# Nucleon in a periodic magnetic field

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AA, Ulf-G. Meißner and A. Rusetsky, Phys. Rev. D **95**, 031502 (2017)

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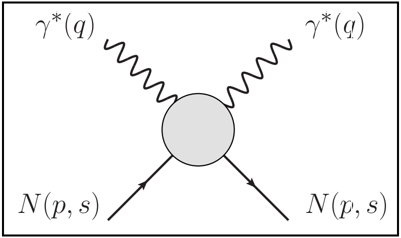
# Outline

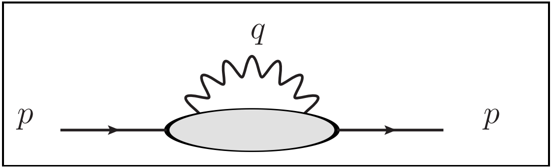
- Introduction: Compton scattering
- Nucleon in an external field
- Energy shift
- Work in progress
- Outlook

# Compton scattering




- Two invariant amplitudes: $T_1(\nu, q^2)$ and $T_2(\nu, q^2)$, $\quad \nu \equiv p \cdot q/m$
- Why important?
  - ▷ the proton-neutron mass difference
  - ▷ Lamb shift in the muonic hydrogen
  - ▷ the existence of a fixed pole in the Regge theory

# Cottingham formula




▷ Electromagnetic contribution to the proton-neutron mass difference:

$$(m_p - m_n)_{\mathrm{em}} = \frac{ie^2}{2m(2\pi)^4} \int^{\Lambda} d^4q D(q^2)\{3q^2\, T_1 + (2\nu^2 + q^2)\, T_2\} + \text{counter terms}$$

▷ $D(q^2)$ - photon propagator

- Electroproduction cross sections $\rightarrow$ $T_1$, $T_2$ $\quad (Q^2 = -q^2 > 0)$

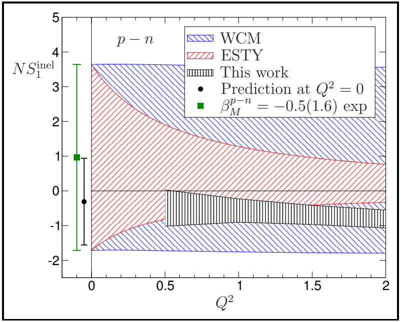
# Information on $T_1$

- $T_1(\nu, Q^2) \to$ once-subtracted dispersion relation (fixed $Q^2$)
- A problem: $S_1(Q^2) \equiv T_1(0, Q^2)$ is not fixed by experiment

$$S_1(Q^2) = S_1^{el}(Q^2) + S_1^{inel}(Q^2)$$

- $S_1^{el}(Q^2) \to$ Born terms ( + nucleon FFs)
- Low-energy theorem: $S_1^{inel}(0) = -\frac{\kappa^2}{4m^2} - \frac{m}{\alpha_{\rm em}}\,\beta_M$ Scherer, Tarrach
  - ▷ $\beta_M$ – magnetic polarizability, $\kappa = F_2(0)$, $\alpha_{\rm em} \approx 1/137$
- OPE: $S_1(Q^2 \to \infty) = \frac{C}{Q^4}$, $C$ – known coefficient Collins, Hill
- Region $0 < Q^2 \lesssim 2$ GeV$^2$: $S_1^{inel}(Q^2)$ is unknown

# Theoretical approaches

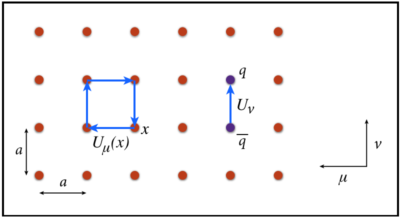



Source: J. Gasser et al., Eur. Phys. J. C **75**, 375 (2015)

- Chiral EFTs, NRQED — BKM, Hill, Pineda . . .
- Phenomenological Ansatzes — Pachucki, Walker-Loud, . . .
- Reggeon dominance hypothesis — Gasser, Leutwyler, . . .
- Lattice QCD $\rightarrow$ devoid of any model dependence

