

Explicit Delta degree of freedom in Chiral two- and three-nucleon forces

A. M. Gasparyan, Ruhr-Universität Bochum

in collaboration with

H. Krebs, E. Epelbaum,
D. Siemens, V. Bernard, Ulf-G. Meißner

April 5, 2017, Bochum

Outline

- Introduction & Motivation
- 2-N forces with explicit Δ
- 3-N forces with explicit Δ
- π N scattering with explicit Δ
- Summary and Outlook

EFT with explicit $\Delta(1232)$

→ Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\text{MeV}$

EFT with explicit $\Delta(1232)$

→ Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\text{MeV}$

→ Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$

Hemmert, Holstein, Kambor '98

EFT with explicit $\Delta(1232)$

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\text{MeV}$
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$
Hemmert, Holstein, Kambor '98
- Δ gives a large contribution to LEC (c_3, c_4) via resonance saturation
Bernard, Kaiser, Meißner '97



EFT with explicit $\Delta(1232)$

→ Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\text{MeV}$

→ Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$

Hemmert, Holstein, Kambor '98

→ Δ gives a large contribution to LEC (c_3, c_4) via resonance saturation

Bernard, Kaiser, Meißner '97



→ Explicit decoupling of Δ makes comparison with Δ -less theory more transparent




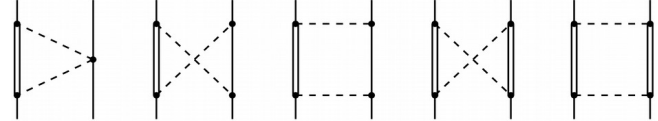

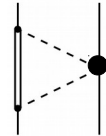

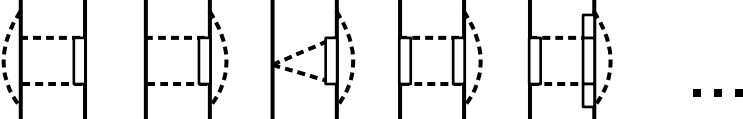

Bernard, Fearing, Hemmert, Meißner '98

finite parts of LECs can be always chosen such that

Appelquist, Carrazone '74 (Decoupling theorem)

$$\lim_{\Delta \rightarrow \infty} = \Delta\text{-less}$$

Small scale expansion of 2NF

	Δ -less theory	Δ -full theory: additional graphs
LO		
NLO		 <p>Keiser, Gerstendorfer, Weise '98</p>
N ² LO		 <p>...</p> <p>Krebs, Epelbaum, Meißner '07</p>
N ³ LO		 <p>...</p>
N ⁴ LO		

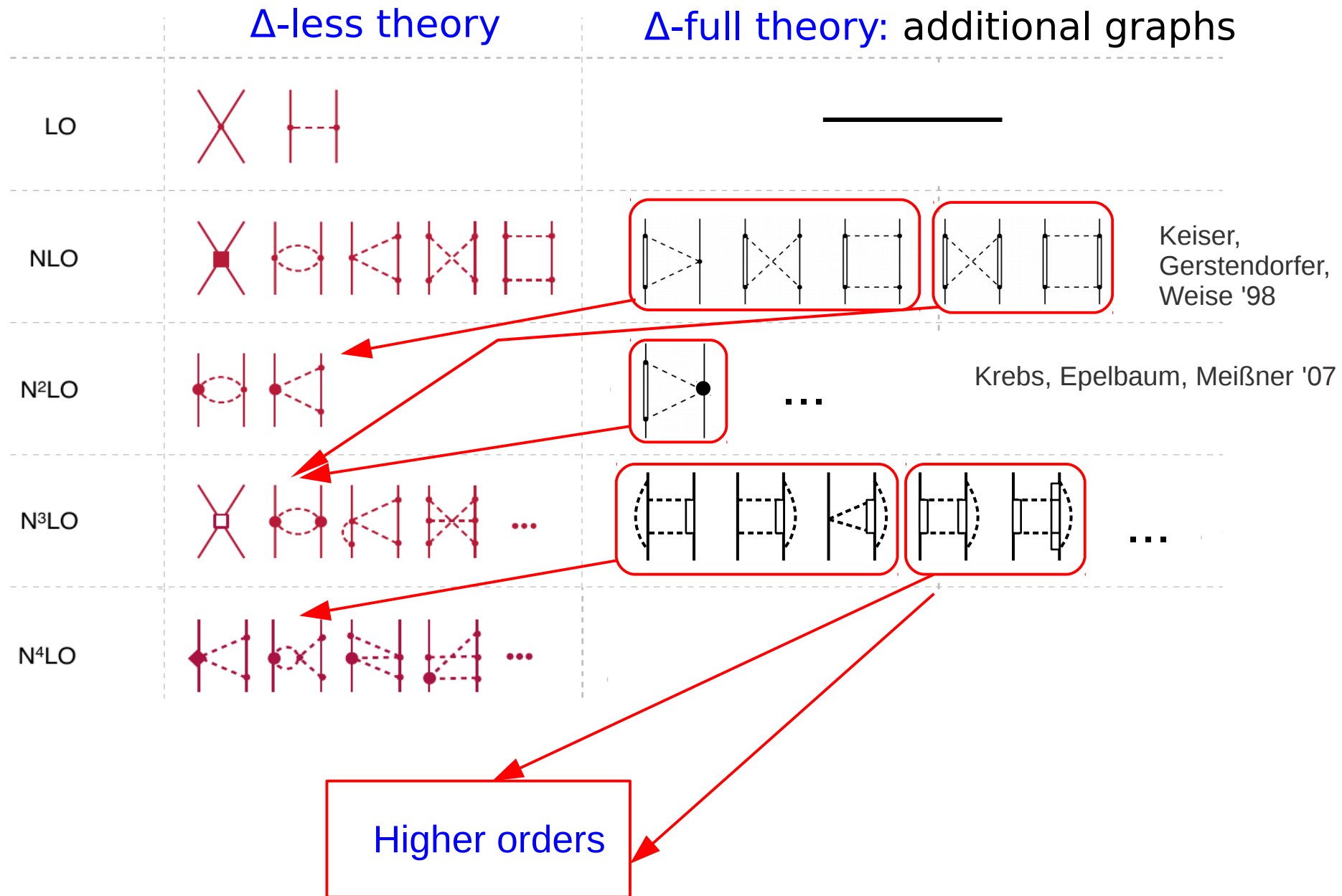
Small scale expansion of 2NF

	Δ -less theory	Δ -full theory: additional graphs
LO		
NLO		 Keiser, Gerstendorfer, Weise '98
N ² LO		 Krebs, Epelbaum, Meißner '07
N ³ LO		
N ⁴ LO		

Small scale expansion of 2NF

	Δ -less theory	Δ -full theory: additional graphs
LO		
NLO		 Keiser, Gerstendorfer, Weise '98
N ² LO		 Krebs, Epelbaum, Meißner '07
N ³ LO		
N ⁴ LO		

Small scale expansion of 2NF



Preliminary results for N³LO 2N forces with explicit Δ

- Only 2-pion-exchange contribution are considered (the long range part)
- $1/m_N$ corrections are not yet included
- Results for peripheral phases, no refitting of LEC's, no cut offs
- No additional parameters, h_A and g_1 ($\pi N\Delta$ and $\pi\Delta\Delta$) are extracted from the fit to πN scattering

