Reactions in three- and four-body systems

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supported by Alexander von Humboldt-Stiftung

in collaboration with A. C. Fonseca

Outline

- 4-particle scattering equations
- application to 4N reactions

$$\left.\begin{array}{l} p + {}^3\mathrm{H} \\ n + {}^3\mathrm{He} \\ d + d \end{array}\right\} \rightarrow \left\{\begin{array}{l} p + n + d \\ p + {}^3\mathrm{H} \\ n + {}^3\mathrm{He} \\ d + d \\ 2p + 2n \end{array}\right.$$

- 3-body nuclear reactions including core excitation

$$\left.\begin{array}{l} p + (nA) \\ d + A \end{array}\right\} \rightarrow \left\{\begin{array}{l} n + (pA) \\ p + (nA) \\ d + A \\ p + n + A \end{array}\right.$$

Scattering: wave function vs transition operator

- Schrödinger equation

$$(H_0 + v)|\psi\rangle = E|\psi\rangle$$

+ impose asymptotic boundary conditions explicitly

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wave function $|\psi\rangle = |\mathbf{k}\rangle + G_0 v|\psi\rangle$

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wave function $$|\psi\rangle = |\mathbf{k}\rangle + G_0 v|\psi\rangle$$

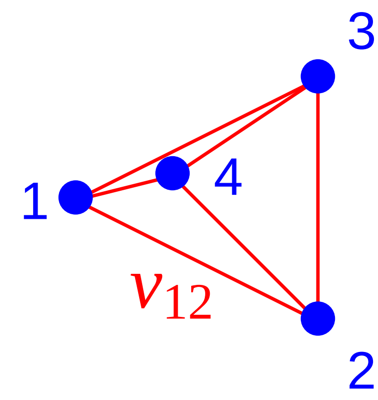
$$G_0 = (E + i0 - H_0)^{-1}$$

transition matrix $$T|\mathbf{k}\rangle = v|\psi\rangle$$

$$T = v + vG_0T$$

$$|\psi\rangle = |\mathbf{k}\rangle + G_0T|\mathbf{k}\rangle$$

4N scattering



Hamiltonian $$H_0 + \sum_{i>j} v_{ij}$$

- Wave function:
  Schrödinger equation (HH + Kohn VP, r-space)
  [M. Viviani, A. Kievsky, L. E. Marcucci, S. Rosati, L. Girlanda]
- Wave function components:
  Faddeev-Yakubovsky equations (r-space)
  [R. Lazauskas, J. Carbonell]
- Transition operators:
  Alt-Grassberger-Sandhas equations (p-space)
  [AD, A. C. Fonseca]

4-body scattering: AGS equations

4-body transition operators

$$t_i = v_i + v_i G_0 t_i$$

$$G_0 = (E + i0 - H_0)^{-1}$$

$$U_\gamma^{jk} = G_0^{-1} \bar{\delta}_{jk} + \sum_i \bar{\delta}_{ji} t_i G_0 U_\gamma^{ik}$$

$$\mathcal{U}_{\beta\alpha}^{ji} = (G_0 t_i G_0)^{-1} \bar{\delta}_{\beta\alpha} \delta_{ji} + \sum_{\gamma k} \bar{\delta}_{\beta\gamma} U_\gamma^{jk} G_0 t_k G_0 \, \mathcal{U}_{\gamma\alpha}^{ki}$$

i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

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i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
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wave function

$$|\Psi_\alpha\rangle = |\Phi_\alpha\rangle + \sum_{\gamma jki} G_0 t_j G_0 U_\gamma^{jk} G_0 t_k G_0 \mathcal{U}_{\gamma\alpha}^{ki} |\phi_\alpha^i\rangle$$

$$|\Phi_\alpha\rangle = \sum_i |\phi_\alpha^i\rangle, \qquad |\phi_\alpha^i\rangle = G_0 \sum_j \bar{\delta}_{ij} t_j |\phi_\alpha^j\rangle$$

4-body scattering amplitudes

two-cluster reactions:

$$\langle \Phi_\beta | T_{\beta\alpha} | \Phi_\alpha \rangle = \sum_{ji} \langle \phi_\beta^j | \mathcal{U}_{\beta\alpha}^{ji} | \phi_\alpha^i \rangle$$

three-cluster breakup:

$$\langle \Phi^j | T_\alpha^j | \Phi_\alpha \rangle = \sum_{\beta k i} \langle \Phi^j | U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

four-cluster breakup:

$$\langle \Phi_0 | T_{0\alpha} | \Phi_\alpha \rangle = \sum_{\beta jki} \langle \Phi_0 | t_j \, G_0 \, U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

[PRC 75, 014005; PRA 85, 012708]

Symmetrized AGS equations

$$t = v + vG_0 t$$

$$G_0 = (E + i\varepsilon - H_0)^{-1}$$

$$U_j = P_j G_0^{-1} + P_j t G_0 U_j$$

$$3+1: \quad P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$2+2: \quad P_2 = P_{13} P_{24}$$

$$\mathcal{U}_{11} = (G_0 t G_0)^{-1} \zeta P_{34} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{11} + U_2 G_0 t G_0 \, \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 t G_0)^{-1} (1 + \zeta P_{34}) + (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 t G_0)^{-1} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{12} + U_2 G_0 t G_0 \, \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{12}$$

$\zeta = -1 \ (+1)$ for fermions (bosons)

basis states partially symmetrized

Scattering amplitudes: $E + i\varepsilon \to E + i0$

2-cluster reactions:

$$T_{fi} = s_{fi} \langle \phi_f | \mathcal{U}_{fi} | \phi_i \rangle$$

$$|\phi_j\rangle = G_0 t P_j |\phi_j\rangle$$

$$|\Phi_j\rangle = (1 + P_j)|\phi_j\rangle$$

3-cluster breakup/recombination:

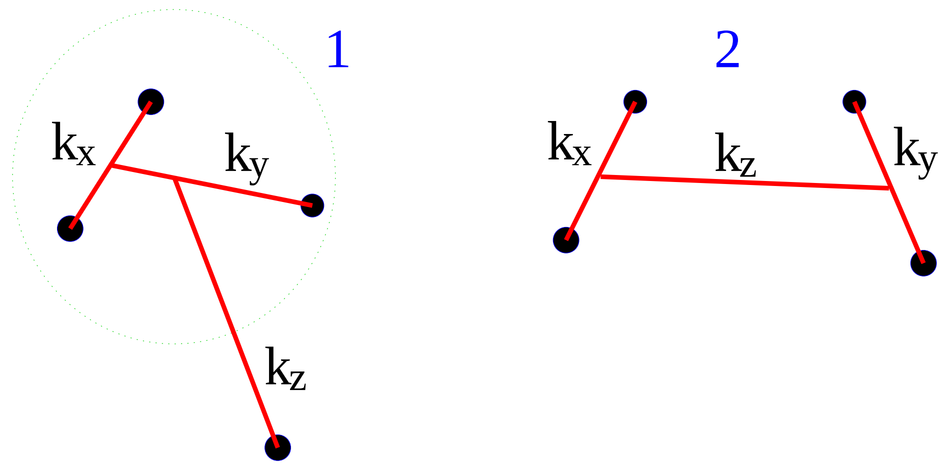
$$T_{3i} = s_{3i} \langle \phi_3 | [(1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{1i} + U_2 G_0 t G_0 \, \mathcal{U}_{2i}] | \phi_i \rangle$$

4-cluster breakup/recombination:

$$T_{4i} = s_{4i} \{ \langle \phi_4 | [1 + (1 + P_1) \zeta P_{34}] (1 + P_1) t G_0 U_1 G_0 t G_0 \, \mathcal{U}_{1i} | \phi_i \rangle$$
$$+ \langle \phi_4 | (1 + P_1)(1 + P_2) t G_0 U_2 G_0 t G_0 \, \mathcal{U}_{2i} | \phi_i \rangle \}$$

Solution of 4N AGS equations

$$\mathcal{U}_{11}|\phi_1\rangle = -G_0^{-1}P_{34}P_1|\phi_1\rangle - P_{34}U_1G_0\,t\,G_0\,\mathcal{U}_{11}|\phi_1\rangle + U_2G_0\,t\,G_0\,\mathcal{U}_{21}|\phi_1\rangle$$

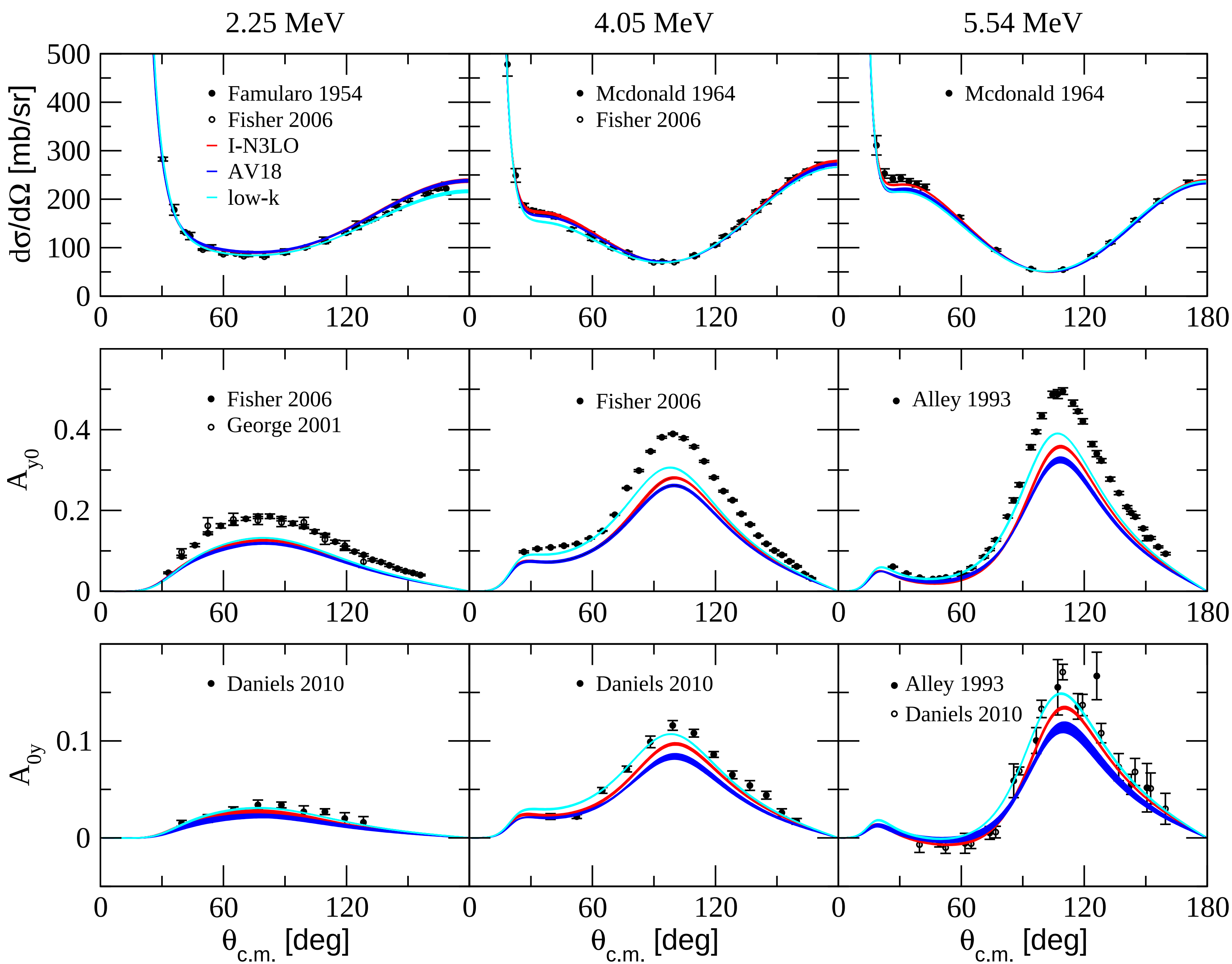


- momentum-space partial-wave basis
  $|k_x k_y k_z [l_z(\{l_y[(l_x S_x) j_x s_y] S_y\} J_y s_z) S_z] JM, [(T_x t_y) T_y t_z] T M_T\rangle_1$
  $|k_x k_y k_z [l_z\{(l_x S_x) j_x [l_y (s_y s_z) S_y] j_y\} S_z] JM, [T_x (t_y t_z) T_z] T M_T\rangle_2$
- large system (up to 30000) of coupled 3-variable integral equations with integrable singularities
- Coulomb interaction: screening and renormalization [PRC 75, 014005, PRL 98, 162502]

