Hadron-Hadron Scattering from Lattice QCD
$\pi - \pi$, $K - K$ and $\pi - N$

Carsten Urbach

HISKP, University of Bonn
Part of the Sino-German CRC 110
European Twisted Mass Collaboration

Bochum, April 2017

Particle Scattering in Lattice QCD

- non-perturbative determination of resonance and scattering properties from first principles highly valuable

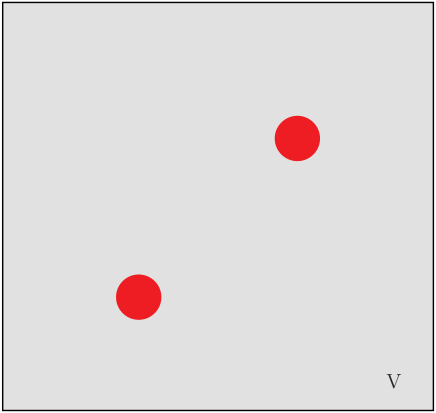
- aim: go beyond spectroscopy of QCD stable states

$\Rightarrow$ Lattice QCD

- unfortunately: direct determinations in Euclidean time very difficult (impossible)

  [Maiani and Testa (1990)]

$\Rightarrow$ use Lüschers finite size method the finite volume as vehicle...



- for $V \to \infty$:

$\Rightarrow$ interaction probability very low

$\Rightarrow$ $E_{2p}(p = 0) = 2M_{1p}$

- for finite V:

$\Rightarrow$ interaction probability rises

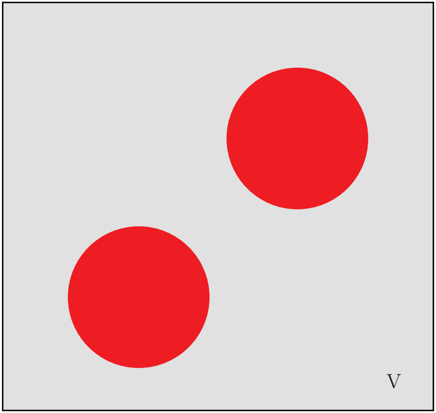
$\Rightarrow$ $E_{2p}(p = 0)$ receives corrections $\propto 1/V$

- Lüscher: correction in $1/V$ related to scattering properties!

[Lüscher, 1986]

Lüscher Method

the finite volume as vehicle...



- for $V \to \infty$:

$\Rightarrow$ interaction probability very low

$\Rightarrow$ $E_{2p}(p = 0) = 2M_{1p}$

- for finite V:

$\Rightarrow$ interaction probability rises

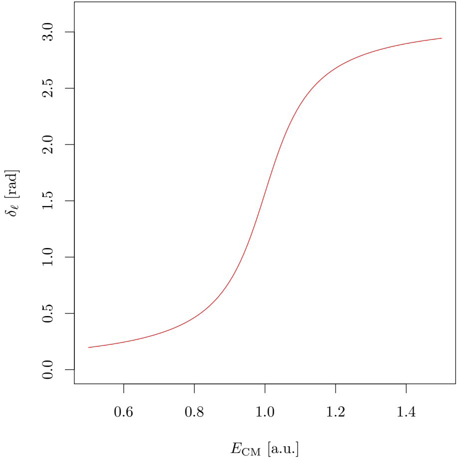
$\Rightarrow$ $E_{2p}(p = 0)$ receives corrections $\propto 1/V$

- Lüscher: correction in $1/V$ related to scattering properties!

[Lüscher, 1986]

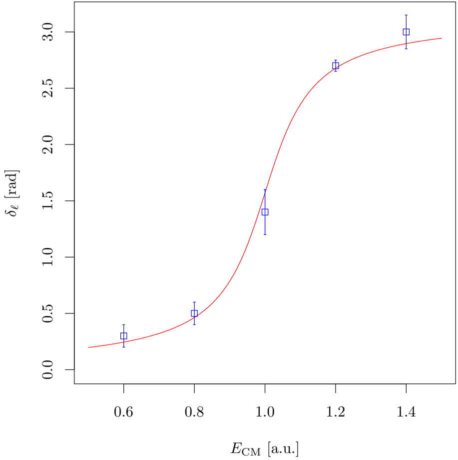
Goal

- would like to map out phase shift
- then extract mass and width (or pole position)
- however: Lüscher method will give only discrete points
- need various volumes
- can also use different reference frames