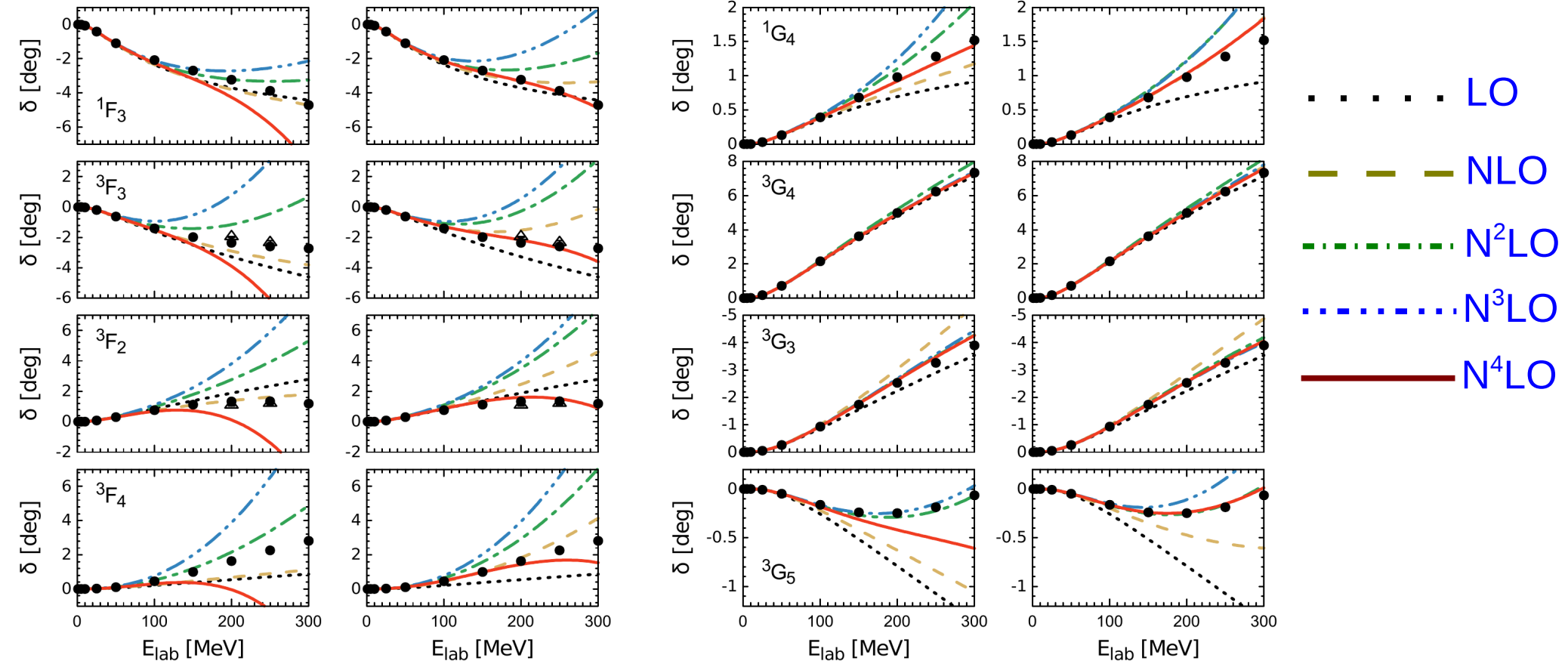
F and G waves

Δ -less

Δ -full

Δ -less

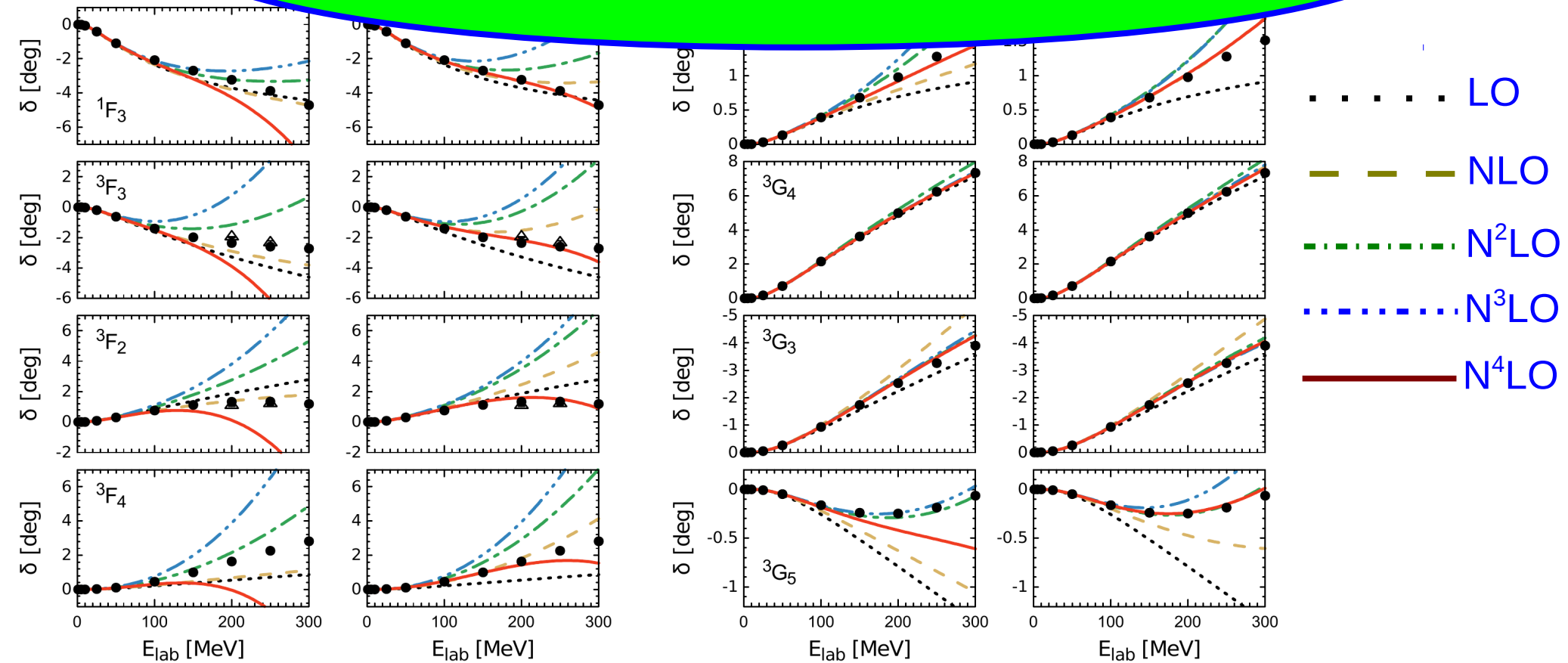
Δ -full



Data: Nijmegen PWA

F and G waves

Δ - F-waves might be sensitive to the short-range physics

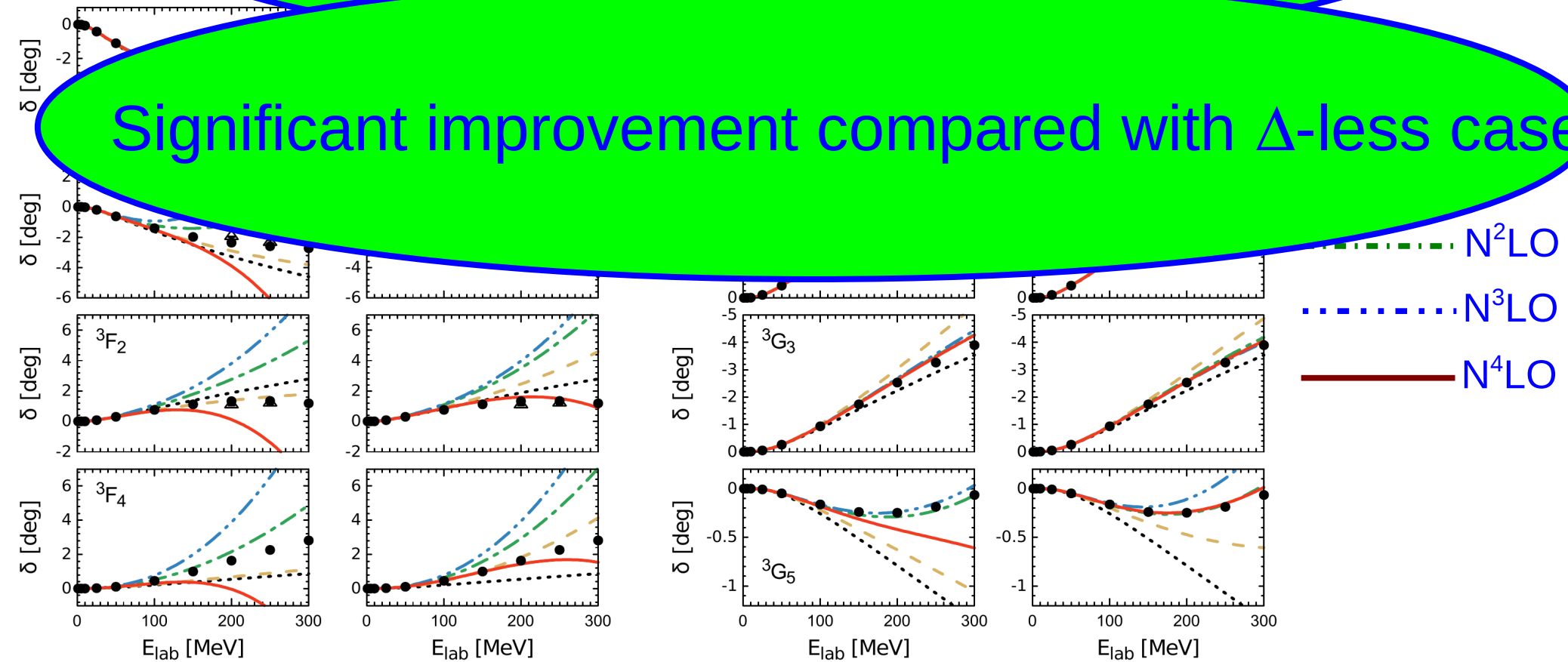


Data: Nijmegen PWA

F and G waves

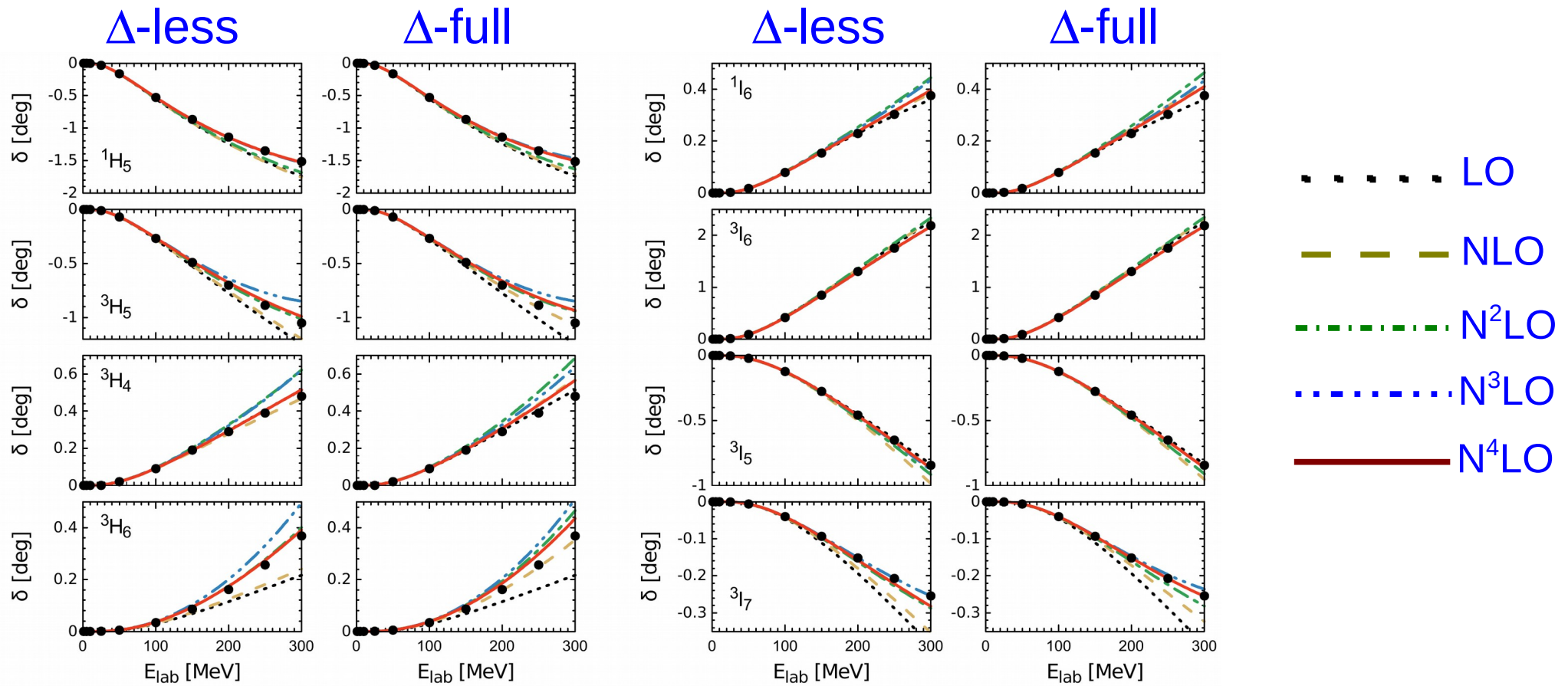
Δ - F-waves might be sensitive to the short-range physics