Realistic NN potentials

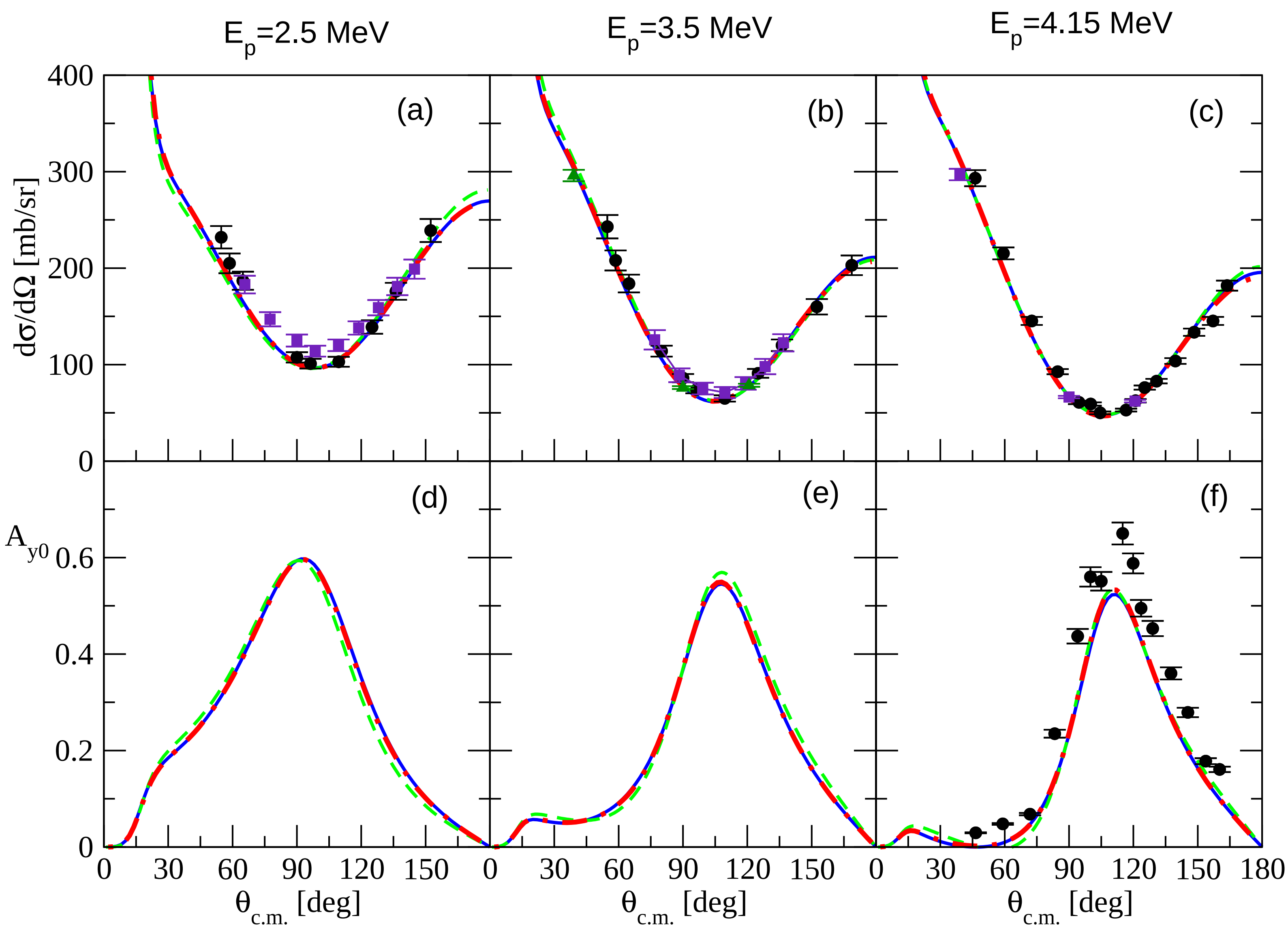
2NF	3NF	4NF	$B_{^3\mathrm{H}}$ (MeV)
INOY04			8.49
CD Bonn + Δ	1Δ	1Δ	8.28
CD Bonn			8.00
N3LO			7.85
AV18			7.62

Benchmark: p-^{3}He scattering



AGS/HH/FY (Lisbon/Pisa/Strasbourg, PRC 84, 054010)

Benchmark: p-^{3}H scattering (N3LO)



AGS/HH/FY (PRC 95, 034003)

Singularities of 4N AGS equations

^{3}H, ^{3}He, or d+d bound state poles

$$G_0 U_j G_0 \rightarrow \frac{P_j |\phi_j\rangle s_{jj} \langle\phi_j| P_j}{E + i\varepsilon - E_j^b - k_z^2/2\mu_j}$$

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deuteron bound state poles

$$t \rightarrow \frac{v|\phi_d\rangle\langle\phi_d|v}{E + i\varepsilon - e_d - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

Singularities of 4N AGS equations

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$$t \to \frac{v|\phi_d\rangle\langle\phi_d|v}{E + i\varepsilon - e_d - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

free resolvent

$$G_0 \to \frac{1}{E + i\varepsilon - k_x^2/2\mu_j^x - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

Treatment of singularities above breakup

Complex-energy method:

1. solve for $\mathcal{U}_{fi}(E + i\varepsilon)$ with finite $\varepsilon = \varepsilon_1, \ldots, \varepsilon_n$
2. extrapolate to $\varepsilon \to 0$ for physical amplitudes $\mathcal{U}_{fi}(E + i0)$

[L. Schlessinger, PR 167, 1411 (1968)]

[H. Kamada *et al*, Prog. Theor. Phys. 109, 869L (2003)]

Integration with special weights

accuracy & efficiency of the complex-energy method is greatly improved by a special integration

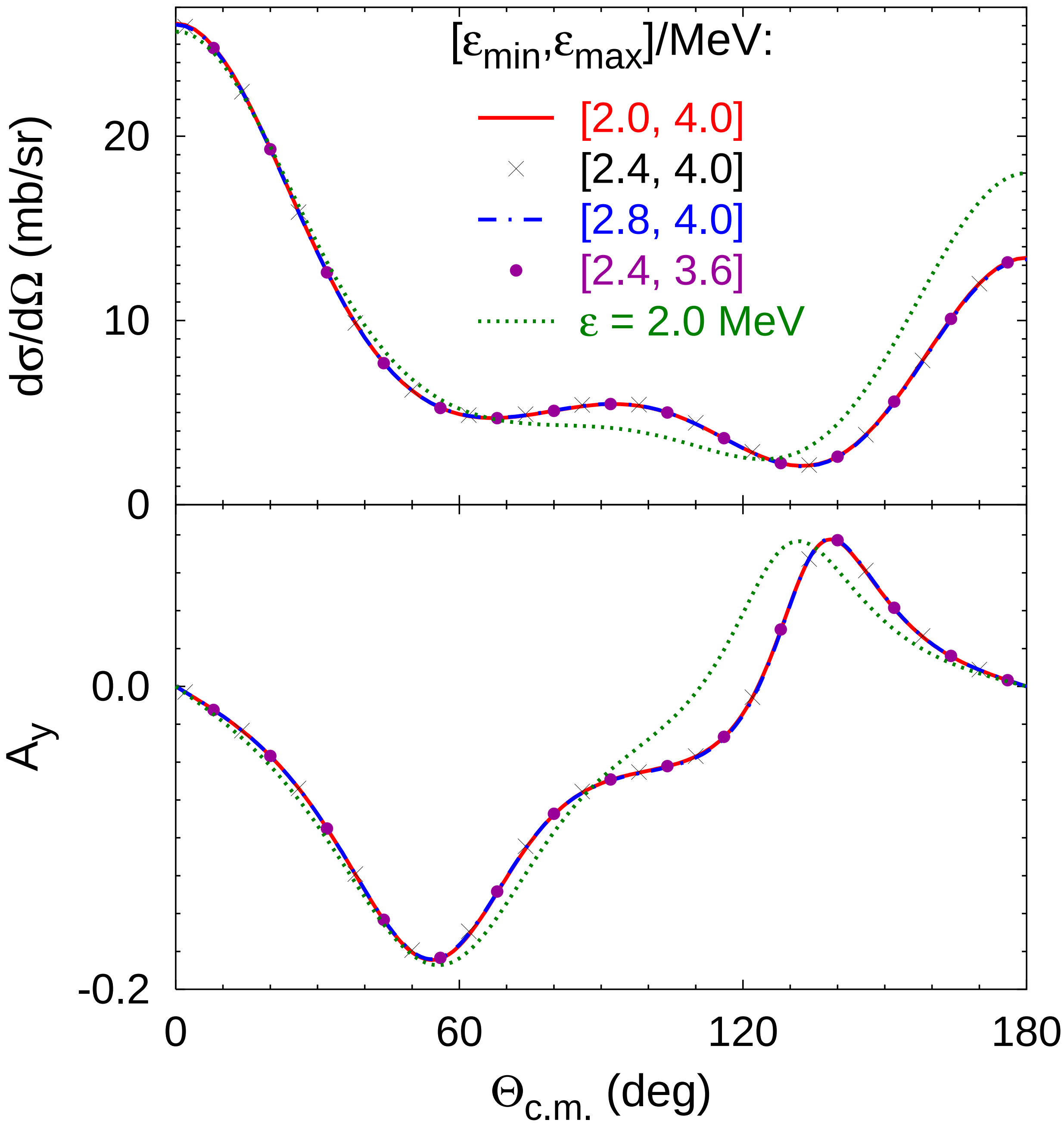
$$\int_a^b \frac{f(x)}{x_0^n + iy_0 - x^n} dx \approx \sum_{j=1}^{N} f(x_j) w_j(n, x_0, y_0, a, b)$$

