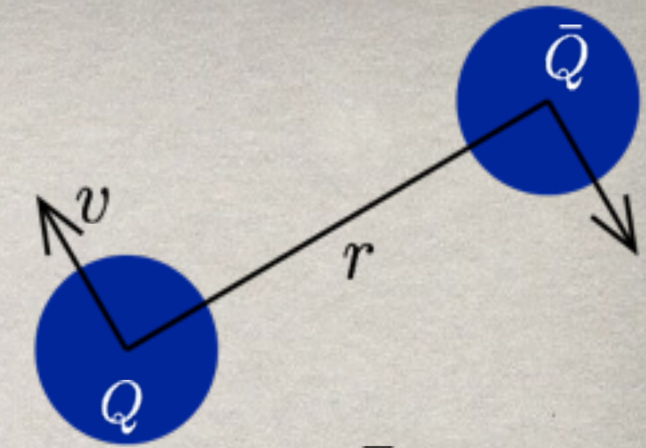
Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

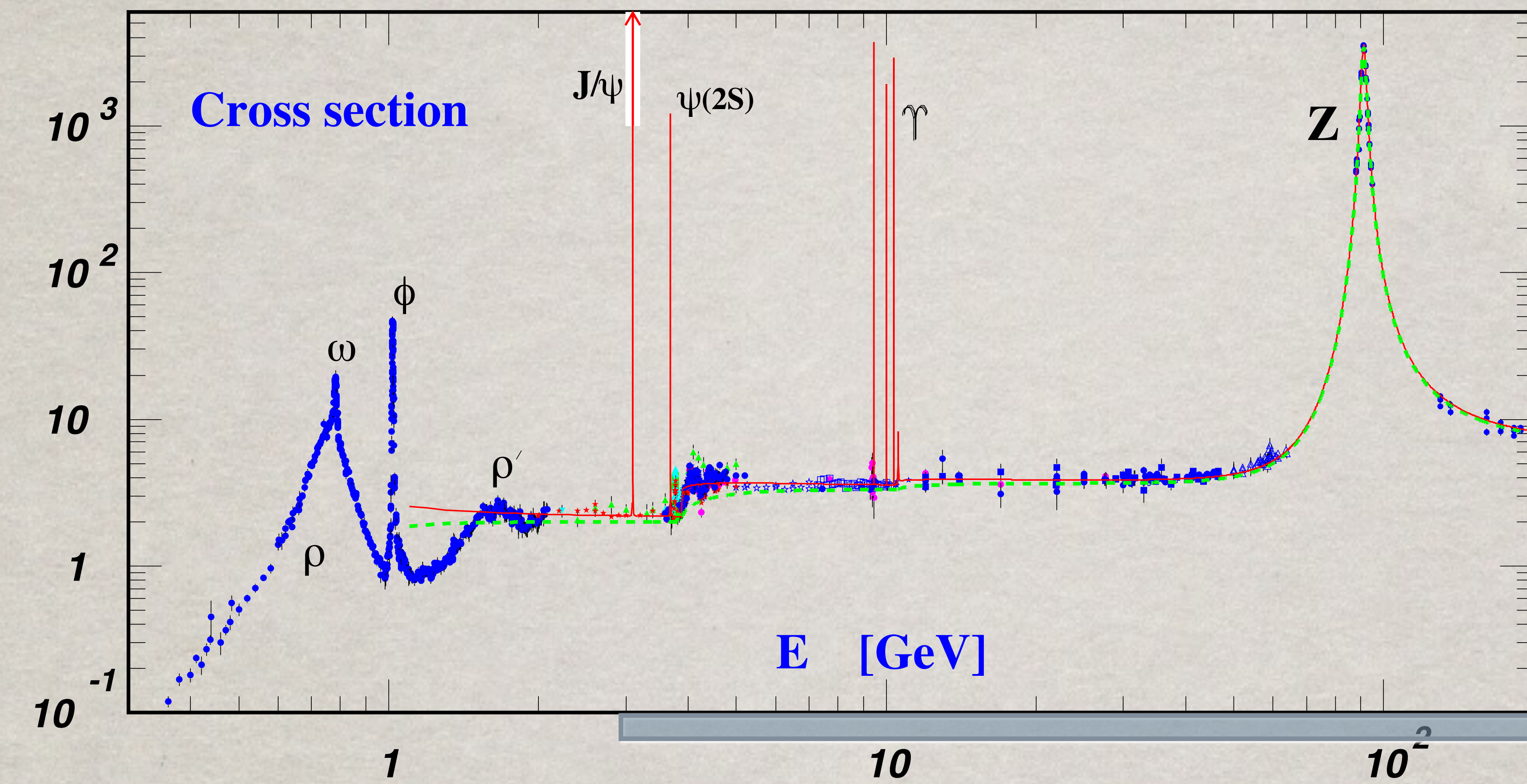
$m_c \sim 1.5$GeV

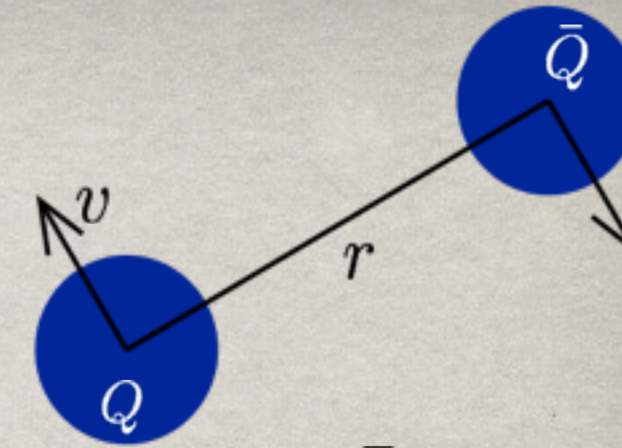
$m_b \sim 5$GeV

$m_t \sim 170$GeV

A large scale $m_Q \gg \Lambda_{QCD}$ $\alpha_s(m_Q) \ll 1$

Heavy quarks offer a privileged access





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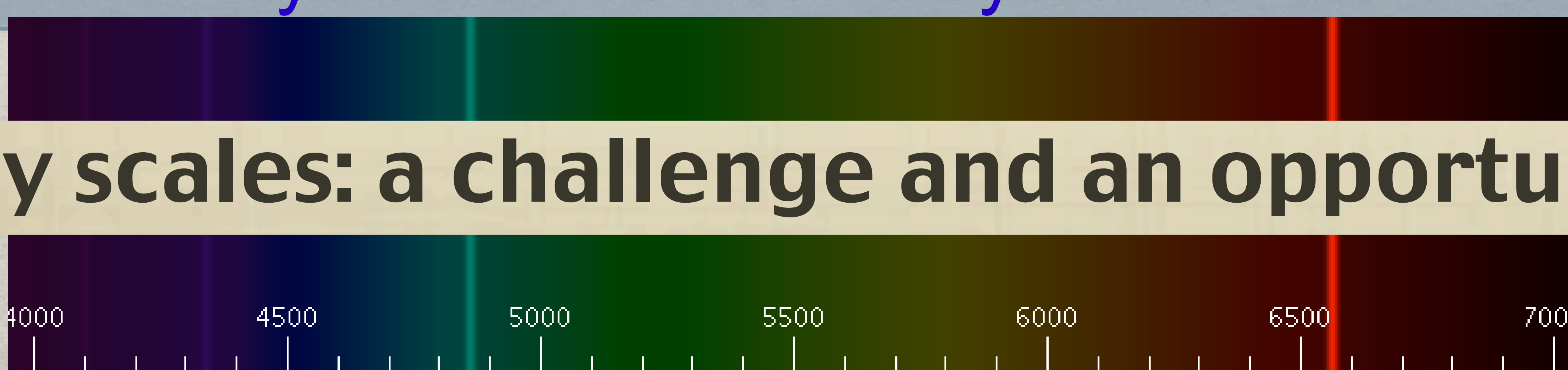
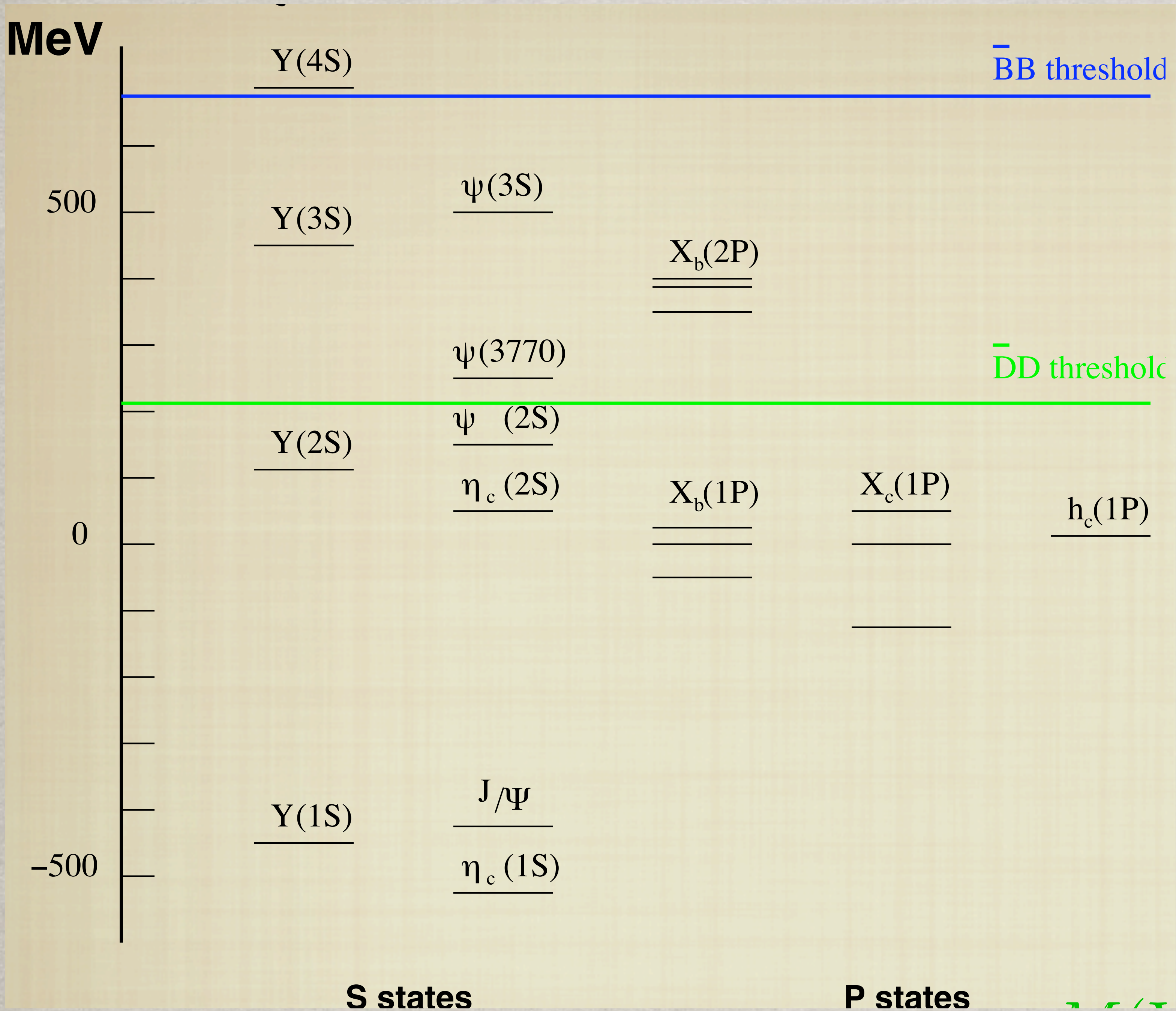
Heavy quarkonia are nonrelativistic bound systems: multiscale systems



many scales: a challenge and an opportunity

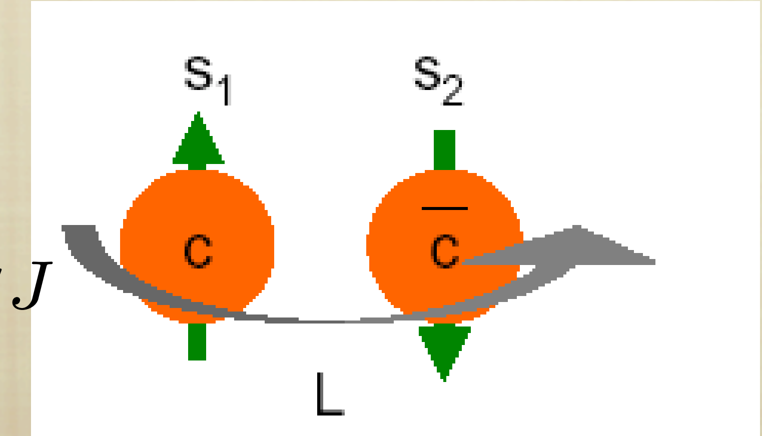
Electromagnetic bound states: atoms, molecules,

Quarkonium scales

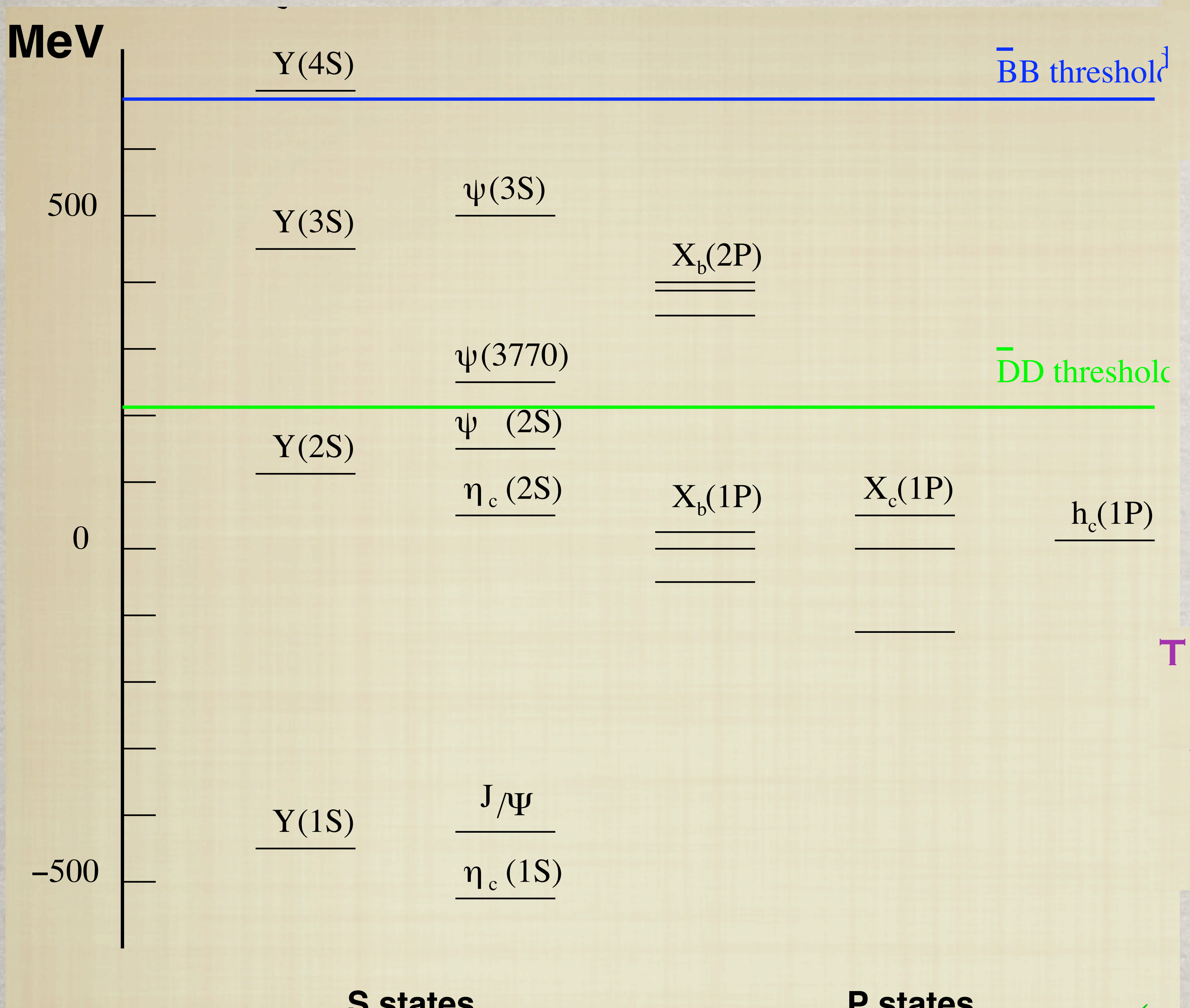


Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$



Quarkonium scales



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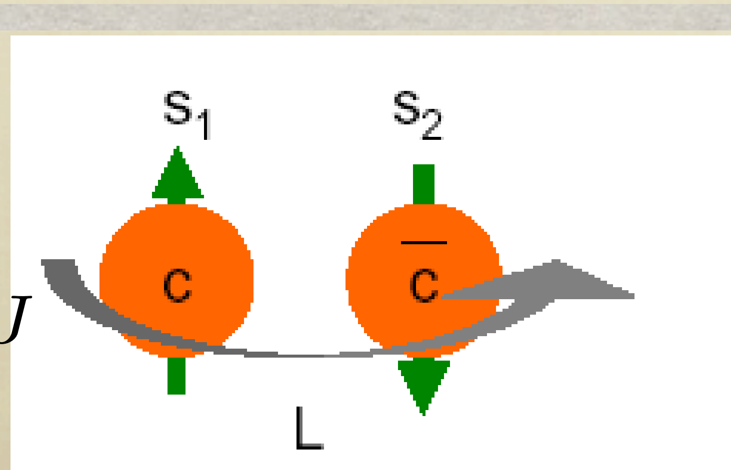


NR BOUND STATES HAVE AT LEAST 3 SCALES

$$m \gg mv \gg mv^2 \quad v \ll 1$$

$$mv \sim r^{-1}$$

and $\Lambda_{\rm QCD}$

THE SYSTEM IS NONRELATIVISTIC(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

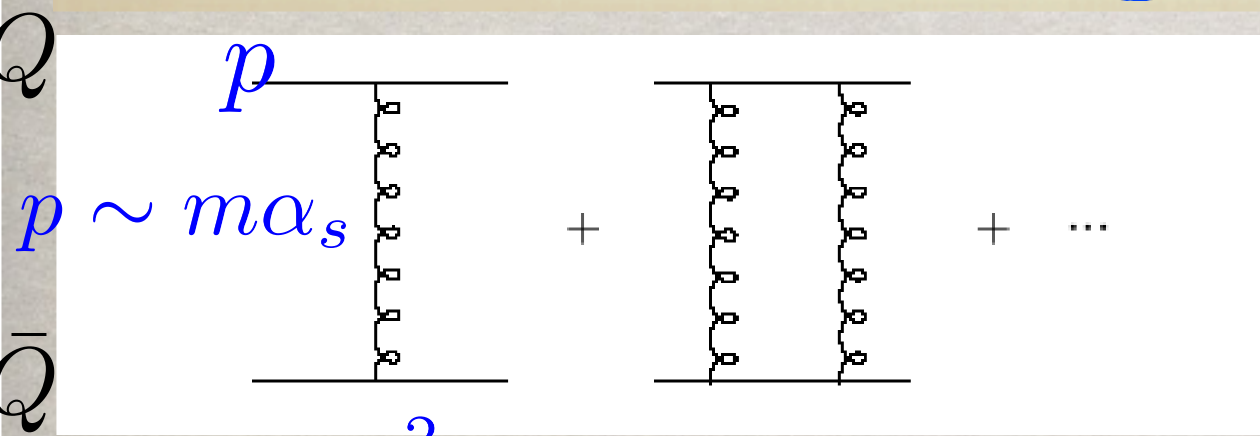
THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,{\rm GeV}; m_c \simeq 1.5\,{\rm GeV}$$

QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$



$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

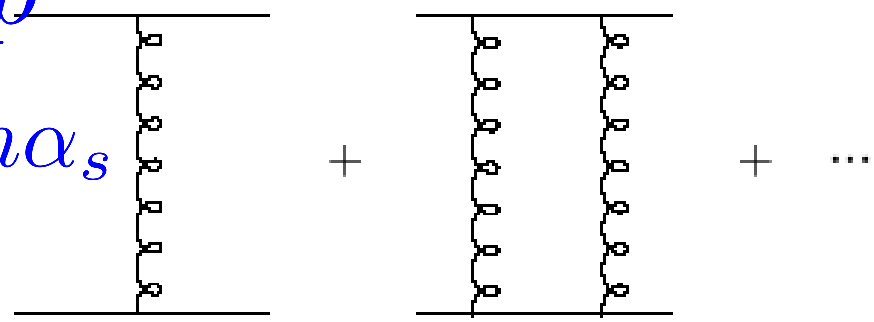
$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$

Q p

$p \sim m\alpha_s$



$\bar{Q}$

$$\frac{g^2}{p^2}\left(1+\frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E-\left(\frac{p^2}{m}+V\right)}$$

- From $(\frac{p^2}{m}+V)\phi = E\phi \to p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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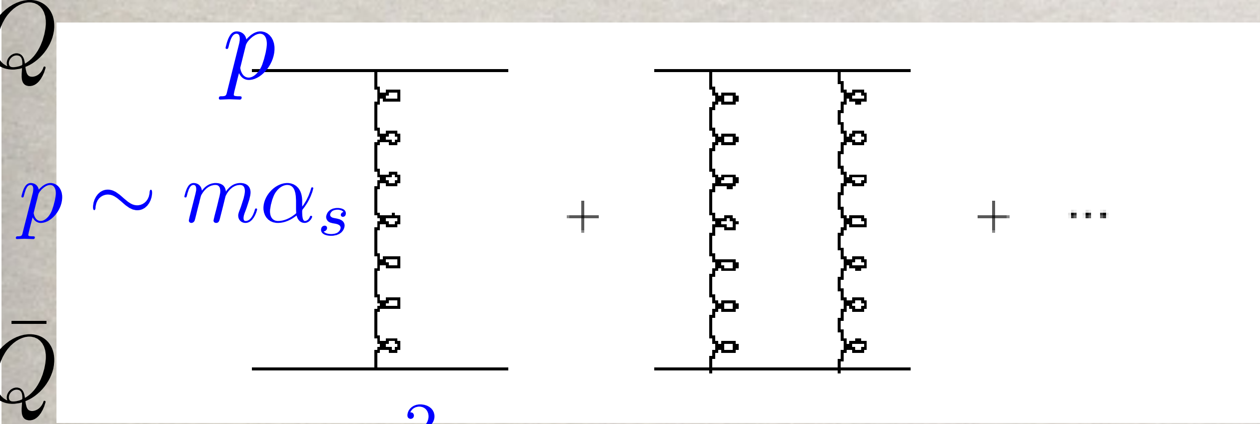

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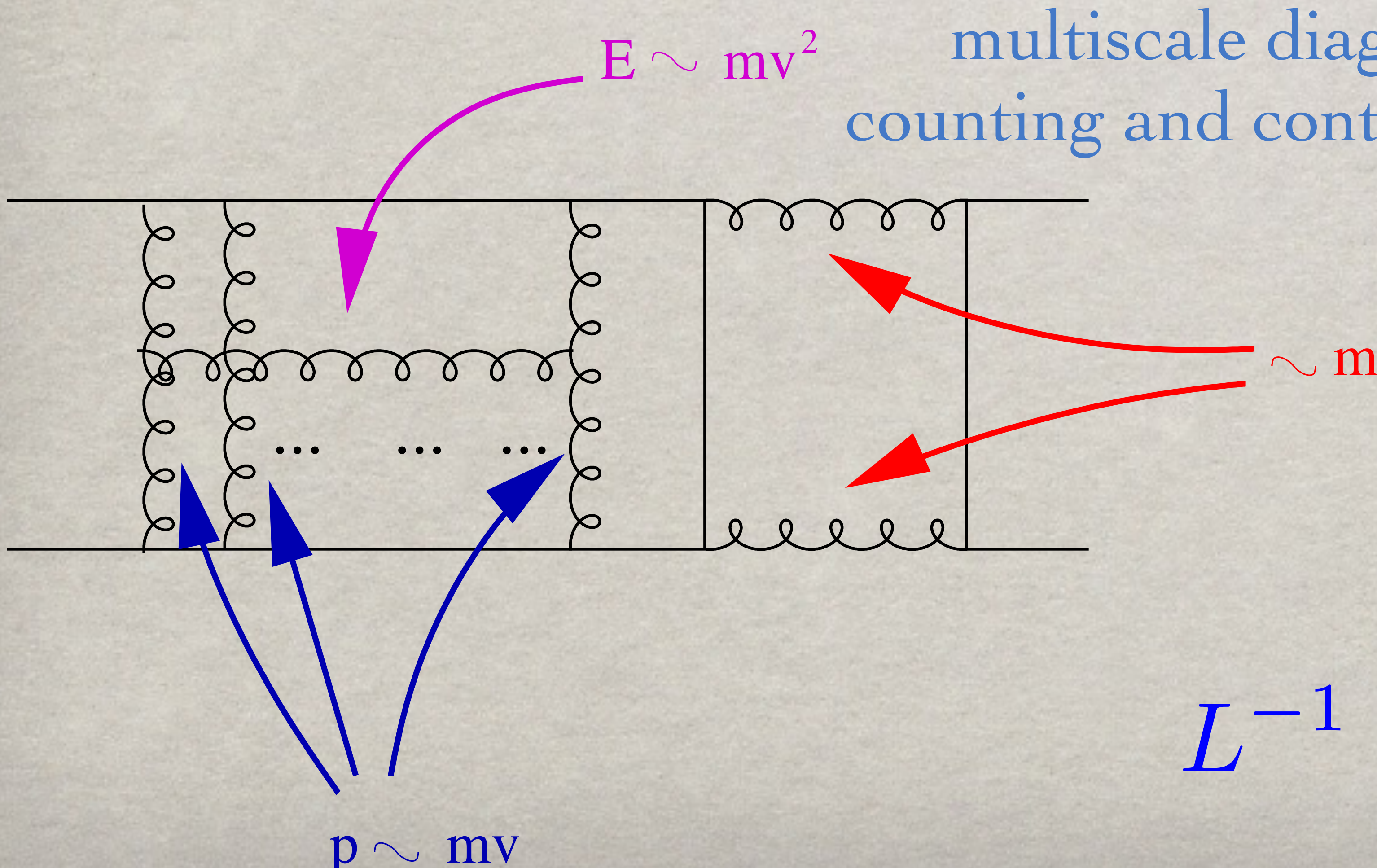
multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