Significant improvement compared with Δ -less case



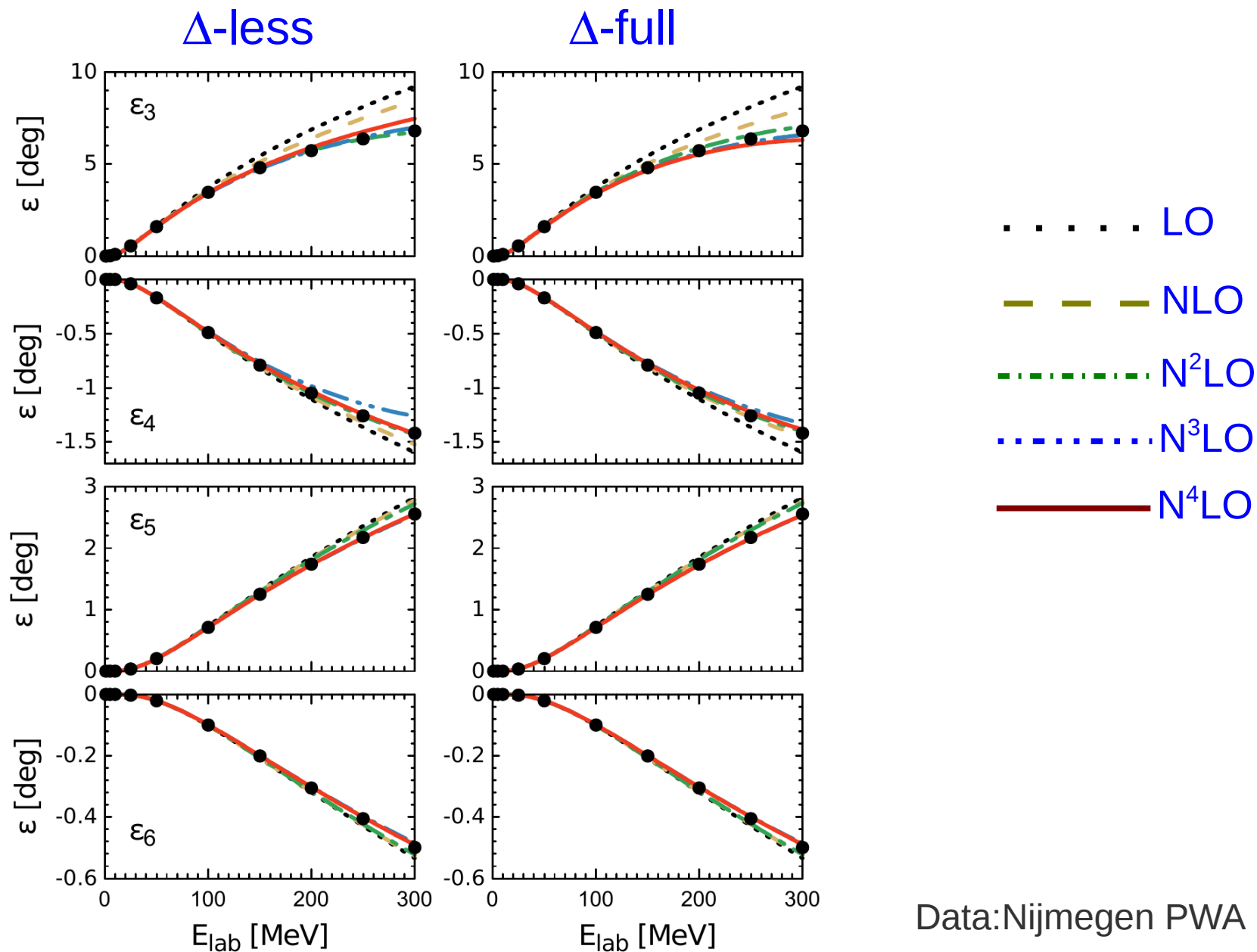
Data: Nijmegen PWA

H and I waves



Data: Nijmegen PWA

Mixing angles $\varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$



Small scale expansion of 3NF

	Δ -less theory	Δ -full theory: additional graphs
NLO		
N ² LO		
N ³ LO		
N ⁴ LO		

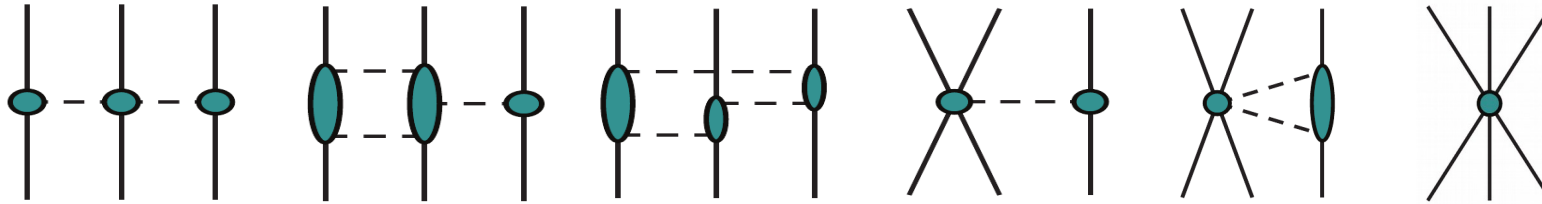
Small scale expansion of 3NF

	Δ -less theory	Δ -full theory: additional graphs
NLO		
N ² LO		
N ³ LO		
N ⁴ LO		

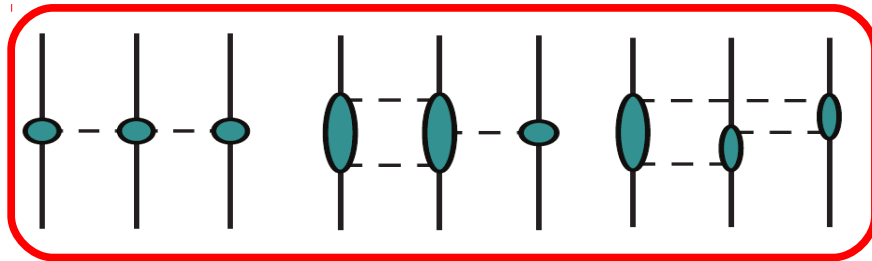
Small scale expansion of 3NF

	Δ -less theory	Δ -full theory: additional graphs
NLO		
N ² LO		
N ³ LO		
N ⁴ LO		<div data-bbox="1251 1061 1932 1295" style="border: 1px solid red; padding: 5px;"> <p>Large contributions to the ring and $2\pi-1\pi$ topologies saturating some of the N^{5,6}LO graphs in the Δ-less theory</p> </div>

Long-range 3NF



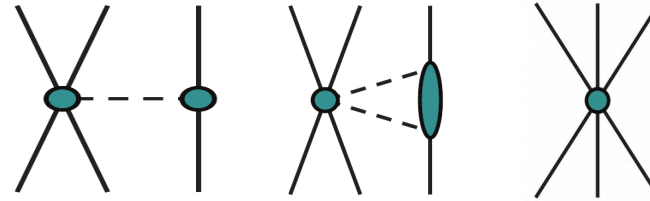
Long-range 3NF



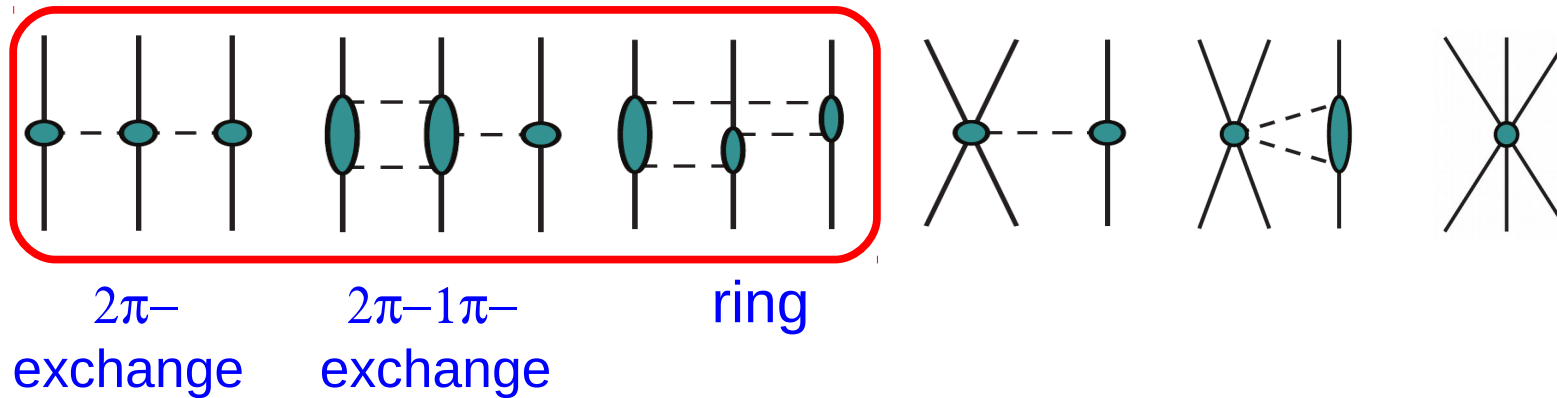
$2\pi-$
exchange

$2\pi-1\pi-$
exchange

ring



Long-range 3NF



- Only the long range part considered (coordinate space)
- Scheme independent
- No unknown parameters

Most general structure of a local 3NF

Krebs, AG, Epelbaum '13
Schat, Phillips '13
Epelbaum, AG, Krebs, Shat '15

Up to N^4 LO all considered contribution are local

Constraints:

- Locality
- Isospin symmetry
- Parity and time-reversal invariance

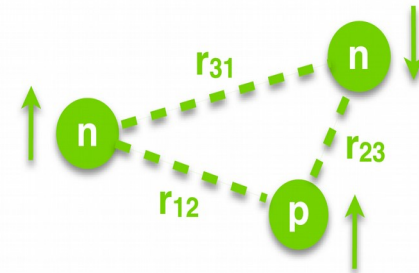
Most general structure of a local 3NF

Krebs, AG, Epelbaum '13
Schat, Phillips '13
Epelbaum, AG, Krebs, Shat '15

Up to N^4 LO all considered contribution are local

Constraints:

- Locality
- Isospin symmetry
- Parity and time-reversal invariance



Most general structure of a local 3NF

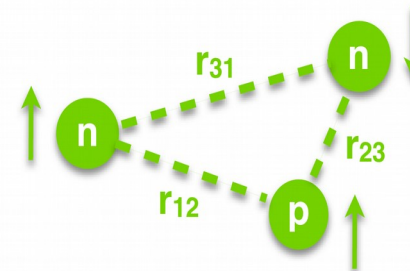
Krebs, AG, Epelbaum '13
 Schat, Phillips '13
 Epelbaum, AG, Krebs, Shat '15

Up to N^4 LO all considered contribution are local

$$\begin{aligned}
 \tilde{\mathcal{G}}_1 &= 1 \\
 \tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \\
 \tilde{\mathcal{G}}_3 &= \vec{\sigma}_1 \cdot \vec{\sigma}_3, \\
 \tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3, \\
 \tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
 \tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3) \\
 \tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \\
 \tilde{\mathcal{G}}_8 &= \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{23} \cdot \vec{\sigma}_3 \\
 \tilde{\mathcal{G}}_9 &= \hat{r}_{23} \cdot \vec{\sigma}_3 \hat{r}_{12} \cdot \vec{\sigma}_1 \\
 \tilde{\mathcal{G}}_{10} &= \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_3 \\
 \tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{23} \cdot \vec{\sigma}_2 \\
 \tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_2 \\
 \tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{12} \cdot \vec{\sigma}_1 \hat{r}_{23} \cdot \vec{\sigma}_2 \\
 \tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{12} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_2 \\
 \tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \hat{r}_{13} \cdot \vec{\sigma}_1 \hat{r}_{13} \cdot \vec{\sigma}_3 \\
 \tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{12} \cdot \vec{\sigma}_2 \hat{r}_{12} \cdot \vec{\sigma}_3 \\
 \tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_3 \\
 \tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \\
 \tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \hat{r}_{23} \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \\
 \tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \hat{r}_{23} \vec{\sigma}_3 \cdot \hat{r}_{12} \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})
 \end{aligned}$$

Constraints:

- Locality
- Isospin symmetry
- Parity and time-reversal invariance

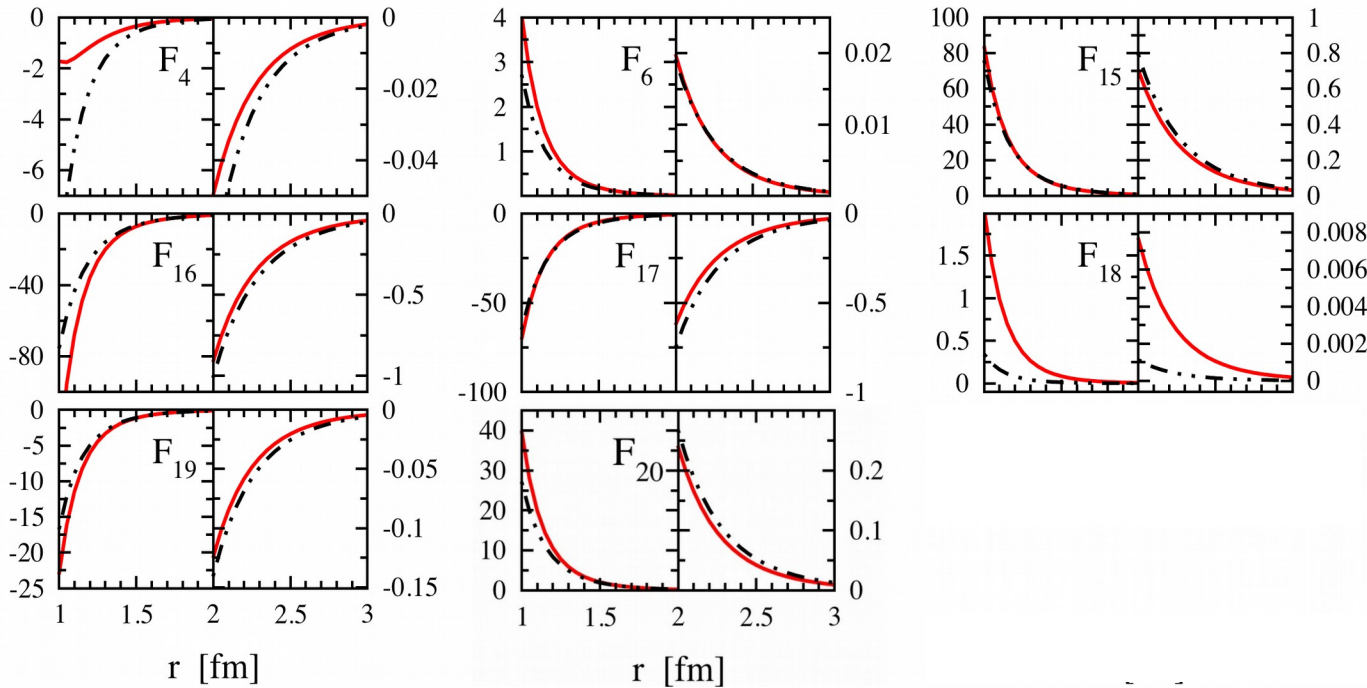
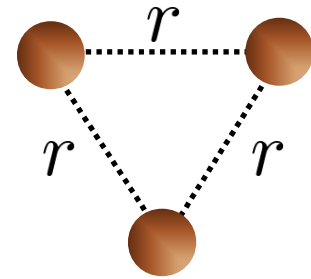


$$V_{3N} = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i F_i(r_{12}, r_{23}, r_{31}) + 5\text{perm}$$

Two-pion-exchange 3NF in Δ -full and Δ -less approach (preliminary)