where the quasi-singular factor is absorbed into special weights

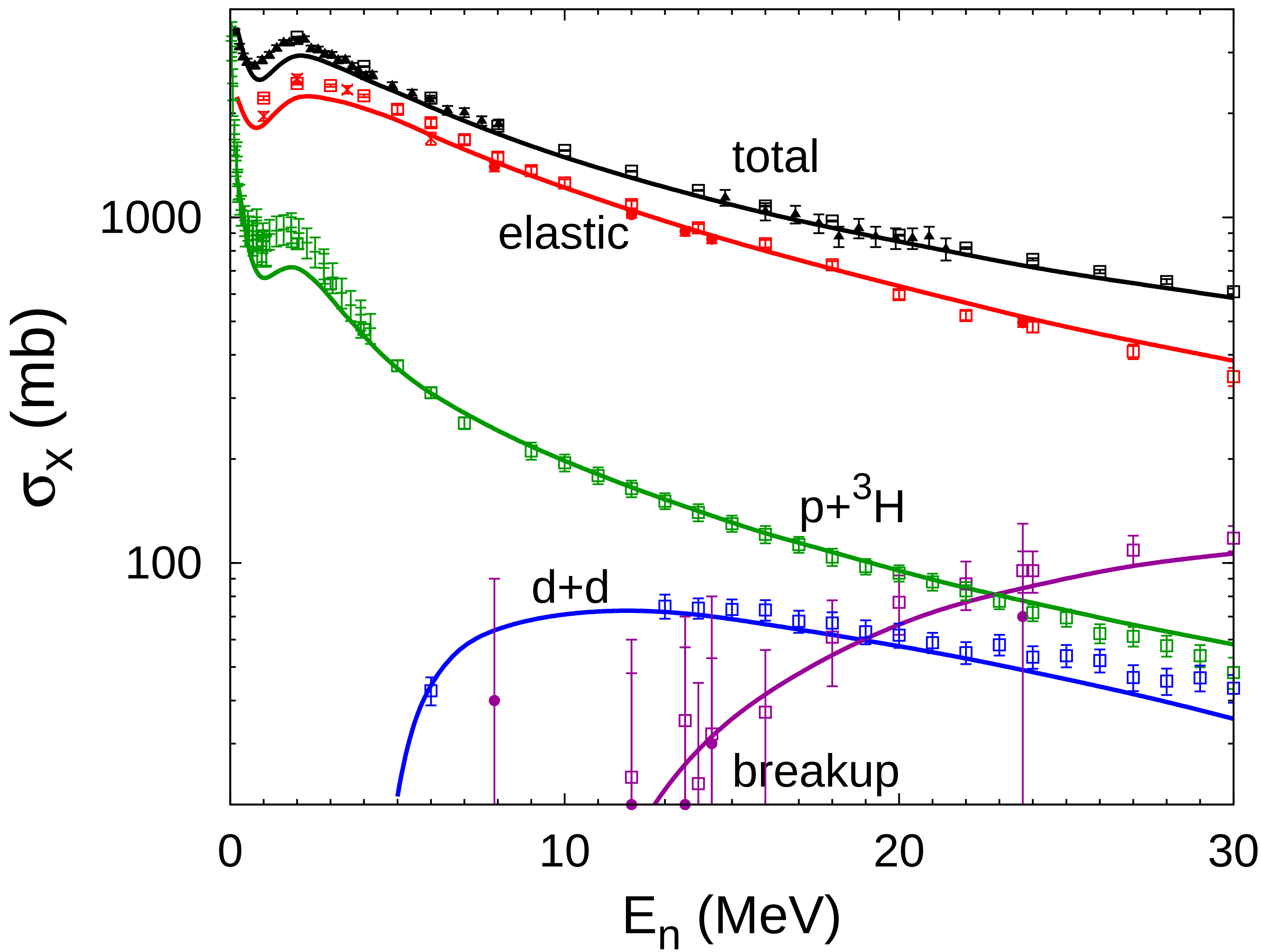
$$w_j(n, x_0, y_0, a, b) = \int_a^b \frac{S_j(x)}{x_0^n + iy_0 - x^n} dx$$

that may be calculated using spline functions $\{S_j(x)\}$ for standard Gaussian grid $\{x_j\}$ [PRC 86, 011001]

Extrapolation $\varepsilon \to 0$: $^3\mathrm{H}(p,n)^3\mathrm{He}$ **at 24 MeV**

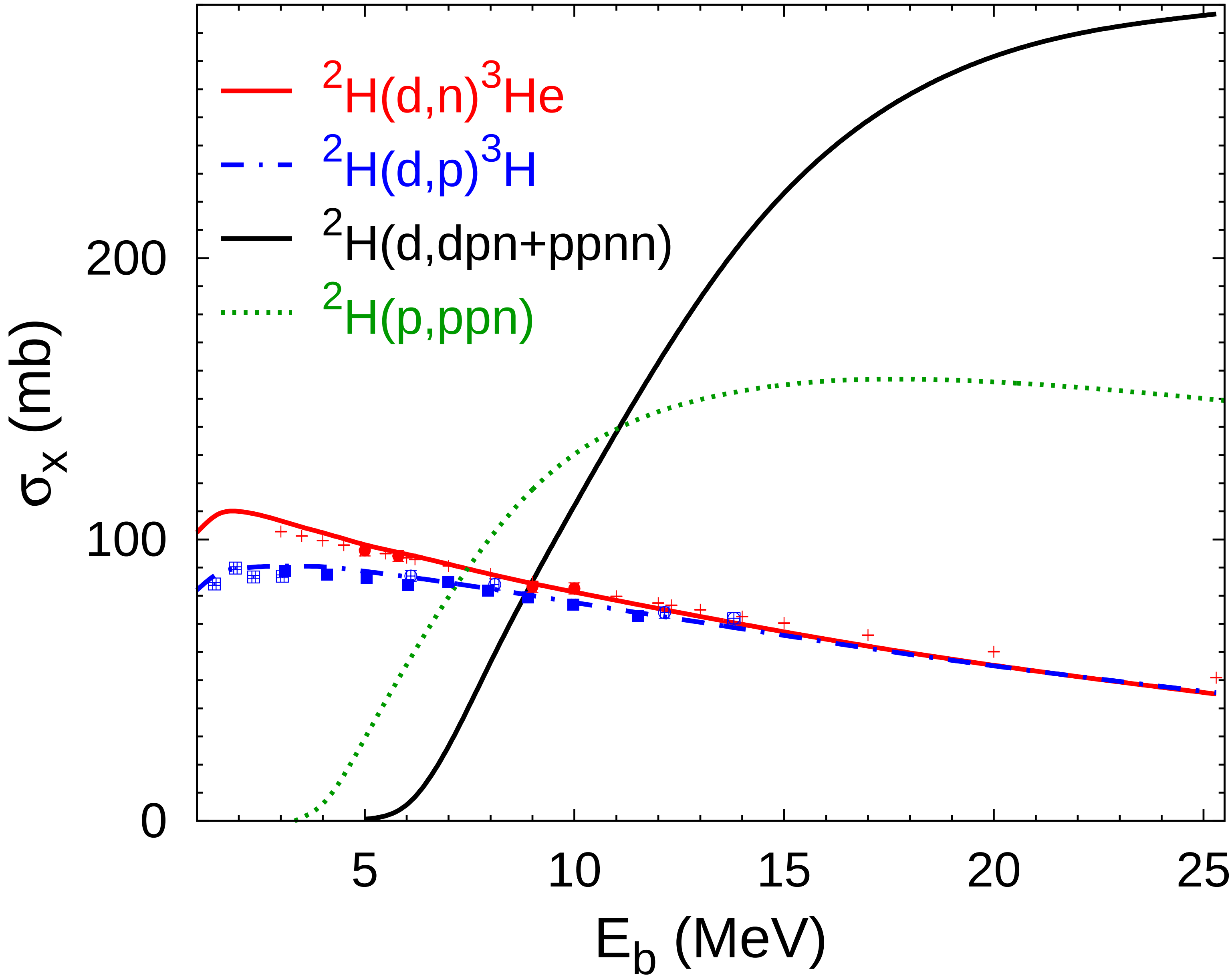


n+^{3}He total and partial cross sections



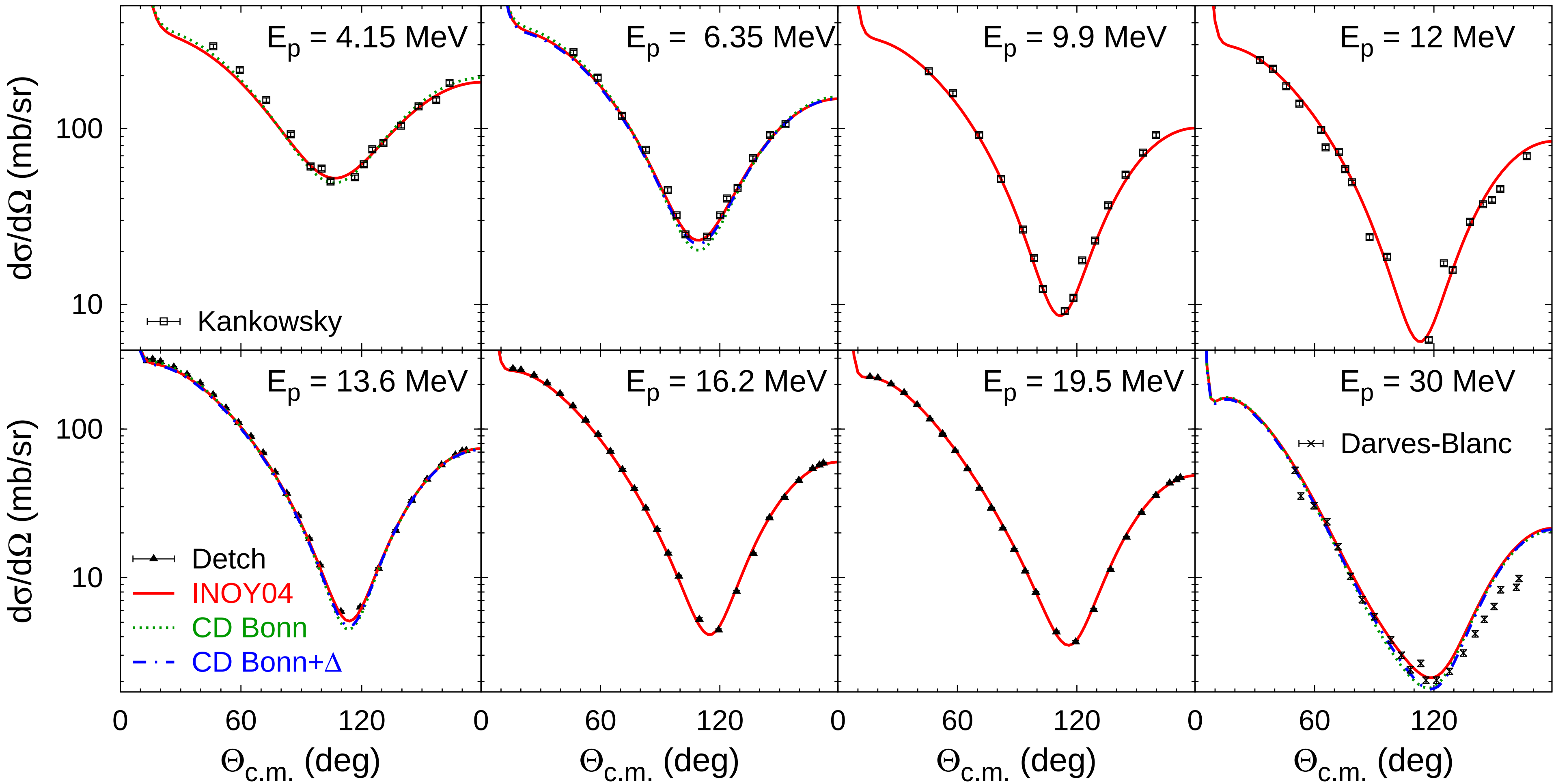
[PRL 113, 102502; PRC 90, 044002]

d+d transfer and breakup cross sections



[PLB 742, 285; PRC 92, 024001; PRC 95, 024003]

p+^{3}H elastic scattering



[PRC 91, 034001]

