Partial-Wave Analysis of NN scattering data at fifth order in chiral EFT

Patrick Reinert

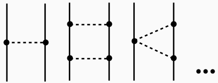
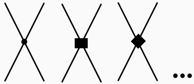
April 5, 2017

Institut für Theoretische Physik II, Ruhr-Universität Bochum

What are we doing?

- NN potential derived from ChPT up to N^4LO (Q^5)

- Potential consists of:

long-range pion exchanges	short-range nucleon-nucleon (NN) contact interactions
	

- have to fix LECs with empirical data
  - Step 1: πN LECs are extracted from πN scattering
  - **Step 2: NN LECs are fitted** $\leftarrow$

- up to now, NN LECs have been fitted to Nijmegen Partial Wave Analysis (NPWA) via phase shifts
  - NPWA phases include model-dependent assumptions
  - since 1993, the NN scattering database has been extended
  - new PWAs yield slightly different results

<u>Goal:</u> Increase accuracy by directly fitting to experimental data

Electromagnetic Corrections

Long-range EM interactions

use interactions employed by Nijmegen group (Phys. Rev. C 48, 792) :

- *np* amplitudes in Born Approximation
  - magnetic moment (MM) interaction
- *pp* amplitudes in Coulomb Distorted Wave Born Approximation
  - "relativistic" Coulomb interaction
    $V_{C1}(r) = \frac{\alpha'}{r}, \quad \alpha' = \alpha\left(1 + \frac{2q^2}{m_p^2}\right)\left(1 + \frac{q^2}{m_p^2}\right)^{-\frac{1}{2}}$
  - additional relativistic and recoil corrections $V_{C2}(r) \approx -\frac{\alpha\alpha'}{m_p^2}\frac{1}{r^2}$
  - magnetic moment (MM) interaction
  - vacuum polarization interaction

Short-range EM interactions

- are included implicitly in NN contact LECs

Data

- SAID database contains scattering data from 50ies to present
- In total 5009 np and 3178 pp individual measurements
- Grouped in measurement data sets (857 np, 360 pp)
- Each data set gives:
  - Observable values O_i^{exp}
  - Statistical errors δO_i^{exp}
  - Normalization error δ_{sys}

Comparison between theory and experiment via standard χ^2 approach:

$$\chi_j^2 = \sum_{i=1}^{n_j} \left(\frac{O_i^{exp} - Z O_i^{theo}}{\delta O_i} \right)^2 + \left(\frac{Z-1}{\delta_{sys}} \right)^2$$

- Normalization Z is estimated to minimize χ_j^2

Data Selection

Problem: Not all data are mutually compatible

- np differential cross sections notoriously difficult to measure
- Sometimes not accounted for all systematic errors

Result:

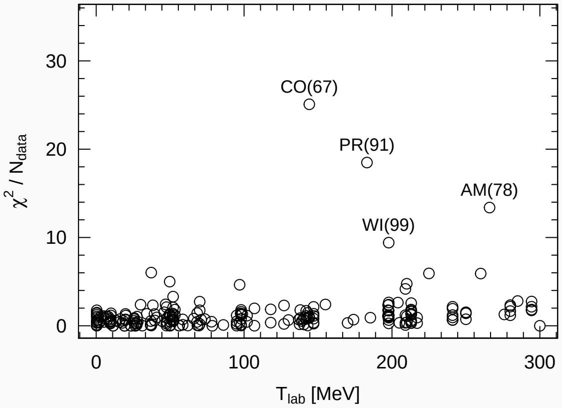
- leads to bad fit, high χ^2
- data not normal-distributed $\rightarrow$ applicability of χ^2 estimation questionable

Solution:

- use 2013 Granada database (Phys. Rev. C 88.064002)
- uses "3σ-criterion" to reject non-normal-distributed data
- self-consistency was checked by Granada group
- we use 2727 np- and 2158 pp- measurements up to $T_{lab} = 300$ MeV

Outliers

χ^2/N_{Data} per dataset for *pp* observables at N^{4}LO:



Unfortunately, χ^2 is very sensitive to outliers:

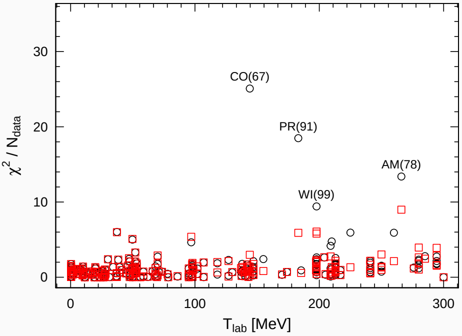
T_{lab} [MeV]	with outliers	without outliers
0-100	0.86	0.86
100-200	1.93	1.24
200-300	1.73	1.71
0-300	1.44	1.23

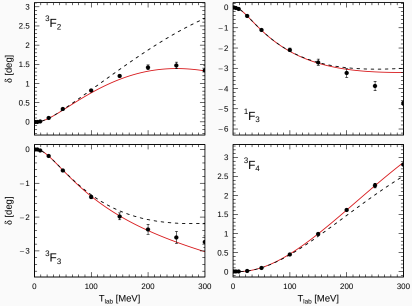
- deviation lies within estimated theoretical uncertainty
- but *parametrized* phase shifts are actually quite good

$\Rightarrow$ check (at N^{4}LO) unparametrized partial waves (F-Waves and higher)

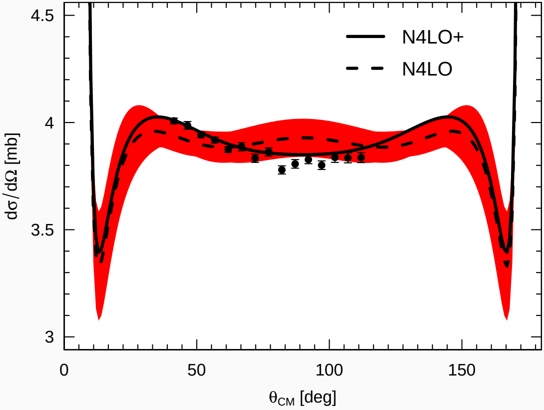
Outliers

We thus add the N^5LO NN contact interactions in F-waves and $^3D_3 - {}^3G_3$ mixing angle to the N^4LO potential (N^4LO^+).





Outliers - Example CO(67)



About this dataset:

- differential cross section $d\sigma/d\Omega$
- $T_{lab} = 144.1$ MeV
- experimental errors $\sim 0.5\%$

$\Rightarrow$ Need accurate F-Waves (in particular $^3F_2$) at energies ~ 150 MeV to describe such observables well.

Theoretical Error Estimation

- ChPT is low-momentum expansion in $Q = \max(m_\pi/\Lambda, q/\Lambda)$ and thus becomes less accurate for higher energies
- Want to account for that in the fit, i.e. increase energy range without worsening description at low energies

Theoretical error estimation:

- estimate theoretical error of observable X:

$$\delta X^{(0)} = Q^2|X^{(0)}|$$

$$\delta X^{(\nu)} = \max_{2\leq i\leq \nu}\left(Q^{\nu+1}|X^{(0)}|, Q^{\nu+1-i}|\Delta X^{(i)}|\right)$$

with $\Delta X^{(2)} = X^{(2)} - X^{(0)}, \qquad \Delta X^{(i)} = X^{(i)} - X^{(i-1)}\,, \quad i \geq 3$

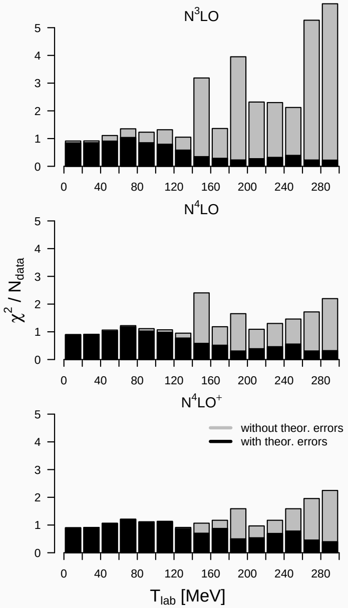
- correction from higher orders (if available):

$$\delta \tilde{X}^{(\nu)} = \max_{\nu\leq i\leq j}\left(\delta X^{(\nu)}, |X^{(i)} - X^{(j)}|\right)$$