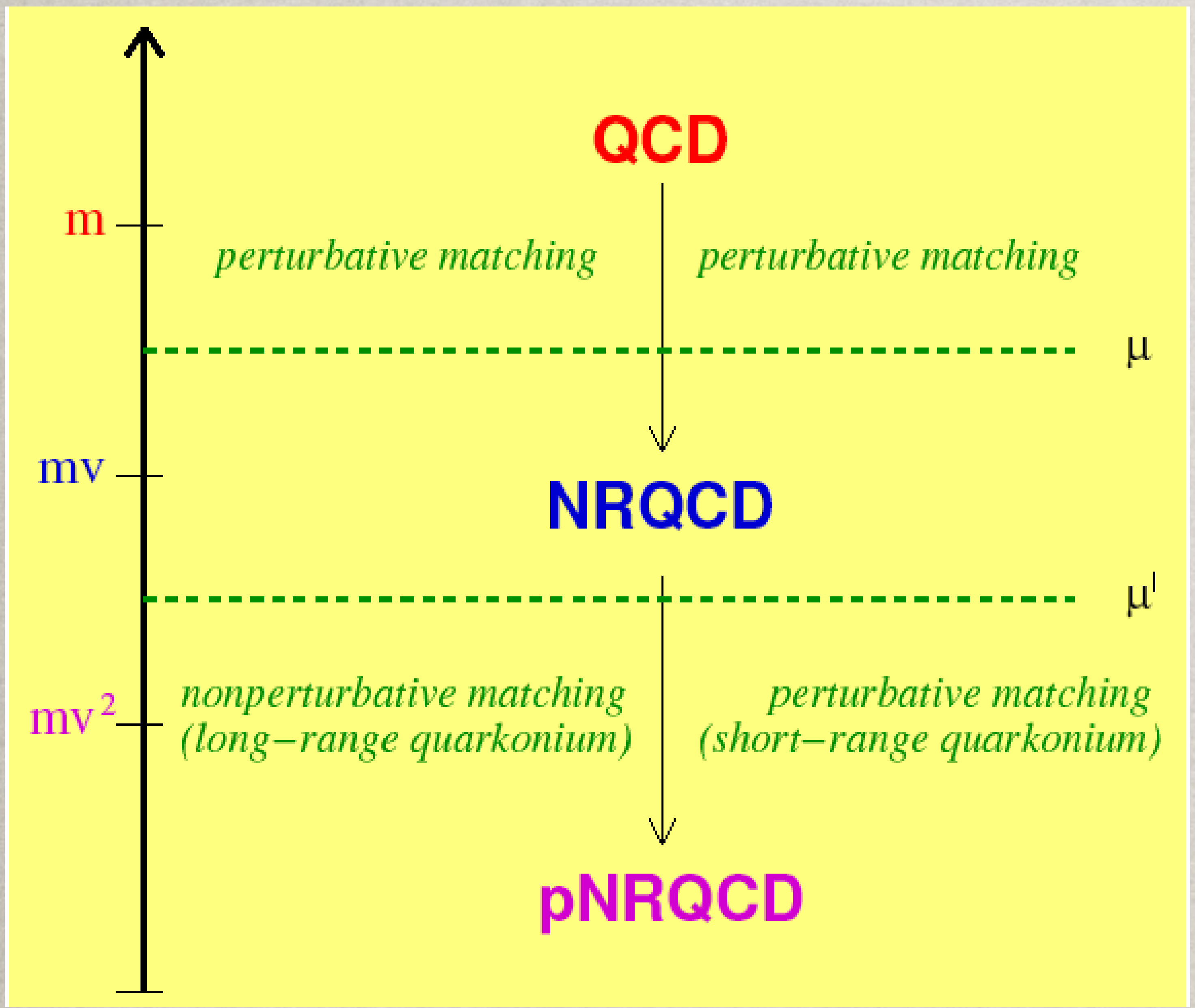


Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

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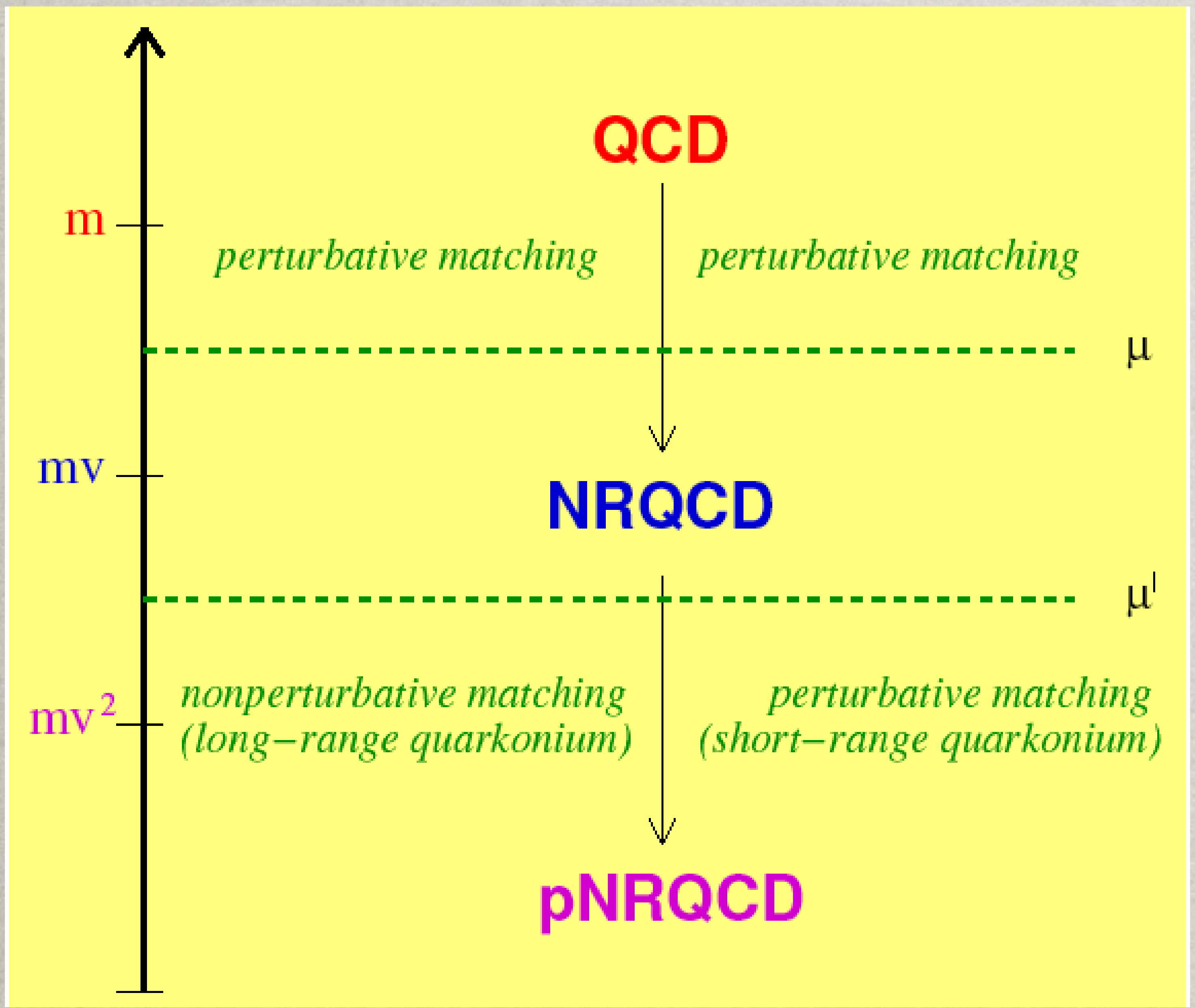

Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft (relative momentum)

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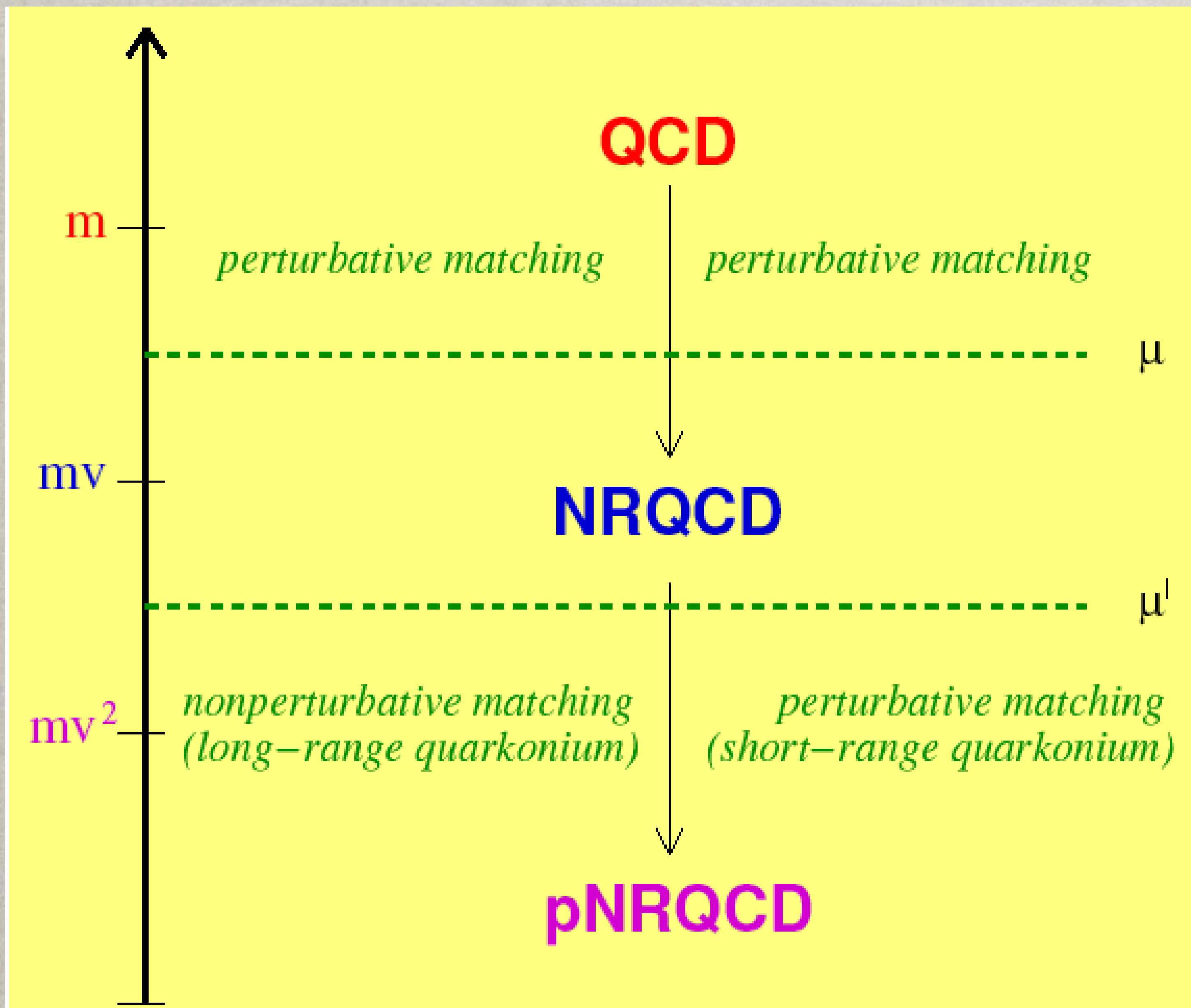

Hard

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Soft (relative momentum)

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

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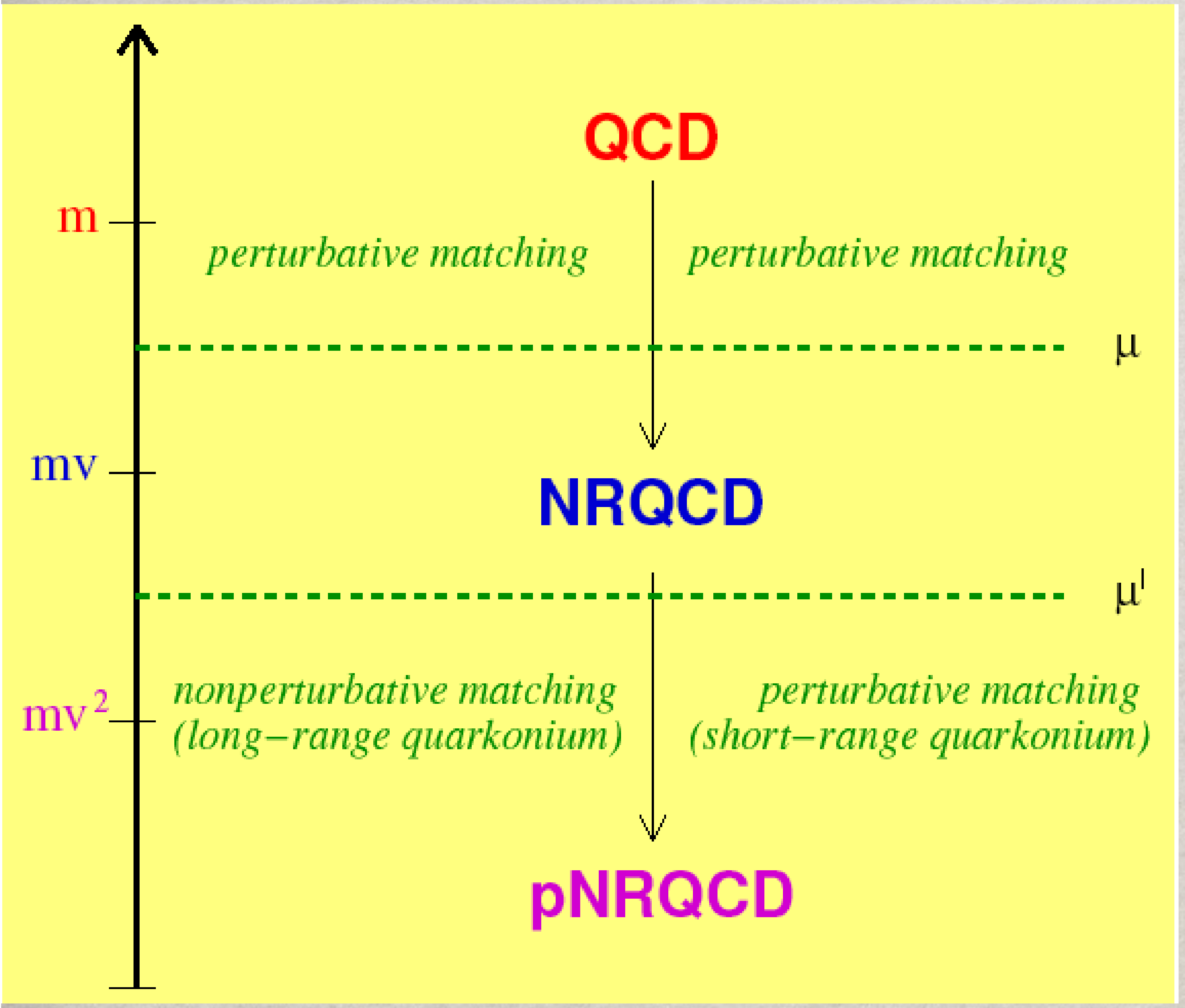
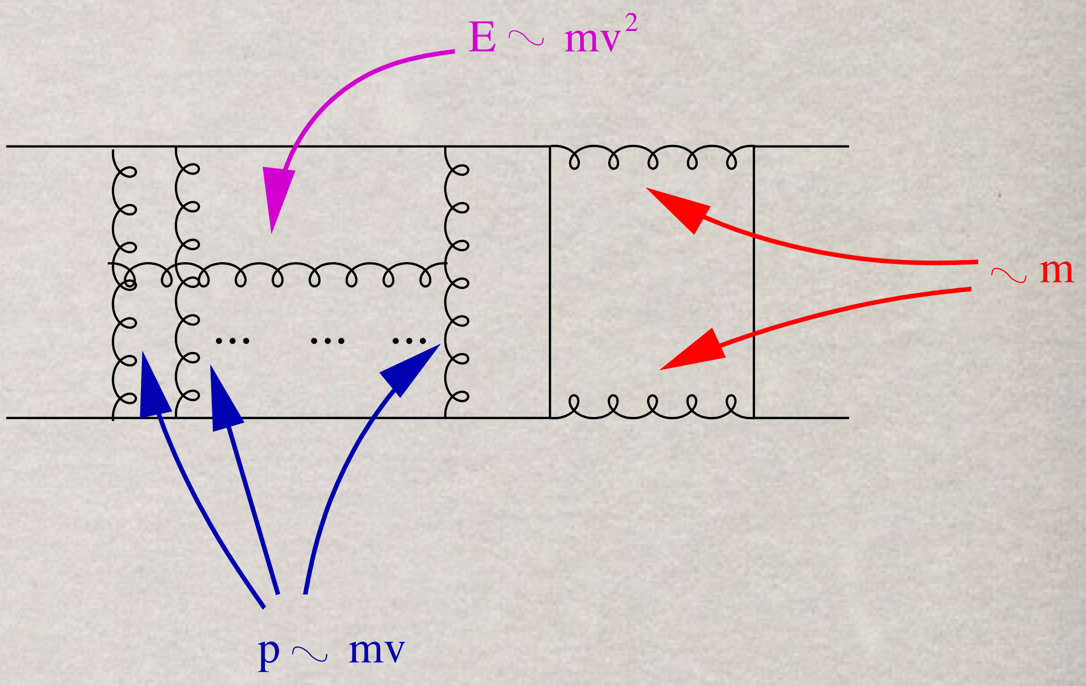
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)





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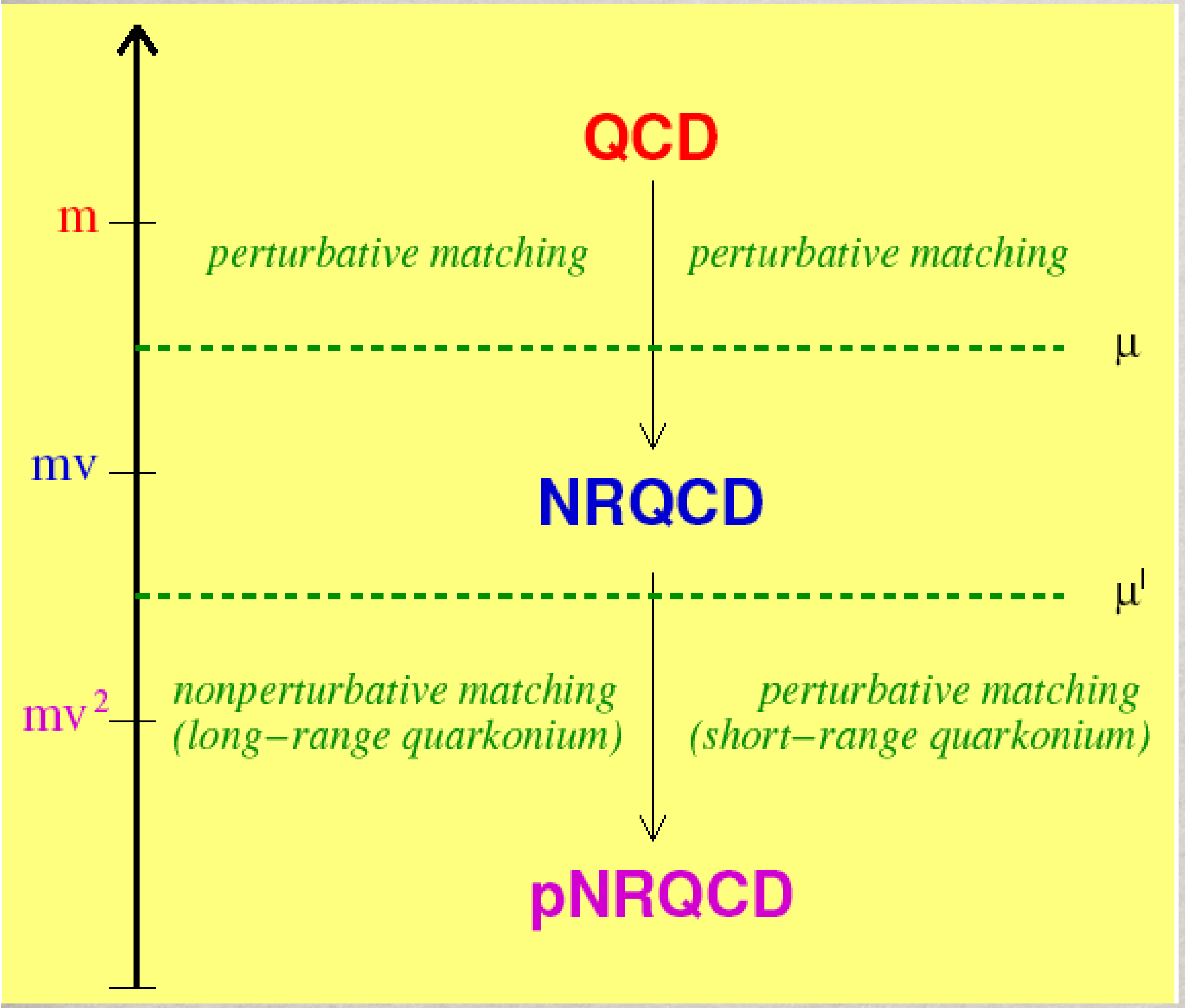