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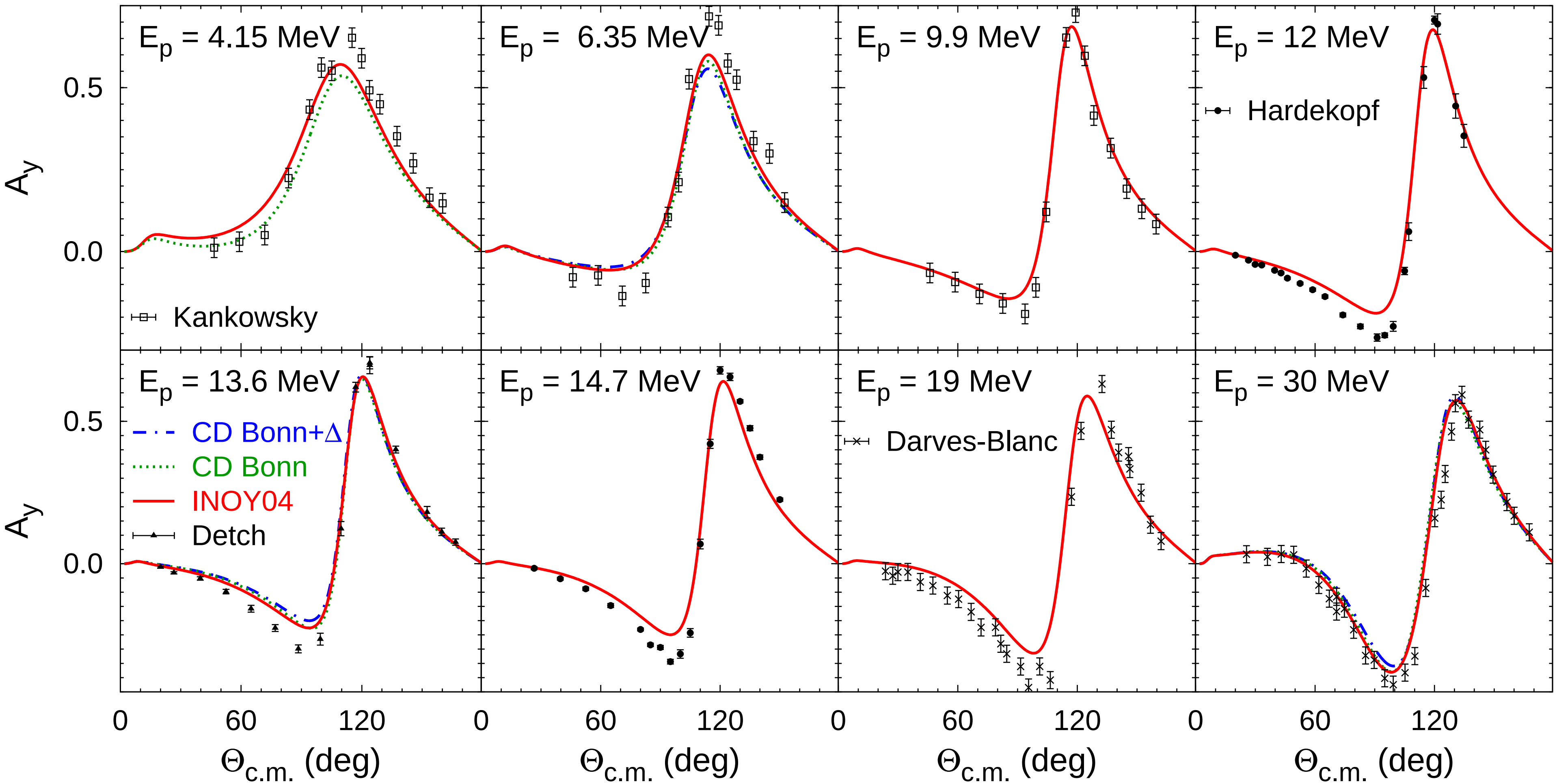
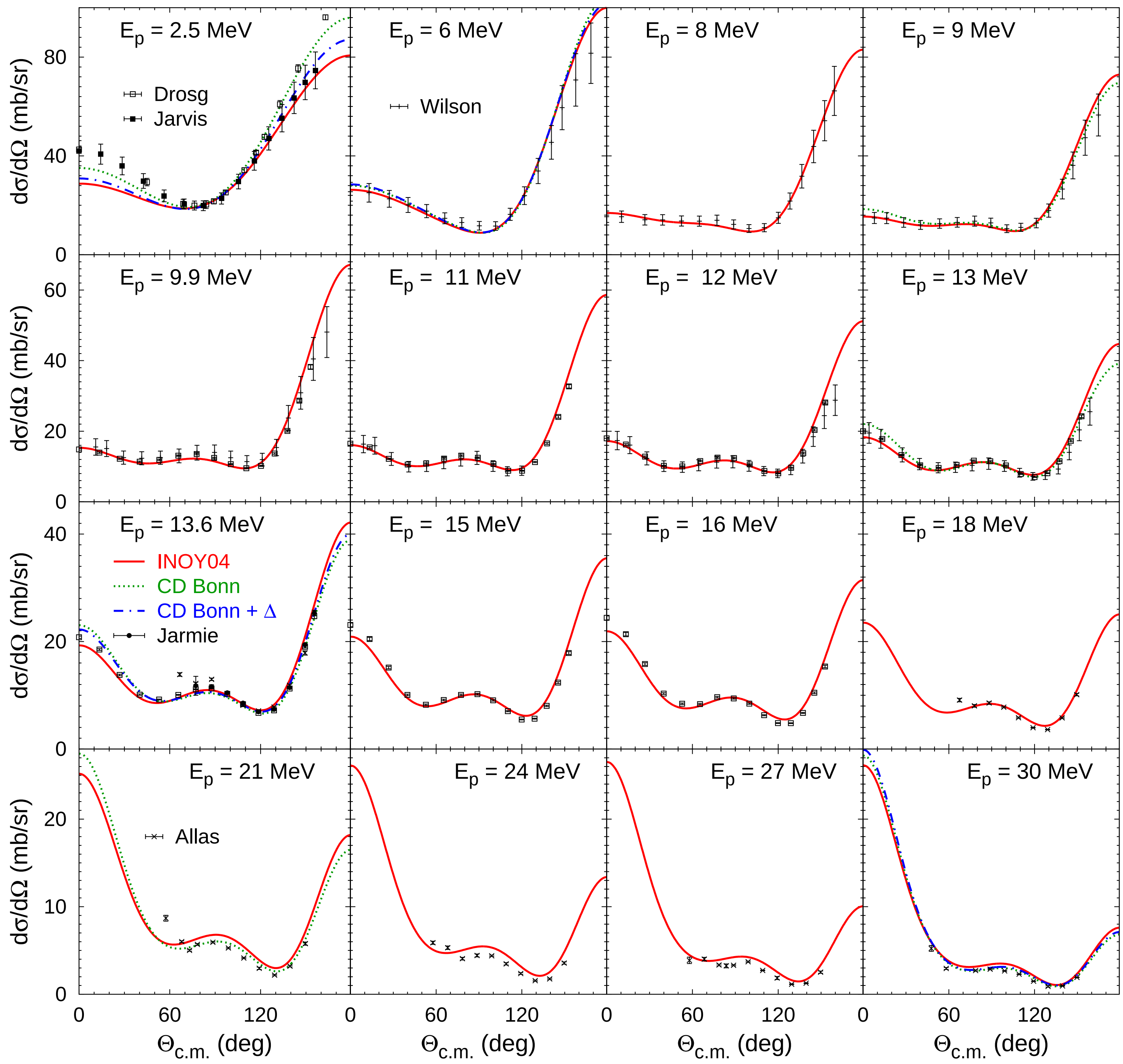
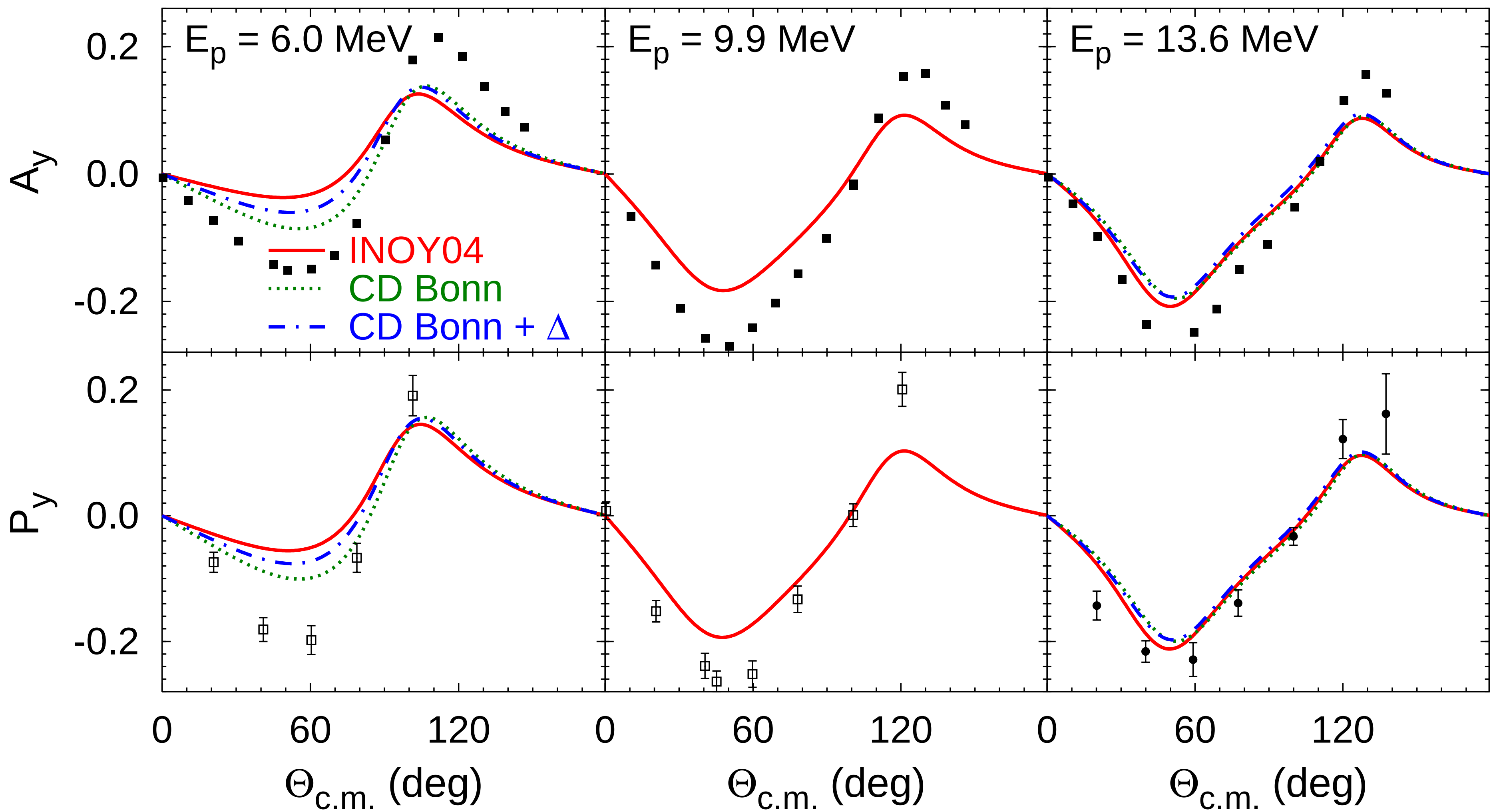