Goal

- would like to map out phase shift
- then extract mass and width (or pole position)
- however: Lüscher method will give only discrete points
- need various volumes
- can also use different reference frames



Systematic Uncertainties

- . . . we need to control systematic errors:
  - lattice spacing effects $\Rightarrow$ continuum limit, lattice spacing $a \to 0$,
    $\Rightarrow$ remove leading order lattice artefacts
  - finite size effects $\Rightarrow$ thermodynamic limit, physical volume $L^3 \to \infty$,
    $\Rightarrow$ use chiral effective field theories.
  - chiral effects $\Rightarrow$ chiral limit, $m_{\text{PS}} \to m_\pi$,
    $\Rightarrow$ use chiral effective field theories.
    or simulate directly at the physical point!

$\Rightarrow$ be aware: subtle interplay of limits

- from experience: we need

$$\begin{aligned} a &< 0.1\,\text{fm}, \\ L &> 2\,\text{fm}, \\ m_{\text{PS}} &< 300\,\text{MeV}. \end{aligned}$$

Ensemble-Details

- $2+1+1$ quark flavour ensembles from ETM Collaboration
  $m_u = m_d < m_s < m_c$ Wilson twisted mass fermions

  [Frezzotti, Rossi, (2004); ETMC, R. Baron et. al., JHEP 06 111 (2010)]

- improved scaling: $\propto \mathcal{O}(a^2)$
  note: flavour symmetry broken at finite lattice spacing values

- charged pion masses range from $\approx 230\ \mathrm{MeV}$ to $\approx 500\ \mathrm{MeV}$

- $L \geq 3\ \mathrm{fm}$ and $M_\pi \cdot L \geq 3.5$ for most ensembles

- bare m_s and m_c fixed for each lattice spacing

- three lattice spacings (A, B and D ensembles):
  $a_A = 0.086\ \mathrm{fm}$, $a_B = 0.078\ \mathrm{fm}$ and $a_D = 0.061\ \mathrm{fm}$

- special smearing method: stochastic Laplacian Heaviside

  [Peardon et al, (2009), Morningstar et al, (2011)]

$\pi - \pi$ Scattering with $I = 2$

$\pi - \pi$ Scattering for $I = 2$

- weakly repulsive channel
- very interesting check of chiral perturbation theory
- at small momenta $k \to 0$ use effective range expansion

$$k^{2\ell+1} \cot \delta_\ell \;=\; \frac{1}{a_\ell} + \mathcal{O}(k^2)$$

  scattering length a_ℓ

- only S-waves ($\ell = 0$) contribute (to a good approximation)

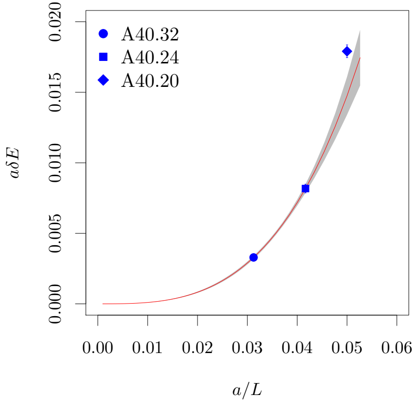
$\Rightarrow$ benchmark quantity for lattice QCD

$\pi - \pi$ Scattering with $I = 2$: Finite Volume Dependence

Lüscher formula (known constants c_i)

$$\delta E = E_{2p} - 2E_{1p} = -\frac{4\pi a_0}{M_\pi L^3}\left(1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2}\right) + \mathcal{O}(L^{-6})\,,$$

- valid, if other FS corrections small
- three ensembles with identical parameters but L
- smallest L deviates a few sigma
- smallest L too small
- all other ensembles have comparably larger L-values

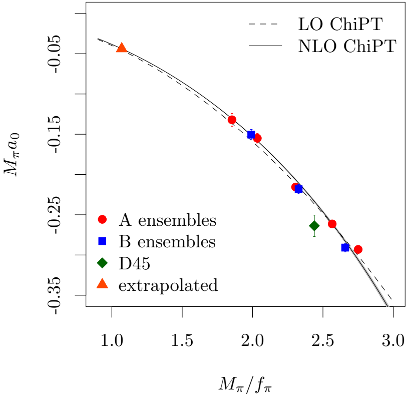


$\pi - \pi$ Scattering with $I = 2$: ChiPT Fits

- ChiPT formula at NLO [Beane et al, (2005,2007)]