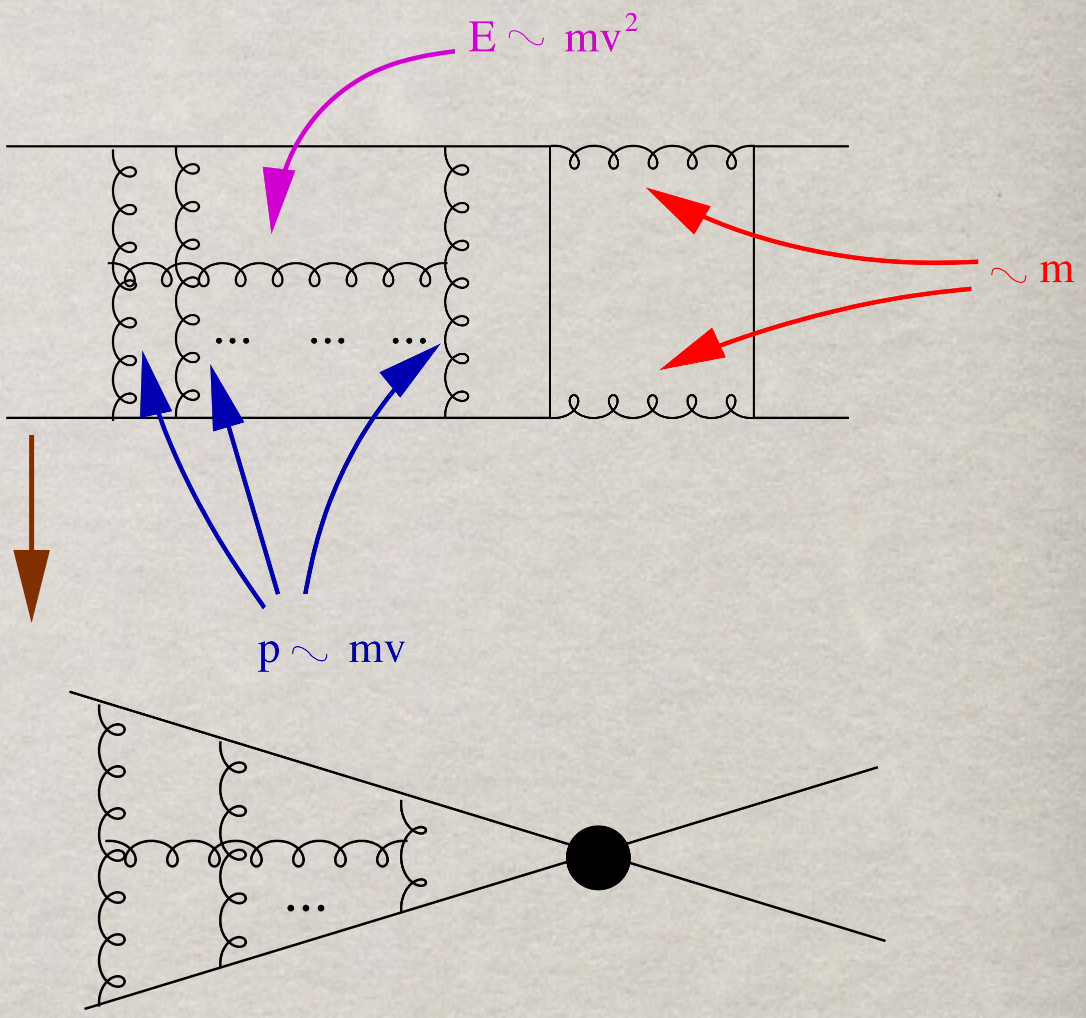

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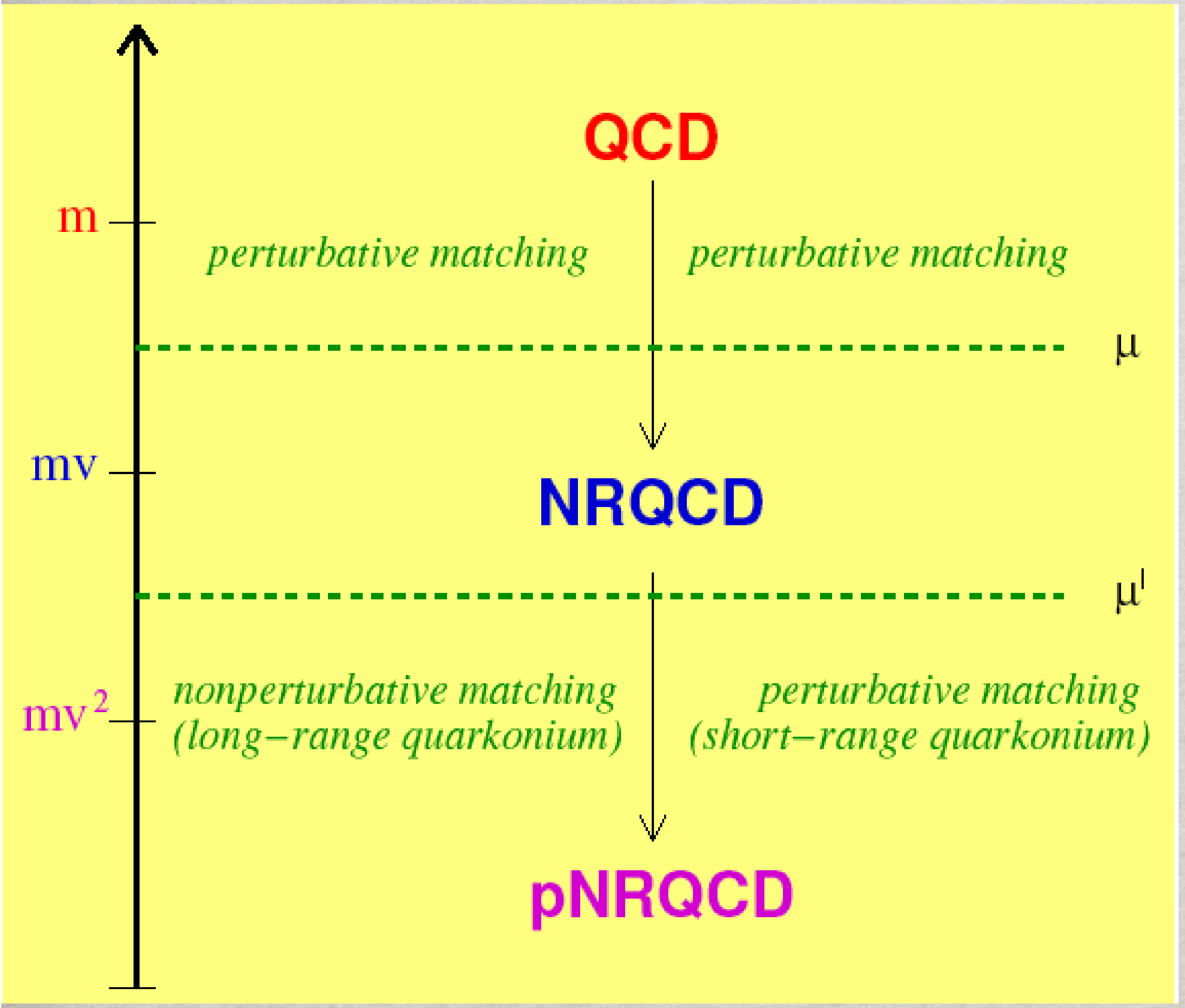
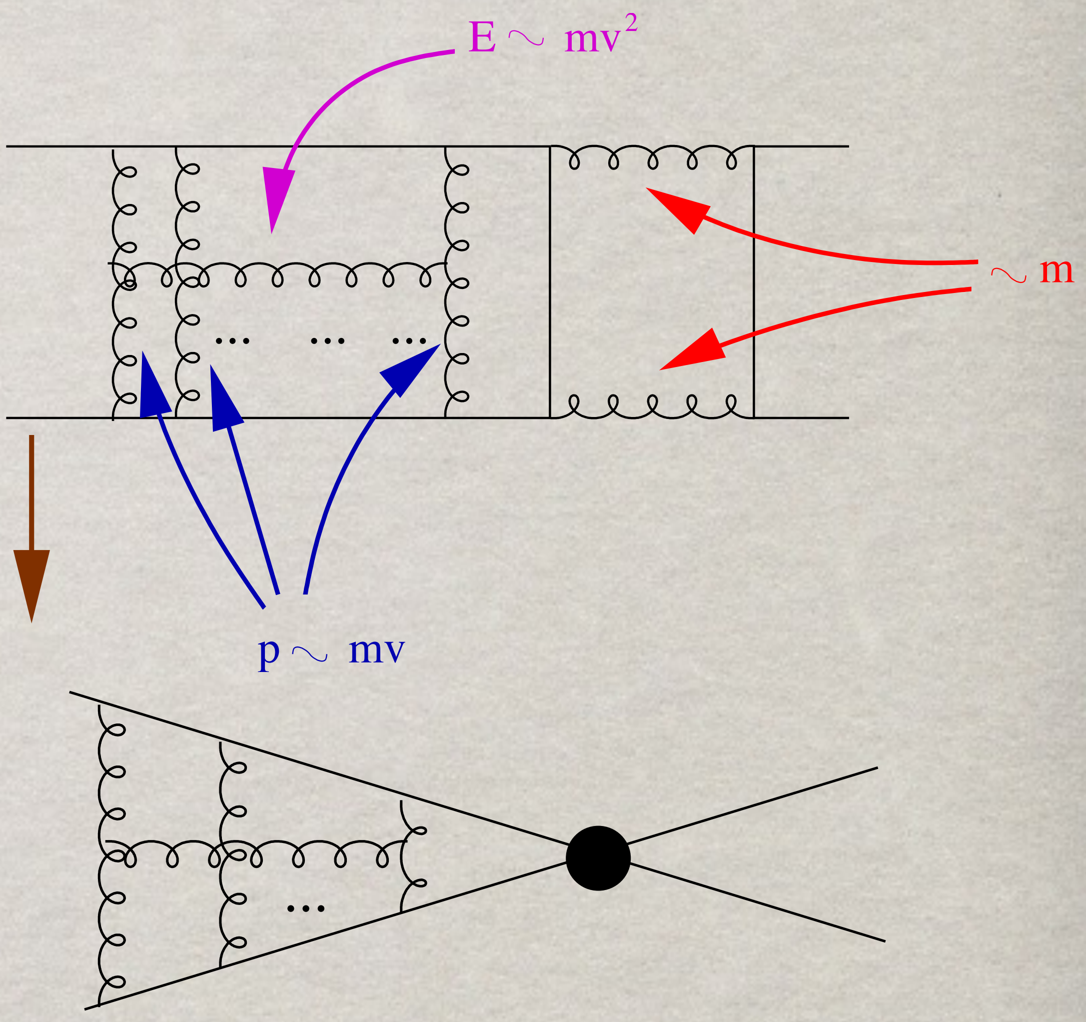
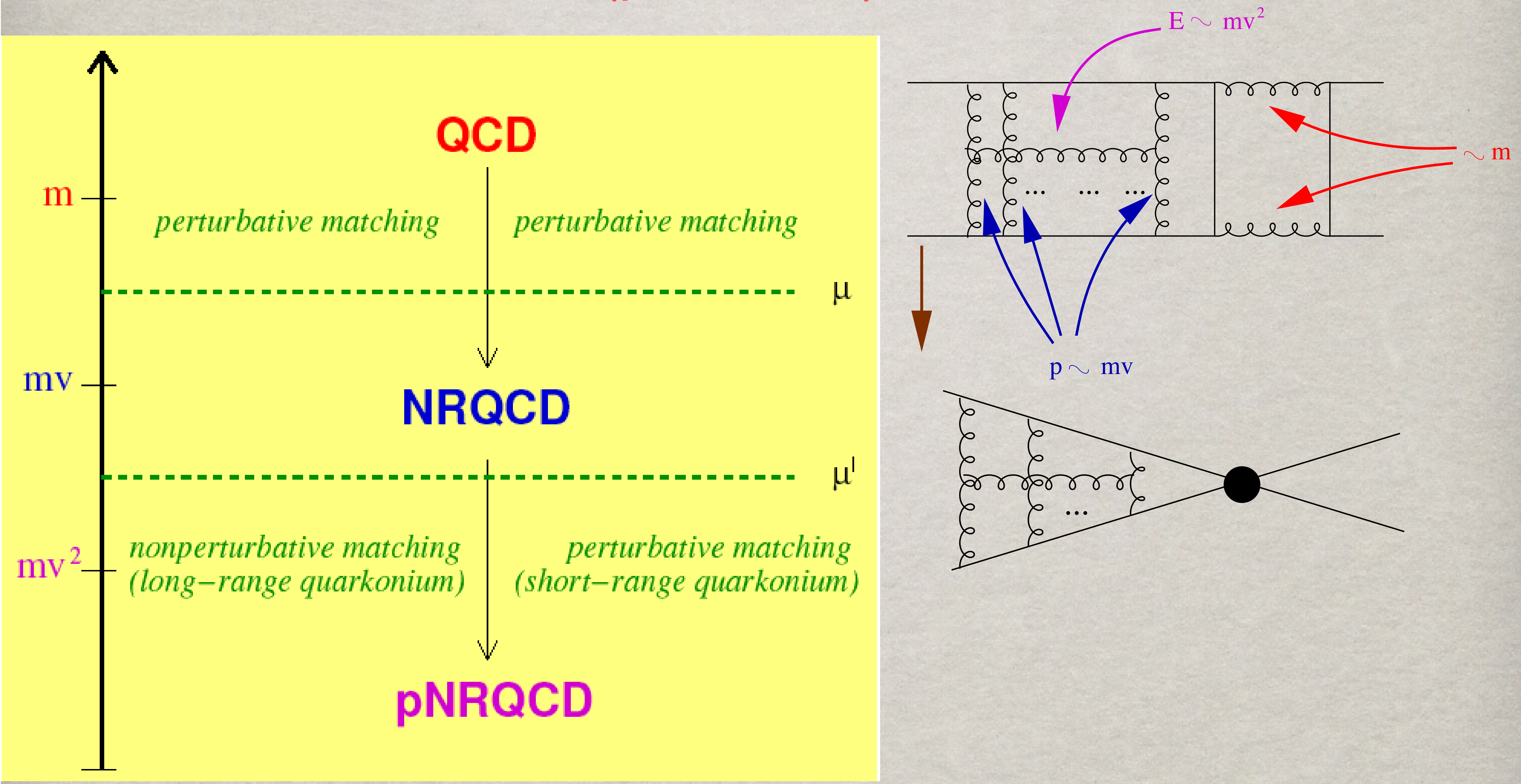




$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)



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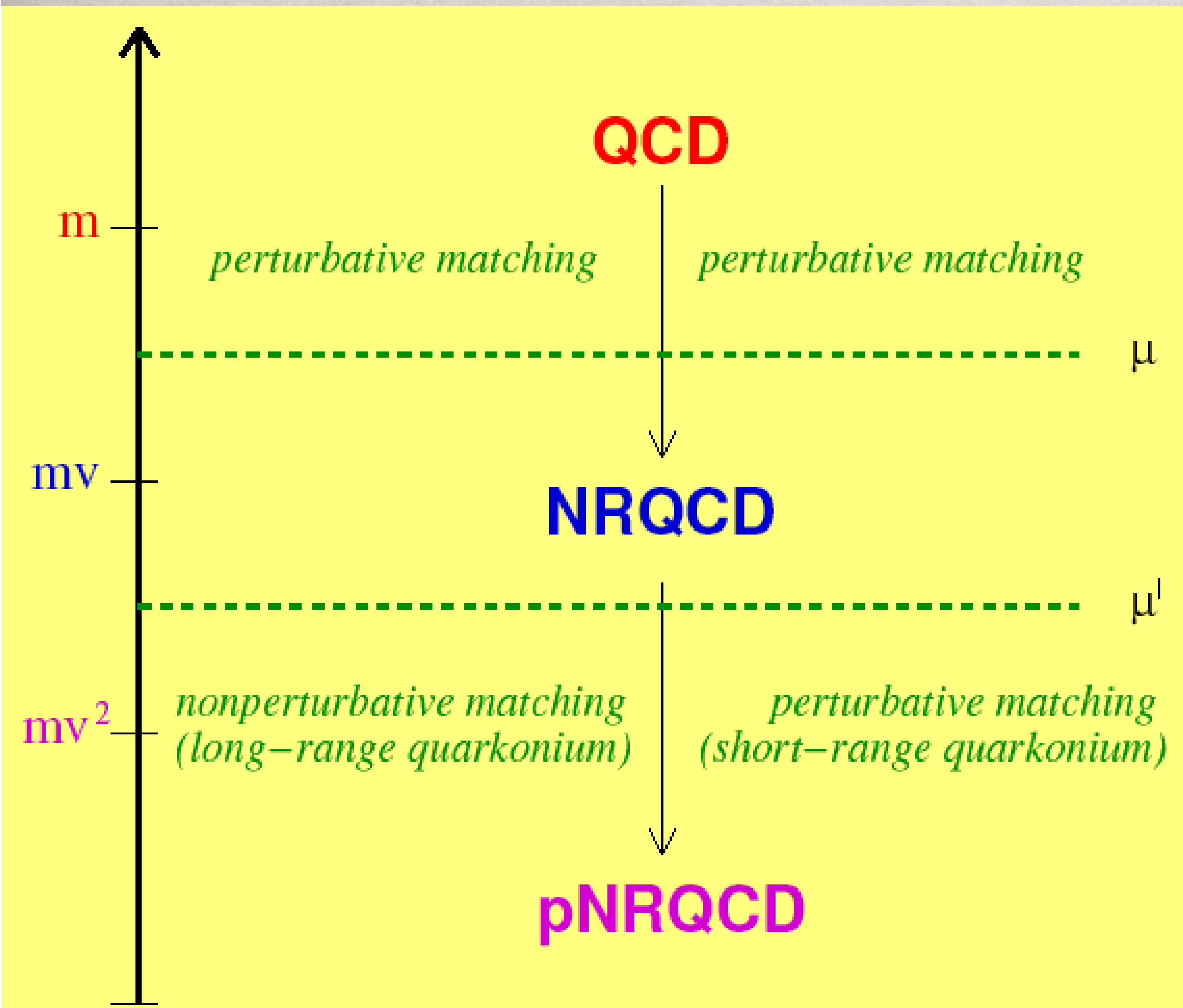

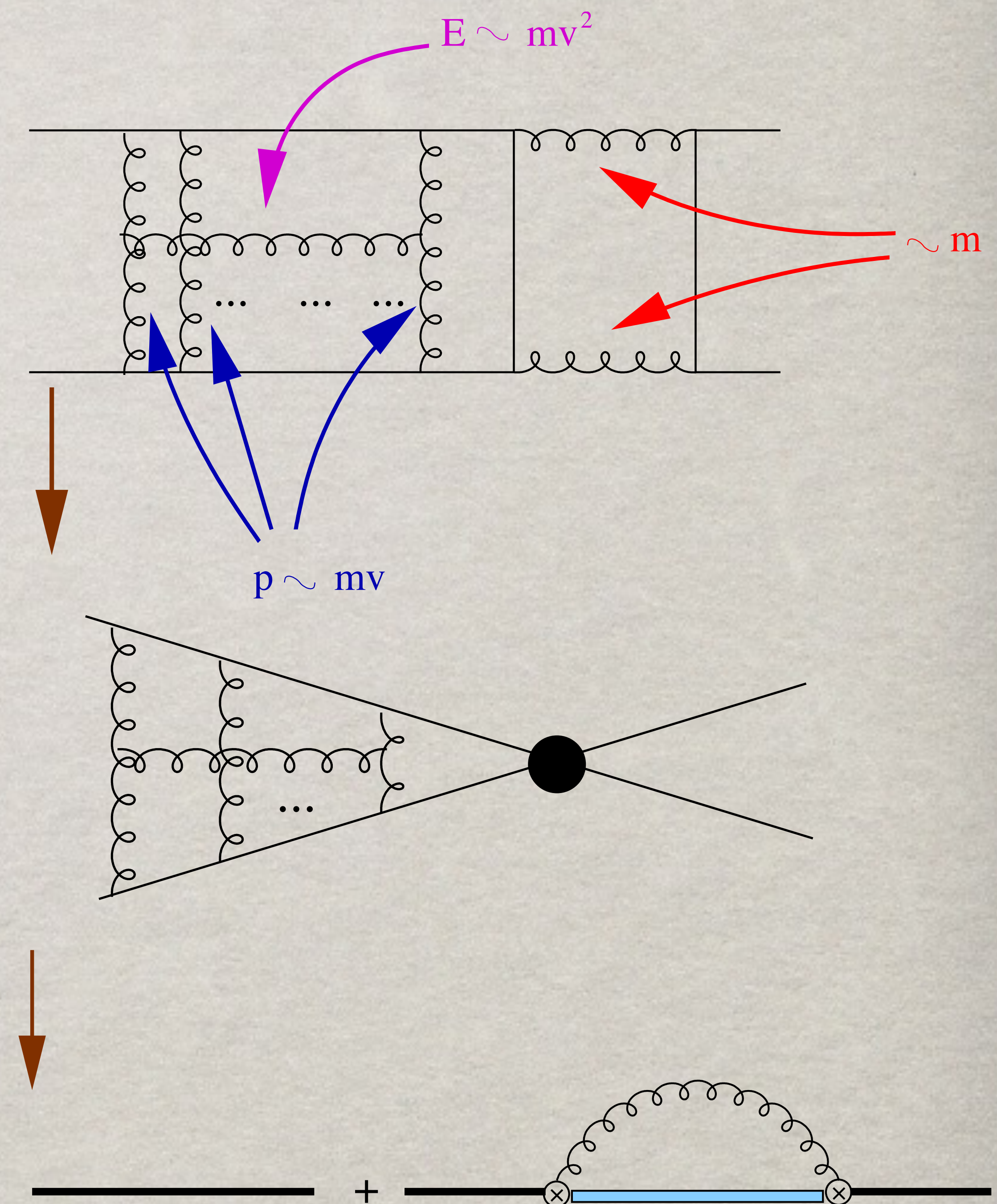

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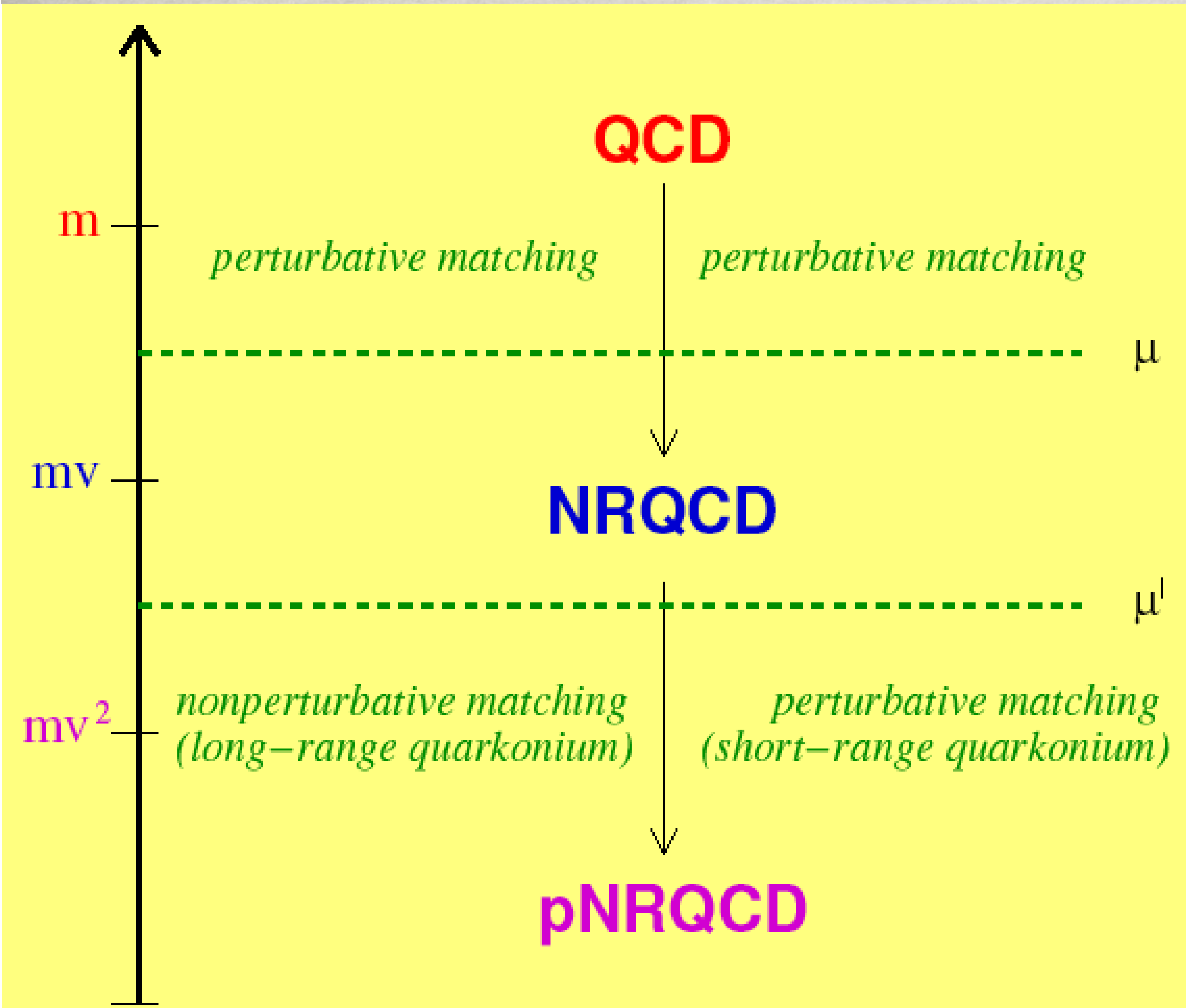
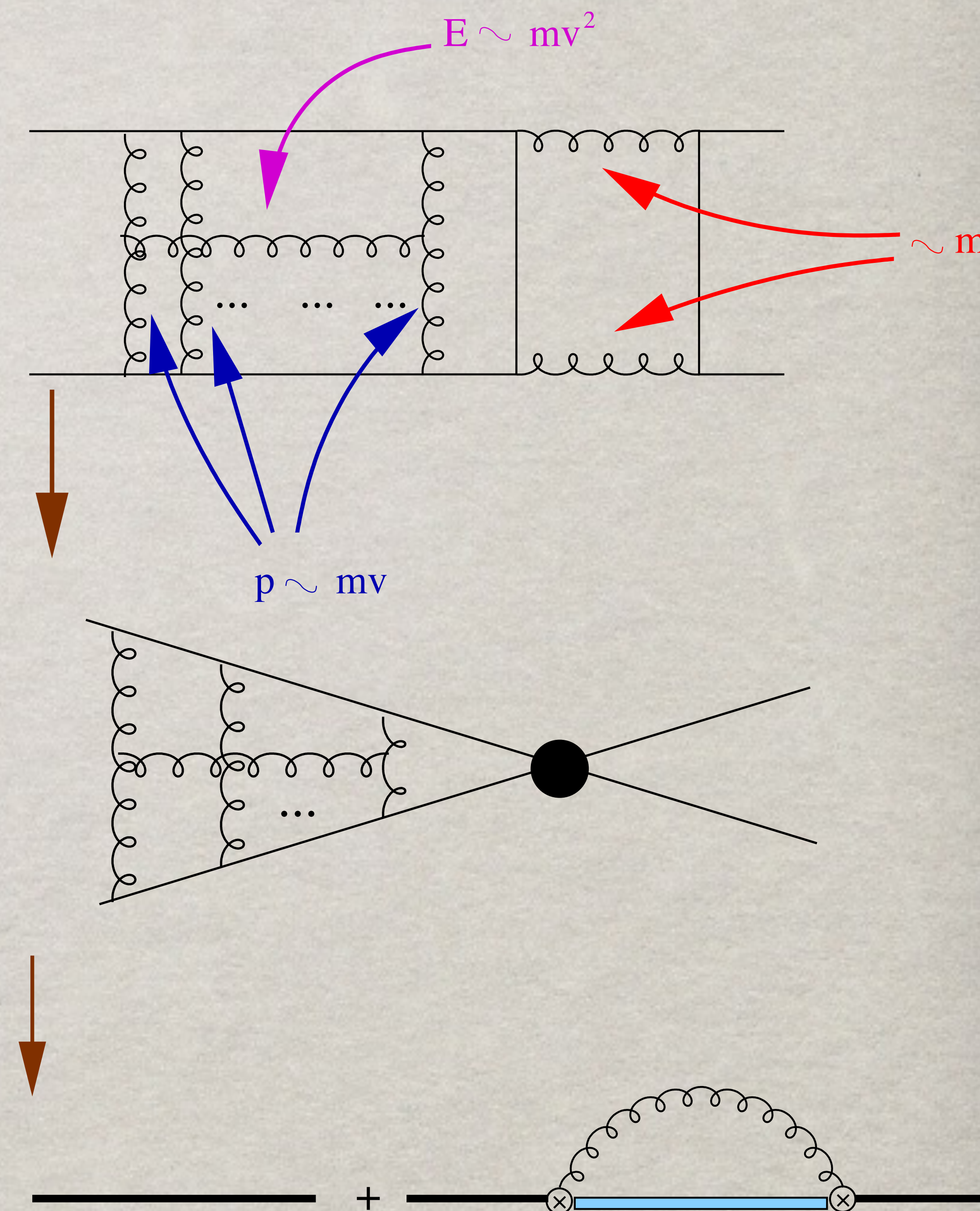
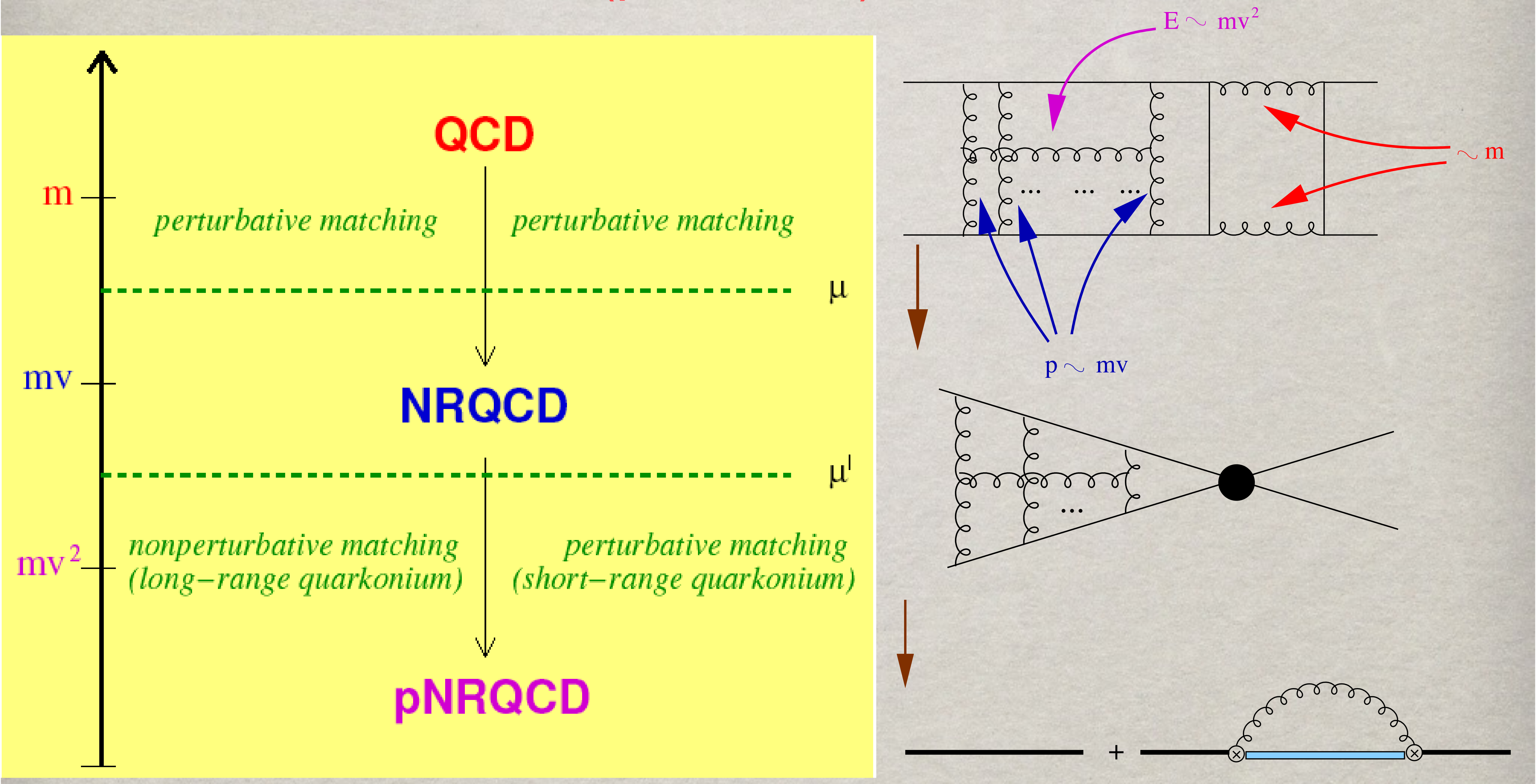