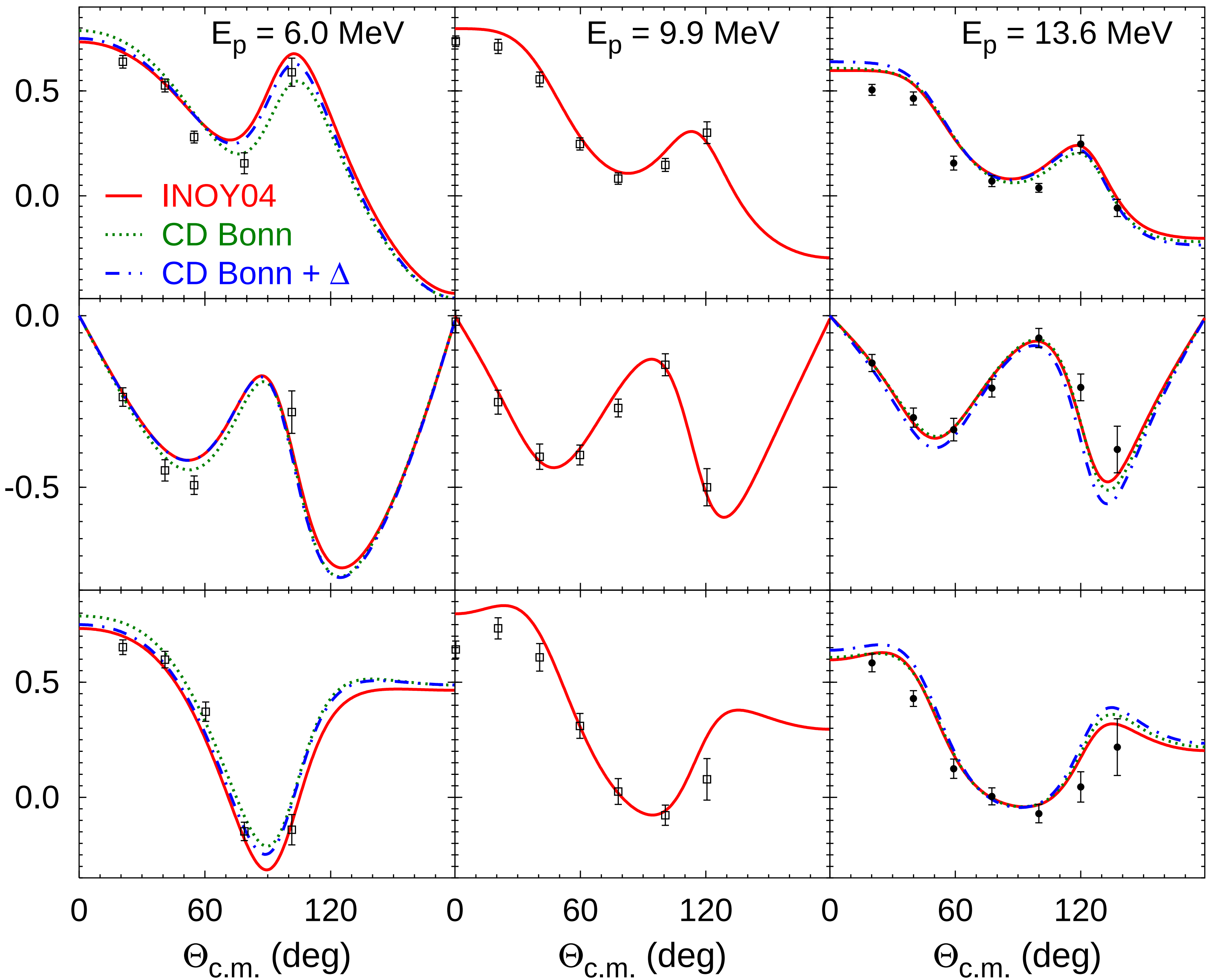
Charge exchange reaction $^3H(p,n)^3He$



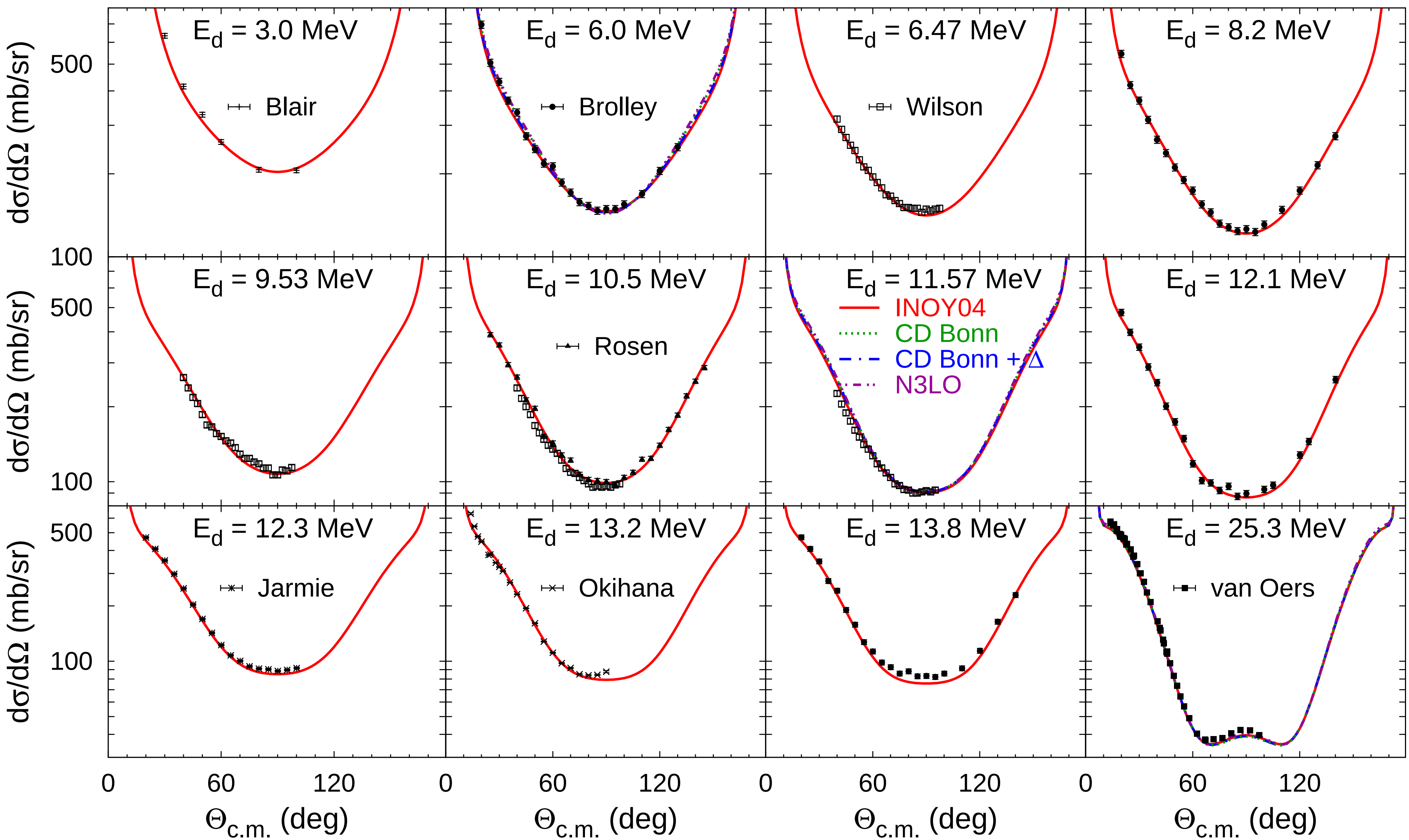
Nucleon polarization in $^3\text{H}(p,n)^3\text{He}$



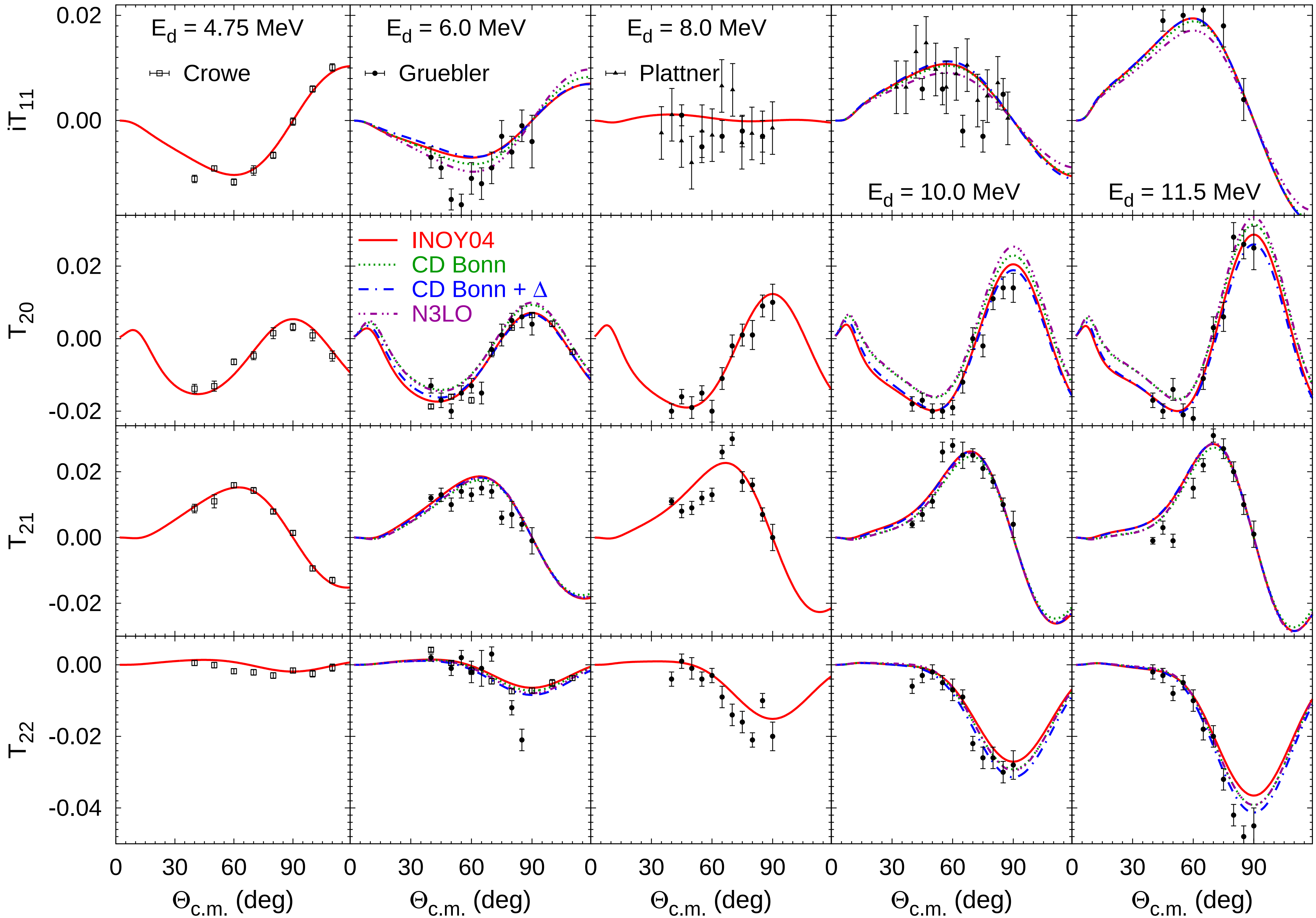
Spin transfer in $^3\mathrm{H}(p,n)^3\mathrm{He}$