Krebs, AG, Epelbaum, in preparation

TPE “structure functions” F_i in MeV”
in equilateral-triangle configuration

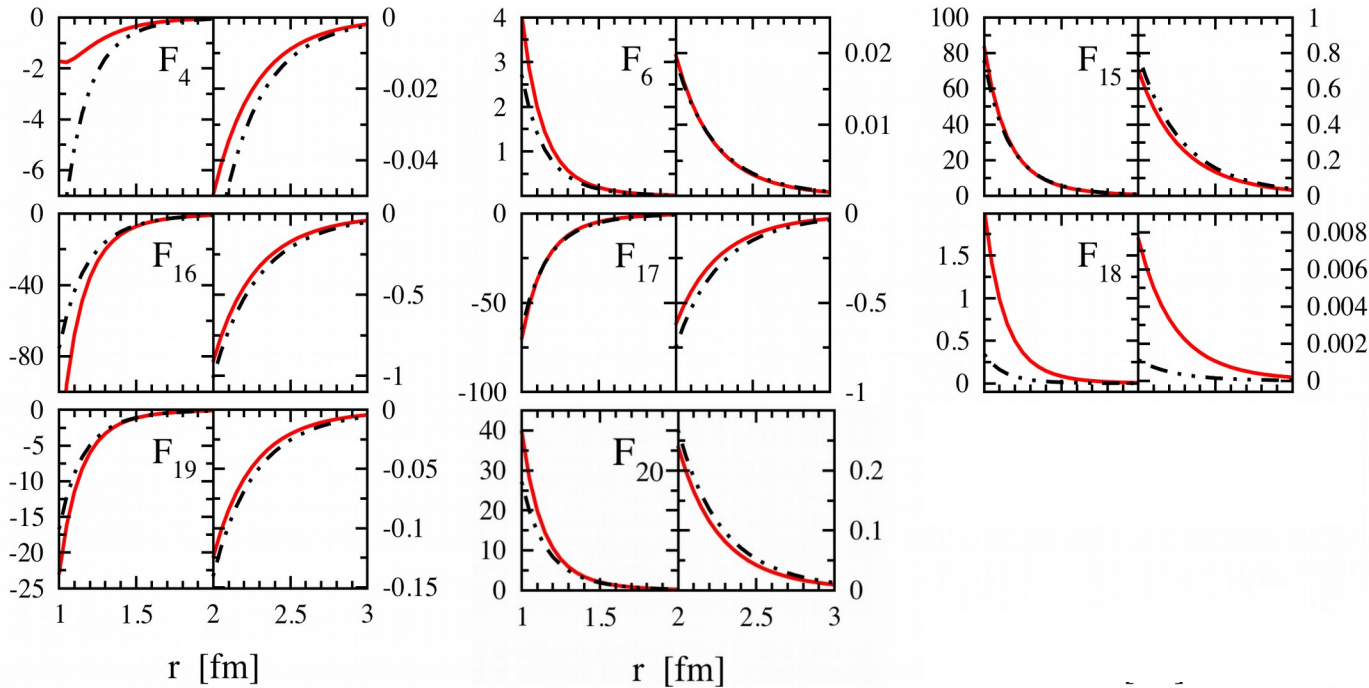
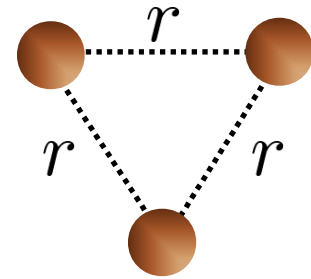


— $N^4LO \Delta$ -less
- - - $N^3LO-\Delta$

Two-pion-exchange 3NF in Δ -full and Δ -less approach (preliminary)

Krebs, AG, Epelbaum, in preparation

TPE “structure functions” F_i in MeV
in equilateral-triangle configuration



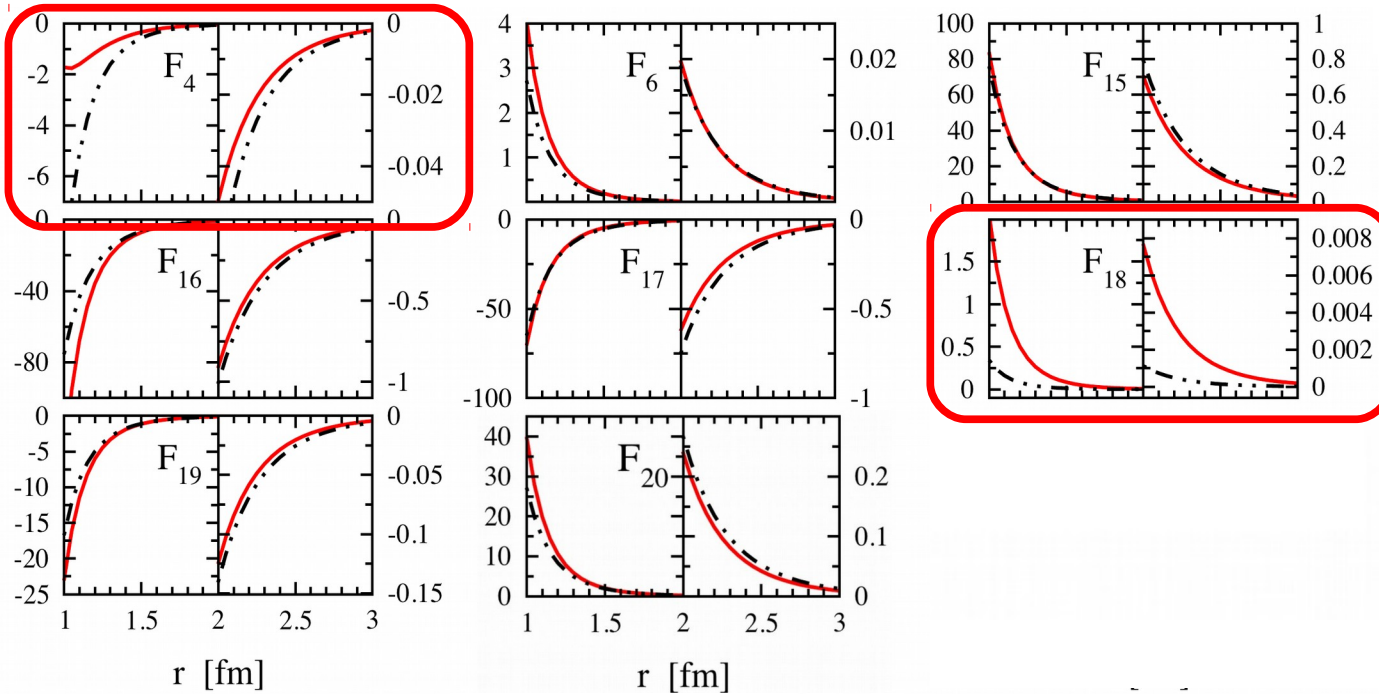
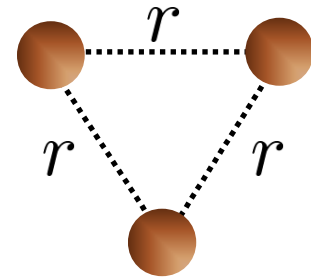
— $N^4LO \Delta$ -less
- - - $N^3LO-\Delta$

→ similar results for large contributions

Two-pion-exchange 3NF in Δ -full and Δ -less approach (preliminary)

Krebs, AG, Epelbaum, in preparation

TPE “structure functions” F_i in MeV
in equilateral-triangle configuration

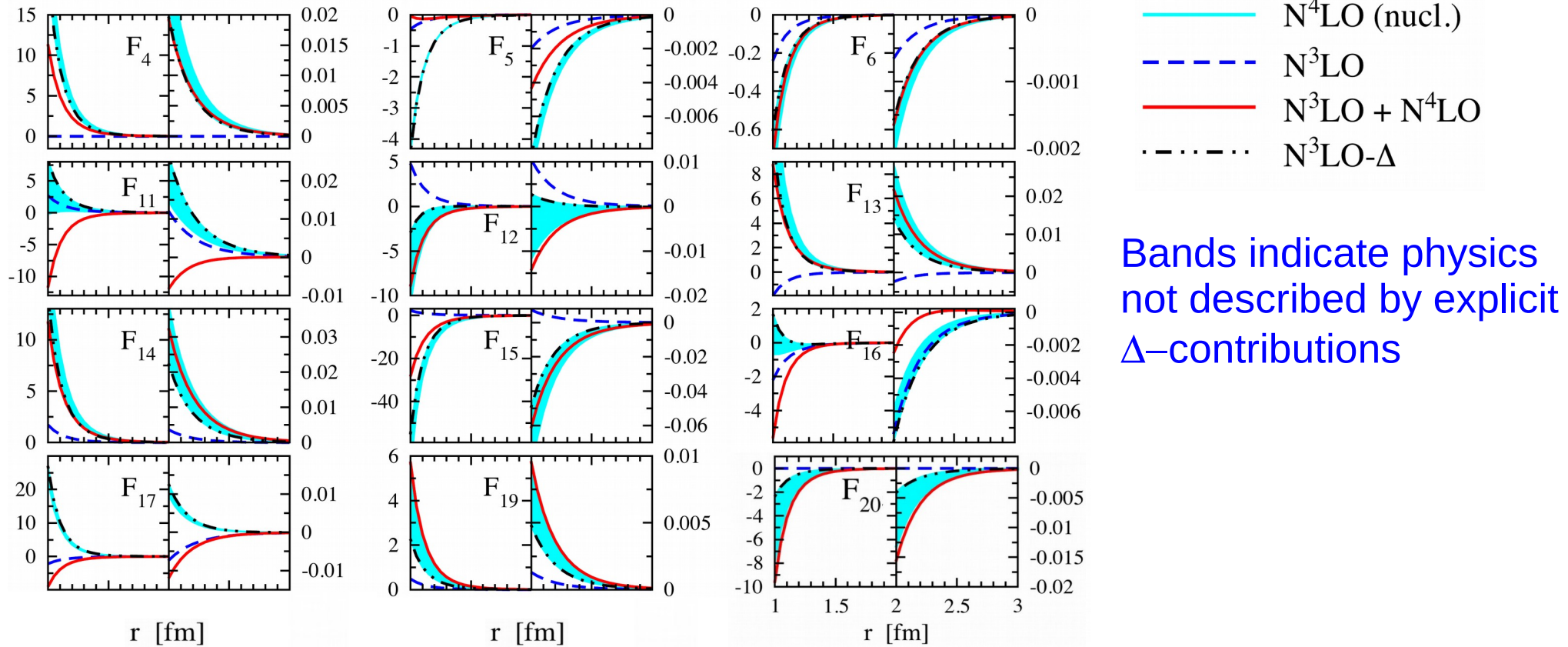


— $N^4LO \Delta$ -less
- - - $N^3LO-\Delta$

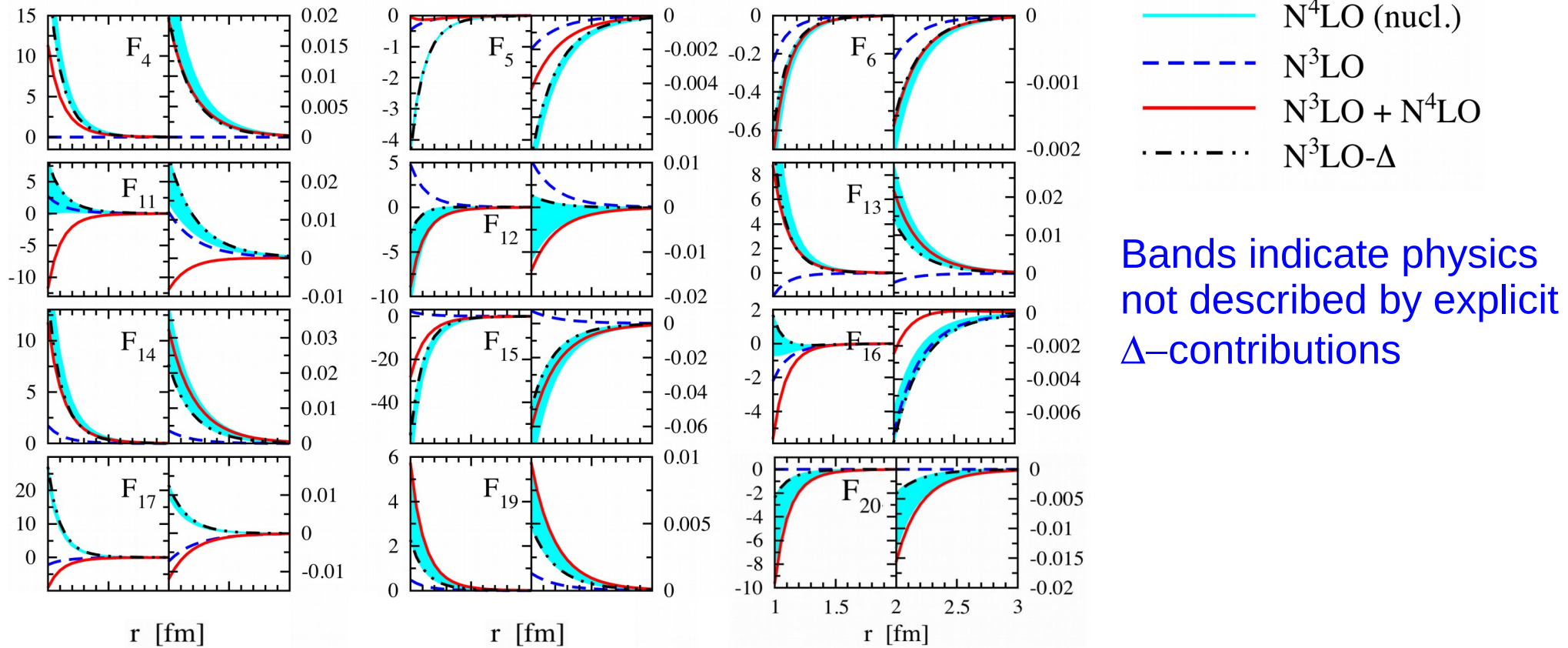
→ similar results for large contributions