$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_{\rm s}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)



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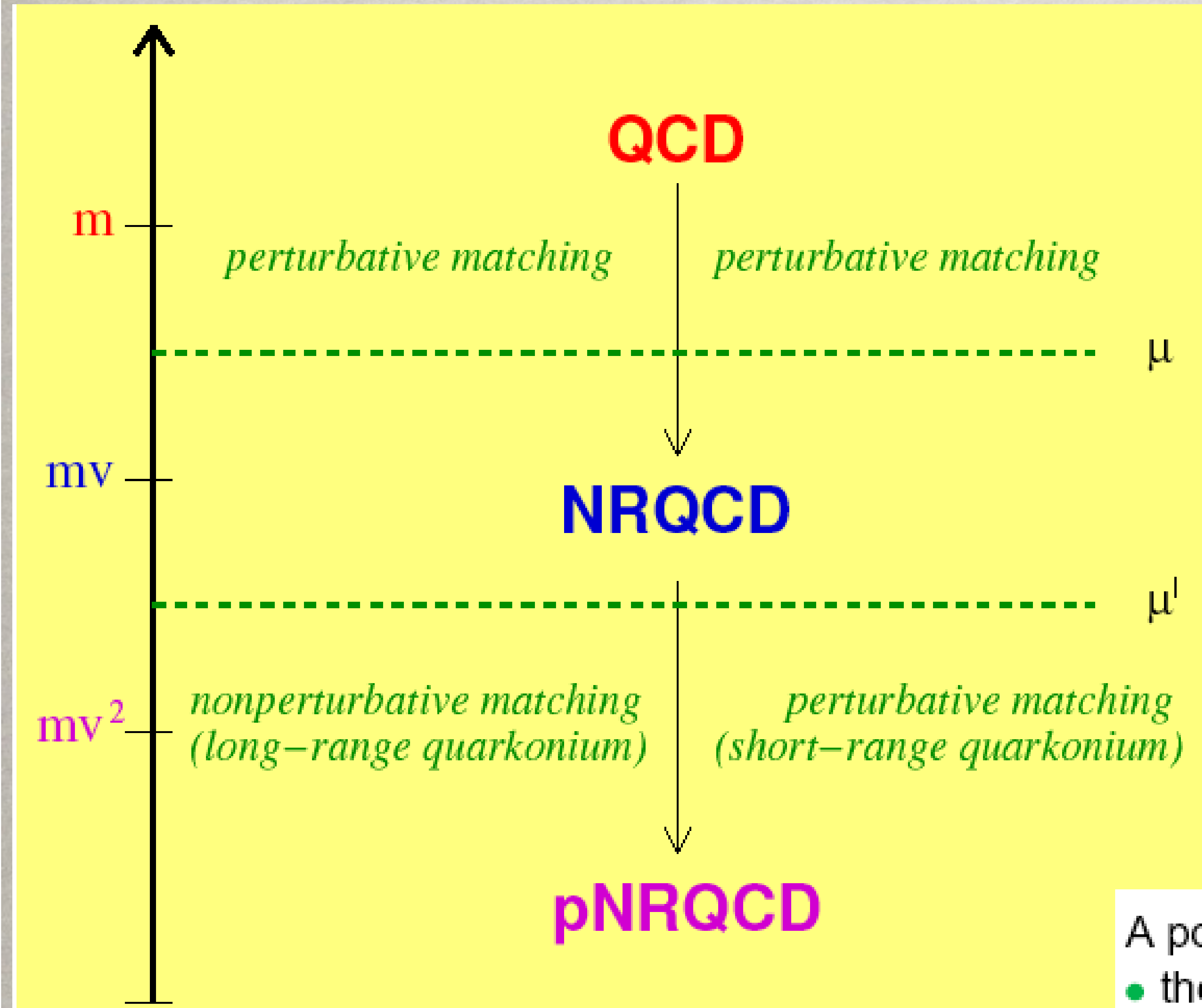
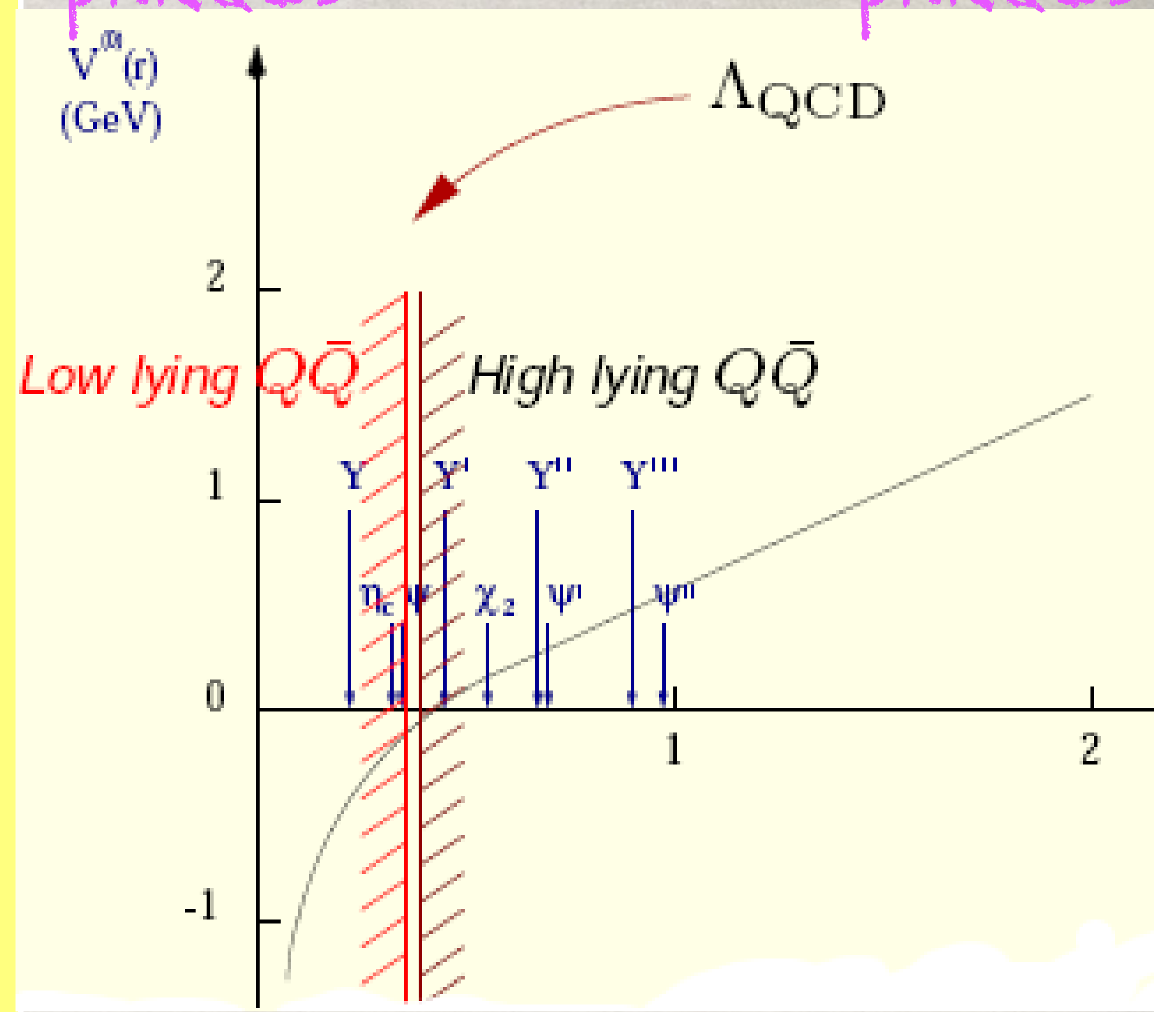
Quarkonium with NR EFT: pNRQCD



weakly coupled pNRQCD

strongly coupled pNRQCD



A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

In QCD another scale is relevant Λ_{QCD}

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

today we have a complete theory description for quarkonia states away from the strong decay threshold

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The EFT has been constructed pNRQCD

*Work at calculating higher order perturbative corrections in v and alpha_s
*Resumming the log
*Calculating/extracting nonperturbatively the low energy quantities
*Extending the theory (electromagnetic effect, 3 bodies)

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The issue here is precision physics and the study of confinement

- Precise and systematic high order calculations allow the extraction of precise determinations of standard model parameters like the quark masses and alpha_s

- The eft has allowed to systematically factorize and to study the low energy nonperturbative contributions

weakly coupled pNRQCD

$r \ll \Lambda_{\rm QCD}^{-1}$

Singlet static potential

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}$$

LO in r

Octet static potential

$$+V_A {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} \right\}$$
$$+\frac{V_B}{2} {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O}\, \mathbf{r}\cdot g\mathbf{E} \right\}$$
$$+\cdots$$

NLO in r

S singlet field

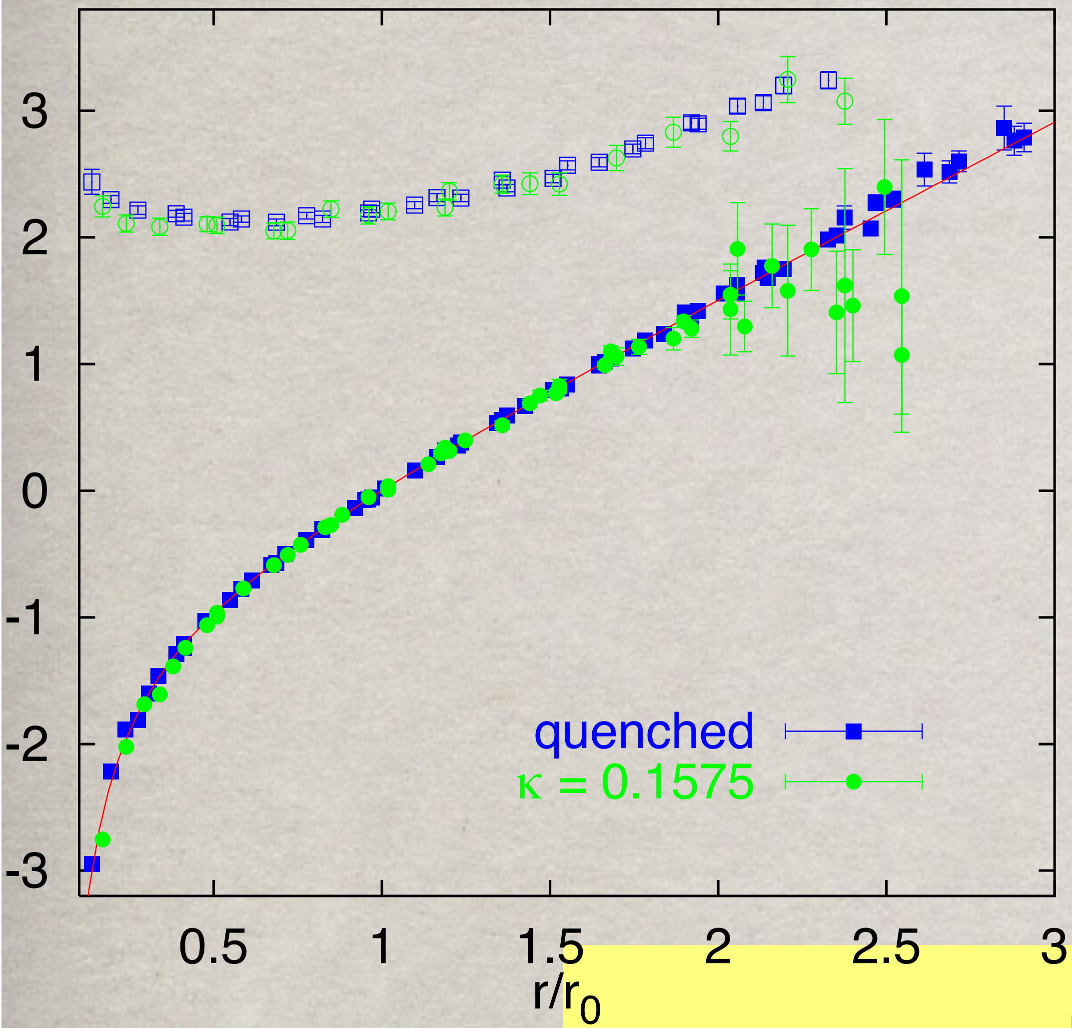
O octet field

——— singlet propagator

═══ octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$ $mv \sim \Lambda_{QCD}$



- integrate out all scales above mv^2
- gluonic excitations develop a gap Λ_{QCD} and are integrated out

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

Brambilla Pineda Soto V

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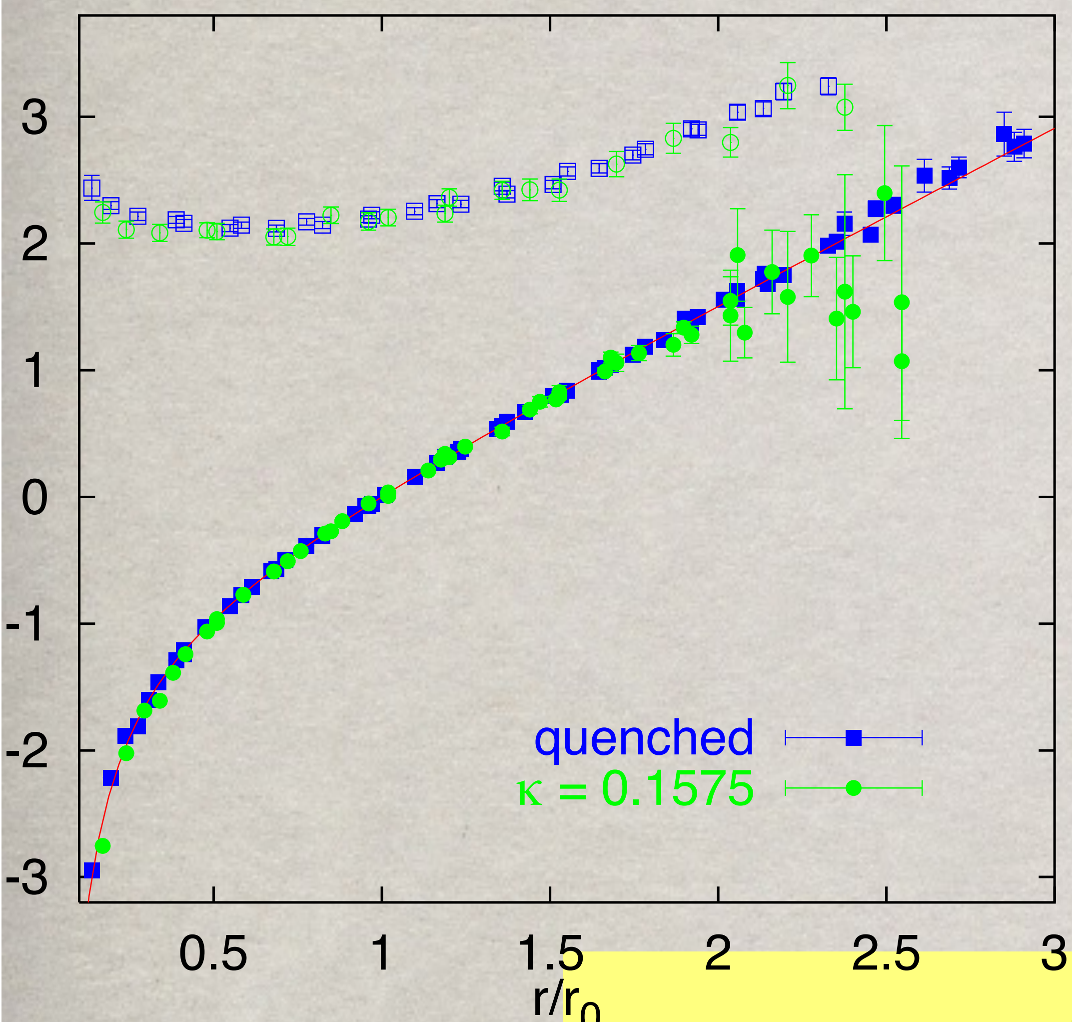

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- A potential description emerges from the EFT

Brambilla Pineda Soto V

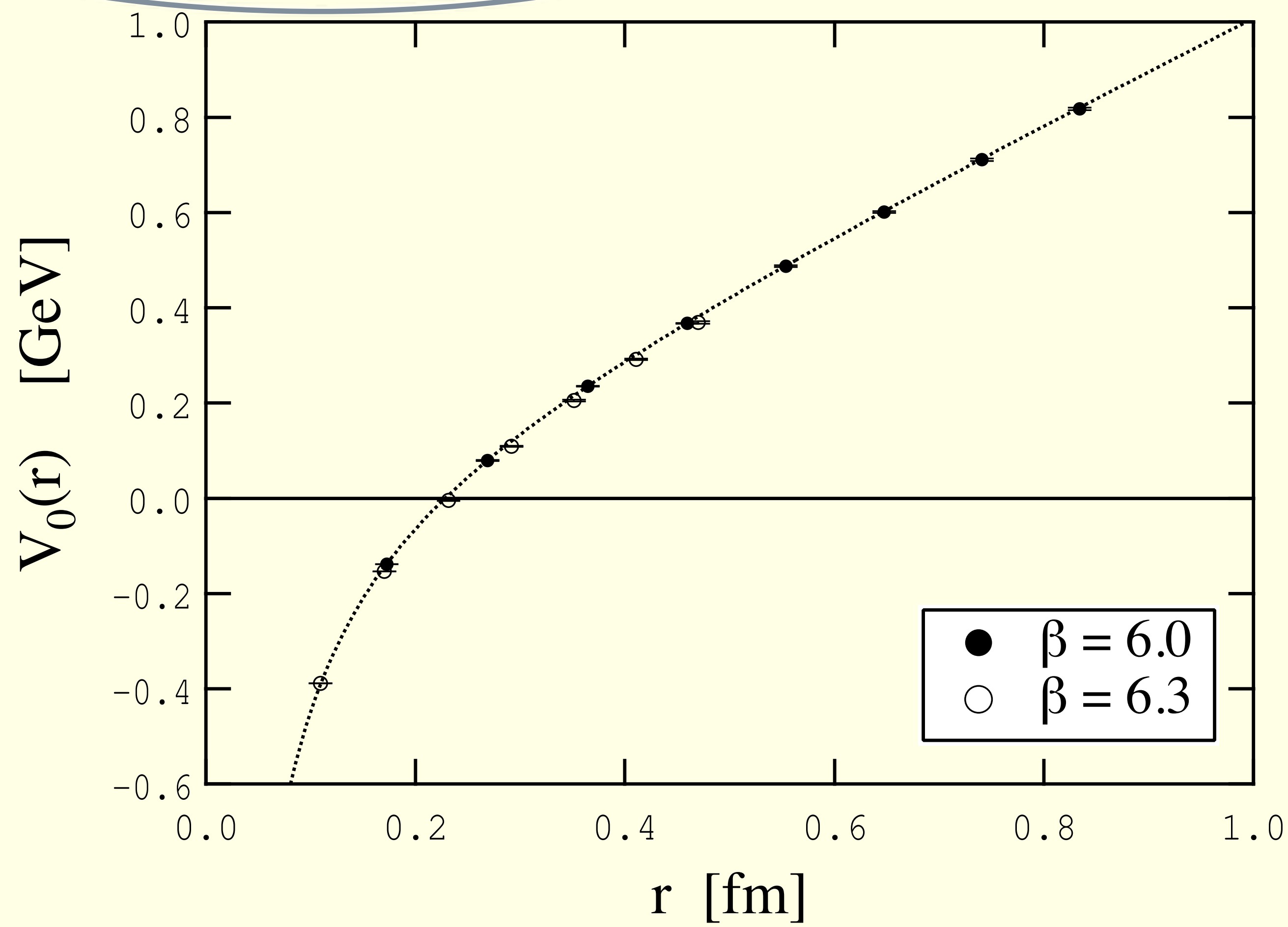
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- V to be calculated on the lattice or in QCD vacuum models

Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

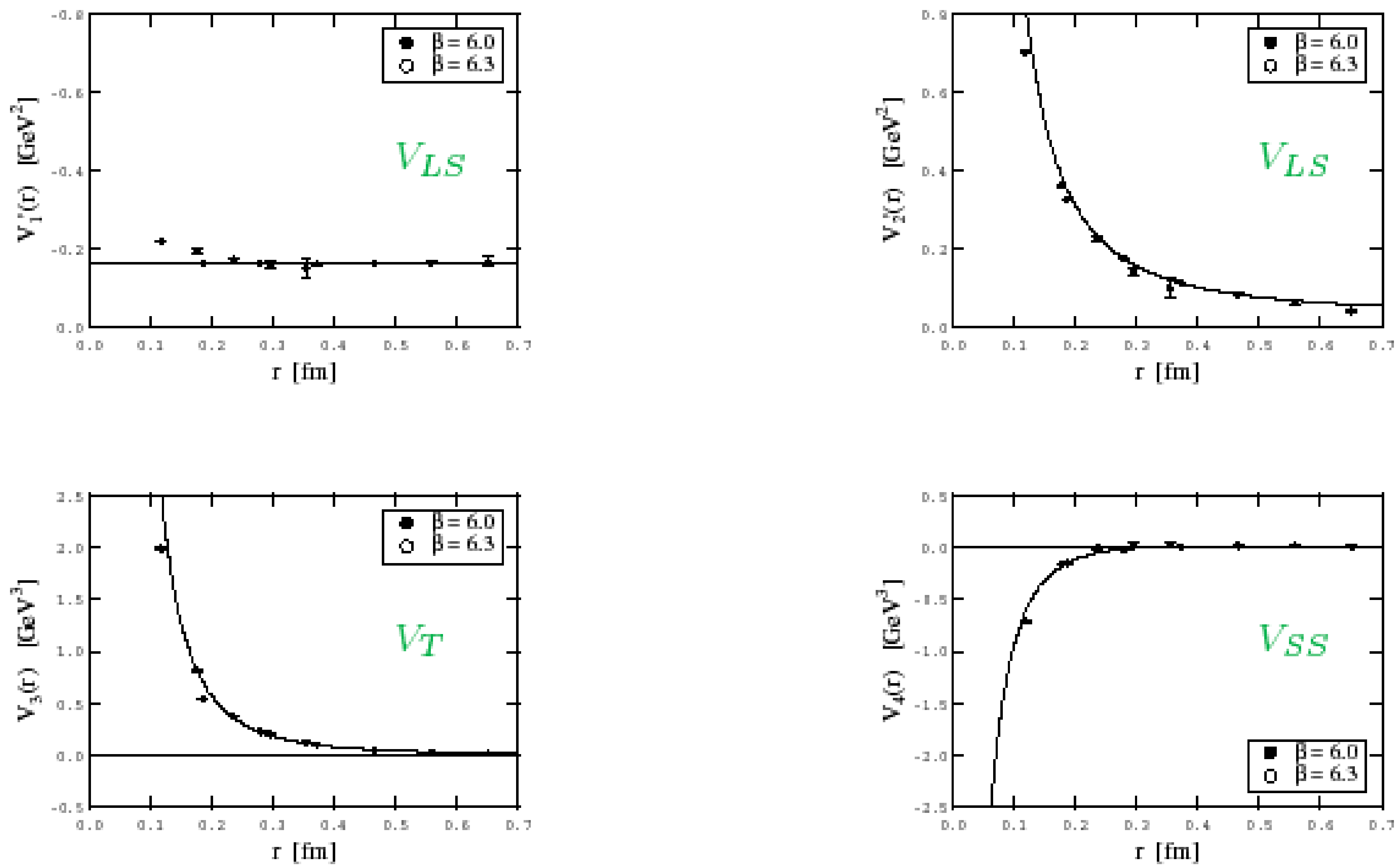
$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r \times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle$$

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$



○ Koma Koma NPB 769(07)79

Spin dependent potentials

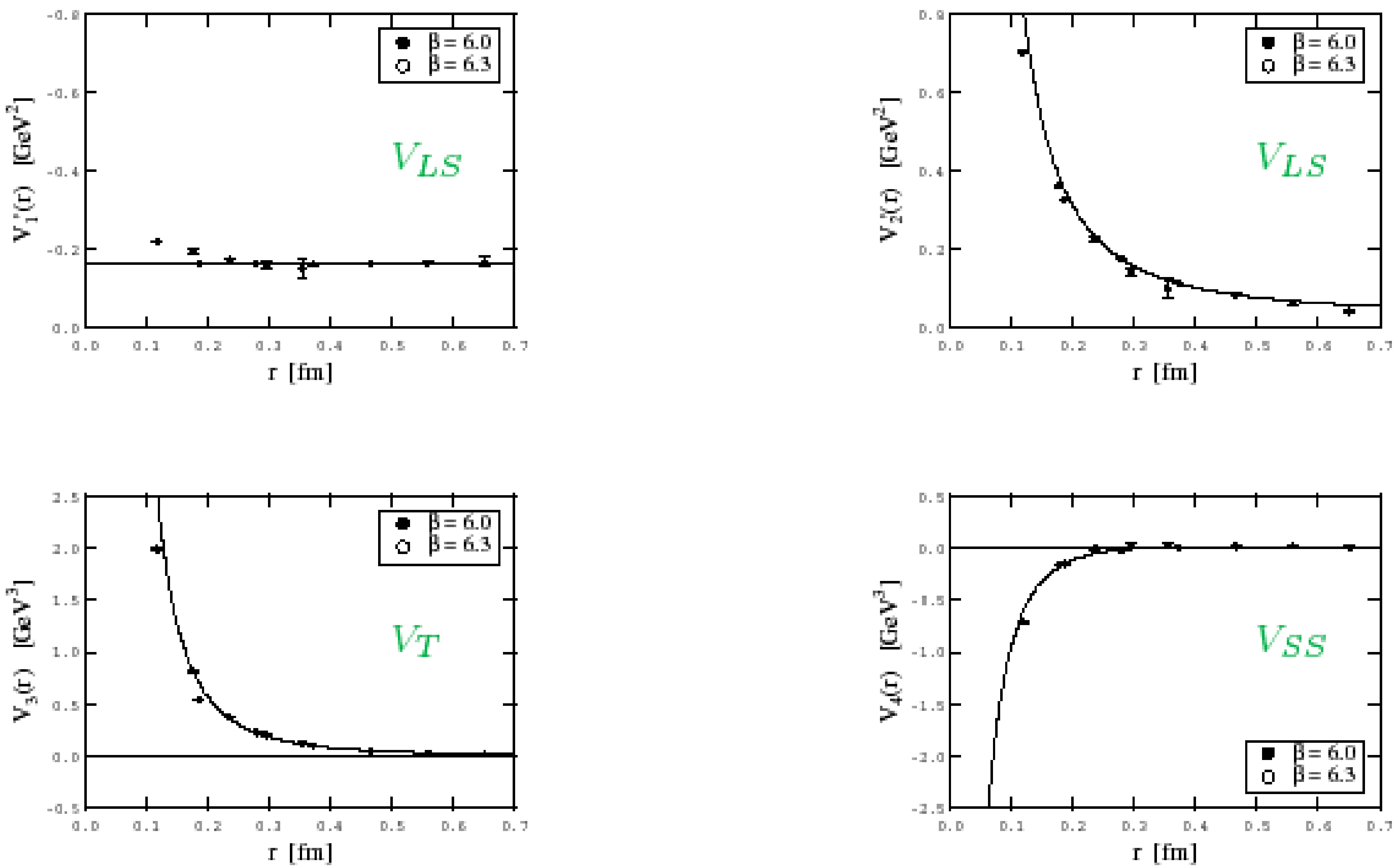


Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

N. B., Martinez, Vairo 2014

Spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

N. B., Martinez, Vairo 2014

For states close or above the strong decay threshold the situation is much more complicated.

there is no mass gap between quarkonium and the creation of a heavy-light mesons couple

$$m_{Q\bar{q}} + m_{\bar{Q}q} = 2m + 2\Lambda_{QCD}$$

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Many phenomenological models exist

States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...

- Molecular states, i.e. states built on the pair of heavy-light mesons.
  - Tornqvist PRL 67(91)556

- Pairs of heavy-light baryons.
  - Qiao PLB 639 (2006) 263

- The usual quarkonium states, built on the static potential, may also give rise to molecular states through the interaction with light hadrons (hadro-quarkonium).
  - Dubynskiy Voloshin PLB 666 (2008) 344

- Tetraquark states.

MAIANI, PICCININI, POLOSA ET AL. 2005--

  - Jaffe PRD 15(77)267
  - Ebert Faustov Galkin PLB 634(06)214

Vijande, Valcarce, Richard

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

  - Alexandrou et al. PRL 97(06)222002
  - Fodor et al. PoS LAT2005(06)310

States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...

- Molecular states, i.e. states built on the pair of heavy-light mesons.
  - Tornqvist PRL 67(91)556

- Pairs of heavy-light baryons.
  - Qiao PLB 639 (2006) 263

- The usual quarkonium states, built on the static potential, may also give rise to molecular states through the interaction with light hadrons (hadro-quarkonium).
  - Dubynskiy Voloshin PLB 666 (2008) 344

- Tetraquark states.

MAIANI, PICCININI, POLOSA ET AL. 2005--

- Jaffe PRD 15(77)267
- Ebert Faustov Galkin PLB 634(06)214

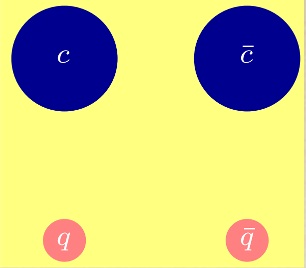
Vijande, Valcarce, Richard

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

- Alexandrou et al. PRL 97(06)222002
- Fodor et al. PoS LAT2005(06)310

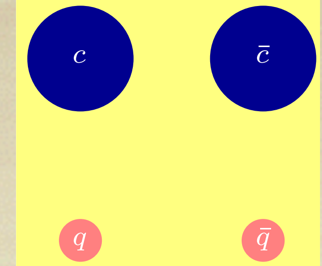
choosing one of these degrees of freedom and an interaction originates a model for exotics.

$X(3872)$: interpretations



4-quark state with $J^{PC} = 1^{++}$

$X(3872)$: interpretations



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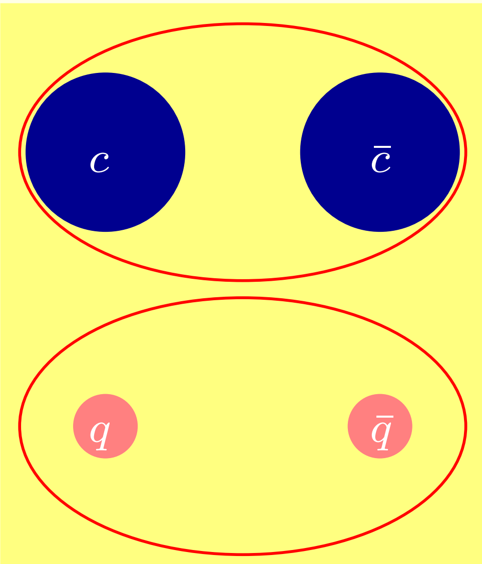
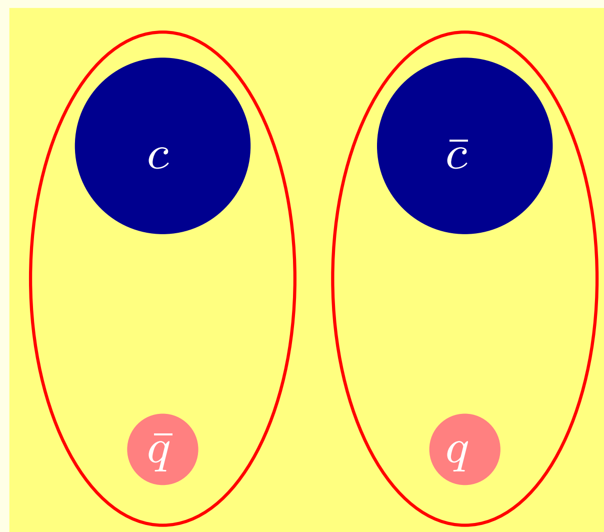


Høgassen et al 05

$$X \sim (c\bar{c})^8_{S=1} \otimes (q\bar{q})^8_{S=1}$$
$$\sim (c\bar{q})^1_{S=0} \otimes (q\bar{c})^1_{S=1} + (c\bar{q})^1_{S=1} \otimes (q\bar{c})^1_{S=0}$$

Predictions based on the phenomenological $H = -\sum_{ij} C_{ij}\, T^a \otimes T^a\, \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}$;

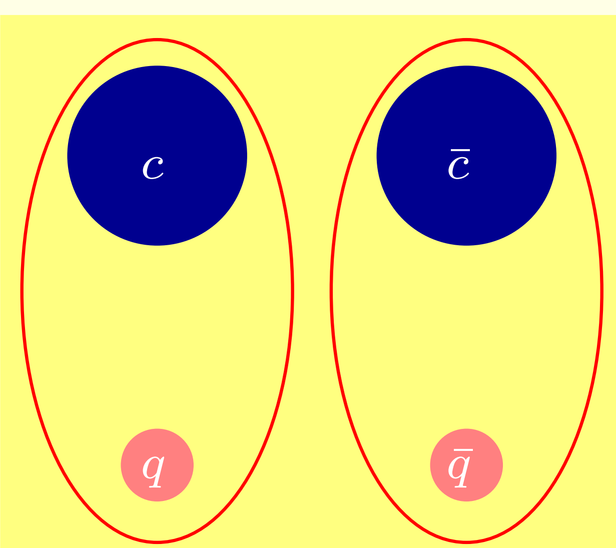
Molecular model



Törnqvist 93, Swanson 04

$$X \sim (c\bar{q})^1_{S=0} \otimes (q\bar{c})^1_{S=1} + (c\bar{q})^1_{S=1} \otimes (q\bar{c})^1_{S=0}$$
$$\sim D\,\bar{D}^* + D^*\,\bar{D}$$

This is assumed to be the dominant long-range Fock component; short-range components of the type $(c\bar{c})^1_{S=1} \otimes (q\bar{q})^1_{S=1}$ $\sim J/\psi\; \rho, \omega$ are assumed as well.



Maiani et al 04

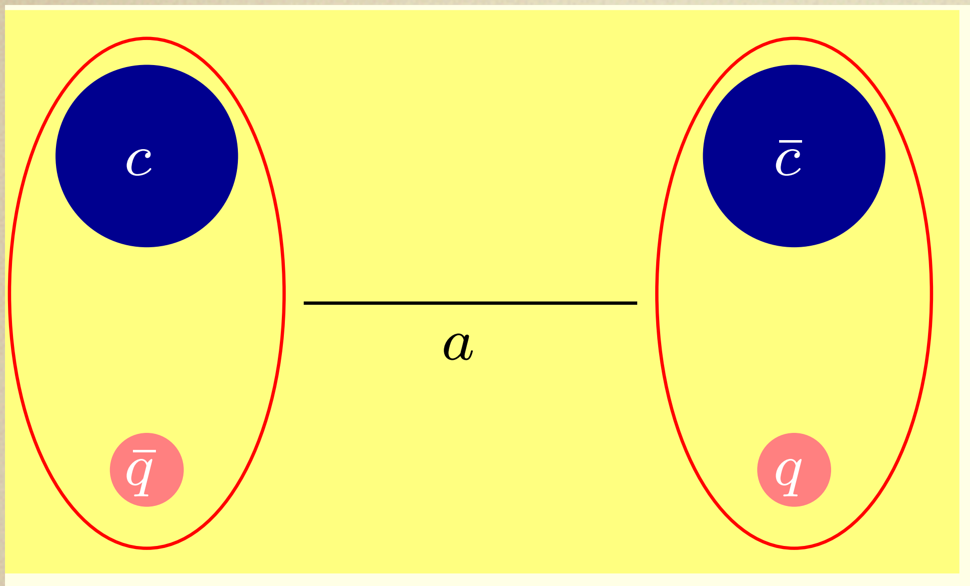
$$X \sim (cq)^{\bar{3}}_{S=1} \otimes (\bar{c}\bar{q})^{3}_{S=0} + (cq)^{\bar{3}}_{S=0} \otimes (\bar{c}\bar{q})^{3}_{S=1}$$

the dynamical assumption (there is no scale separation like in the doubly heavy baryons) is that quark pair cluster in tightly bound color triplet diquarks (see 1-gluon exchange); the difficulty in breaking the system explains the narrow width.

Tetraquark model

Predictions based on the phenomenological Hamiltonian: $H = \sum_{ij} \kappa_{ij}\; \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}$; the

In some cases it is possible to develop an EFT owing to special dynamical condition



this happens if the state is sufficiently close to a threshold and if it has S-wave coupling to the threshold—> loosely bound molecule with universal properties

- An example is the $X(3872)$ intepreted as a $D^0\,\bar{D}^{*\,0}$ or $\bar{D}^0\,D^{*\,0}$ molecule. In this case, one may take advantage of the hierarchy of scales:

$$\Lambda_{\rm QCD} \gg m_\pi \gg m_\pi^2/M_{D^0} \approx 10 \text{ MeV} \gg E_{\rm binding} \approx M_X - (M_{D^{*0}} + M_{D^0}) = (0.1 \pm 1.0)\ \text{MeV}$$

Systems with a short-range interaction and a large scattering length have universal properties that may be exploited: in particular, production and decay amplitudes factorize in a short-range and a long-range part, where the latter depends only on one single parameter, the scattering length. An universal property that fits well with the observed large branching fraction of the $X(3872)$ *decaying into* $D^0\bar{D}^0\pi^0$ *is* $\mathcal{B}(X \to D^0\bar{D}^0\pi^0) \approx \mathcal{B}(D^{*\,0} \to D^0\pi^0) \approx 60\%$.

Pakvasa Suzuki 03, Voloshin 03, Braaten Kusunoki 03 Braaten Hammer 06

Situation similar to QED when one approach the interactions among atoms or molecules

e.g. two heavy nuclei at a distance R plus electrons

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in QCD the situation is more complex: scale Lambda_QCD, colour singlet and colour octet degrees of freedom

We need a description of states close or above threshold from QCD

Already the case of QCD without light quark is very interesting. The degrees of freedom are heavy quarkonium, heavy hybrids and glueballs

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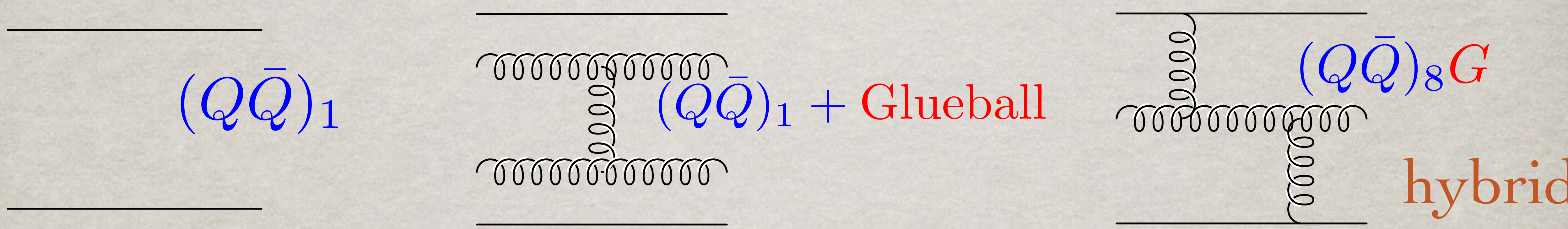

MODELS for HYBRIDS

Constituent gluon picture — Horn and Mandula 1978

- treat hybrids as a three-body system $Q\bar{Q}g$
- add J^{PC} quantum numbers of gluon and quarkonium

Fluxtube model — Isgur and Paton 1983

- gluons assumed to form string between heavy quarks
- hybrids correspond to vibrational excitations of string

Born-Oppenheimer (BO) approximation — Griffiths, Michael and Rakow 1983

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BO picture recently used by Braaten and collaborators

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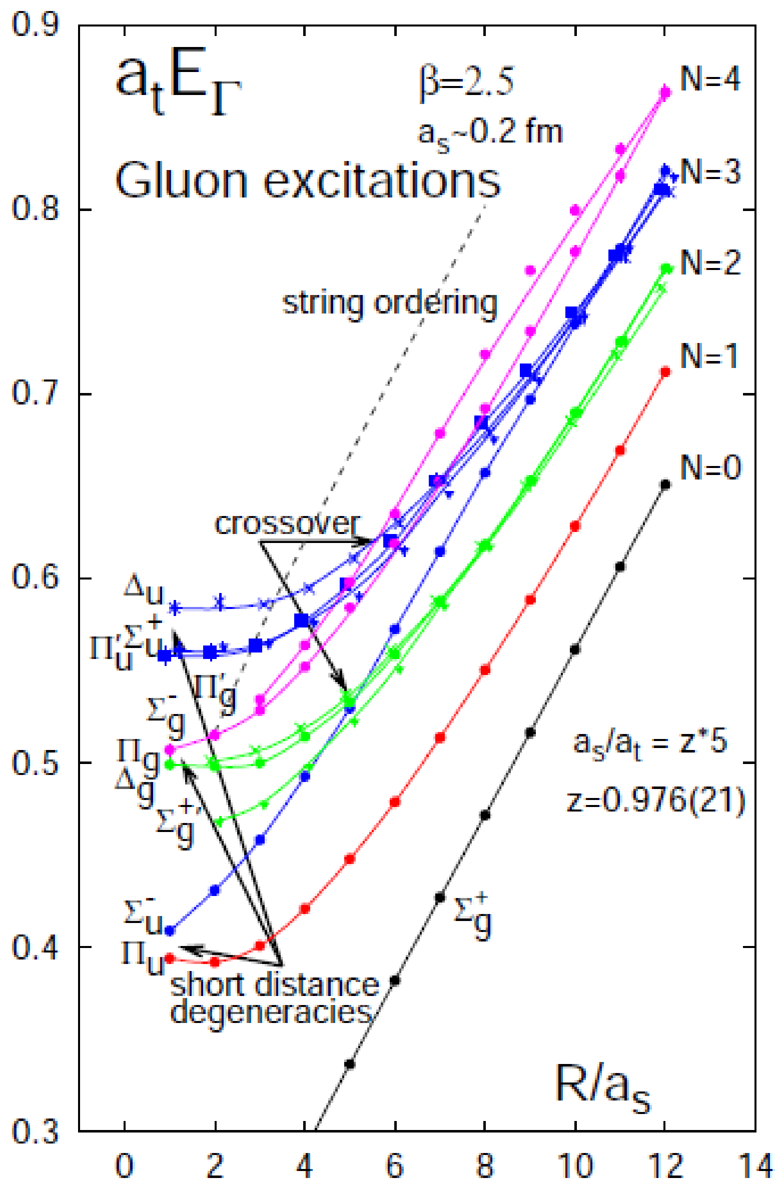
We obtain an EFT description
starting from QCD—> NRQCD—>pNRQCD

Heavy-quark heavy antiquark plus glue

Define the symmetries of the system and the system static energies in NRQCD

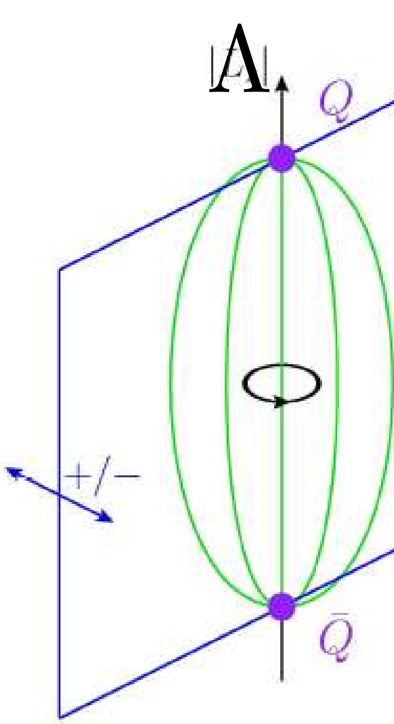
Static
Lattice energies   Juge Kuti Morningstar 2003



Symmetries

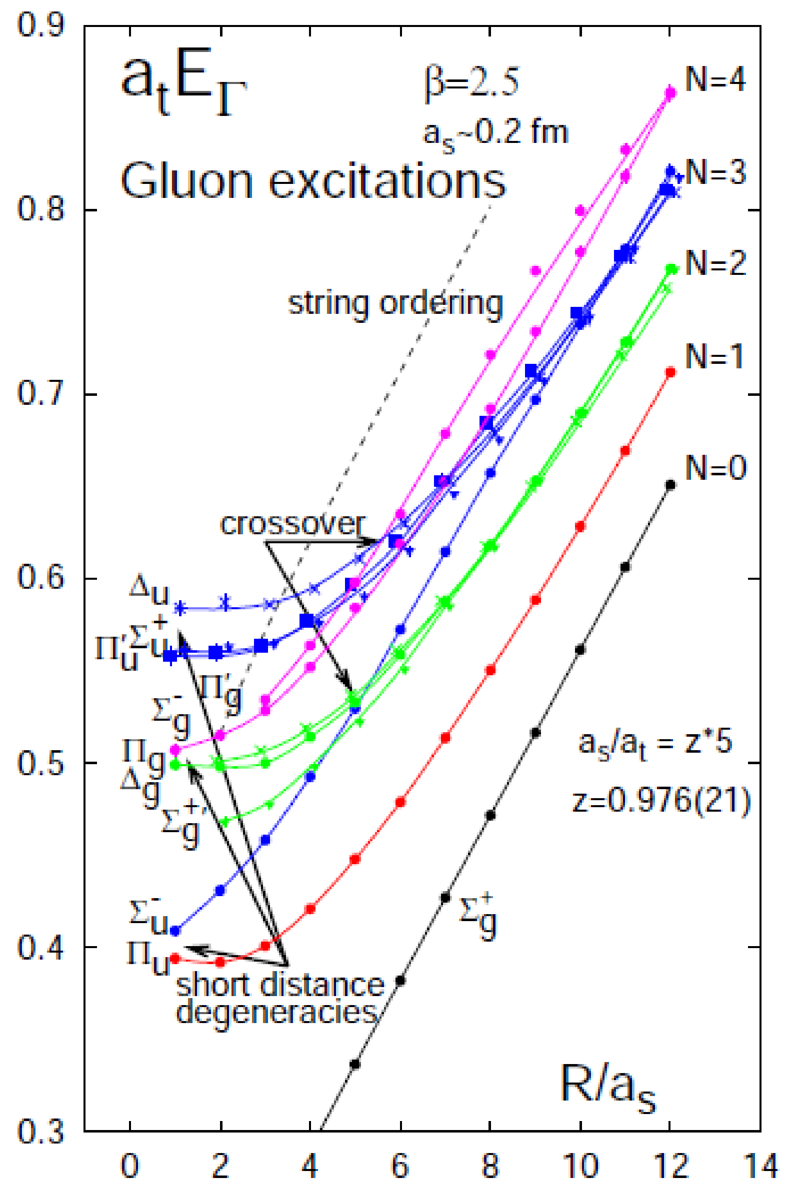
Static states classified by symmetry group $D_{\infty h}$
Representations labeled Λ_η^σ

- Λ rotational quantum number $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP: $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)



- The static energies correspond to the irreducible representations of D_c
- In general it can be more than one state for each irreducible represent $D_{\infty h}$, usually denoted by primes, e.g. Π_u, Π_u', Π_u''...