Theoretical Error Estimation

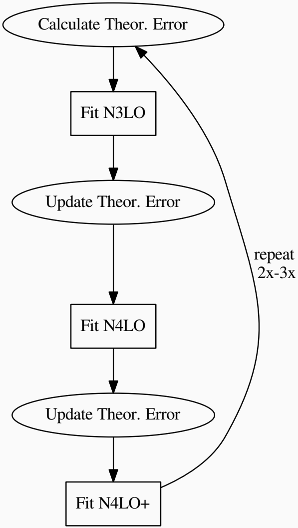
- use breakdown scale
  - $\Lambda = 600$ MeV for R = 0.8 - 1.0 fm
  - $\Lambda = 500$ MeV for R = 1.1 fm
  - $\Lambda = 400$ MeV for R = 1.2 fm
- ideally have $\chi^2/N_{Data} \sim 1$ over whole energy range
- may try larger breakdown scales
- total observable error in χ^2-term:

$$\delta O_i^2 = (\delta O_i^{exp})^2 + (\delta O_i^{theo})^2$$



Fitting Procedure

- non-linear fit due to non-perturbativeness
- Fit N^3LO - N^4LO^+ NN contact LECs to scattering data
  - up to 300 MeV for R=0.8, 0.9, 1.0 fm
  - up to 250 MeV for R=1.1 fm
  - up to 200 MeV for R=1.2 fm
- use fit to Nijmegen PWA as starting values for LECs
- make use of derivative-based optimization algorithms
  - in the future, use derivatives for statistical errors, correlations and error propagation



Fitting Procedure

Impose additional constraints on ${}^3S_1 - {}^3D_1$ coupled channel:

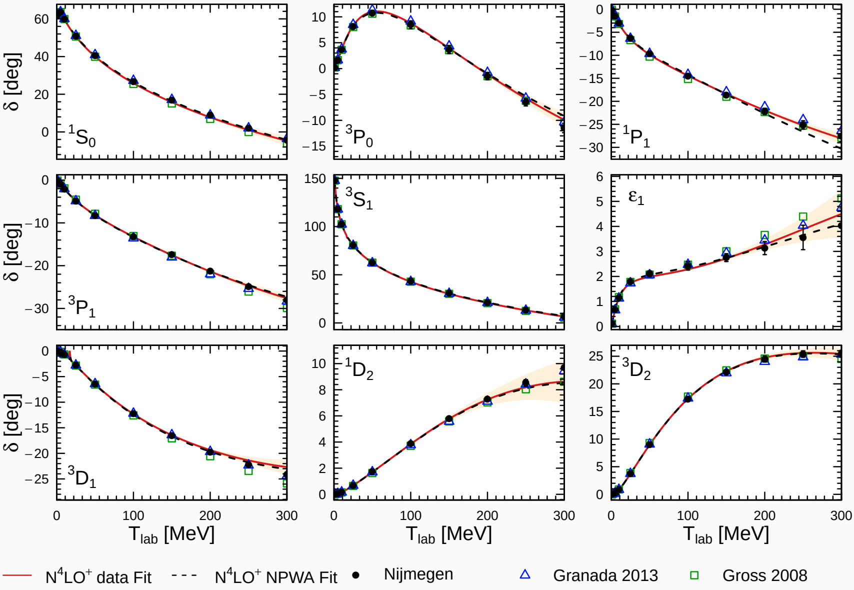
$$\chi^2 = \underbrace{\sum_j \chi_j^2}_{\text{scattering data}} + \underbrace{\left(\frac{E_d^{exp} - E_d^{theo}}{\Delta E_d}\right)^2}_{\substack{\text{Deuteron binding}\\ \text{energy penalty}}} + \underbrace{\left(\frac{P_d^{\text{``}exp\text{''}} - P_d^{theo}}{\Delta P_d}\right)^2}_{\substack{\text{D-state probability}\\ \text{penalty}}} + \underbrace{\left(\frac{\tilde{C}_{1S0}^{np} - \tilde{C}_{1S3}}{\Delta \tilde{C}_{1S3}}\right)^2}_{\substack{\text{Wigner SU(4)}\\ \text{penalty}}},$$