→ slightly different for small contributions

Two-pion-one-pion-exchange 3NF in Δ -full and Δ -less approach (preliminary)

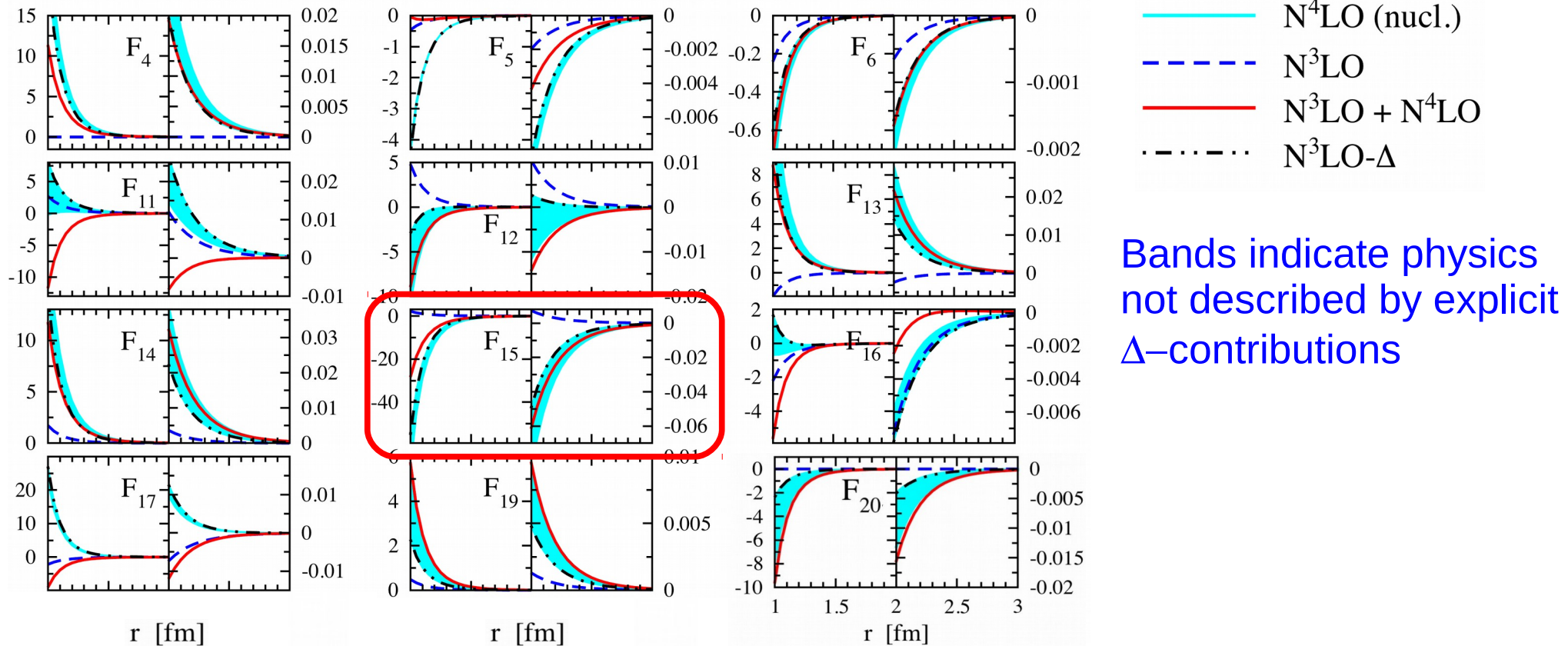


Two-pion-one-pion-exchange 3NF in Δ -full and Δ -less approach (preliminary)



→ Dominant effects come from $N^3\text{LO}-\Delta/N^4\text{LO}$

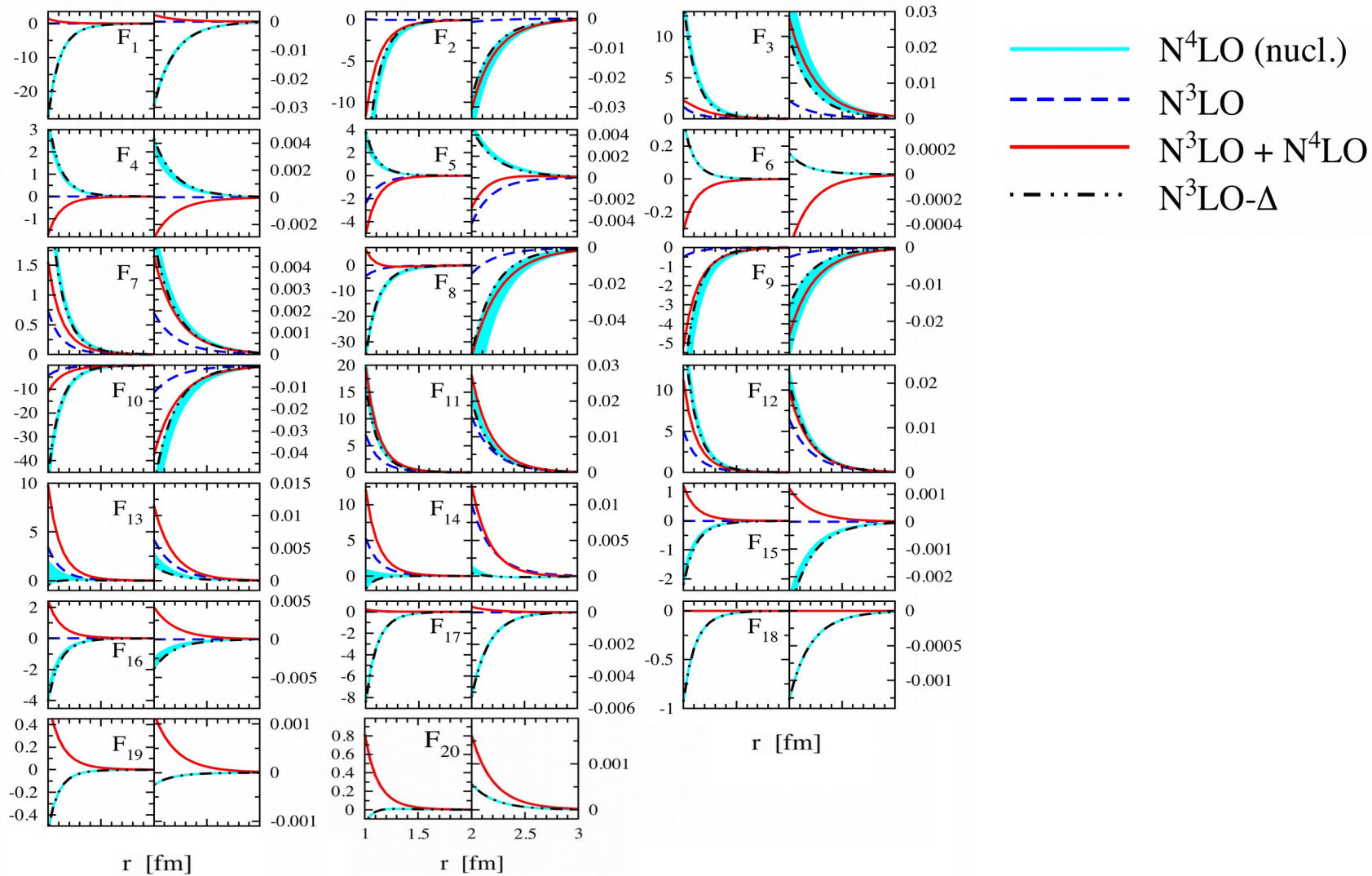
Two-pion-one-pion-exchange 3NF in Δ -full and Δ -less approach (preliminary)



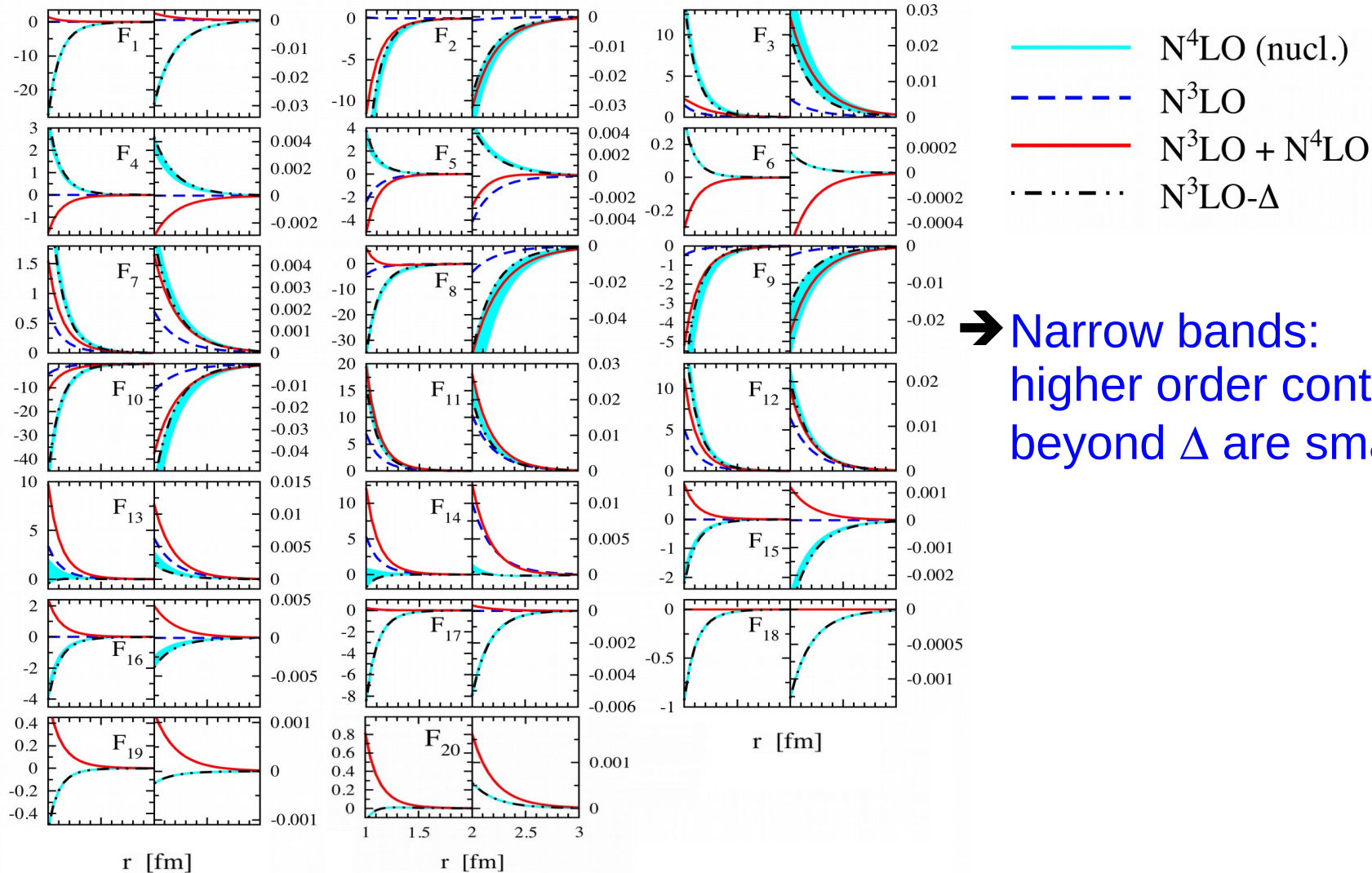
→ Dominant effects come from $N^3\text{LO}-\Delta/N^4\text{LO}$