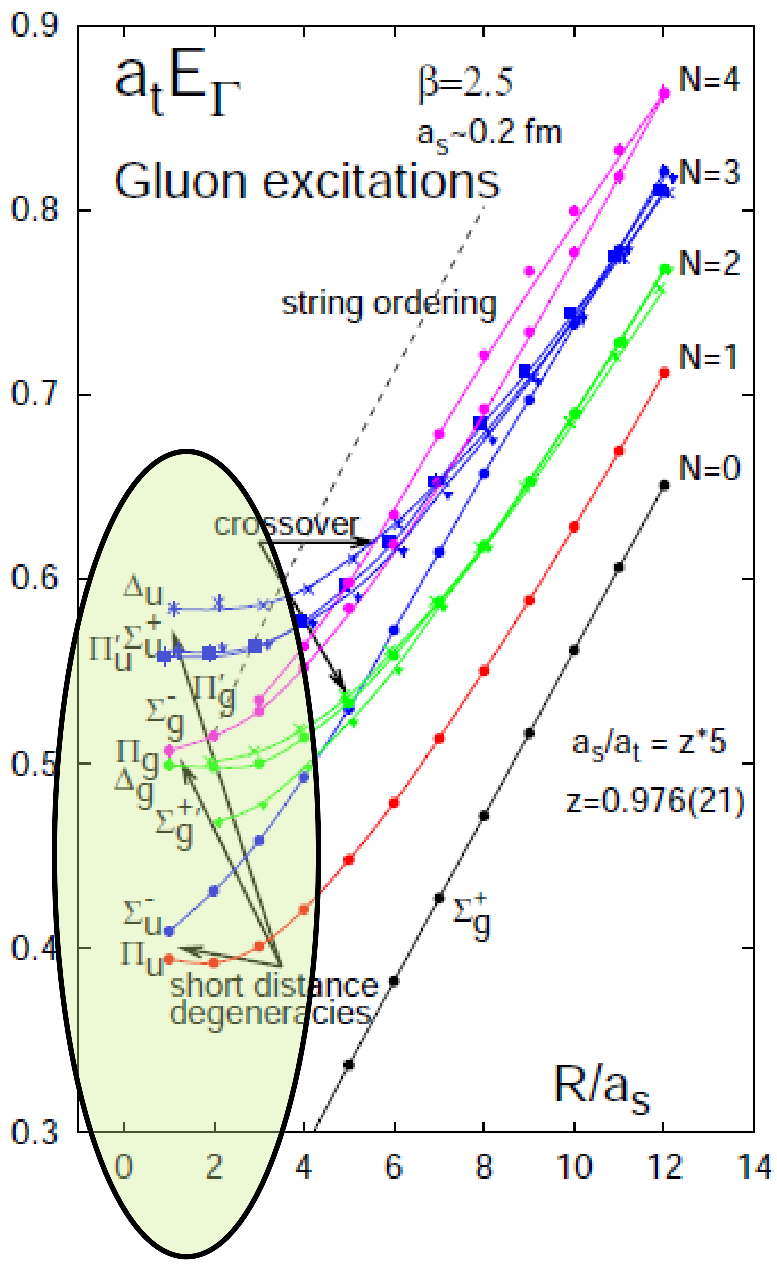
static Lattice energies



- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for Σ_g^+ and Π_u were compared in Bali et al 2000 and good agreement was found below string breaking distance.

- Juge Kuti Morningstar PRL 90 (2003) 161601 even the case without light quark is difficult

static Lattice energies



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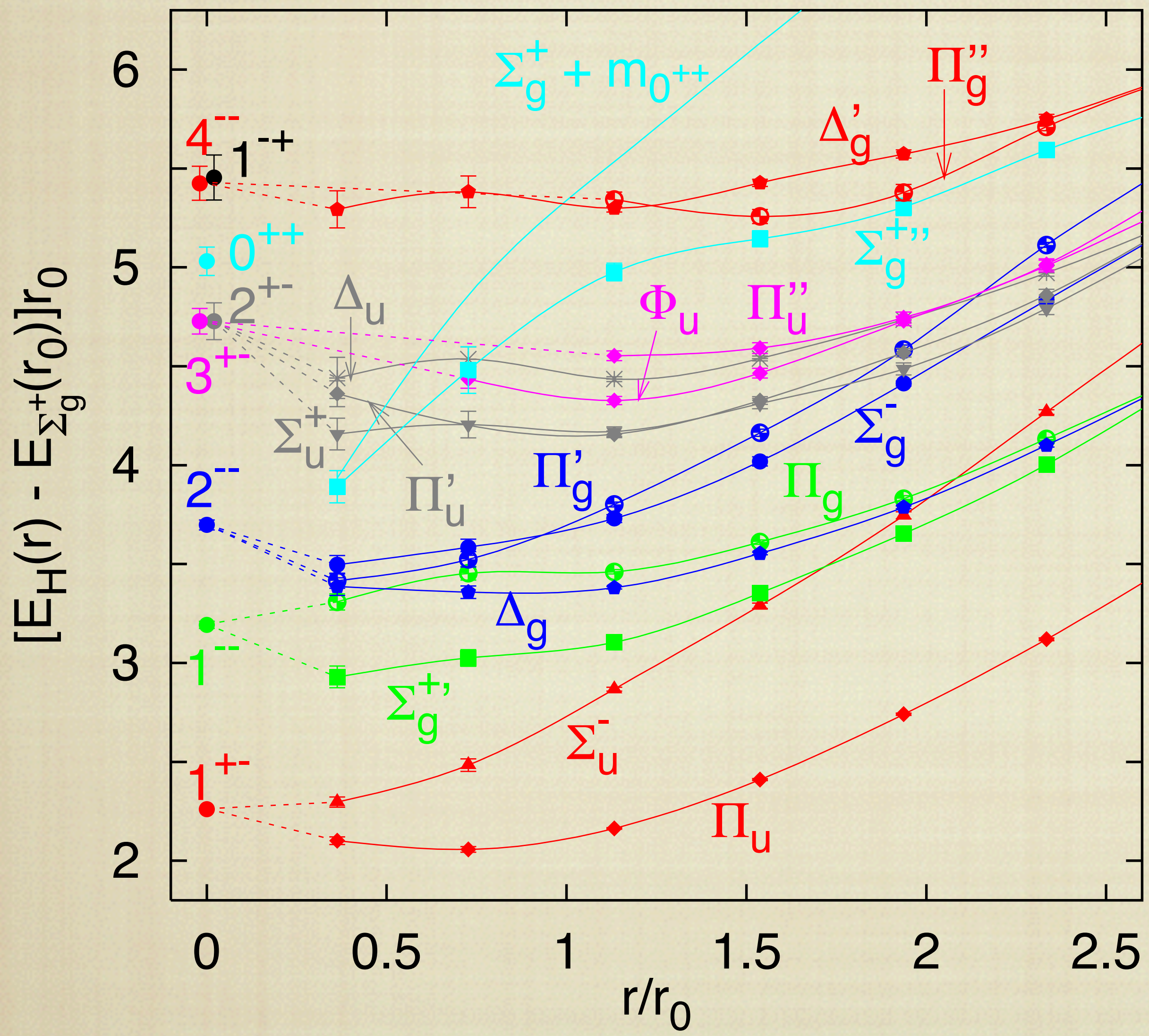
pNRQCD predicts the structure of multiplets at short distance and the ordering

In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

LATTICE



	$L=1$	$L=2$
$\Sigma_g^{+\prime}$	$\mathbf{r}\cdot(\mathbf{E})$	
Σ_g^-		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g	$\mathbf{r}\times(\mathbf{E})$	
Π'_g		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j + (\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Σ_u^-	$\mathbf{r}\cdot\mathbf{B}$	
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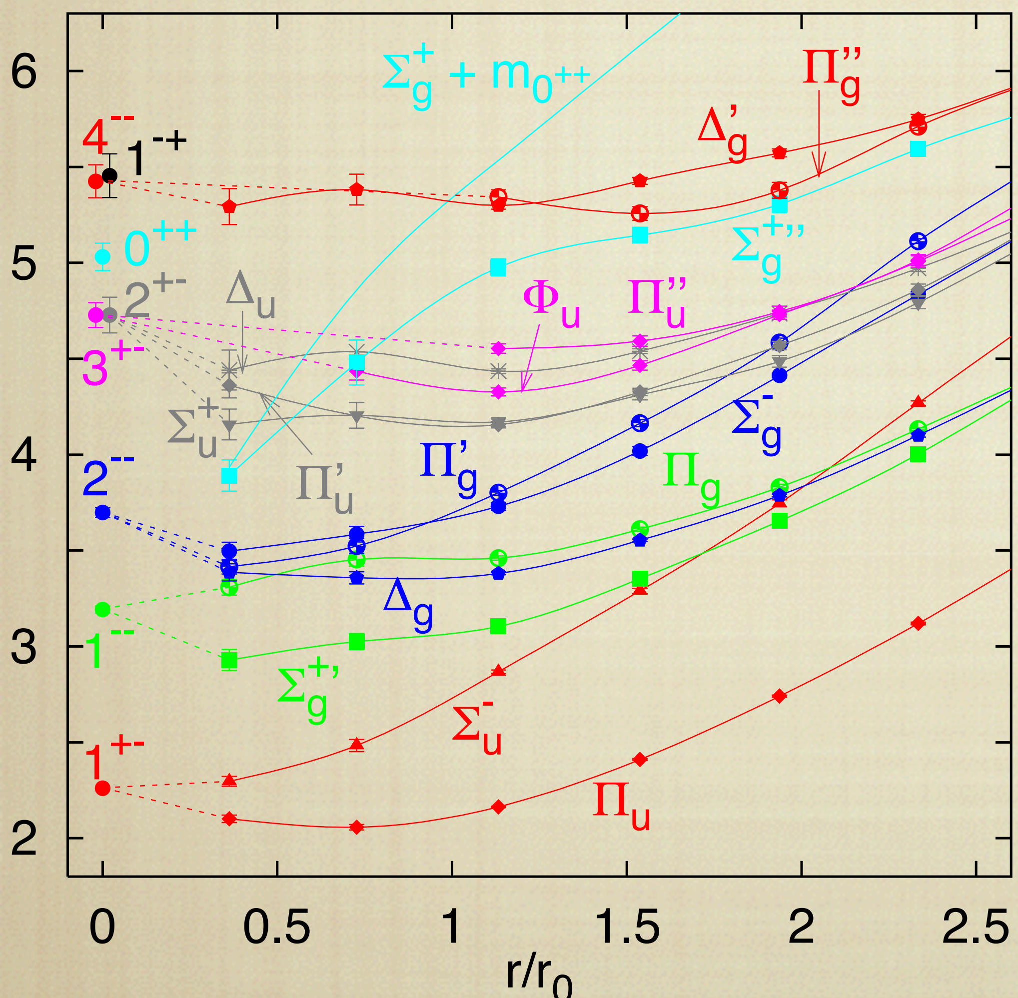
Brambilla Pineda Soto Vairo 0

Match to pNRQCD: one can determine the form of the potential

- *At lowest order in the multipole expansion, the singlet decouples while the octet is still coupled to gluons.*

- *Static hybrids at short distance are called gluelumps and are described by a static adjoint source* (O) *in the presence of a gluonic field* (H):

$$\mathrm{H}(R,r,t) = \mathrm{Tr}\{\mathrm{O}H\}$$



$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T}\ln\langle H^a(\frac{T}{2})\phi^{\mathrm{adj}}_{ab}H^b(-\frac{T}{2})\rangle$$

$$\langle H^a(\frac{T}{2})\phi^{\mathrm{adj}}_{ab}H^b(-\frac{T}{2})\rangle^{\mathrm{np}} \sim h\, e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H + O(r^2)$$

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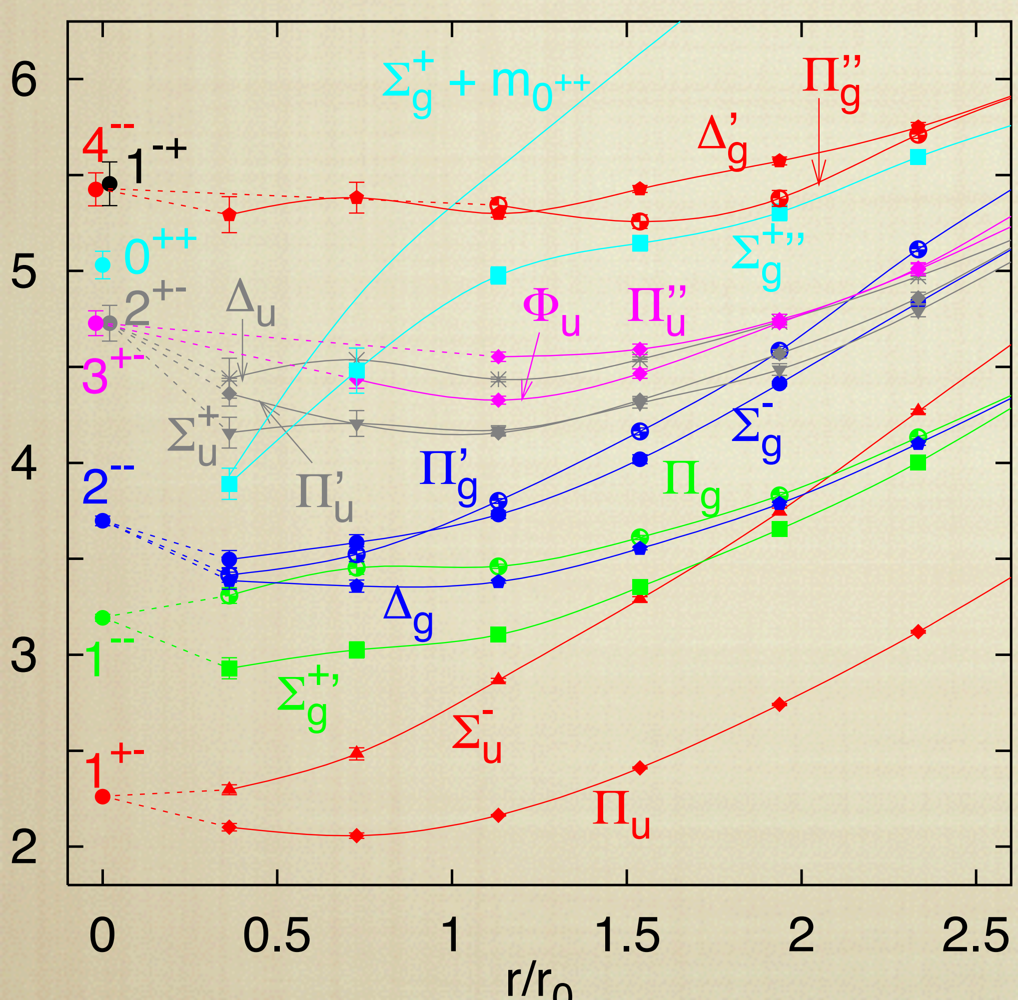

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$$E_H(r) = V_o(r) + \Lambda_H + O(r^2)$$

octet potential

gluelump mass

correction softly bre the symmetry

We define symmetries and states in NRQCD

We match the energy and the states to pNRQCD at order 1/m in the expansion (but no spin for now) and identify coupled Schroedinger equations for Sigma_u and Pi_u

These are nonperturbative and would require lattice calculations of matrix elements

Lacking the lattice calculation, we identify the potentials with a multipole expansion in pNRQCD, solve the coupled equations and get the lowest ccbar, bbar and bcbar muliplets

Lowest energy multiplet Σ_u^-–Π_u

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

- The two lowest laying hybrid static energies are Π_u and Σ_u^-.
- They are generated by a gluelump with quantum numbers 1^{+-} and thus are degenerate at short distances.
- The kinetic operator mixes them but not with other multiplets.
- Well separ[...]

Coupled radial Schrödinger equations

Projection vectors in matrix elements allow for two different solutions (coupled or uncoupled) for the Σ_u^- and Π_u radial wave functions:

1st solution

$$\left[-\frac{1}{2\mu r^2}\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix} = \mathcal{E}\begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2}\partial_r r^2 \partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

- energy eigenvalue $\mathcal{E}$ gives hybrid mass: $m_H = m_Q + m_{\bar{Q}} + \mathcal{E}$
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = (\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$
- Schrödinger equations can be solved numerically

Lowest energy multiplet Σ_u^-–Π_u

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

The Lambda -doubling effect breaks the degeneracy between opposite parity spin-symmetry
multiplets and lowers the mass of the multiplets that get mixed contributions of different static
energies.

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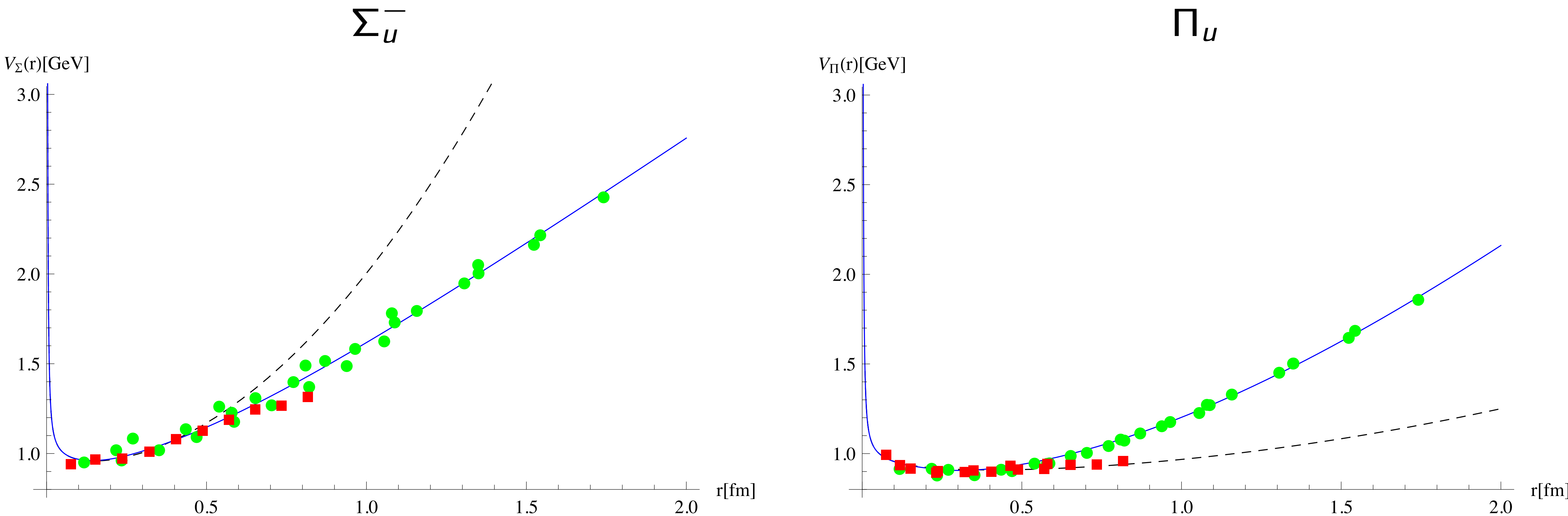


Lattice data: Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.25)}$

- $r \leq 0.25$ fm: pNRQCD potential.
  - Lattice data fitted for the $r = 0 - 0.25$ fm range with the same energy offsets as in $V^{(0.5)}$.

$$b_\Sigma^{(0.25)} = 1.246\,\mathrm{GeV}/\mathrm{fm}^2, \quad b_\Pi^{(0.25)} = 0.000\,\mathrm{GeV}/\mathrm{fm}^2\,.$$

- $r > 0.25$ fm: phenomenological potential.
  - $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.
  - Same energy offsets as in $V^{(0.25)}$.
  - *Constraint:* Continuity up to first derivatives.

Berwein, N.B. , Tarrus, Vairo arXiv:1510.04299

Hybrid state masses from $V^{(0.25)}$

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H_1'	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H_2'	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Consistency test:

1. The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

- As expected the our approach works better in bottomonium than charmonium

- Spin symmetry multiplets

H_1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u
H_6	$\{3^{--}, (2,3,4)^{-+}\}$	Σ_u^-, Π_u
H_7	$\{3^{++}, (2,3,4)^{+-}\}$	Π_u

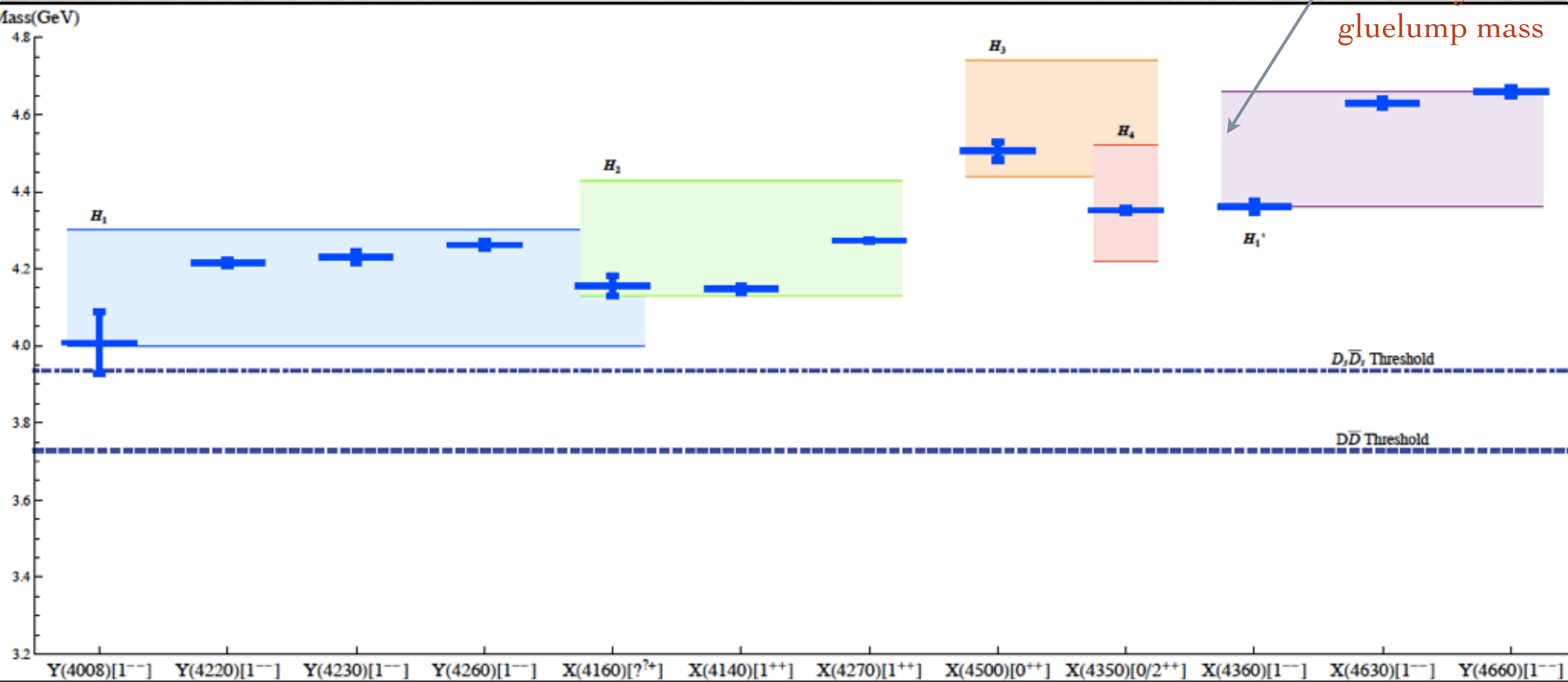
Berwein, N.B. , Tarrus, Vairo arXiv:1510.04299

Identification with experimental states

Most of the candidates have 1^{--} or $0^{++}/2^{++}$ since the main observation channels are production by e^+e^- or $\gamma\gamma$ annihilation respectively.

Charmonium states

error bands come from the uncertainty on the gluelump mass



- Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible H_1 candidate, $m_{H_1} = 10.79 \pm 0.15$.

However, some of the candidates decay modes violate Heavy Quark Spin Symmetry.