# Lattice QCD

- Path integral formulation in Euclidean space-time




- Space-time is discretized and finite ⇒ natural UV cut-off $\sim 1/a$

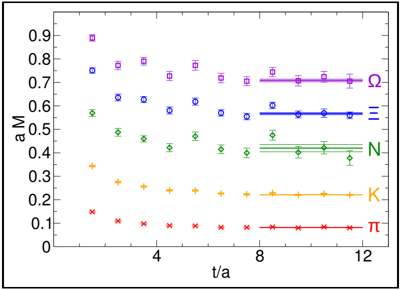
K. G. Wilson, Phys. Rev. D **10** (1974) 2445

- Integration → Monte Carlo methods

- Correlation functions → energy levels, current matrix elements

FLAG: S. Aoki *et al.*, arXiv:1607.00299 (2016)

# Stable hadrons




Source: BMW Collaboration, Science **322**, 1224 (2008)

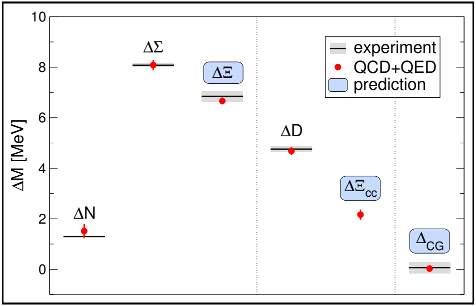
- Two-point correlation function:

$$C(t) = \sum_{\mathbf{x}} \langle 0|O(\mathbf{x},t)O^\dagger(\mathbf{0},0)|0\rangle = |Z_O|^2 e^{-mt}\left[1 + \sum_n |Z_n|^2 e^{-\Delta E_n t}\right]$$

  ▷ $O(\mathbf{x},t)$ - field operator, $Z_O$, $Z_n$ - overlap factors, $\Delta E_n > 0$

- Effective mass: $M(t) = \frac{1}{a}\log\frac{C(t)}{C(t+a)}, \quad M(t) \to m$

# Neutron-proton mass splitting



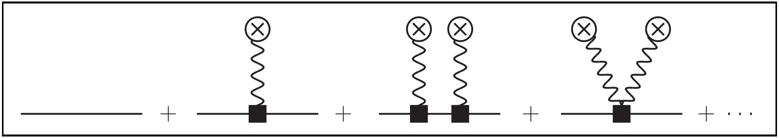

Source: S. Borsanyi et al., Science **347** (2015) 1452

| | $\Delta m$ [MeV] | QCD [MeV] | QED [MeV] |
|---|---|---|---|
| $\Delta N = n - p$ | 1.51(16)(23) | 2.52(17)(24) | -1.00(07)(14) |

- The fully unquenched lattice QCD + QED computation
  - ▷ 4 non-degenerate flavors, $m_\pi \approx 195$ MeV

BMW Collaboration

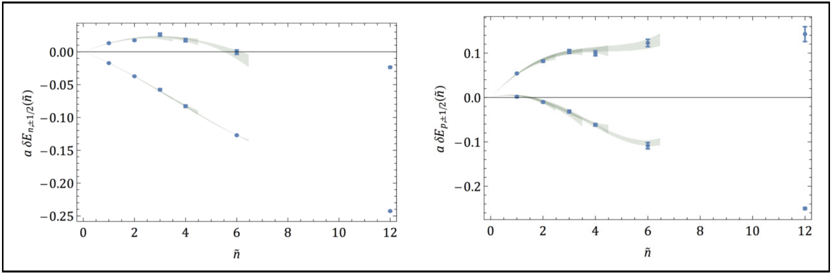
# External field method




Source: W. Detmold et al., Phys. Rev. D **73**, 114505 (2006)

- Two-point function in an external electromagnetic field $A_\mu(x)$
- $O(A^2)$ term $\rightarrow$ Compton scattering amplitude
- Uniform magnetic field $\rightarrow$ polarizabilities NPLQCD
- Static magnetic field $\rightarrow$ energy levels, measured on the lattice

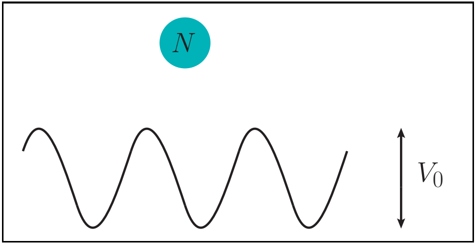
# Magnetic properties




Source: E. Chang *et al.* [NPLQCD Collaboration], Phys. Rev. D **92**, 114502 (2015)