→ The largest $N^4\text{LO}$ contribution is saturated by Δ

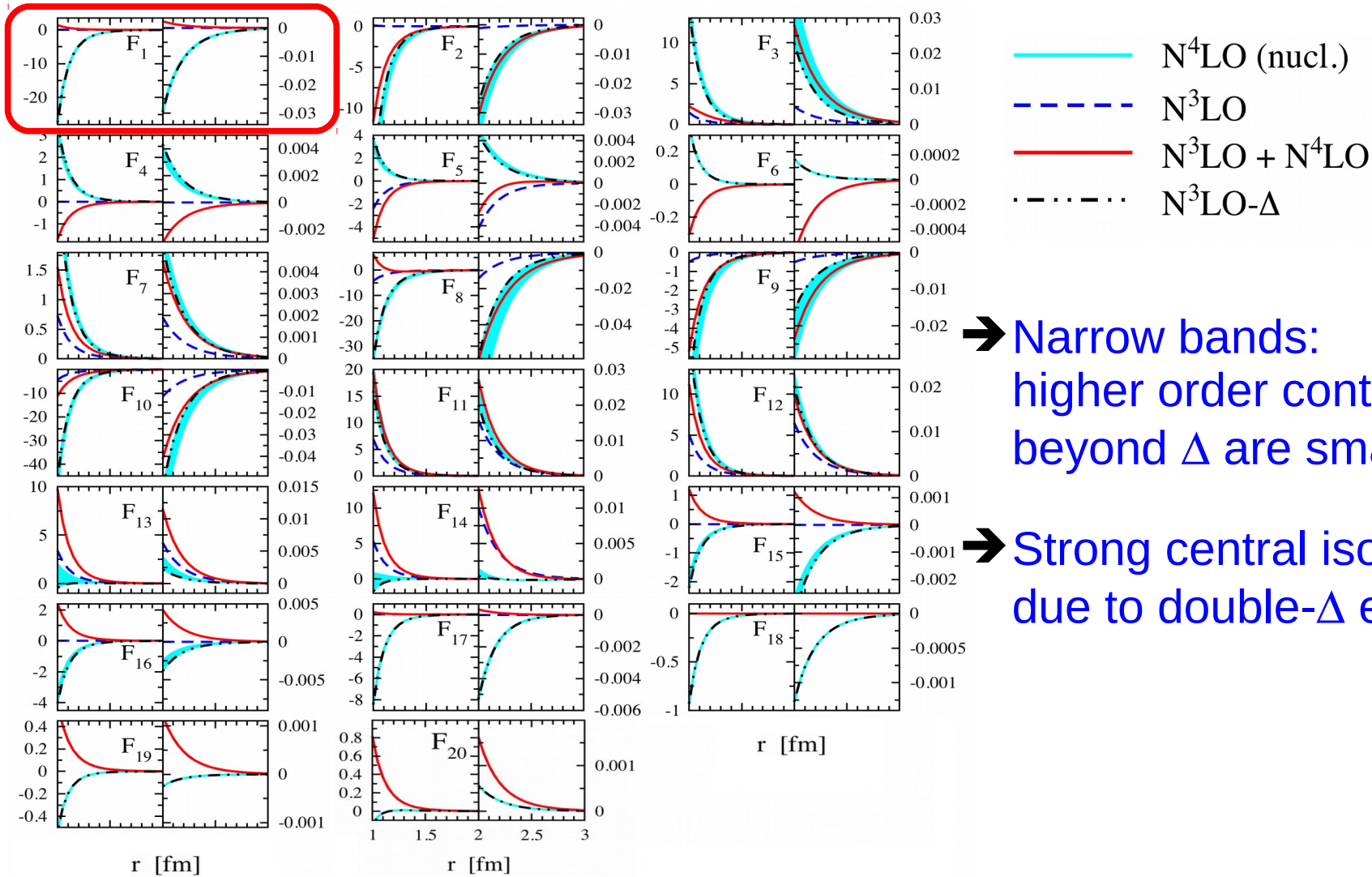
Ring-topology 3NF in Δ -full and Δ -less approach (preliminary)



Ring-topology 3NF in Δ -full and Δ -less approach (preliminary)



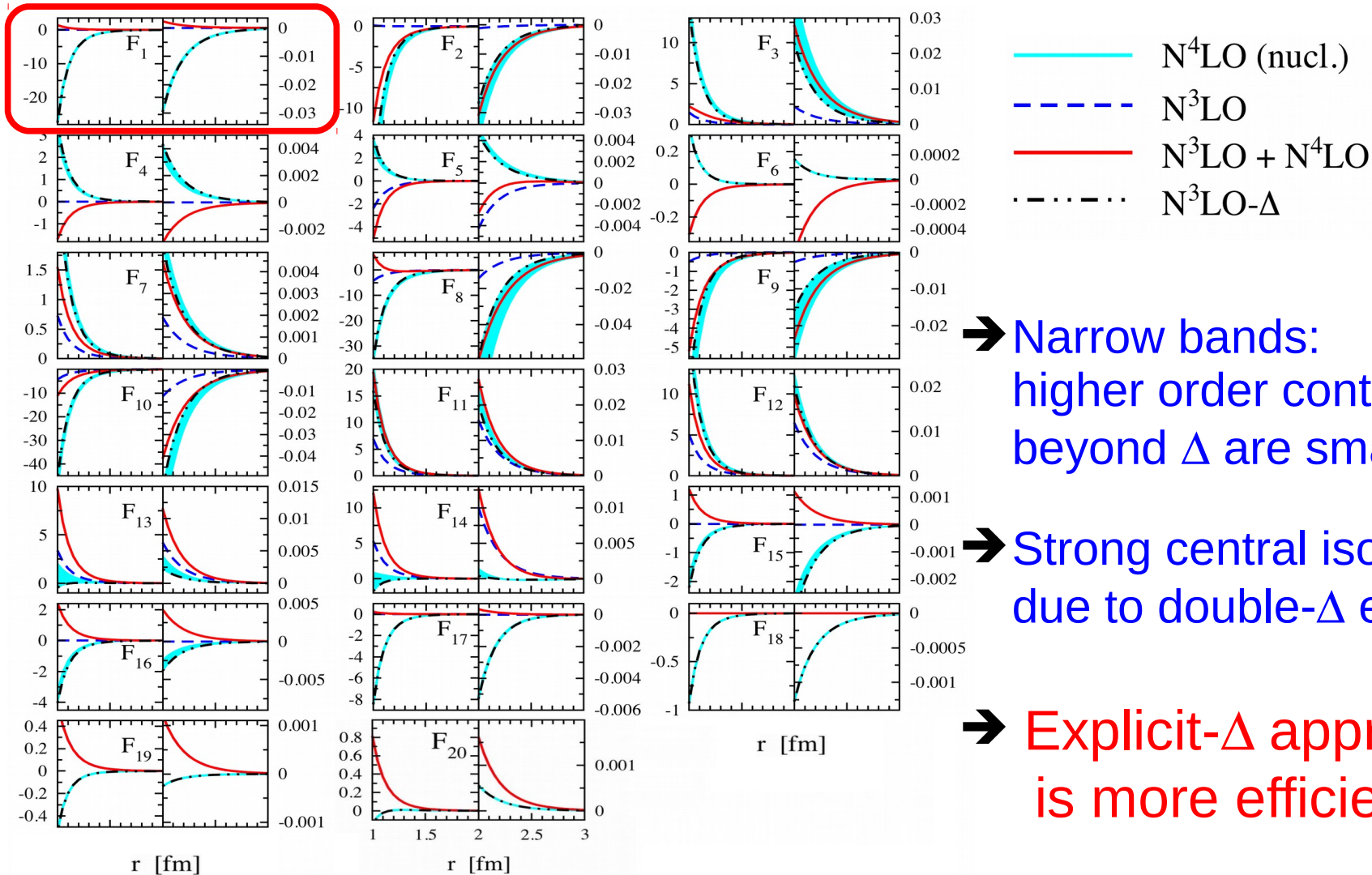
Ring-topology 3NF in Δ -full and Δ -less approach (preliminary)



→ Narrow bands:
higher order contributions
beyond Δ are small

→ Strong central isoscalar 3NF
due to double- Δ excitation

Ring-topology 3NF in Δ -full and Δ -less approach (preliminary)



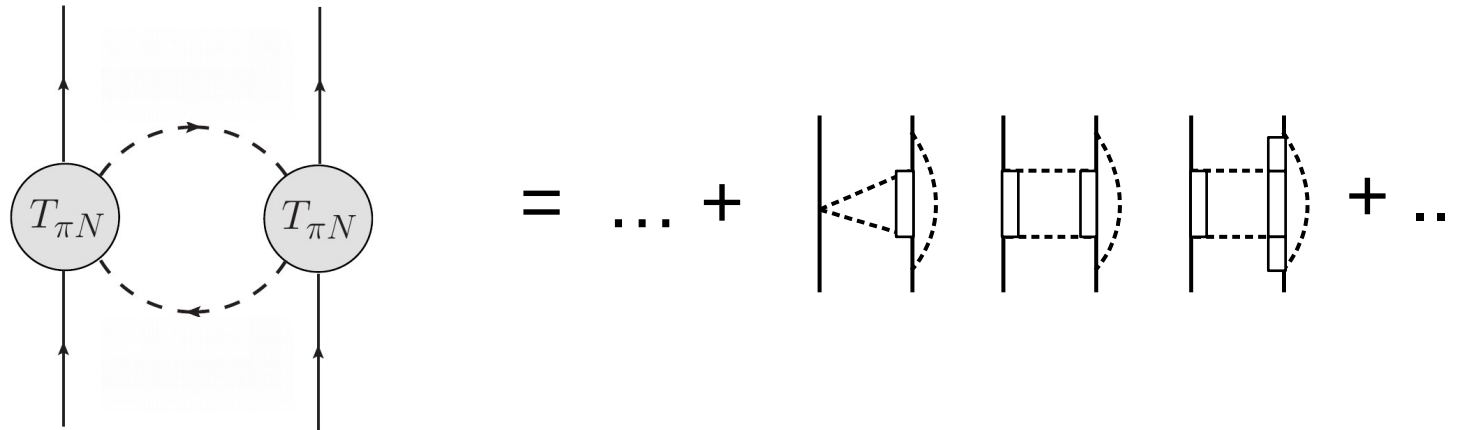
→ Narrow bands:
higher order contributions
beyond Δ are small

→ Strong central isoscalar 3NF
due to double- Δ excitation

→ Explicit- Δ approach
is more efficient !