Berwein, N.B. , Tarrus, Vairo arXiv:1510.04299

Comparison to direct lattice calculations

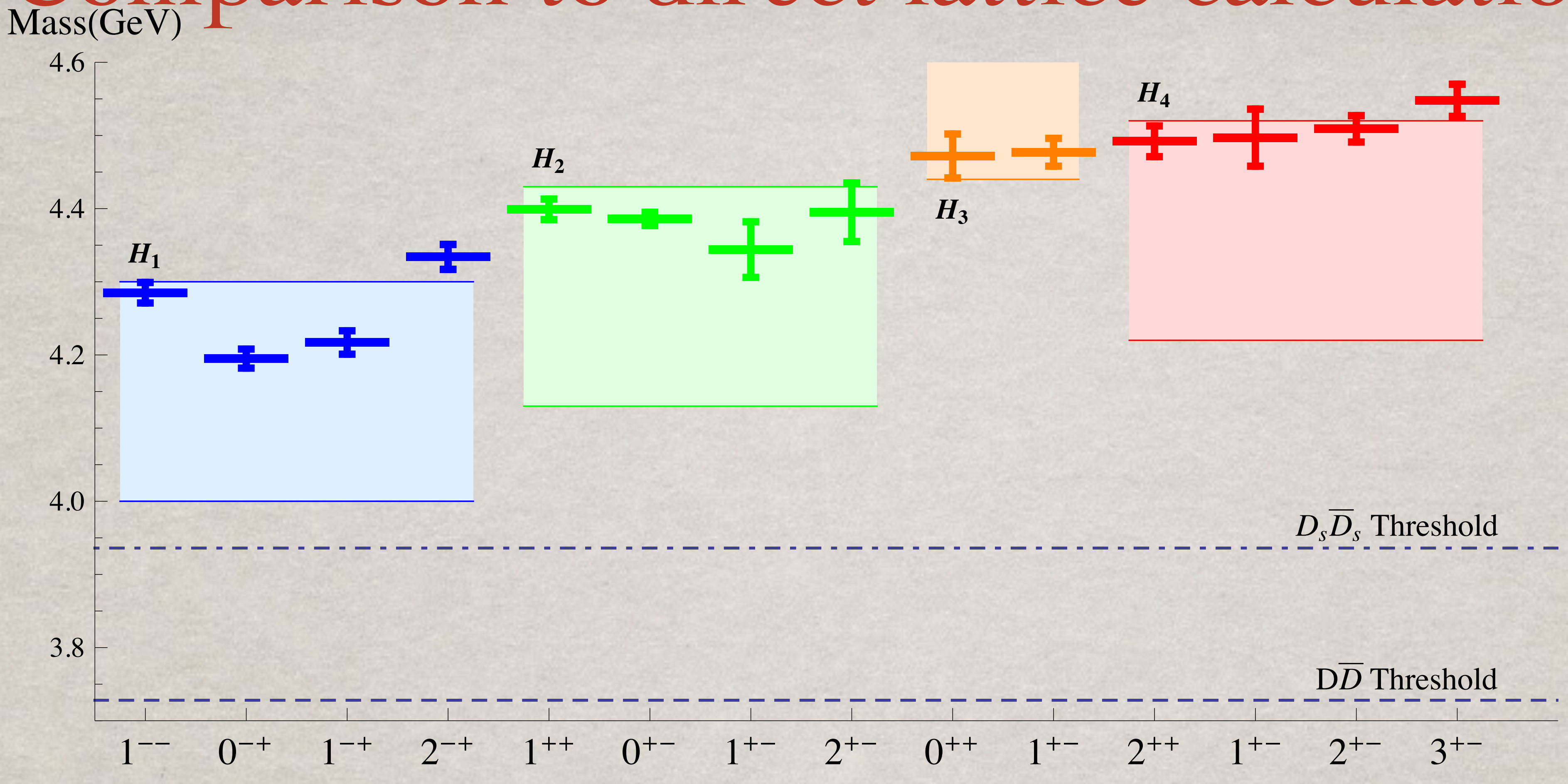


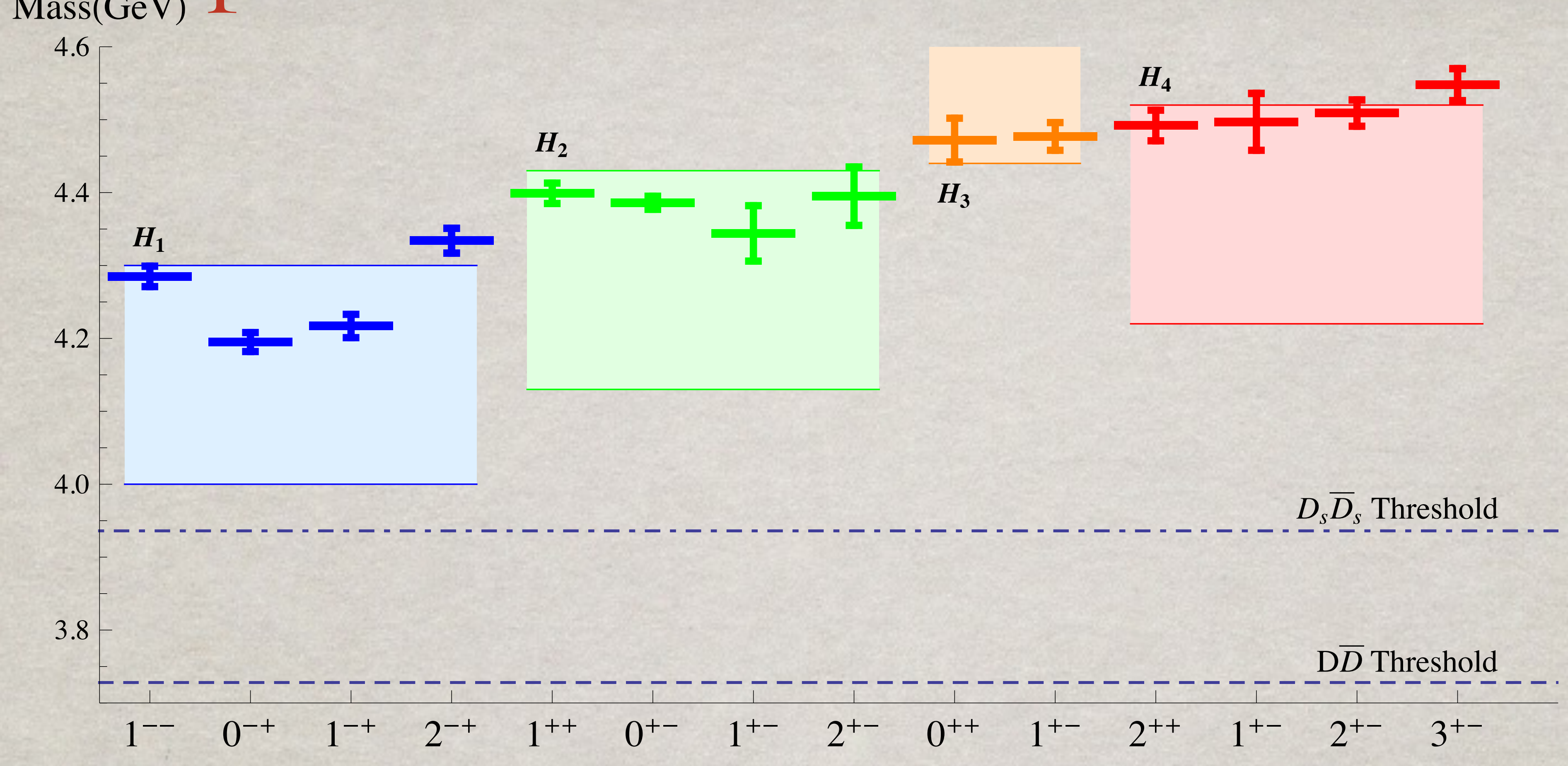
FIG. 5. Comparison of the results from direct lattice computations of the masses for charmonium hybrids [48] with our results using the $V^{(0.25)}$ potential. The direct lattice mass predictions are plotted in solid lines with error bars corresponding the mass uncertainties. Our results for the H_1, H_2, H_3, and H_4 multiplets have been plotted in error bands corresponding to the gluelump mass uncertainty of ± 0.15 GeV.

We observe the same Lambda-doubling pattern in lattice calculations, multiplets that receive mixed contributions from Sigma_u and Pi_u have lower masses then those that remain pure Pi_u states

L. Liu *et al.* [Hadron Spectrum Collaboration], JHEP **1207**, 126 (2012) [arXiv:1204.5425 [hep-ph]].

with pions of 400 MeV and no extrapolation to the continuum

Comparison to direct lattice calculations



support the result of the pNRQCD and BO approaches that the hybrid states appear in three distinct multiplets (H_2, H_3, and H_4) as compared to the constituent gluon picture, where they are assumed to form one supermultiplet together (cf. also the discussion in [34]).

uncertainty of ±0.15 GeV.

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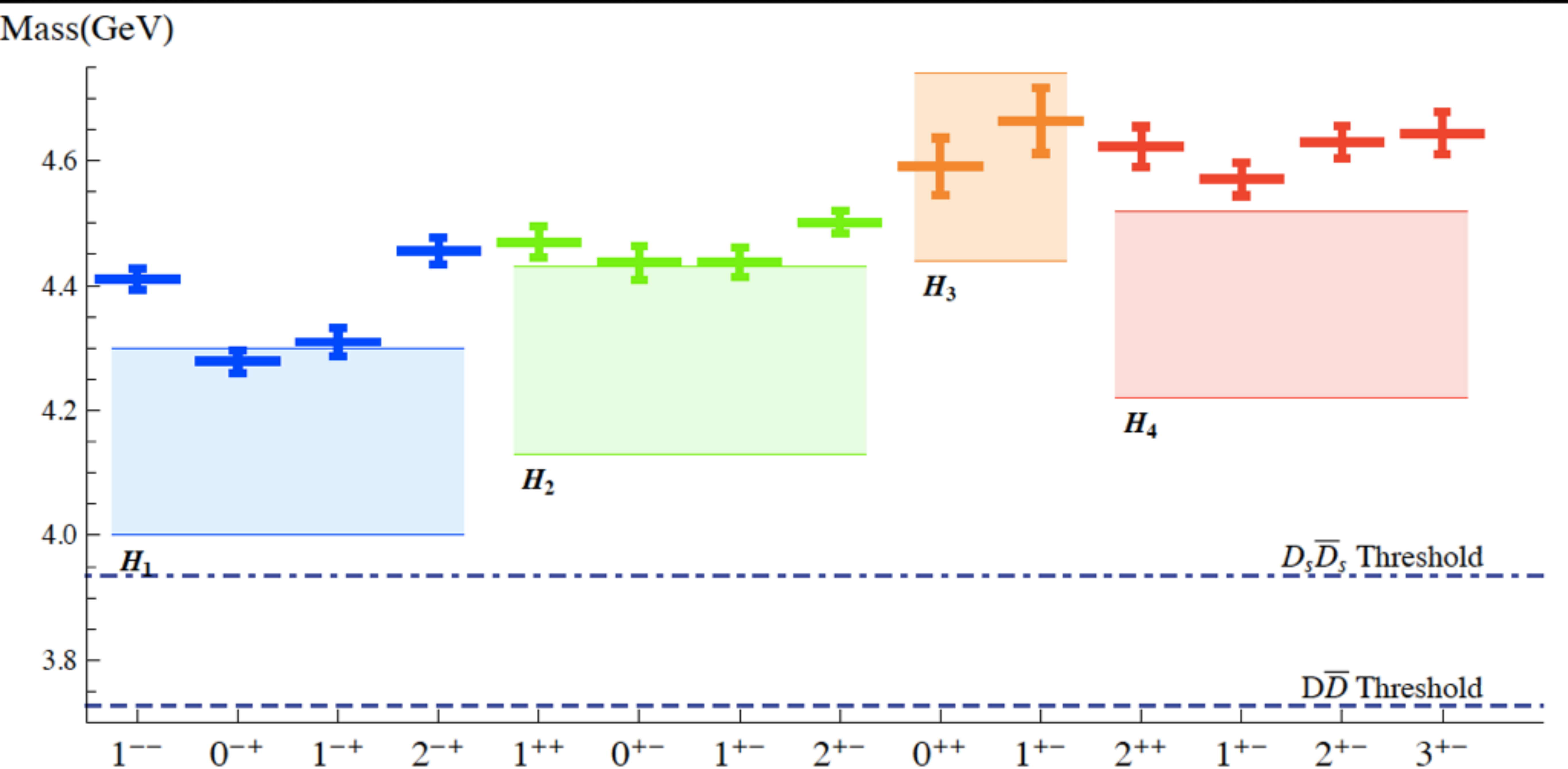


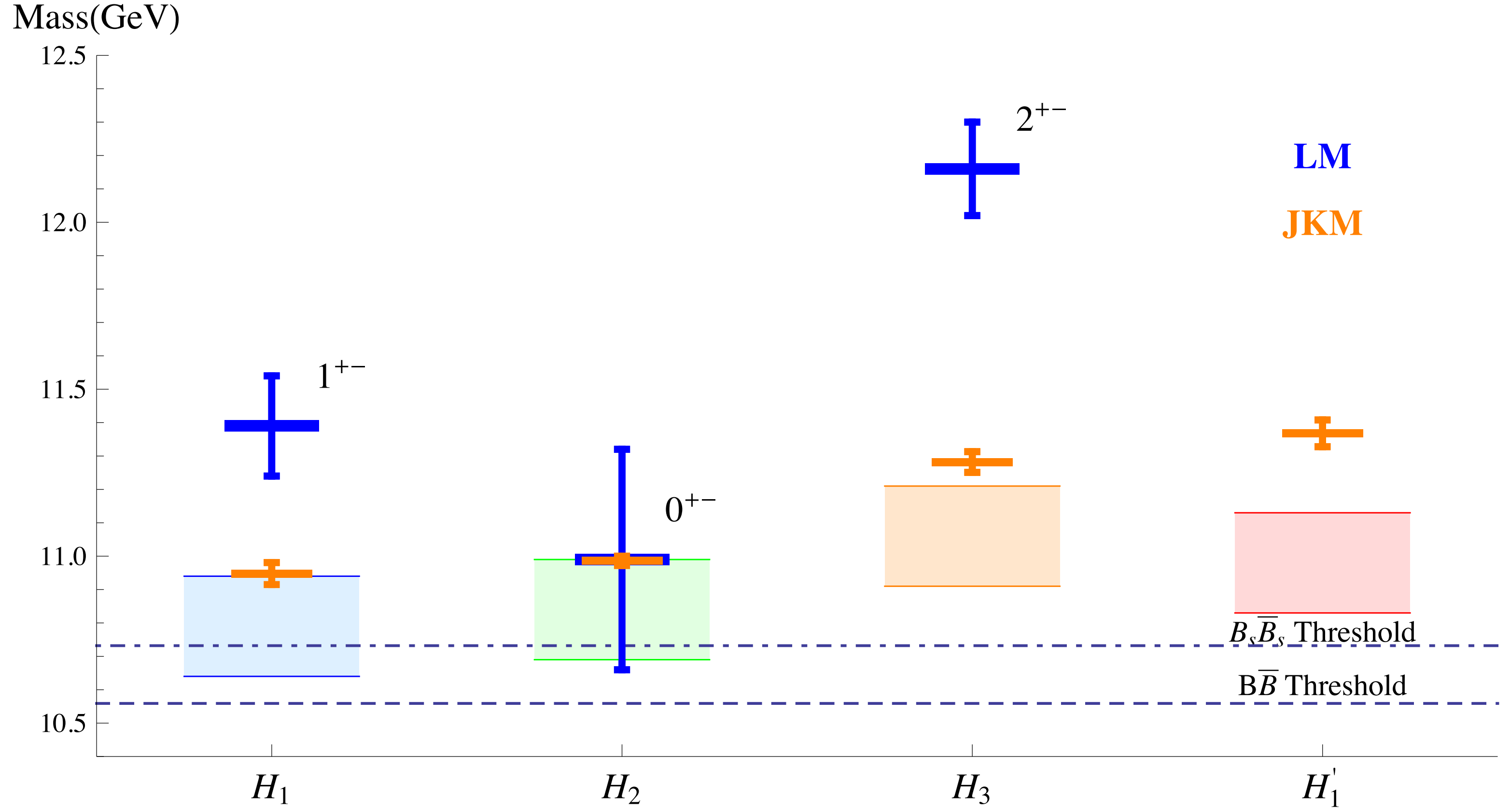
new lattice data from hadron spectrum collaboration

JHEP 1612 (2016) 089 with pion mass 240 MeV but no continuum limit

Comparison with direct lattice computations

Bottomonium sector

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- Our masses are $0.15 - 0.25$ GeV lower except the for the H_1' multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

Comparison to sum rules

arXiv:1304.4522 [hep-ph]

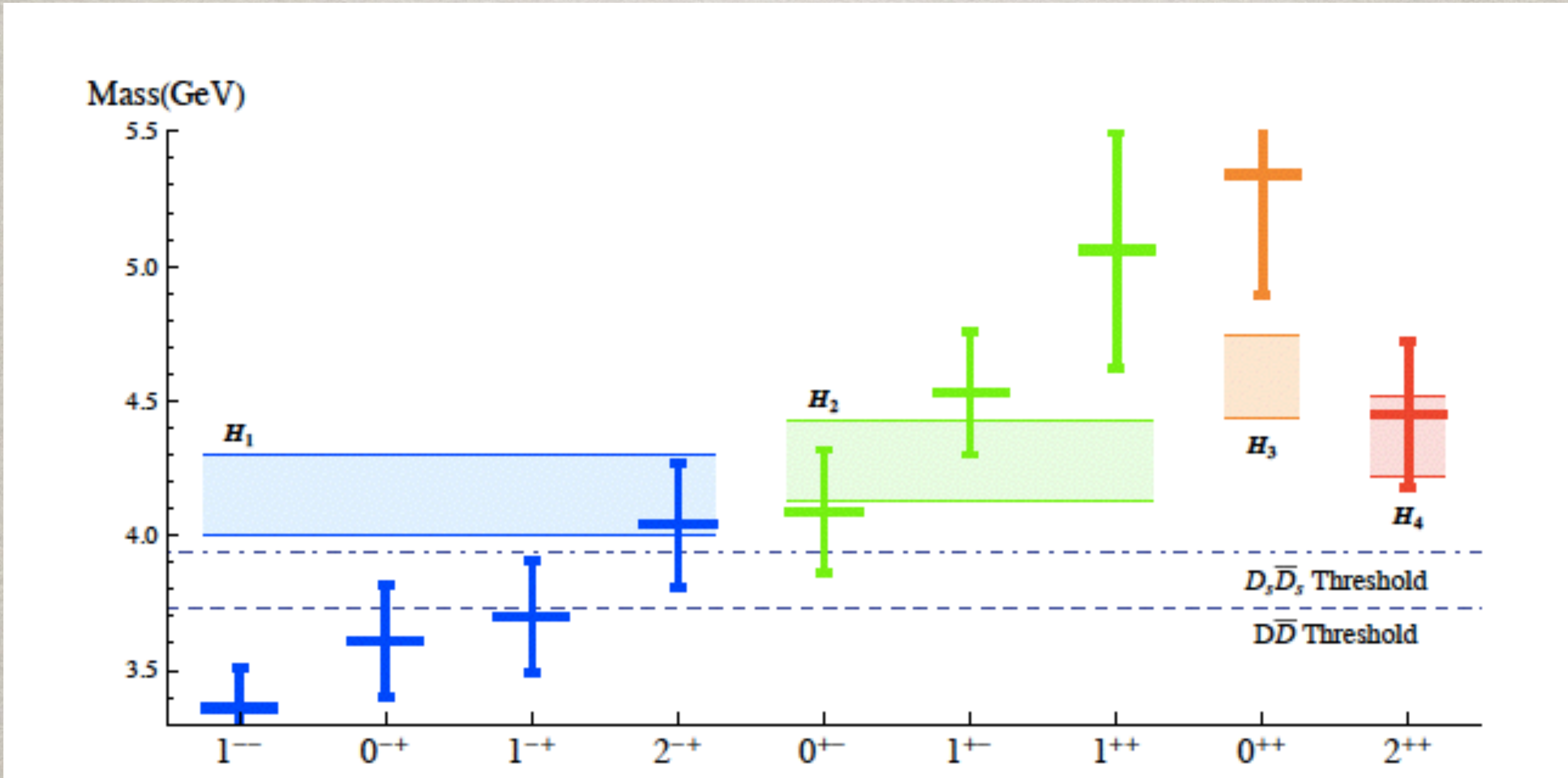


FIG. 7. Comparison of the mass predictions for charmonium hybrids in the upper figure and for bottomonium hybrids in the lower figure, obtained using QCD sum rules [68], with our results using the $V^{(0.25)}$ potential. The solid lines correspond to the QCD sum rules masses with error bars corresponding to their uncertainties. Our results for the H_1, H_2, H_3, and H_4 multiplets have been plotted in error bands corresponding to the gluelump mass uncertainty of ±0.15 GeV.

Currently in consideration:

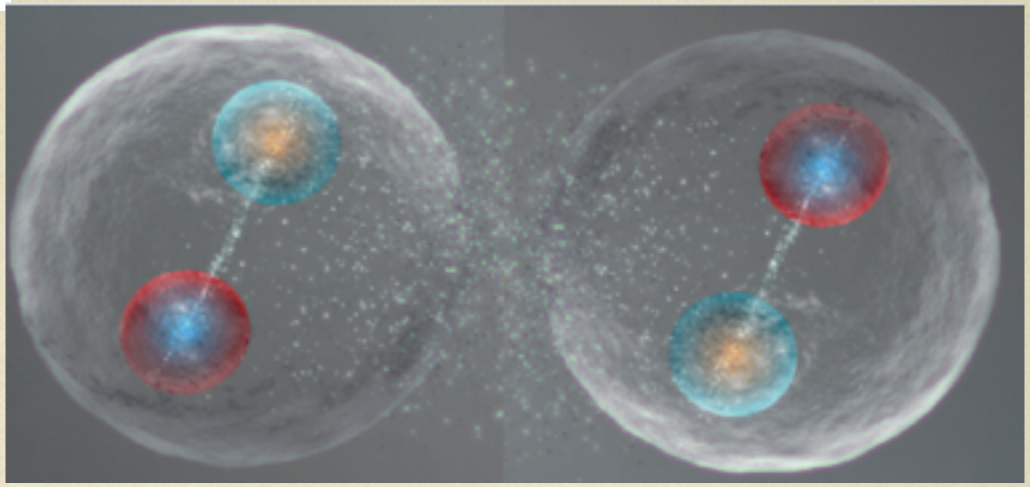
Adding the spin of the heavy quark: interesting multiplet structure coming from the quark-antiquark being in an octet

Mixing and transitions with quarkonium Soto et al 2016

Add light quarks: tetraquarks , need lattice tetra quark potentials

Chromopolarizability & color van der Waals forces

$$\eta_b - \eta_b$$



Interactions between color neutral objects:

Via creation of instantaneous color dipole moments & gluon transitions in virtual color-octet intermediate state

— Polarizability—

— Chromopolarizability of 1S bottomonium;
use pNRQC (potential Nonrelativistic QCD)

N. Brambilla, GK, J. Tarrús-Castellà, A. Vairo, PRD 93, 054002 (2016)

— Chromopolarizability of 1S bottomonium;

use pNRQC (potential Nonrelativistic QCD)

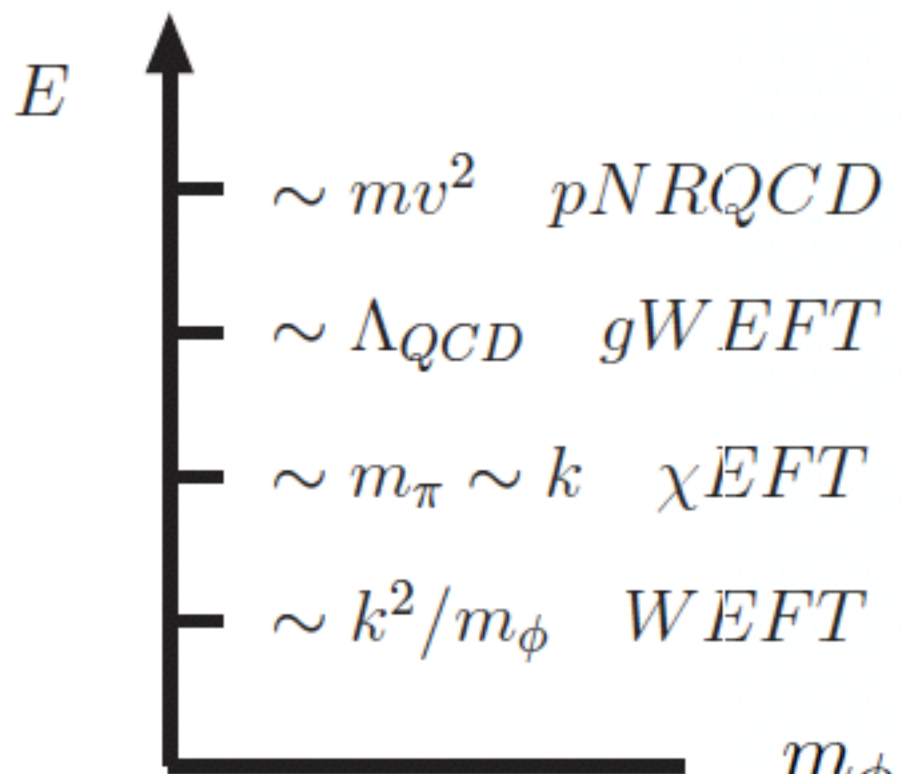
— van der Waals force between two bottomonia;

use QCD trace anomaly to match pNRQC to a chiral EFT

QCD ⟶ NRQCD ⟶ pNRQCD

pNRQCD ↓ gWEFT ⟶ χEFT ⟶ WEFT

Polarizability (gWEFT) — van der Waals (χEFT ⟶ WEFT)



m : bottom mass, v : relative velocity

$$m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$$

m_ϕ : mass bottomonium, $r_{\phi\phi} \sim 1/m_\pi$: relative distance

$$\mathbf{k}^2_{\phi\phi}/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$

Chromopolarizability

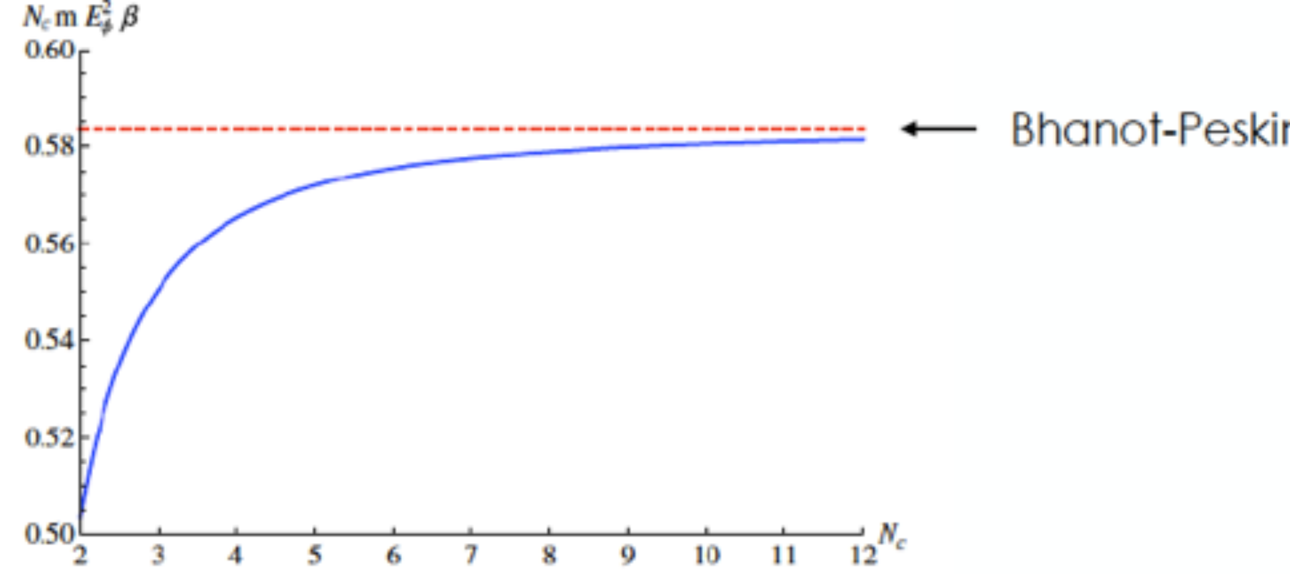


FIG. 3. The dependence of the polarizability on the number of colors. The dashed line at the constant value 7/12 corresponds to the large-N_c limit computed in Ref. [13].