- Lattice calculation at one SU(3)-symm. point, $m_\pi \approx 806$ MeV
- Constant magnetic field: $\mathbf{B} = \frac{6\pi}{eL^2}\tilde{n}\,\mathbf{e}_z, \quad \tilde{n} \in \mathbb{Z}$ 't Hooft

# $\gamma^* N \to \gamma^* N$: field configuration




- Static periodic magnetic field $\mathbf{B} = (0, 0, B_3)$:

$$A_1 = \frac{B}{\omega}\sin(\omega x_2), \qquad A_0 = A_2 = A_3 = 0$$

- Frequency $\omega \neq 0$ $\to$ photon virtuality $q^2 = -\omega^2$
- Magnetic flux is quantized $\Rightarrow \omega = \frac{2\pi N}{L}, \qquad N \in \mathbb{Z}\backslash\{0\}$

# Energy shift

- For $\nu = 0$ and $q^2 < 0 \to S_1(q^2)$ is real
- If $V_0 = e^2 B^2 / 2m\omega^2$ is "small" $\Rightarrow$ perturbation theory
- The free energy spectrum:

$$w(\mathbf{k}_n) = \sqrt{m^2 + \mathbf{k}_n^2}, \qquad \mathbf{k}_n = \frac{2\pi \mathbf{n}}{L}, \qquad \mathbf{n} \in \mathbb{Z}^3$$

- Spin-averaged energy shift of the ground state ($\mathbf{k}_n = 0$):

$$\delta E = \frac{1}{2} \sum_s \delta E_s = \frac{B^2}{4m} S_1(-\omega^2) + O(B^3)$$

A remarkably simple expression!

# Method I

$$\mathcal{L}_{\rm eff} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \cdots \,, \qquad \mathcal{L}_n = O(A^n)$$

- Non-relativistic EFT (nucleons $\psi$ + photons $A_\mu$)
  - ▷ The 2-pt. function at $O(A^2)$ in the infinite volume
  - ▷ Matching: LECs are fixed $\leftrightarrow$ FFs, Compton amplitude

$$H = H_0 + H_1 + H_2 + \cdots \,, \qquad H_n = O(A^n)$$

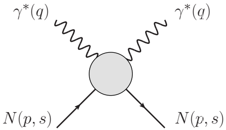
- Non-relativistic Hamiltonian from $\mathcal{L}_{\rm eff}$
  - ▷ Perturbation theory at $O(A^2) \Rightarrow$ energy shift in a finite volume

$\hookrightarrow$ details in Phys. Rev. D **95**, 031502 (2017)

# Method II

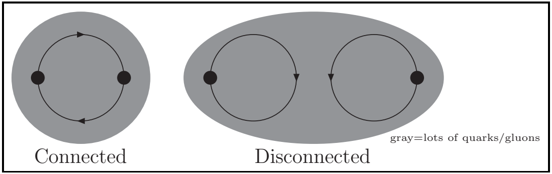
$$E_s = m + \xi_s B + \eta_s B^2 + O(B^3)$$

▷ $E_s$ – energy level ($s = \pm 1/2$), $\xi_s, \eta_s$ – unknown parameters

- Directly from 2-pt. function $C(B, t)$ at $O(A^2)$
  - ▷ LHS: $C(B,t) = C(0,t)[1 - \xi_s B \cdot t - \eta_s B^2 \cdot t + \cdots]$
  - ▷ RHS: $C(B,t) = C(0,t) + O(B) \cdot t + O(B^2) \cdot t + \cdots$
- First-order term $O(B) = 0$
- Second-order term $O(B^2) =$ 

- Comparison: $\xi_s = 0$, $\eta_s \to$ Compton amplitude

# Disconnected contributions




- Computationally very demanding (noisy)
- How to estimate them?
  - ▷ Large $N_c$? $\gamma\gamma \to \gamma\gamma$ Bijnens,...
  - ▷ Partially quenched $\chi$PT Golterman, Sharpe,...