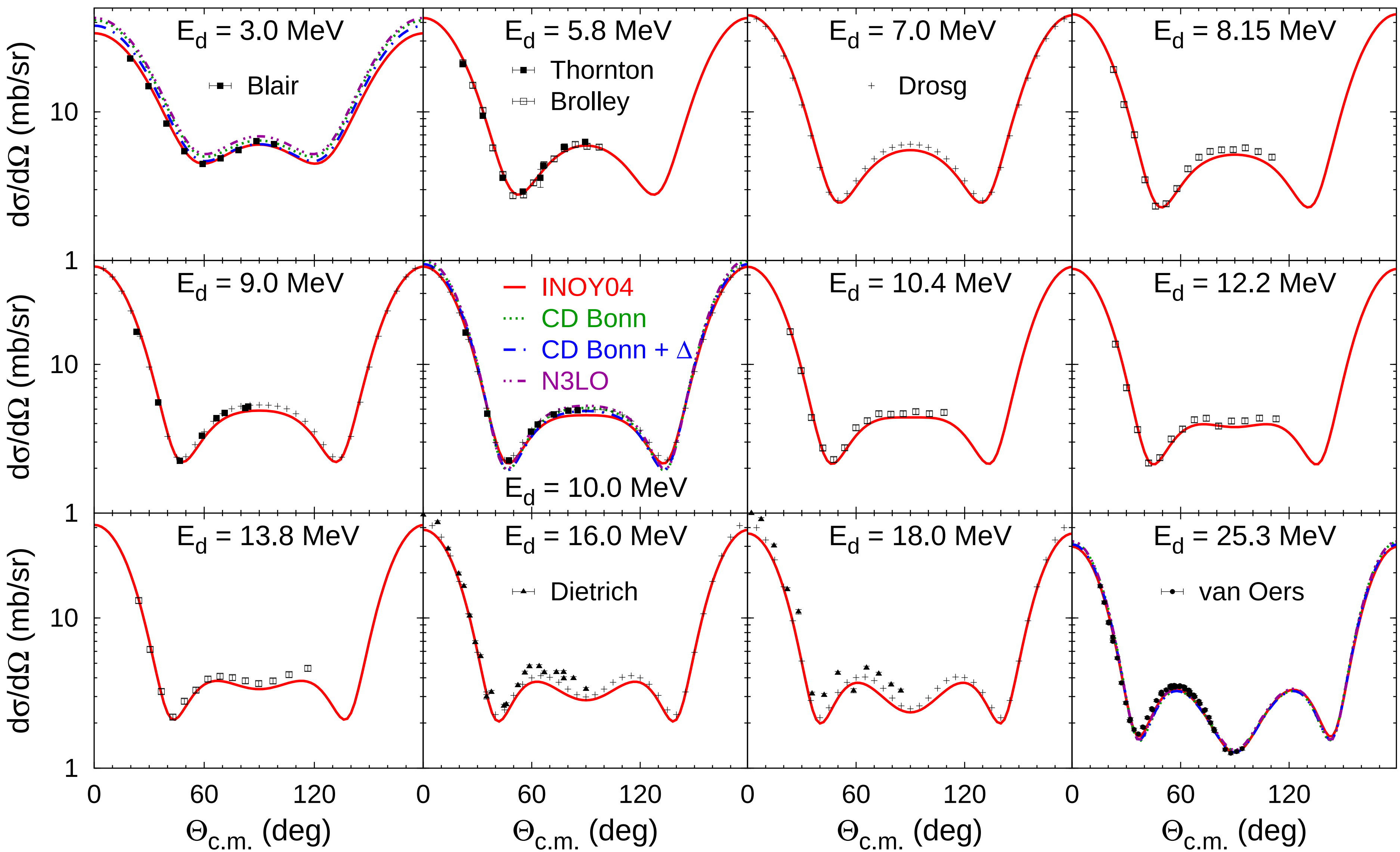
d+d elastic scattering



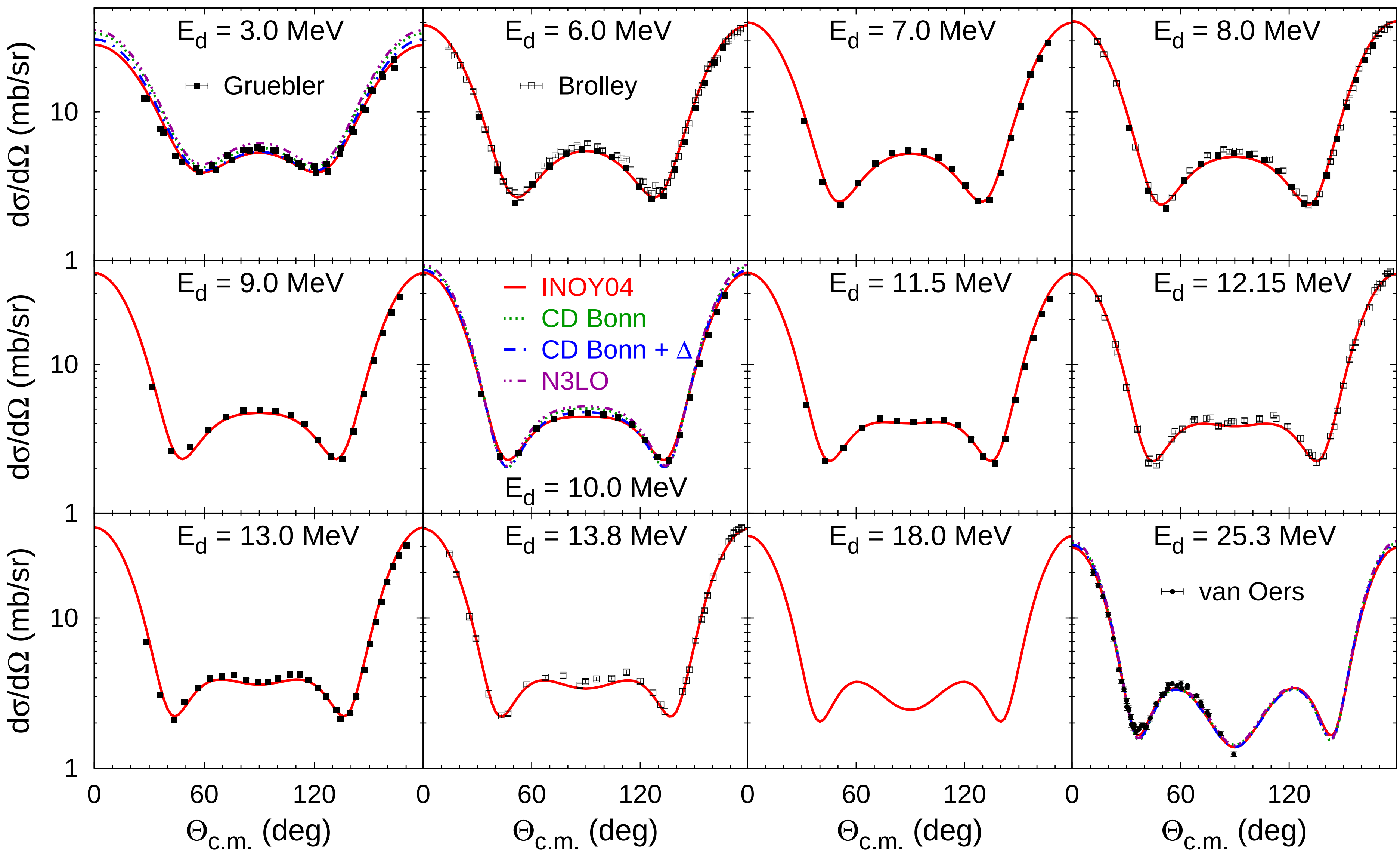
d+d elastic scattering: analyzing powers



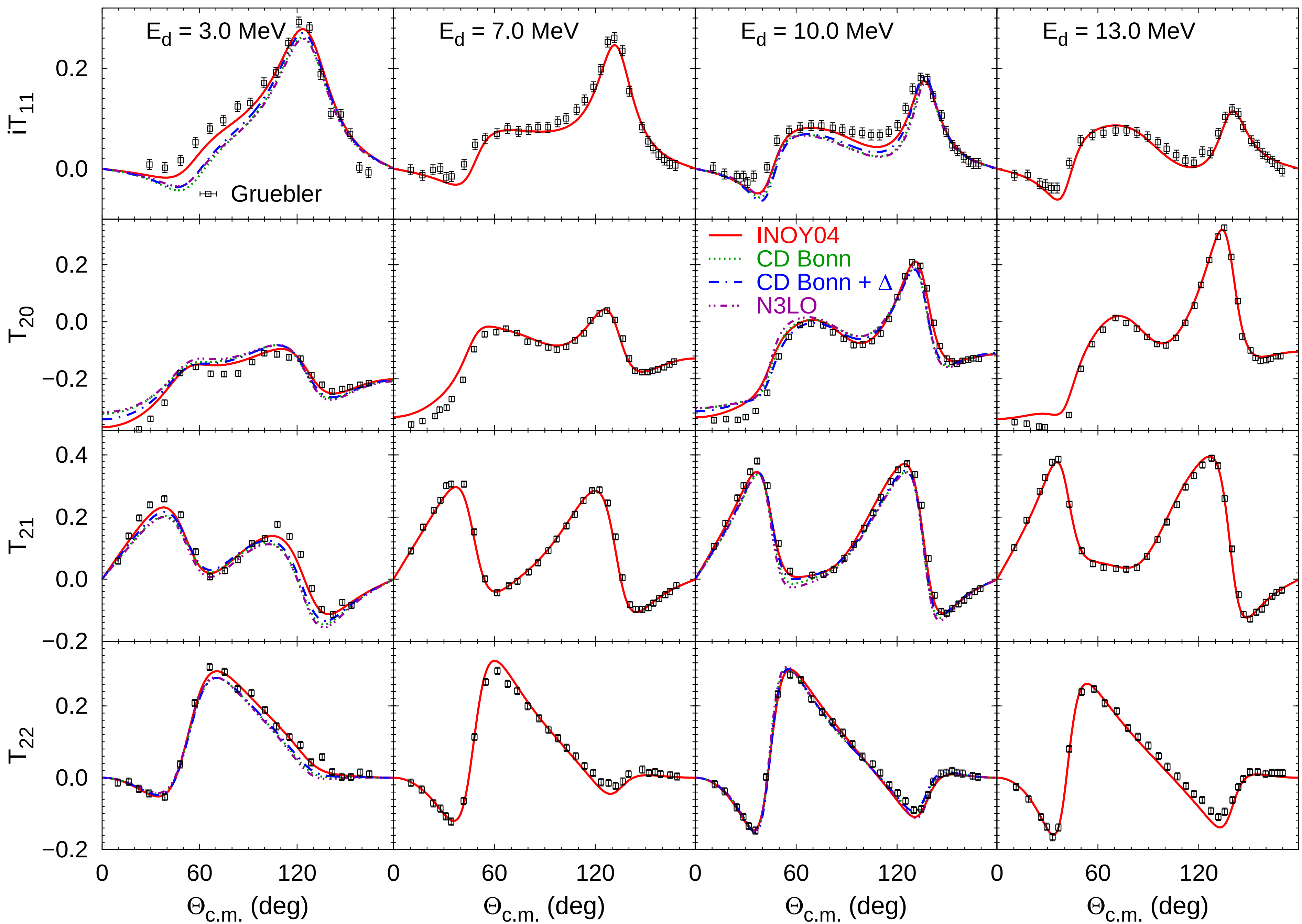
Transfer reaction $^2\mathrm{H}(d,n)^3\mathrm{He}$