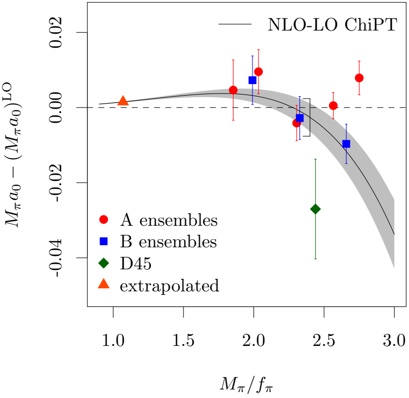
$$M_\pi a_0 = -\frac{M_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{M_\pi^2}{16\pi^2 f_\pi^2}\left[3\ln\frac{M_\pi^2}{f_\pi^2} - 1 - \ell_{\pi\pi}^{I=2}(\mu_R = f_{\pi,\mathrm{phys}})\right]\right\}$$

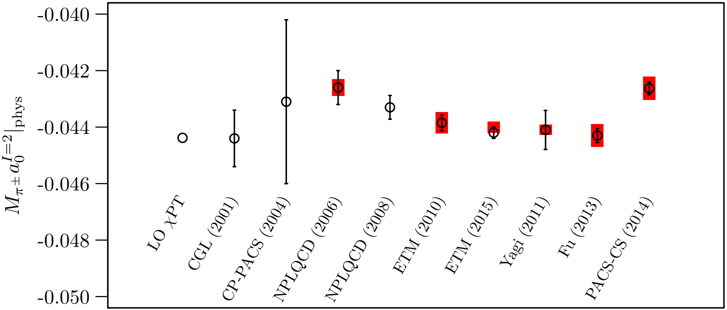
- functional form highly constraining
- surprisingly small deviations from LO ChiPT
- lattice artefacts small (in fact $\mathcal{O}(a^2 m_q)$)
- see JHEP 1509 (2015) 109



$\pi - \pi$ Scattering with $I = 2$: ChiPT Fits

- LO ChiPT (parameter-free) subtracted
- any single point not significantly different from 0
- explains the large uncertainty on $\ell_{\pi\pi}^{I=2}$
- significantly higher precision needed to sort out





- result:

$$M_\pi a_0^{I=2} \;=\; -0.0442(2)_{\rm stat}\left({}^{+4}_{-0}\right)_{\rm sys}\,, \qquad \ell_{\pi\pi}^{I=2} \;=\; 3.79(0.61)_{\rm stat}\left({}^{+1.34}_{-0.11}\right)_{\rm sys}$$

[ETMC, Helmes, CU, et al, (2015)]

K^+K^+ Scattering with $I = 1$

K^+K^+ Scattering with $I = 1$: Motivation

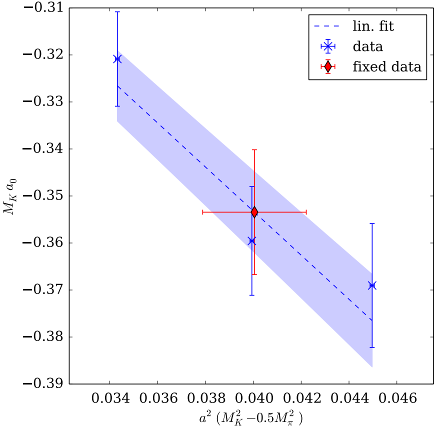
- at STAR or ALICE experiments: numerous light hadrons created
- kaons carry on average much lower momentum than pions
- kaons much more likely to interact elastically
- lattice computation of KK scattering valuable input
- theoretically interesting: does chiral perturbation theory still work for KK?

K^+K^+ Scattering with $I=1$: Strange Quark Mass

- value of sea strange quark mass up to 10% off
- corrected for by varying the valence strange quark mass

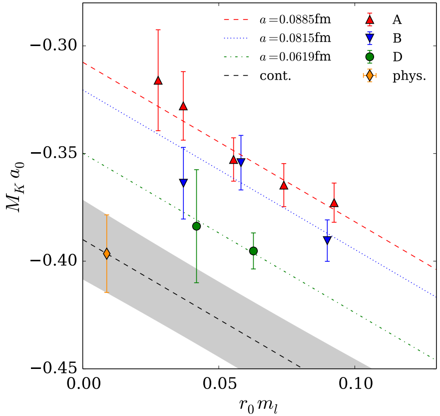
$\Rightarrow$ small unknown systematic uncertainty

- interpolate linearly in $M_K^2 - 0.5M_\pi^2$
- input: M_K, M_π and lattice spacing
- now work at fixed strange quark mass value



at fixed strange quark mass:

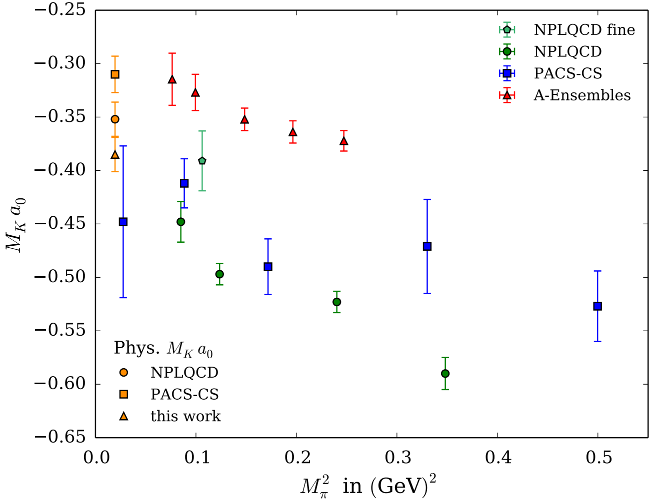
- extrapolate in light quark mass
- and lattice spacing a^2
- combined fit of all data simultaneously
- first continuum extrapolation of this quantity



- result

$$M_K a_0 = -0.385(16)_{\mathrm{stat}}(^{+0}_{-12})_{m_s}(^{+0}_{-5})_{Z_P}(4)_{r_f}$$

K^+K^+ Scattering with $I = 1$: Comparison



[NPLQCD (2008), PACS-CS (2014), ETMC (2017)]

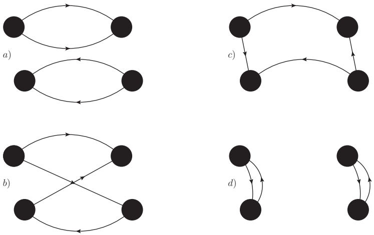
$\pi - \pi$ Scattering with $I = 0$

- two-particle operator for $I = 0$ (with $\Gamma = \gamma_5, 1$):

$$\mathcal{O}_\Gamma^{I=0} = \frac{1}{\sqrt{3}}(\mathcal{O}_\Gamma^+ \mathcal{O}_\Gamma^- + \mathcal{O}_\Gamma^- \mathcal{O}_\Gamma^+ + \mathcal{O}_\Gamma^0 \mathcal{O}_\Gamma^0)\,, \quad \text{e.g.} \quad \mathcal{O}_{\gamma_5}^{\pm,0} \equiv \pi^{\pm,0}$$

- isospin limit: only four diagrams contribute

$$C_{\pi\pi}(t-t') = \langle \mathcal{O}_{\gamma_5}^{I=0}(t')(\mathcal{O}_{\gamma_5}^{I=0})^\dagger(t)\rangle$$
$$= D(t) + \frac{1}{2}X(t) - 3B(t) + \frac{3}{2}V(t)$$



- in the elastic region sufficient to determine δE from $C_{\pi\pi}$!

- $a_0^{I=0}$ in principle from same analysis as for $I = 2$

$I = 0$ in Twisted Mass Lattice QCD

- Twisted Mass Lattice QCD explicitly breaks isospin symmetry at finite lattice spacing values

$\Rightarrow$ cannot project to states with $I = 0$

see also [Buchoff et al., (2009)]

- way out $\Rightarrow$ valence action with explicit isospin symmetry

- con: have to deal with lattice artefacts from unitarity violations
  here: mixing with lower lying states (due to vacuum diagram)

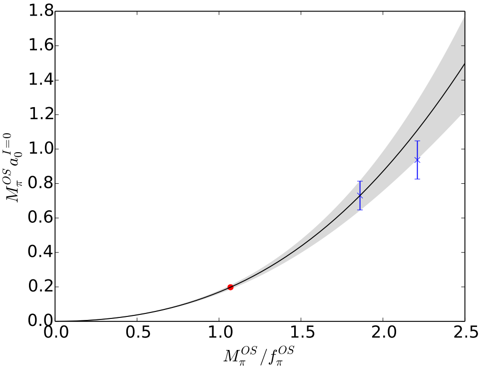
- use different discretisation with reduced isospin splitting

- apply generalised eigenvalue problem (GEVP) to identify state of interest

[Michael and Teasdale (1983); Lüscher and Wolff (1990)]