# Partially quenched $\chi$PT

$$\langle \mathcal{O}(q,\bar{q})\rangle \propto \int \mathcal{D}G_\mu \det(D+m_{\rm sea})e^{-S[G]}\frac{1}{D+m_{\rm valence}}\cdots\frac{1}{D+m_{\rm valence}}$$

▷ $\mathcal{O}(q,\bar{q})$ - multi-local, gauge-invariant operator

- $m_{\rm sea} \neq m_{\rm valence} \to$ partially quenched QCD (PQQCD):

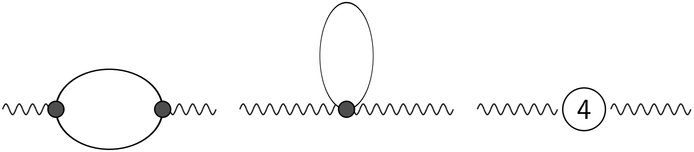
$$\text{sea } (q, N_f) + \text{valence } (q_v, N) + \text{ghost } (N)$$

- Idea ($m_{\rm sea} = m_{\rm valence}$): $\left\langle \bar{q}(y)\Gamma' q(y)\bar{q}(x)\Gamma q(x)\right\rangle_{\rm QCD} \equiv$

$$\equiv \underbrace{\left\langle \bar{q}(y)\Gamma' q_v(y)\bar{q}_v(x)\Gamma q(x)\right\rangle_{\rm PQQCD}}_{\text{connected}} + \underbrace{\left\langle \bar{q}(y)\Gamma' q(y)\bar{q}_v(x)\Gamma q_v(x)\right\rangle_{\rm PQQCD}}_{\text{disconnected}}$$

- PQQCD $\xrightarrow{\rm SSB}$ PQ$\chi$PT: $SU(N_f+N|N)$ graded group (coset space)

# An example: HVP

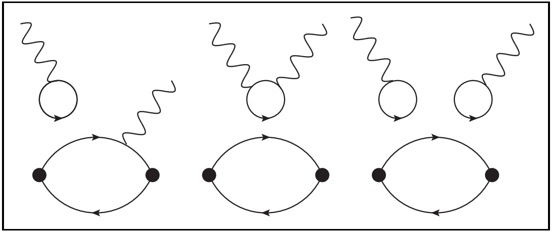



$$a_\mu^{had} = 4\pi^2 \left(\frac{\alpha_{\rm em}}{\pi}\right)^2 \int_0^\infty dq^2 f(q^2) \left(\Pi(q^2) - \Pi(0)\right) ,$$

▷ $\Pi(q^2)$ – hadronic VP, $f(q^2)$ – known function

- Ratio $R = [\Pi(q^2) - \Pi(0)]_{\rm Disc}/[\Pi(q^2) - \Pi(0)]_{\rm Conn}$ Bijnens, Jüttner...
  - ▷ $R = -1/10$ ($N_f = 2$, $m_u = m_d$)
  - ▷ $R = 0$ ($N_f = 3$, $m_u = m_d = m_s$)
- NLO prediction, low-$q^2$ region

# $\gamma^*\pi^+ \to \gamma^*\pi^+$



- Meson sector $\rightarrow$ more simple to study
- Provides a testing ground
- Disconnected pieces start at $O(A^2)$
- In progress...

# Limit $\omega \to 0$

- Frequency $\omega \to 0$: $\mathbf{B} = \mathrm{const}, \quad V_0 \to \infty$
  - ▷ Energy spectrum $\to$ Landau levels – non-perturbative!
  - ▷ $S_1^{el}(-\omega^2)$ explodes: $S_1^{el}(-\omega^2) \sim 1/\omega^2$ (propagator)
- Lower bound: $\omega^2_{min} = 0.13\ \mathrm{GeV}^2$ ($p$), $\omega^2_{min} = 0.06\ \mathrm{GeV}^2$ ($n$)
- $\omega = \frac{2\pi N}{L}$ – quantized: $L \to \infty$ – inconvenient
  - ▷ Alternative: $B = \frac{6\pi N}{eL^2}\frac{\omega L}{\sin(\omega L)}, \quad \omega$ – arbitrary

# Outlook

- Test of the formula on the lattice
  - ▷ nucleons and pions
- Disconnected contributions
- The limit of zero frequency
- Finite-volume effects