$E_d = -2.224575$ MeV , $P_d = 5 \pm 1\%$, $\Delta \tilde{C}_{1S3} = \Delta \tilde{C}_{1S3}/4$

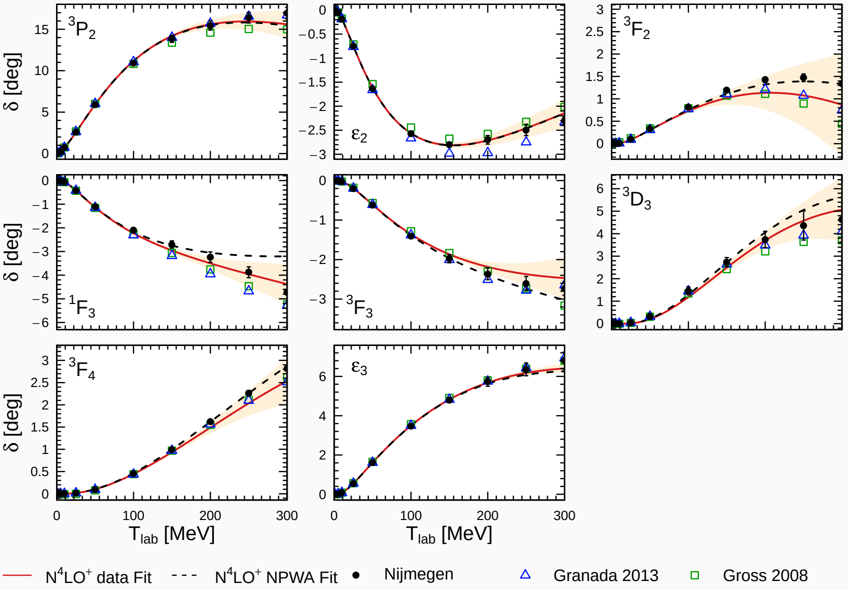
Under investigation:

- due to high precision of E_d: use specialized algorithms for constrained optimization

Phaseshifts (preliminary)



Phaseshifts (preliminary)



χ^2 before/after Fit

np scattering data at R = 0.9 fm:

T_{lab} [MeV]	N^3LO		N^4LO		N^4LO^+	
	before	**after**	before	**after**	before	**after**
0-100	1.09	**1.07**	1.08	**1.07**	1.08	**1.08**
0-200	1.19	**1.11**	1.08	**1.06**	1.08	**1.08**
0-300	1.55	**1.23**	1.17	**1.14**	1.15	**1.11**

pp scattering data at R = 0.9 fm:

T_{lab} [MeV]	N^3LO		N^4LO		N^4LO^+	
	before	**after**	before	**after**	before	**after**
0-100	0.86	**0.82**	0.86	**0.83**	0.86	**0.82**
0-200	1.98	**1.88**	1.32	**1.31**	1.06	**0.93**
0-300	2.80	**2.64**	1.44	**1.38**	1.33	**1.04**

Comparison

How does the newly fitted potential compare to other NN potentials?

np scattering data:

T_{lab} [MeV]	Idaho	CDBONN	NijmI	NijmII	Reid93	N^4LO	N^4LO^+
0-100	1.17	1.08	1.07	1.08	1.09	1.07	1.08
0-200	1.16	1.07	1.06	1.06	1.07	1.06	1.08
0-300	1.23	1.08	1.09	1.10	1.10	1.14	1.11

pp scattering data:

T_{lab} [MeV]	Idaho	CDBONN	NijmI	NijmII	Reid93	N^4LO	N^4LO^+
0-100	0.97	0.84	0.83	0.83	0.81	0.83	0.82
0-200	1.28	0.95	0.96	0.97	0.95	1.31	0.93
0-300	1.36	0.99	1.02	1.03	1.02	1.38	1.04

$\Rightarrow$ N^4LO^+ is on par with high quality phenomenological potentials for $T_{lab} = 0 - 300$ MeV.

Summary

- we have fitted the chiral potential to experimental scattering data
- the parametrization of F-Waves can be important for high accuracy *pp* observables
- at N^4LO^+ the description of scattering data would be on par with phenomenological potentials

For the future...

- calculate statistical properties for LECs (statistical errors, correlations, ...)
- include isospin-breaking effects beyond those of the NPWA

Thank You! .