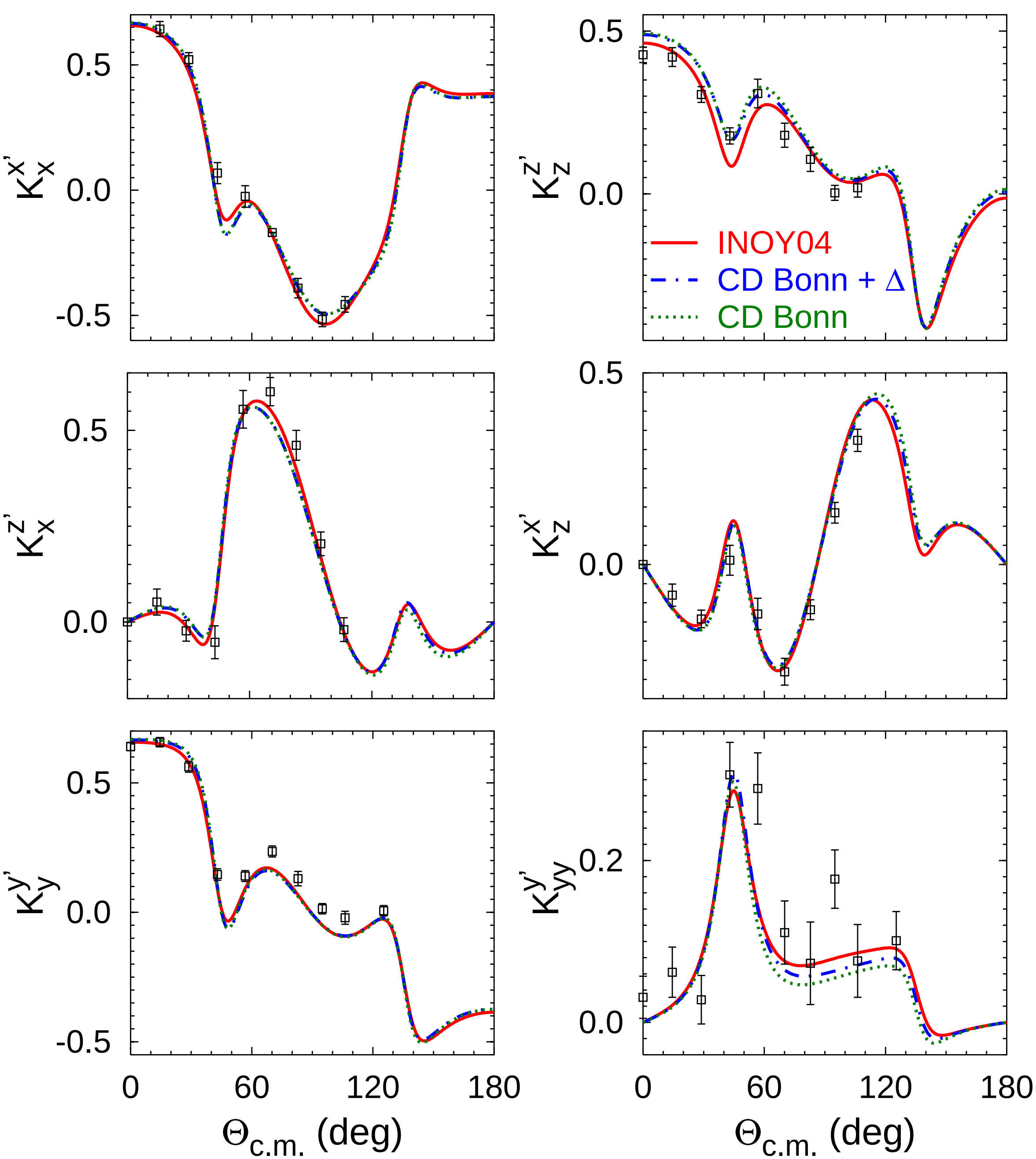
Transfer reaction $^2H(d,p)^3H$



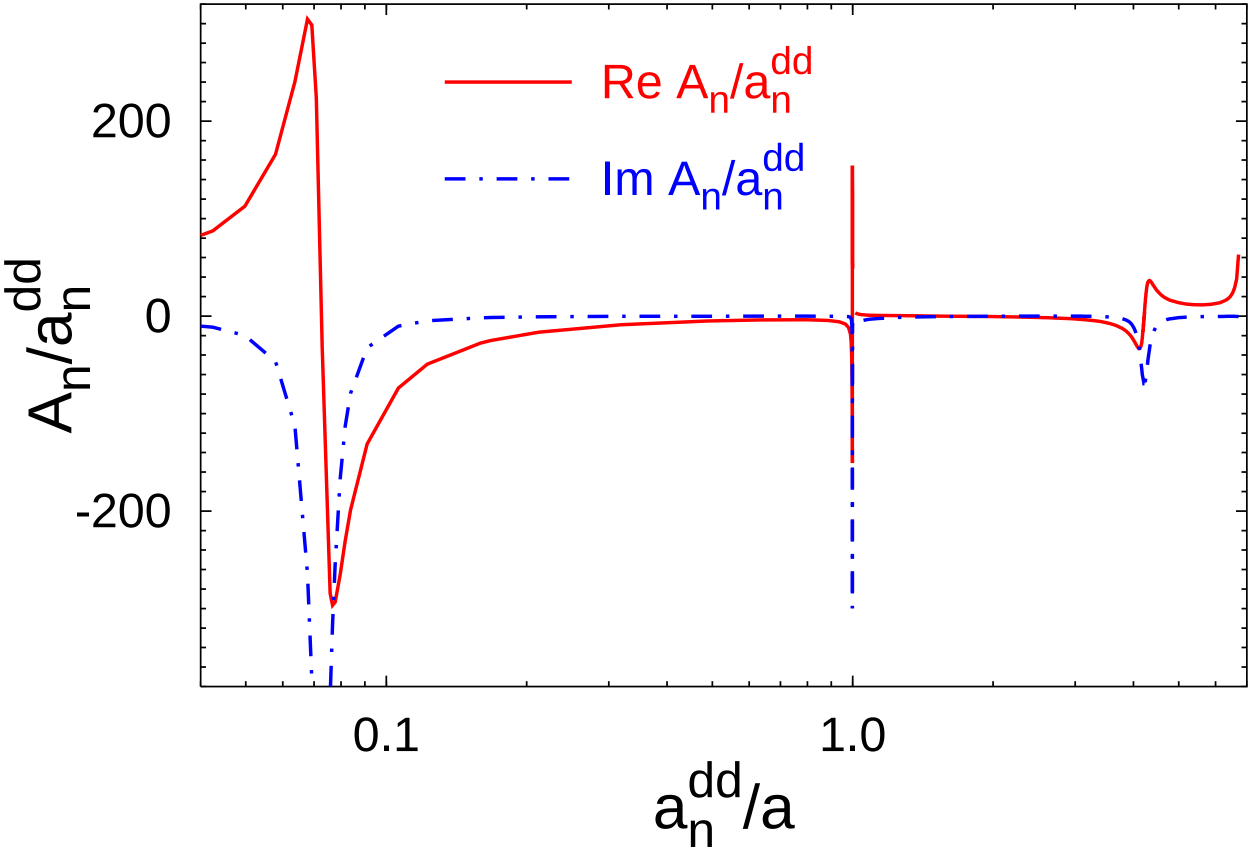
Transfer reaction $^2\mathrm{H}(d,p)^3\mathrm{H}$: analyzing powers

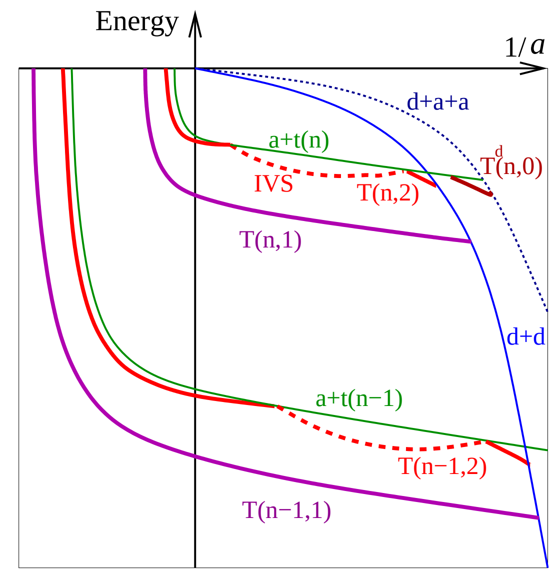


Spin transfer in $^{2}\mathrm{H}(d,n)^{3}\mathrm{He}$ at 10 MeV



Extension: 4-boson universal physics



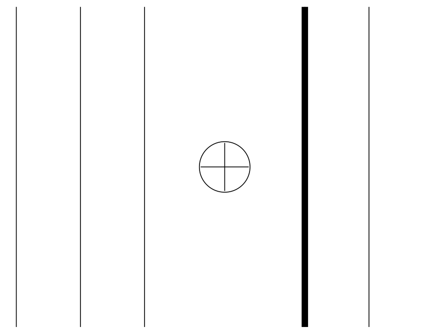


$$a_n^{dd} : b_n = 2b_d$$

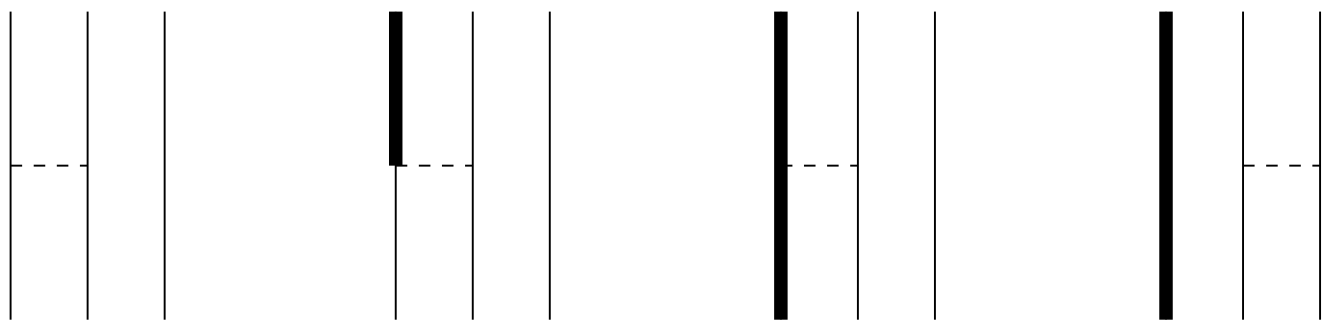
[EPL 95, 43002, PRA 85, 042705]

$A > 4$: 3-body approach including core excitation

$$\mathcal{H} = \mathcal{H}_g \oplus \mathcal{H}_x$$



sector coupling by interaction



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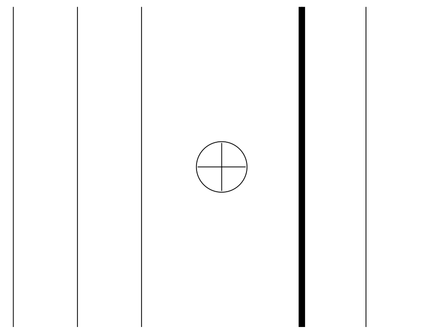

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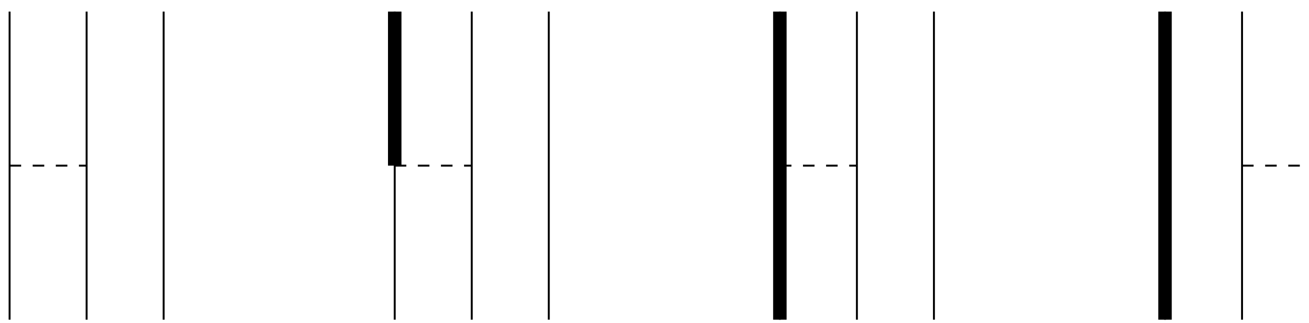


standard form of Alt-Grassberger-Sandhas (AGS)
3-body equations with $H_0 \to H_0 + h_A^{\text{int}}$

$$h_A^{\text{int}} |\mathcal{H}_a\rangle = (m_{A^*} - m_A)\delta_{ax} |\mathcal{H}_a\rangle$$

3-body AGS equations with core excitation (CX)

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$T_{\sigma} = v_{\sigma} + v_{\sigma} G_0 T_{\sigma}$$