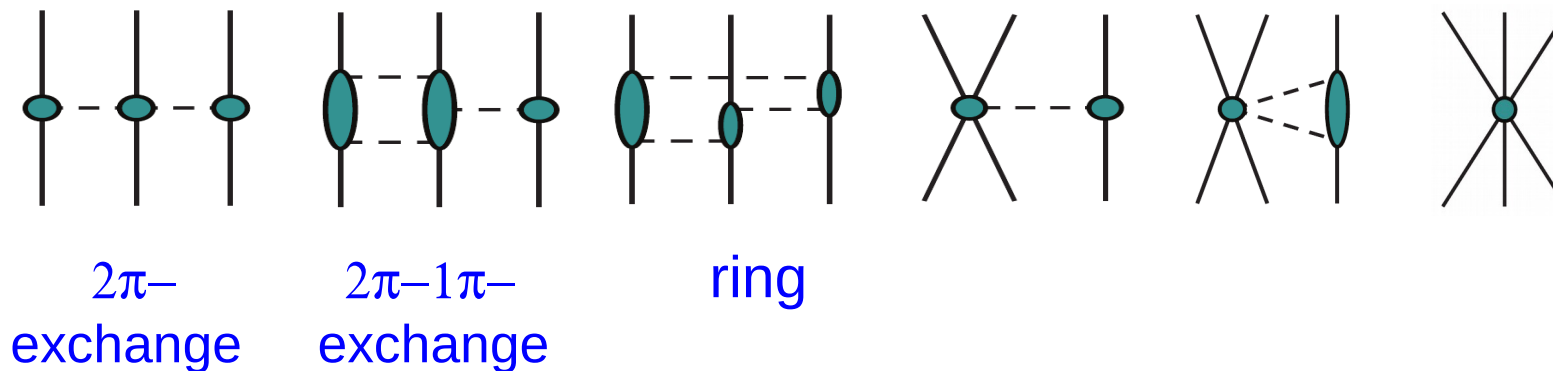
πN input for 2-Nucleon Forces

→ 2-pion exchange contributions



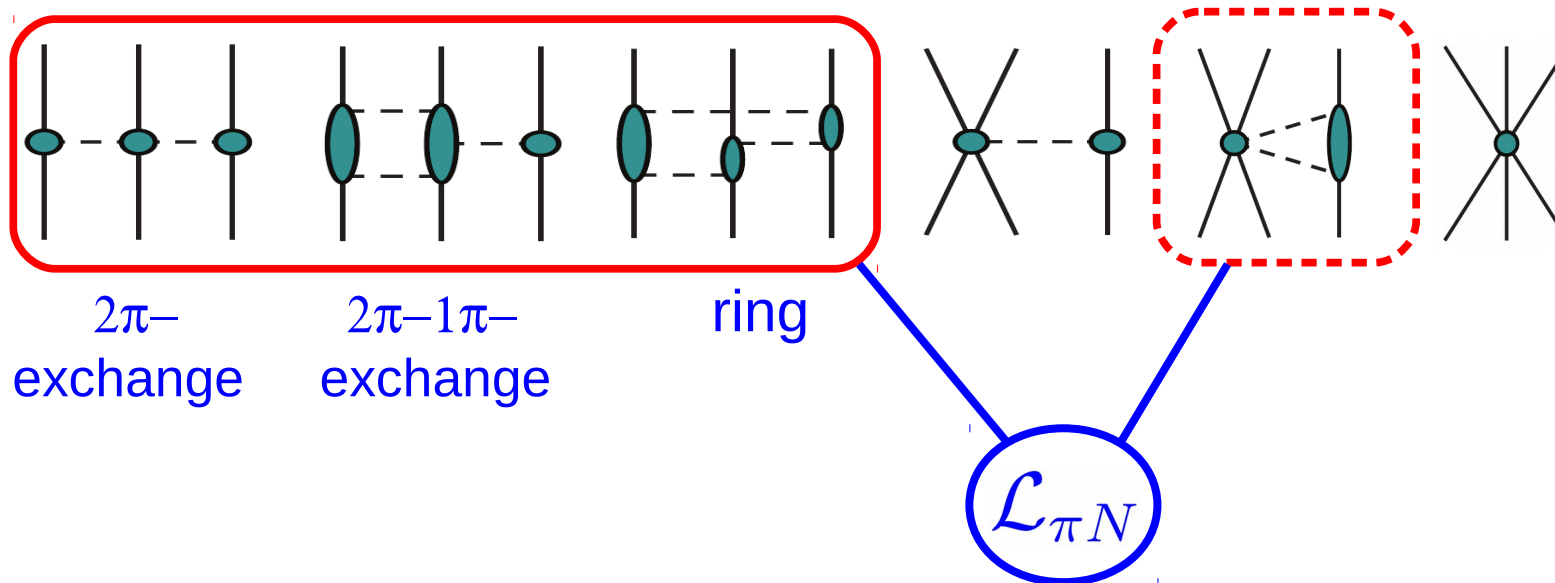
π N input for 3-Nucleon Forces

- Longest-range contributions
- Intermediate-range contributions
- Short-range contributions



πN input for 3-Nucleon Forces

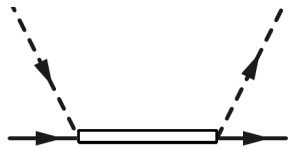
- Longest-range contributions
- Intermediate-range contributions
- Short-range contributions



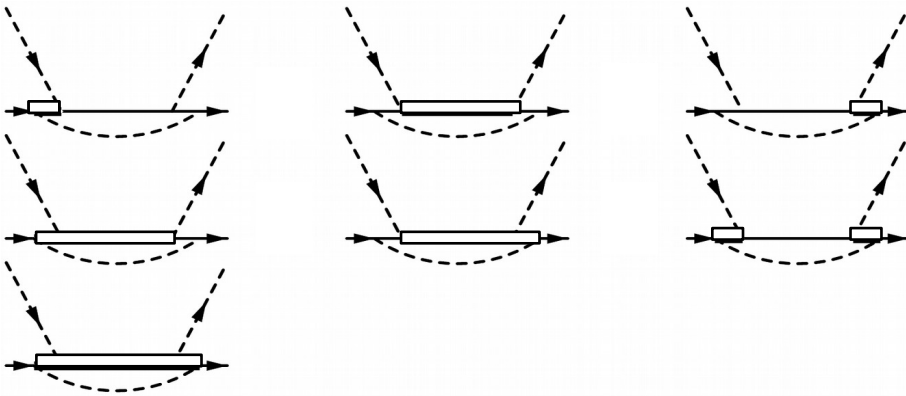
πN scattering up to ε^4

Siemens et al. In preparation

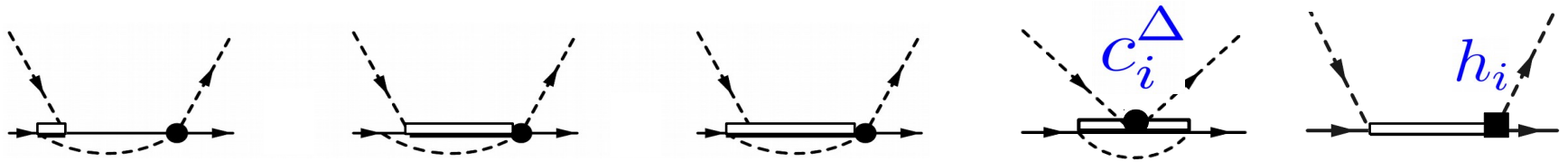
ε^1



ε^3



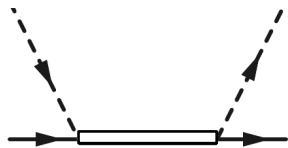
ε^4



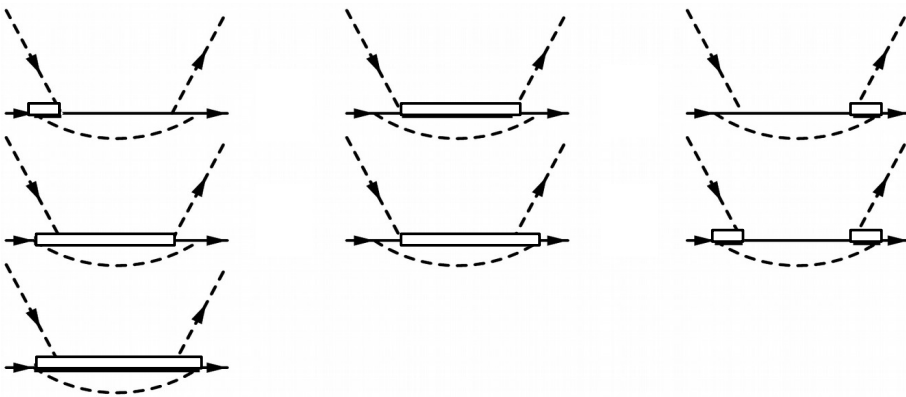
πN scattering up to ε^4

Siemens et al. In preparation

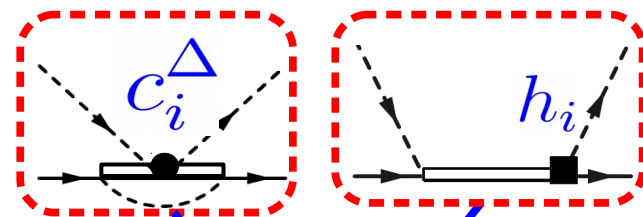
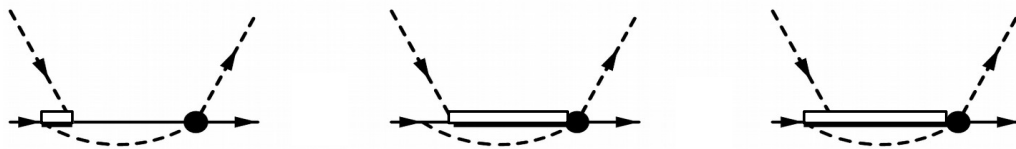
ε^1



ε^3



ε^4

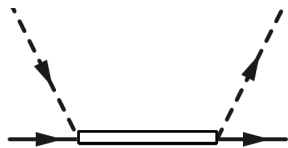


redundant, can be absorbed
by redefining other LEC's

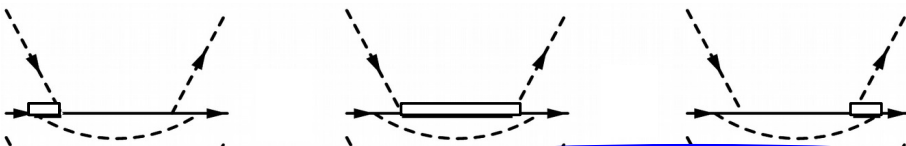
πN scattering up to ε^4

Siemens et al. In preparation

ε^1

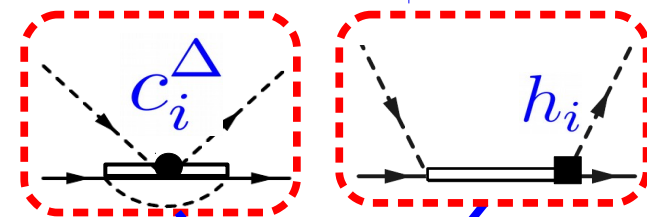
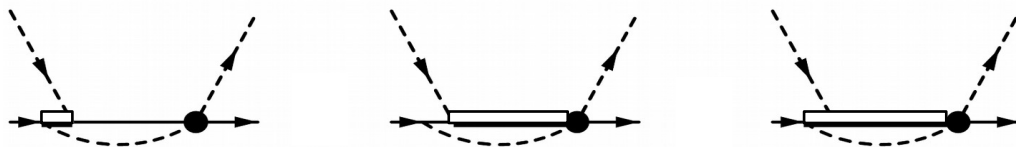


ε^3



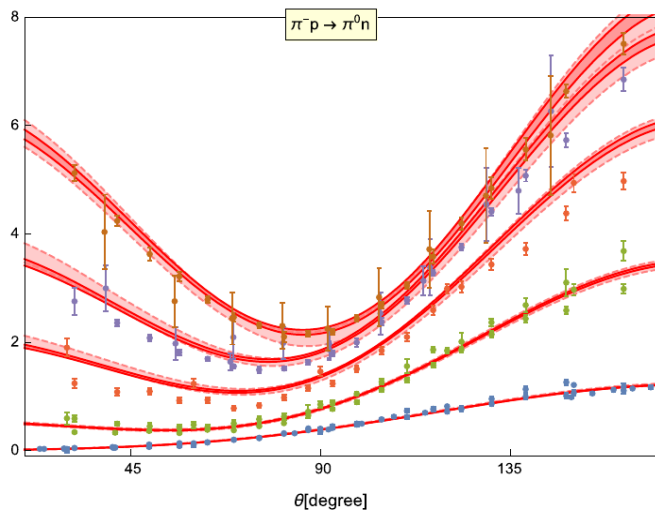
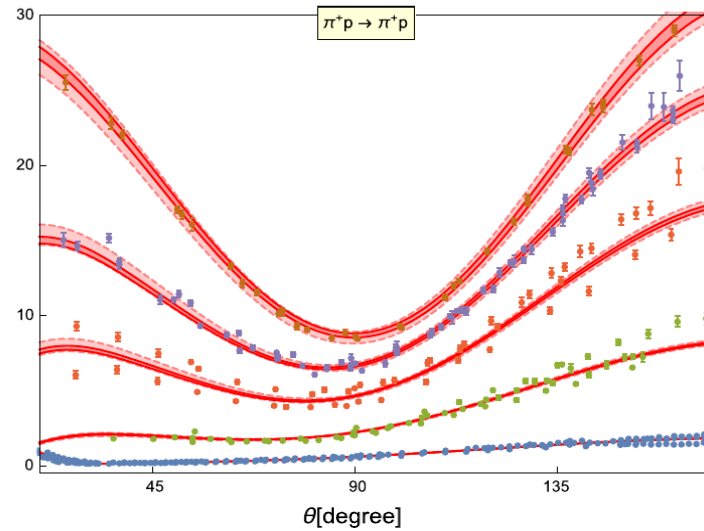
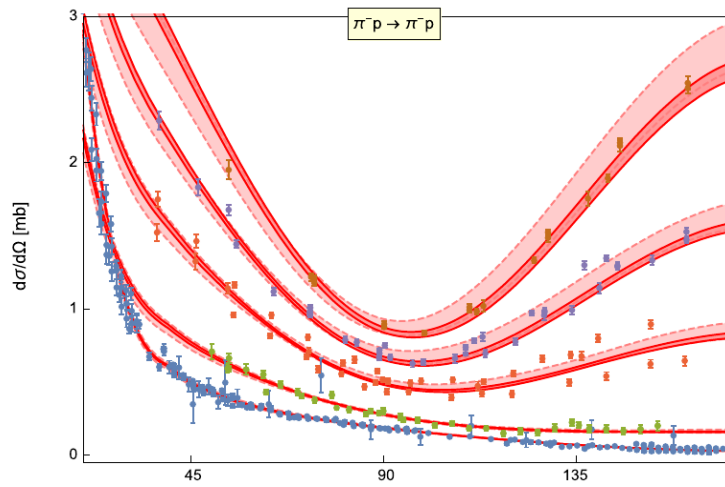
only two new parameters