pNRQCD ⟶ gWEFT

$$L_{\rm gWEFT} = \int d^3\boldsymbol{R} \left\{ \phi^\dagger(t,\boldsymbol{R}) \left[i\partial_0 + E_\phi - \frac{\boldsymbol{\nabla}_R^2}{4m} + \frac{1}{2}\beta g^2 \boldsymbol{E}_a^2 + \cdots \right] \phi(t,\boldsymbol{R}) \right\} + \mathcal{L}_{\rm light}$$

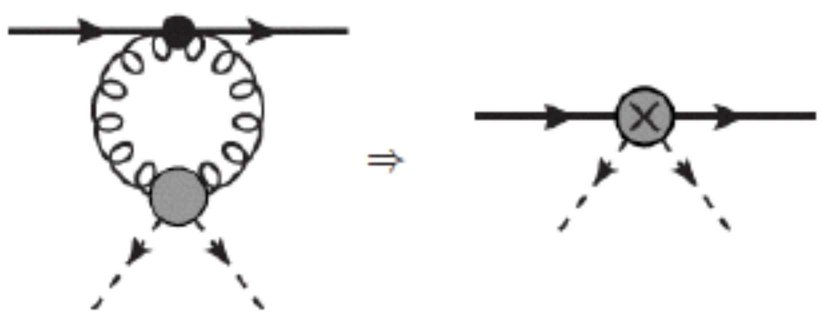
Chromopolarizability

$$\beta = -\frac{2V_A^2 T_F}{3N_c} \langle \phi | \boldsymbol{r} \frac{1}{E_\phi - h_o} \boldsymbol{r} | \phi \rangle$$

$$= -\frac{2V_A^2 T_F}{3N_c} \sum_l \int \frac{d^3 p}{(2\pi)^3} |\langle \phi | \boldsymbol{r} | \boldsymbol{p}\, l \rangle|^2 \frac{1}{E_\phi - \frac{p^2}{m}}$$

van der Waals force

gWEFT ⟶ χEFT



QCD trace anomaly

$$g^2\langle\pi^+(p_1)\pi^-(p_2)|\boldsymbol{E}_a^2|0\rangle = \frac{8\pi^2}{3b}\left((p_1+p_2)^2\kappa_1 + m_\pi^2\kappa_2\right)$$

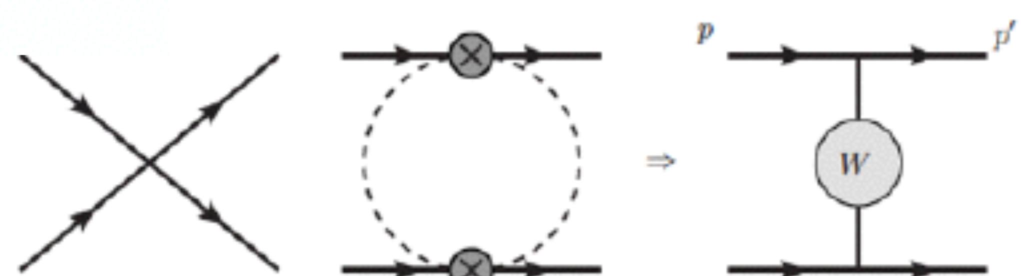
$$\kappa_1 = 1 - 9\kappa/4,\ \kappa_2 = 1 - 9\kappa/2 \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$\kappa = 0.186 \pm 0.003 \pm 0.006 \longleftarrow \psi' \to J/\psi\pi^+\pi^-$$

– integrate out the pion

χEFT ⟶ WEFT

$$r_{\phi\phi} \sim 1/m_\pi \qquad \mathbf{k}_{\phi\phi}^2/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$



— vdW potential

$$W(r) = \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\,\mu\,e^{-\mu r}\,\mathrm{Im}\left[\widetilde{W}(\epsilon - i\mu)\right]$$

$$= -\frac{3\pi\beta^2 m_\pi^2}{8b^2 r^5}\left[\left(4\left(\kappa_2+3\right)^2(m_\pi r)^3 + \left(3\kappa_1^2+43\kappa_2^2+14\kappa_1\kappa_2\right)m_\pi r\right)K_1(2m_\pi r)\right.$$
$$\left. +2\left(2\left(\kappa_2+3\right)\left(\kappa_1+5\kappa_2\right)(m_\pi r)^2 + \left(3\kappa_1^2+43\kappa_2^2+14\kappa_1\kappa_2\right)\right)K_2(2m_\pi r)\right]$$

↓ asymptotic

$$W(r) = -\frac{3(3+\kappa_2)^2\pi^{3/2}\beta^2}{4b^2}\frac{m_\pi^{9/2}}{r^{5/2}}e^{-2m_\pi r}$$

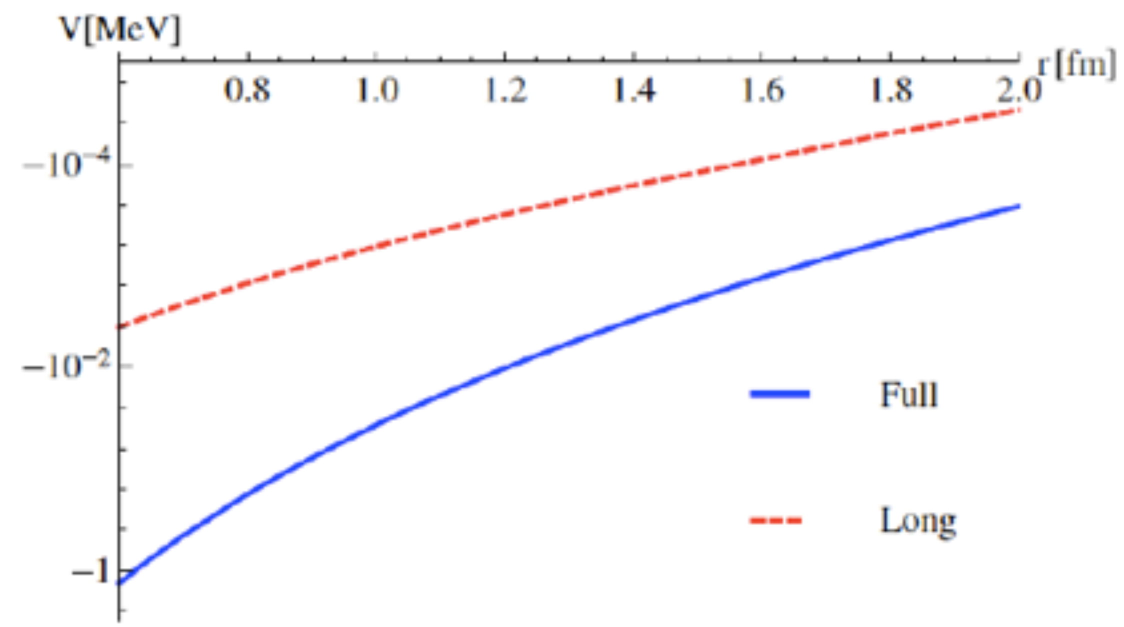


FIG. 9. Comparison of the van der Waals potential (40) (blue line) with its long-range expansion (41) (red line) for $\beta = 0.92$ GeV^{-3} and other parameters like in Fig. 8.

Are there $\eta_b\,\eta_b$ bound-states?

It is likely, but depends somewhat on the medium- and short-range pieces

Conclusions

Quarkonium is a golden system to study strong interactions

For states below threshold non relativistic EFTs provide a systematic tool to investigate a wide range of observables in the real of QCD

For states close or above the strong decay threshold the situation is much more complicated.

Many degrees of freedom show up and the absence of a clear systematic is an obstacle to a universal picture

We have presented results obtained for the hybrid masses in pNRQCD that show a very rich structure of multiplets.

Conclusions

- study of hybrids in EFT framework (NRQCD and pNRQCD)
- study of the spectrum of the static Hamiltonian with non-static corrections
- short distance degeneracy of static energies
- mixing of different static states
- breaking of degeneracy between opposite parity states

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- development of full EFT treatment
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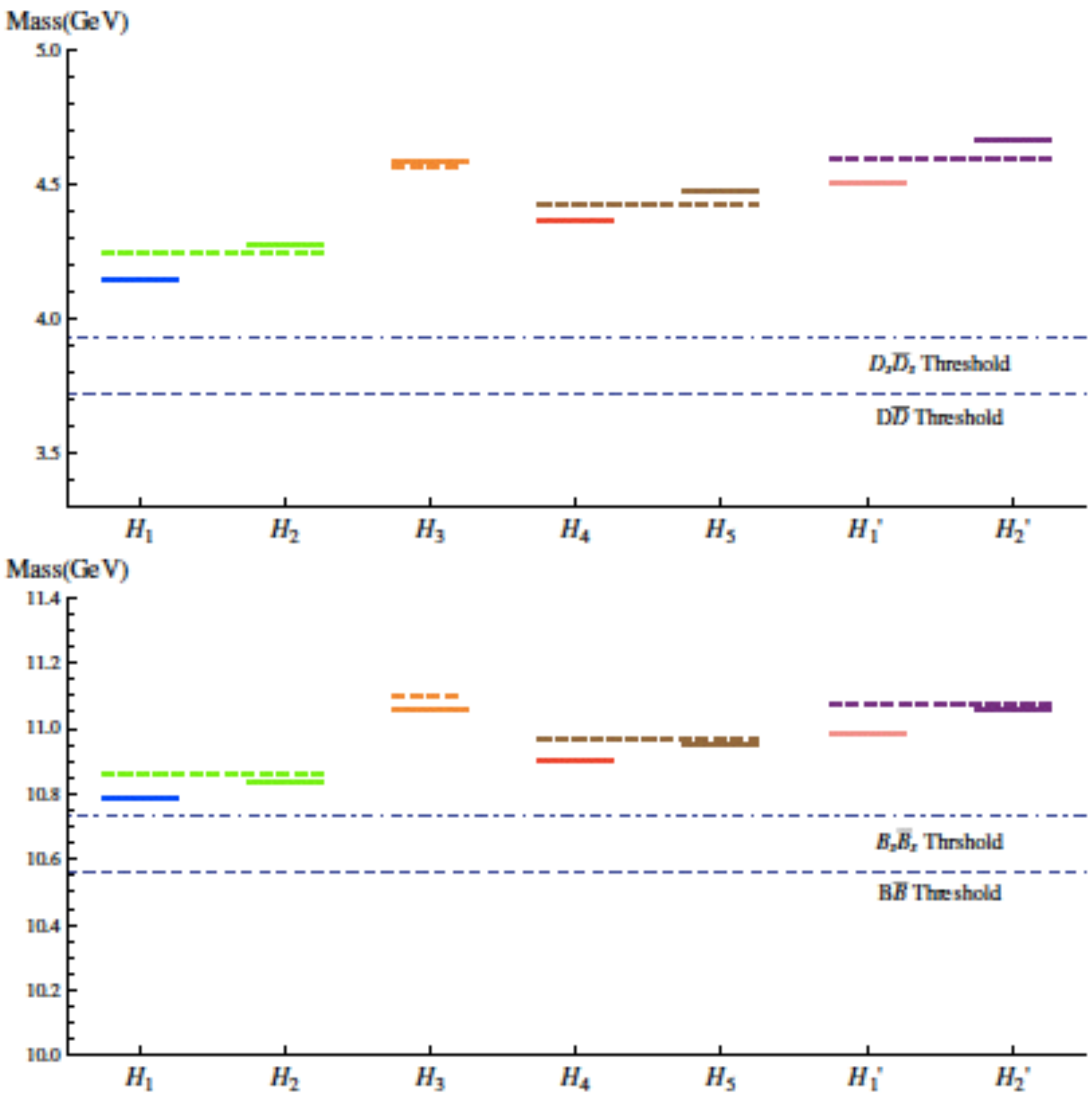
- Fundamental experimental input (like confirmation, quantum numbers, widths and masses) is still crucially missing for some of these states.

Experimental candidates for hybrids

State	M (MeV)	Γ (MeV)	J^{PC}	Decay modes	1st observation
$X(3823)$	3823.1 ± 1.9	< 24	$?^{?-}$	$\chi_{c1}\gamma$	Belle 2013
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$J/\psi\,\pi^+\pi^-$, $J/\psi\,\pi^+\pi^-\pi^0$	Belle 2003
				$D^0\bar{D}^0\pi^0$, $D^0\bar{D}^0\gamma$	
				$J/\psi\,\gamma$, $\psi(2S)\,\gamma$	
$X(3915)$	3917.5 ± 1.9	20 ± 5	0^{++}	$J/\psi\,\omega$,	Belle 2004
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$D\bar{D}$,	Belle 2005
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$D^*\bar{D}$, $D\bar{D}^*$	Belle 2007
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$D\bar{D}$,	Babar 2007
$Y(4008)$	$4008^{+121}_{-\ 49}$	226 ± 97	1^{--}	$J/\psi\,\pi^+\pi^-$,	Belle 2007
$Y(4140)$	4144.5 ± 2.6	$15^{+11}_{-\ 7}$	$?^{?+}$	$J/\psi\,\phi$	CDF 2009
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$D^*\bar{D}^*$	Belle 2007
$Y(4220)$	4216 ± 7	39 ± 17	1^{--}	$h_c(1P)\,\pi^+\pi^-$,	BESIII 2013
$Y(4230)$	4230 ± 14	38 ± 14	1^{--}	$\chi_{c0}\,\omega$,	BESIII 2014
$Y(4260)$	4263^{+8}_{-9}	95 ± 14	1^{--}	$J/\psi\,\pi^+\pi^-$, $J/\psi\,\pi^0\pi^0$	Babar 2005
				$Z_c(3900)\,\pi$,	
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$J/\psi\,\phi$	CDF 2010
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0/2^{++}$	$J/\psi\,\phi$,	Belle 2009
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$\psi(2S)\,\pi^+\pi^-$,	Babar 2007
$X(4630)$	$4634^{+\ 9}_{-11}$	92^{+41}_{-32}	1^{--}	$\Lambda_c^+\Lambda_c^-$,	Belle 2007
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$\psi(2S)\,\pi^+\pi^-$,	Belle 2007
$Y_b(10890)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$\Upsilon(nS)\pi^+\pi^-$	Belle 2010

TABLE V: Neutral mesons above open flavor threshold excluding isospin partners of charged states.

Results I: Comparison to BO approximation



For Hybrid multiplets:

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

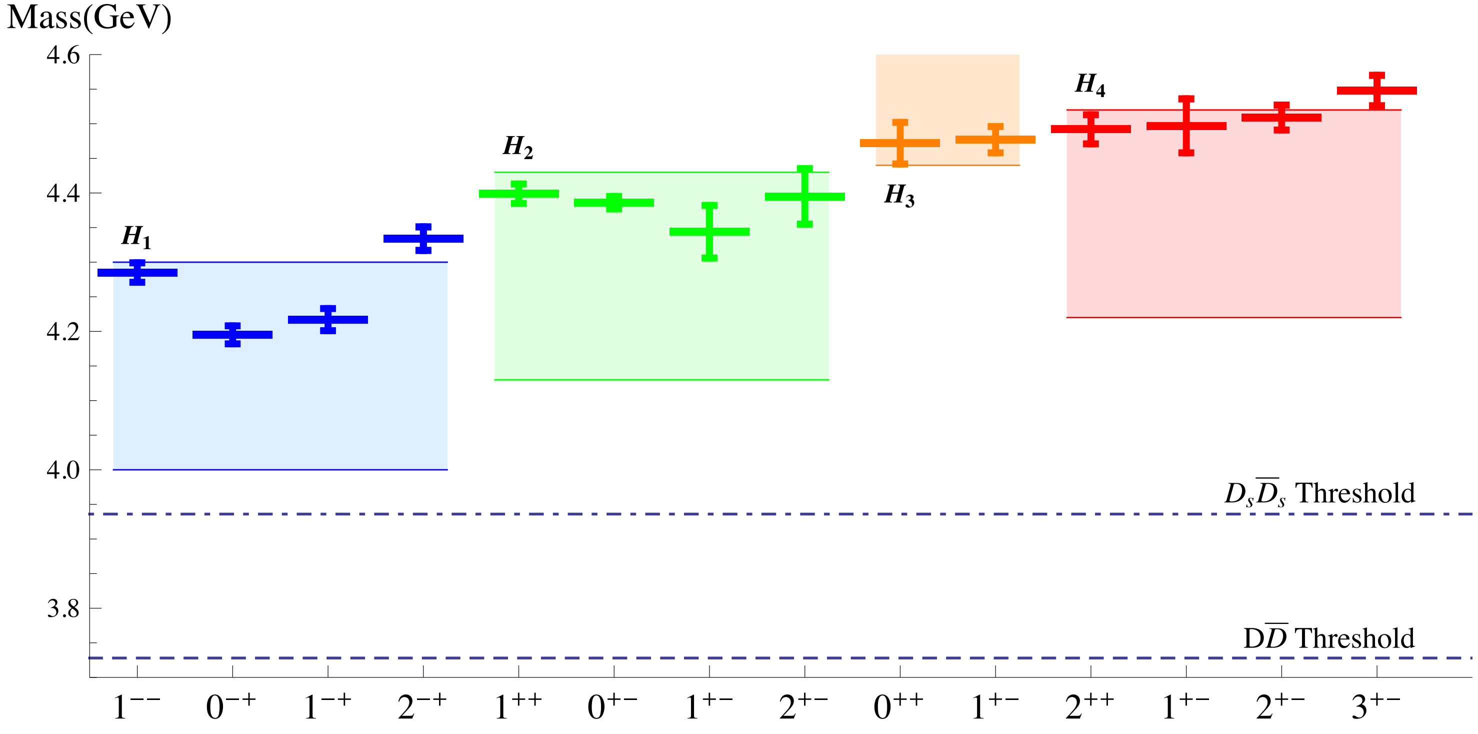
- no distinction between opposite parity states in BO
- mixed states lie lower than pure
- discrepancy for H_2, H_3 and H_5 due to different potential fits

Braaten, Langmack and Smith 2014

Comparison with direct lattice computations

Charmonium sector

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. Liu *et all* 2012
- They worked in the constituent gluon picture, which consider the multiplets H_2, H_3, H_4 as part of the same multiplet.
- Their results are given with the η_c mass subtracted.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- Our masses are $0.1 - 0.14$ GeV lower except the for the H_3 multiplet, which is the only one dominated by Σ_u^-.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

Berwein, N.B. , Tarrus, Vairo 2015

Comparison to sum rules

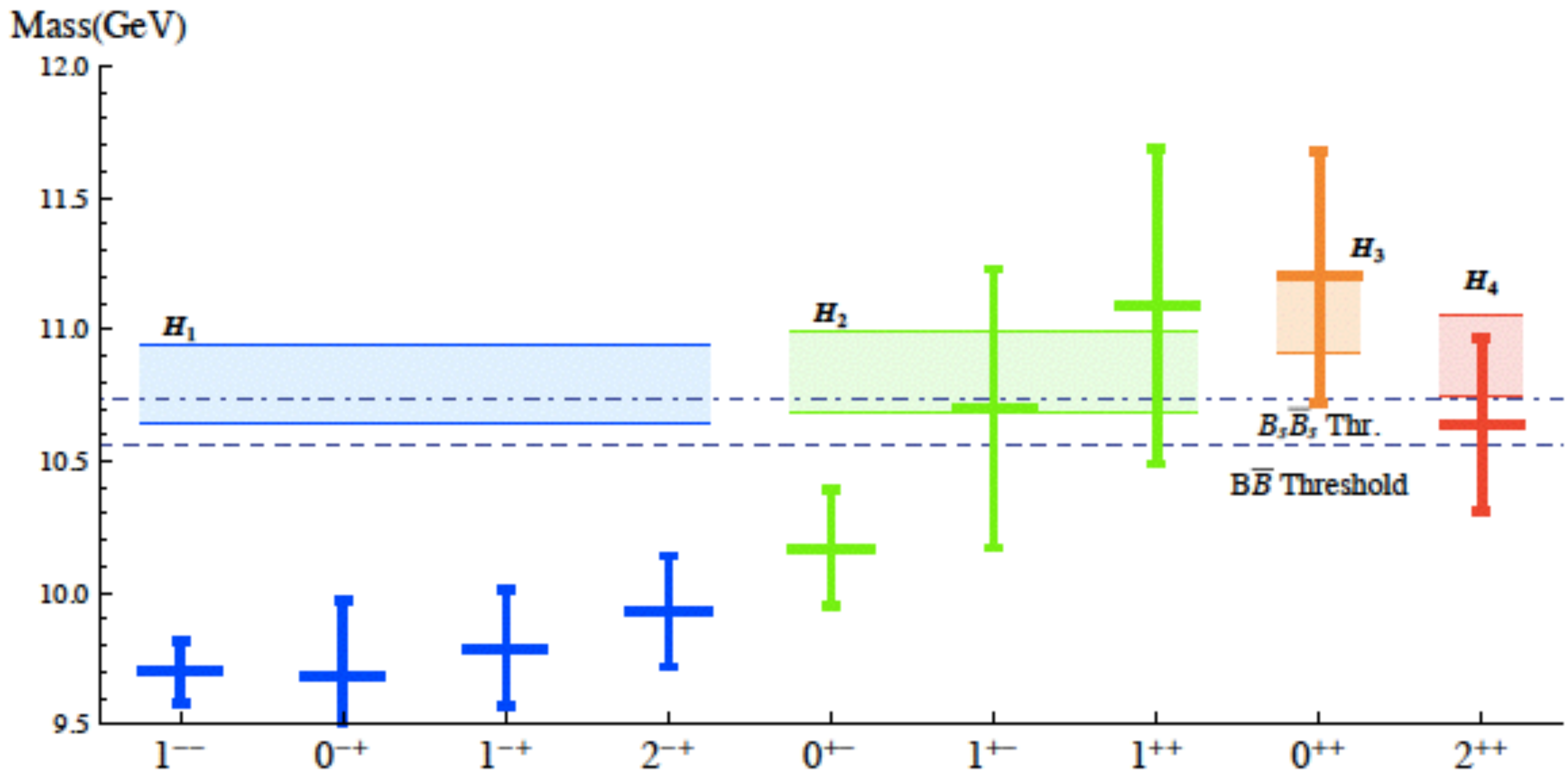


FIG. 7. Comparison of the mass predictions for charmonium hybrids in the upper figure and for bottomonium hybrids in the lower figure, obtained using QCD sum rules [68], with our results using the $V^{(0.25)}$ potential. The solid lines correspond to the QCD sum rules masses with error bars corresponding to their uncertainties. Our results for the H_1, H_2, H_3, and H_4 multiplets have been plotted in error bands corresponding to the gluelump mass uncertainty of ± 0.15 GeV.

- We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the Λ–doubling terms by using coupled Schröringer equations.
- The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.
- A large set of masses for spin symmetry multiplets for $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ has been obtained.
- Λ–doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.
- Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.
- Several experimental candidates for Charmonium hybrids belonging to the H_1, H_2, H_4 and H_1' multiplets.
- One experimental candidate to the bottomonium H_1 multiplet.