Detour: $\pi - \pi$ Scattering with $I = 0$

- much more difficult due to disconnected contributions
- $I = 0$ channel with the σ resonance
- weakly attractive interaction
- only one lattice spacing value



- we obtain (see arXiv:1701.08961)

$$M_\pi a_0^{\mathrm{I=0}} = 0.198(9)_{\mathrm{stat}}(6)_{\mathrm{sys}}$$

- compare to NA48-2 result $M_\pi a_0^{\mathrm{I=0}} = 0.220(3)(2)$

[NA48-2, (2010)]

Outlook: $\pi - N$ Scattering in the Δ channel

$\pi - N$ Scattering in the Δ channel

- very interesting channel
- relevant for many experiments
- there is no lattice result available so far
- reason: pion must be light enough for the Δ to decay
- signal-to-noise ratio decays exponentially

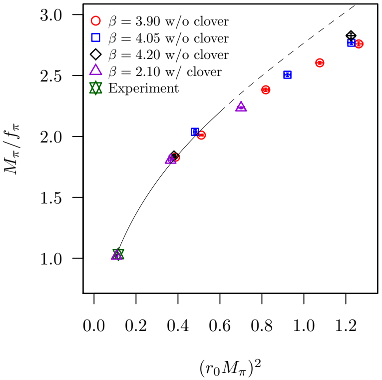
... how does one know to be at the physical point?

- measure e.g. M_π/f_π
  (compare to $1.0337(28)$)

$$\frac{M_{\pi^\pm}}{f_{\pi^\pm}} = 1.0254(31)\,\binom{+26}{-12}$$

- alternatively
  (compare to 6.97):

$$\frac{M_N}{M_\pi} = 7.08(6)$$

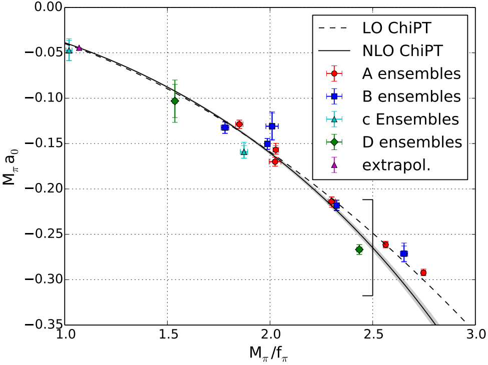


$\Rightarrow$ we are quite confident to be very close to the physical point!

[ETMC, A.Abdel-Rehim et al. (2015)]

$\pi - \pi$ with $I = 2$ at the Physical Point

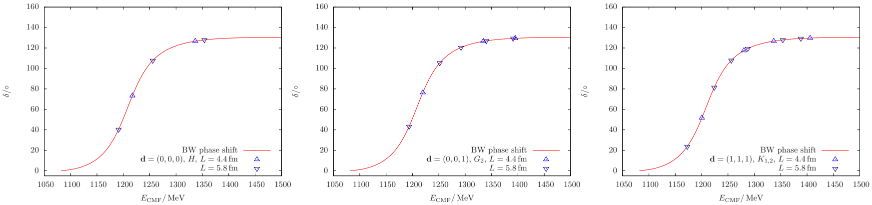
- $N_f = 2$, one lattice spacing
- two volumes
  @$M_\pi = 135$ MeV:
  $L \approx 4$ fm and $L \approx 6$ fm
- more ensembles at larger pion masses



- not extrapolation in M_π needed!
- have to balance statistical versus extrapolation error
- $N_f = 2 + 1 + 1$ currently in production

What to expect for $\pi - N$?

- Compute expected number of points for phaseshift for our volumes
- assuming Breit-Wigner resonance behaviour
- using physical mass values for π, N, Δ as input



- resonance region covered reasonably well

Summary and Outlook

- very precise $N_f = 2 + 1 + 1$ lQCD results for $I = 2$ $\pi - \pi$ scattering
- $I = 0$ $\pi - \pi$ scattering results
- first continuum extrapolated results for K^+K^+ scattering
- ρ meson mass and width
- plan to study $\pi - N$ scattering at physical pion mass

Thanks to ...

- the lattice QCD group in Bonn:
  C. Helmes, C. Jost, B. Knippschild, B. Kostrzewa, L. Liu, M. Oehm,
  K. Ottnad, M. Petschlies, M. Werner

- our friends in Beijing: C. Liu, Z. Wang

- the DFG funding this project in the Sino-German CRC 110

- the ETM collaboration

- ... and for your attention!