ε^4



redundant, can be absorbed
by redefining other LEC's

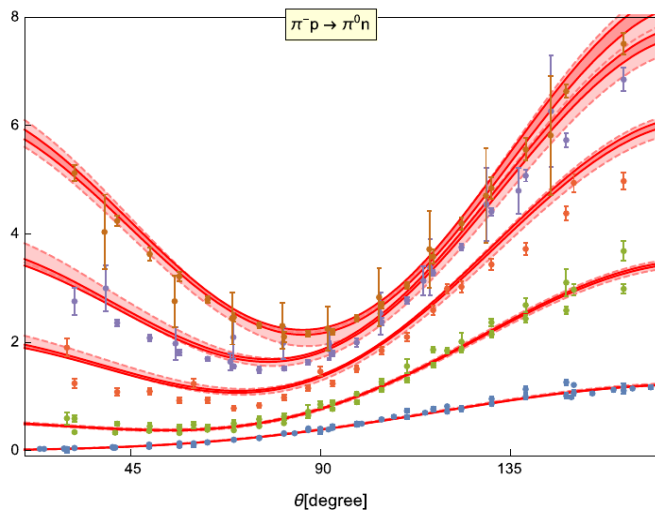
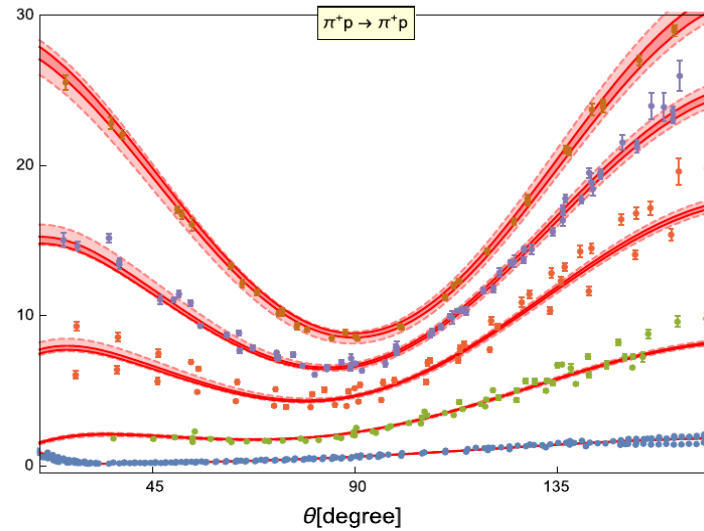
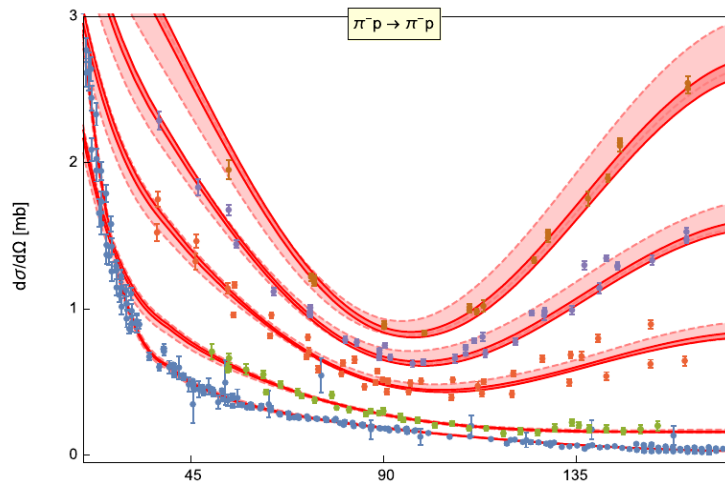
πN differential cross section



- $T_\pi = 167 \pm 5$ MeV
- $T_\pi = 140 \pm 5$ MeV
- $T_\pi = 121 \pm 5$ MeV
- $T_\pi = 90 \pm 5$ MeV
- $T_\pi = 42 \pm 5$ MeV

--- ϵ^3
— ϵ^4

πN differential cross section



- $T_\pi = 167 \pm 5$ MeV
- $T_\pi = 140 \pm 5$ MeV
- $T_\pi = 121 \pm 5$ MeV
- $T_\pi = 90 \pm 5$ MeV
- $T_\pi = 42 \pm 5$ MeV

--- ε^3

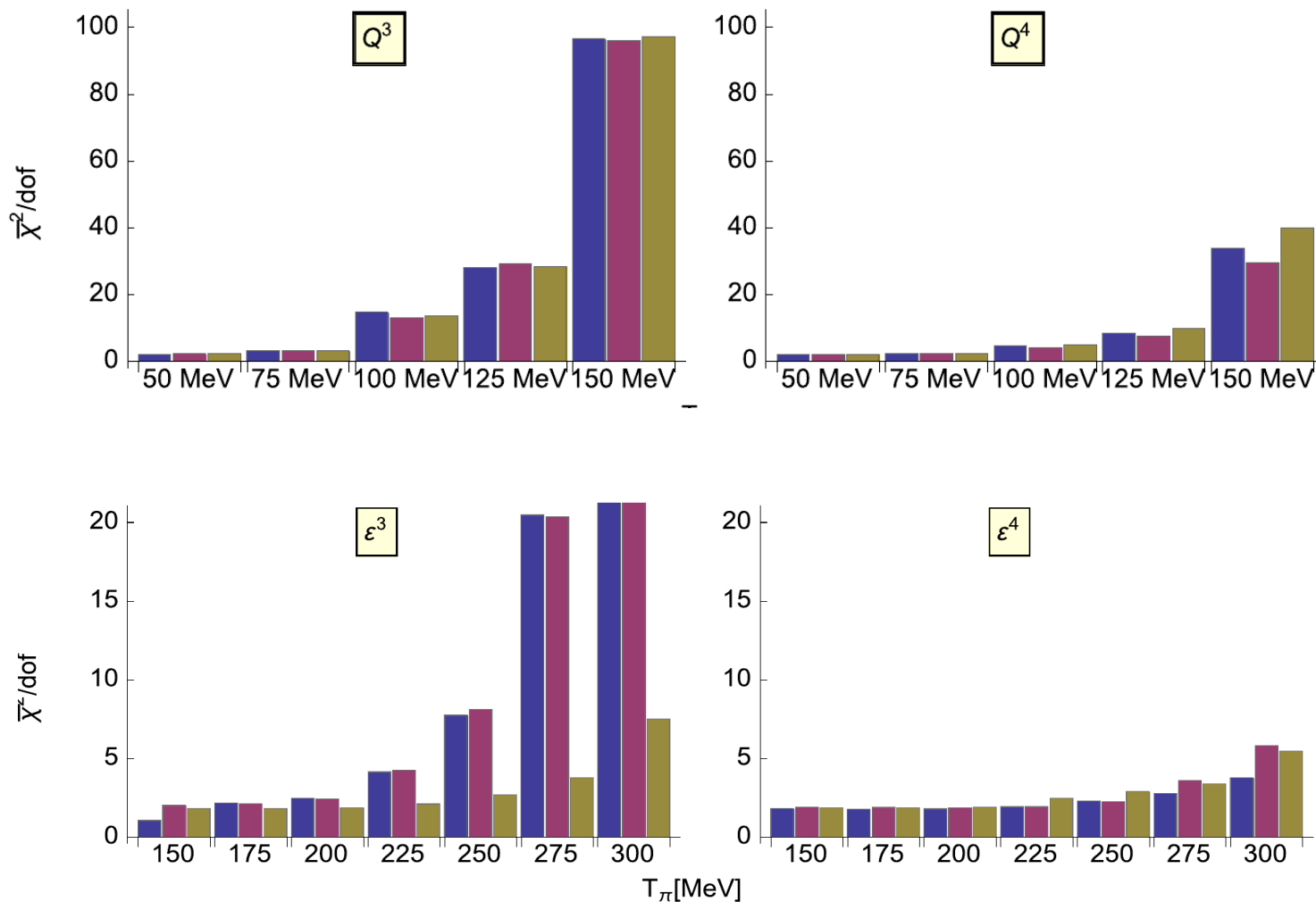
— ε^4

➔ Theoretical error-bands are narrower

Quality of the fit to πN data

in the Δ -less and Δ -full χ PT (without theoretical errors)

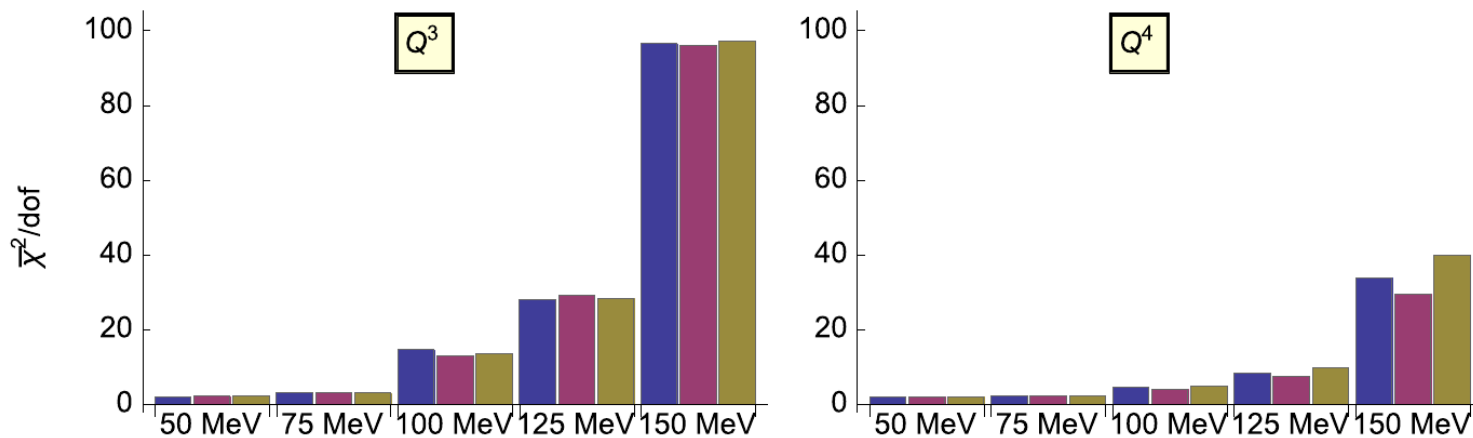
HB-NN HB- πN covariant



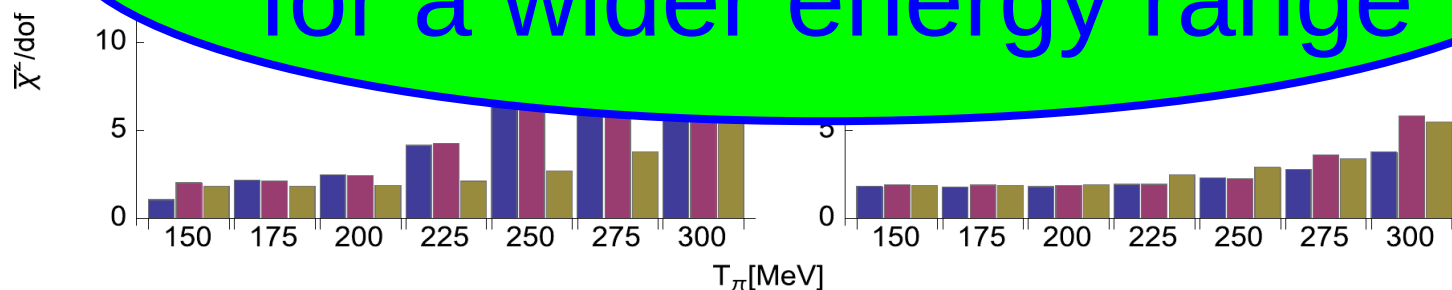
Quality of the fit to πN data

in the Δ -less and Δ -full χ PT (without theoretical errors)

HB-NN HB- πN covariant



sizable reduction of the χ^2
for a wider energy range



Summary

- Preliminary results for Δ -full chiral 2-nucleon and 3-nucleon forces at $N^3\text{LO}$ are presented
- 2-nucleon forces (peripheral phases): significant improvement compared to the Δ -less case
- 3-nucleon forces: indication of a better convergence; sizable Δ -contributions missing in Δ -less $N^4\text{LO}$ 3NF $\sim O(1/\Delta^2)$
- New results for πN scattering at order ε^4 : much better fit to data

Outlook

- Completing construction of Δ -full chiral 2N and 3N forces at $N^3\text{LO}$ and moving forward to even more precise nuclear forces.