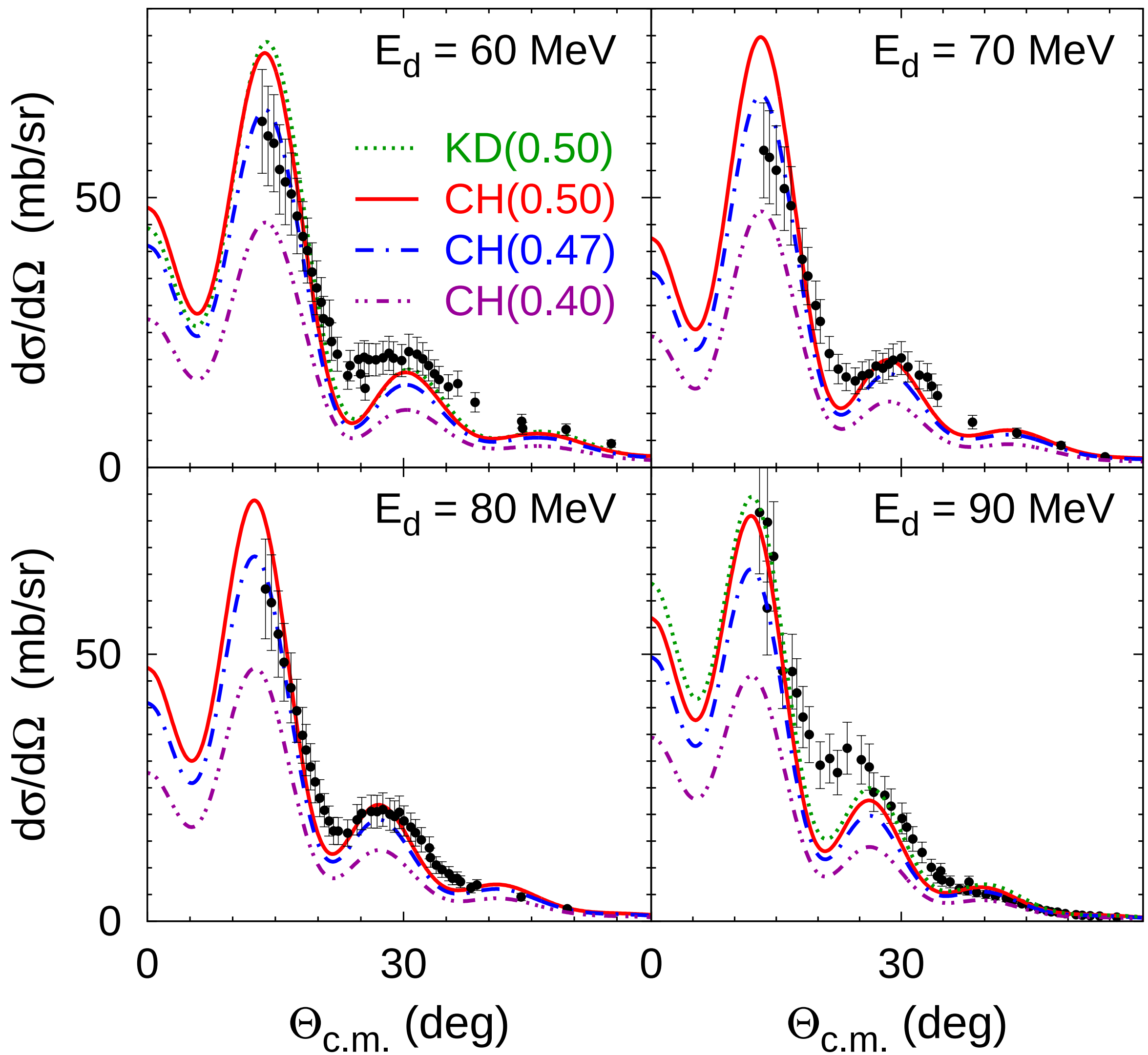
$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $(E - H_0 - v_{\alpha})|\phi_{\alpha}\rangle = 0$

$$H_0 |\mathbf{p}_{\alpha}\mathbf{q}_{\alpha}\rangle_a = [p_{\alpha}^2/2\mu_{\alpha} + q_{\alpha}^2/2M_{\alpha} + (m_{A^*} - m_A)\delta_{ax}] |\mathbf{p}_{\alpha}\mathbf{q}_{\alpha}\rangle_a$$

[PRC 88, 011601(R); PRC 91, 024607; NPA 947, 173]

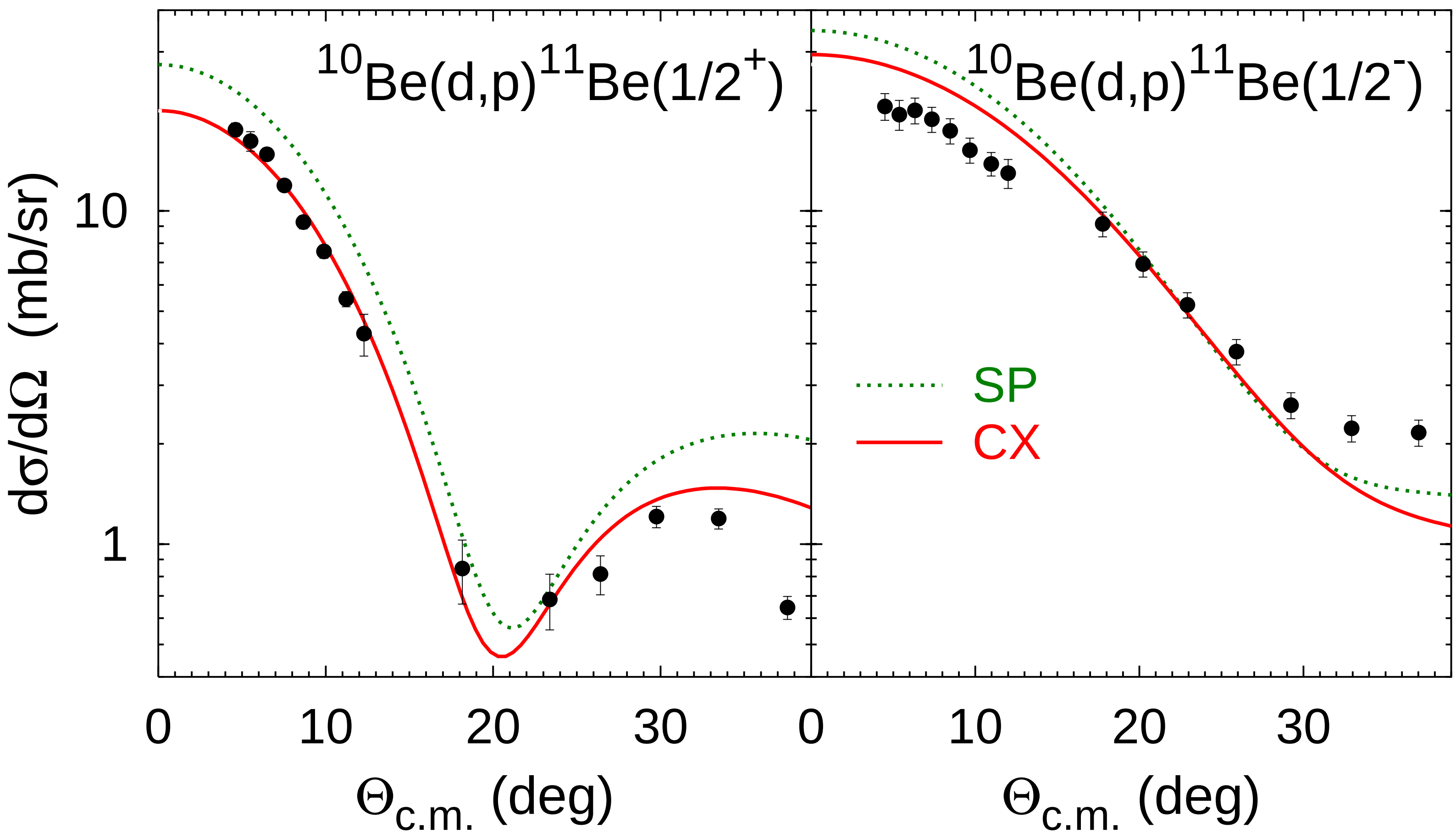
^{24}Mg(d,d')^{24}Mg(2^+) inelastic scattering



Rotational model for V_{NA} with $\beta_2 = 0.4$, 0.47, 0.5

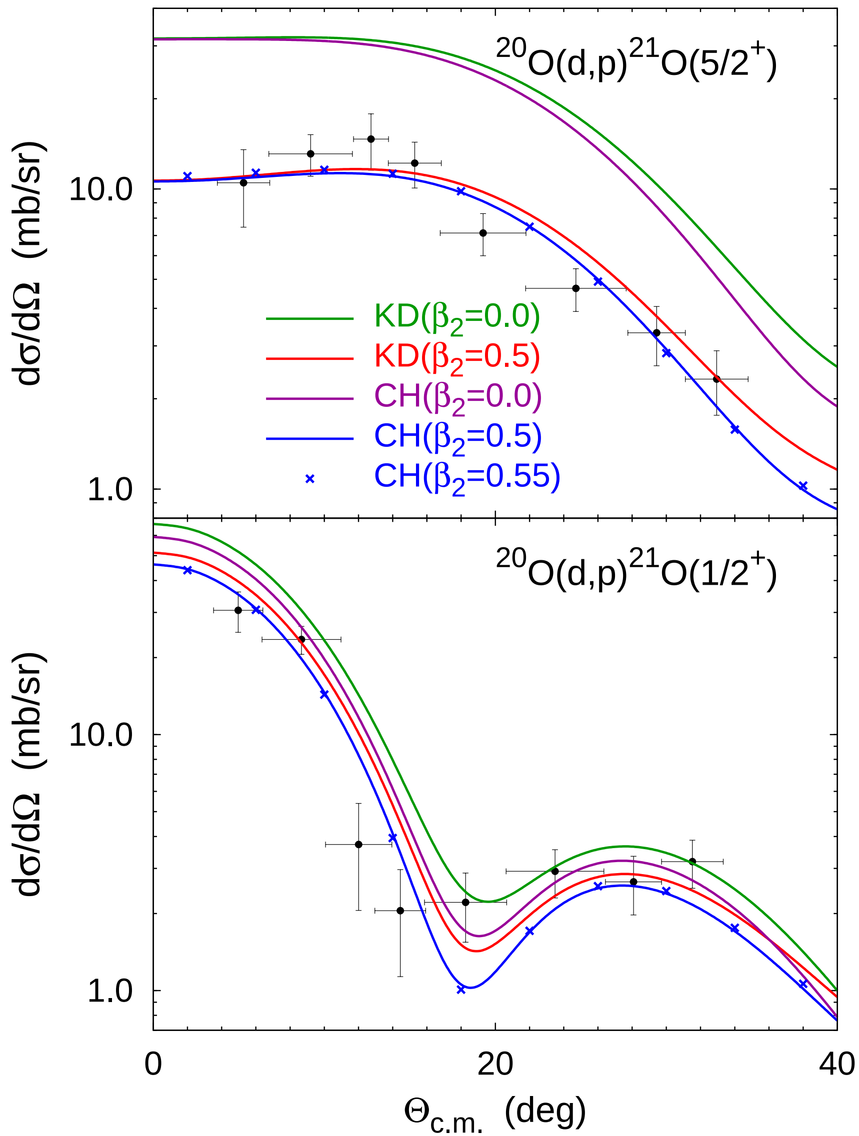
DWBA: $\beta_2 \sim 0.5\ (p,p')$, $\quad \beta_2 \sim 0.4\ (d,d')$

CX effect in ^{10}Be(d,p)^{11}Be at 21.4 MeV



CH89, rotational model for V_{NA} with $\beta_2 = 0.67$

CX effect in $^{20}O(d,p)^{21}O$ at 21 MeV



Vibrational model for V_{NA} with $\beta_2 = 0.5, 0.55$

Few-body reactions

- 4-particle AGS equations in momentum space
- complex-energy method for singularities above breakup threshold

- overall good description: cross sections, analyzing powers, spin transfer coefficients
- discrepancies: minimum of differential cross section, extrema of nucleon analyzing power and polarization

- beyond $A = 4$:
  3-body reactions including core excitation