

Remarks on $P_c(4450)$ and triangle singularity

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Based on:

FKG, U.-G. Meißner, W. Wang, Z. Yang, *Phys. Rev. D* **92**, 071502(R) (2015) [arXiv:1507.04950[hep-ph]]

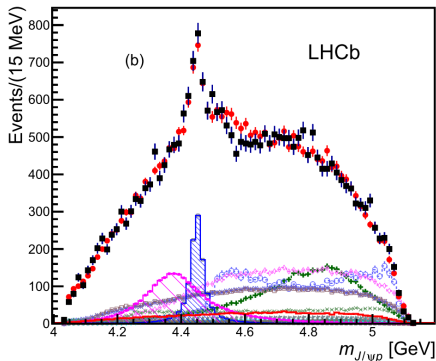
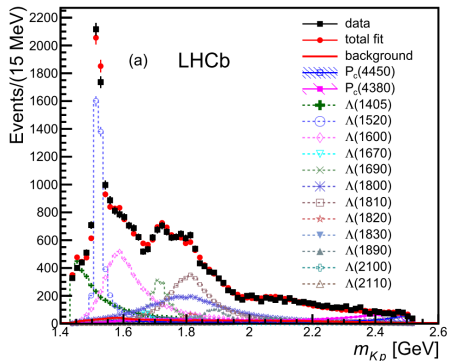
FKG, U.-G. Meißner, J. Nieves, Z. Yang, *Eur. Phys. J. A* **52**, 318 (2016) [arXiv:1605.05113[hep-ph]]

M. Bayar, A. Aceti, FKG, E. Oset, *Phys. Rev. D* **94**, 074039 (2016) [arXiv:1609.04133]

LHCb pentaquark-like structures: Big news in 2015!

PRL115(2015)072001 [arXiv:1507.03414]

submitted to PRL on 13.07, accepted on 24.07; appeared on arXiv on 14.07.2015



$$M_1 = (4380 \pm 8 \pm 29) \text{ MeV},$$

$$M_2 = (4449.8 \pm 1.7 \pm 2.5) \text{ MeV},$$

$$\Gamma_1 = (205 \pm 18 \pm 86) \text{ MeV},$$

$$\Gamma_2 = (39 \pm 5 \pm 19) \text{ MeV}.$$

- Quantum numbers not fully determined, for ($P_c(4380)$, $P_c(4450)$):
($3/2^-, 5/2^+$), ($3/2^+, 5/2^-$), ($5/2^+, 3/2^-$)
- In $J/\psi p$ invariant mass distribution, with hidden charm
 \Rightarrow pentaquarks if they are really hadron states
- Narrow pentaquark-like structures with hidden-charm had been predicted 5 years ago (07.2010):
Prediction of narrow N^ and Λ^* resonances with hidden charm above 4 GeV,*
J. J. Wu, R. Molina, E. Oset, B. S. Zou, Phys. Rev. Lett. **105** (2010) 232001.

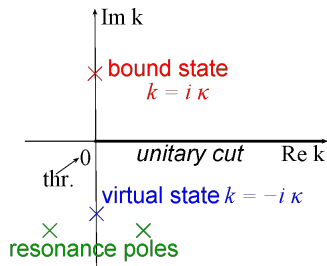
A flood of short papers

- 14.07 The LHCb paper appeared on line, arXiv:1507.03414
- 15.07 R. Chen, X. Liu, X.-Q. Li and S.-L. Zhu, arXiv:1507.03704 [hep-ph].
H.-X. Chen, W. Chen, X. Liu, T. G. Steele and S.-L. Zhu, arXiv:1507.03717 [hep-ph].
- 16.07 L. Roca, J. Nieves and E. Oset, arXiv:1507.04249 [hep-ph].
- 17.07 A. Mironov and A. Morozov, arXiv:1507.04694 [hep-ph].
- 18.07 weekend,
- 19.07 but everybody was working hard (NOT including me)...
- 20.07 F.-K. Guo, U.-G. Meißner, W. Wang and Z. Yang, arXiv:1507.04950 [hep-ph].
L. Maiani, A. D. Polosa and V. Riquer, arXiv:1507.04980 [hep-ph].
- 21.07 J. He, arXiv:1507.05200 [hep-ph]; X. H. Liu, Q. Wang, Q. Zhao, arXiv:1507.05359 [hep-ph].
- 22.07 R. F. Lebed, arXiv:1507.05867 [hep-ph].
- 23.07 Exotic! why no new papers?
- 24.07 M. Mikhasenko, arXiv:1507.06552 [hep-ph].
- 28.07 U.-G. Meißner and J. A. Oller, arXiv:1507.07478 [hep-ph].
- 29.07 V. V. Anisovich *et al.*, arXiv:1507.07652 [hep-ph].
- 30.07 G.-N. Li, M. He, X.-G. He, arXiv:1507.08252 [hep-ph].



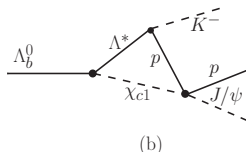
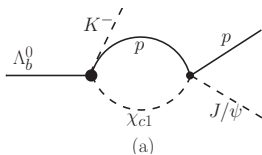
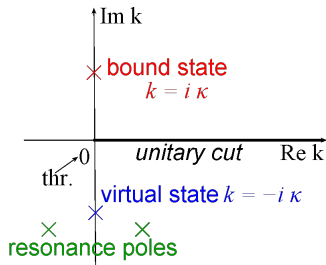
Two kinds of singularities of the S matrix

- Poles in the S -matrix: **dynamics**
 - ☞ **bound states** (real axis, 1st Riemann sheet (RS) of the complex energy plane)
 - ☞ **virtual states** (real axis, 2nd RS)
 - ☞ **resonances** (2nd RS)
- Landau singularities: **kinematics**
 - ☞ (a): **two-body threshold cusp**
 - ☞ (b): **triangle singularity**
 - ...



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$P_c(4450)$ is at the $\chi_{c1}p$ threshold

- Mass: $M = (4449.8 \pm 1.7 \pm 2.5)$ MeV

The LHCb paper says:

the closest threshold is at (4457.1 ± 0.3) MeV [$\Lambda_c(2595)\bar{D}^0$]

\Rightarrow difficult to explain with threshold effect

but, in fact, much more complicated!

- It is located *exactly* at the $\chi_{c1}p$ threshold:

$$M_{P_c(4450)} - M_{\chi_{c1}} - M_p = (0.9 \pm 3.1) \text{ MeV}$$

and at a **triangle singularity** at the same time!

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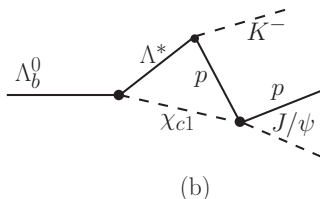
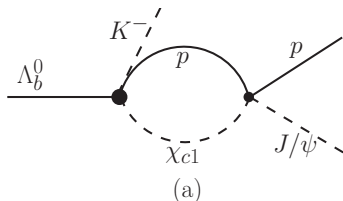
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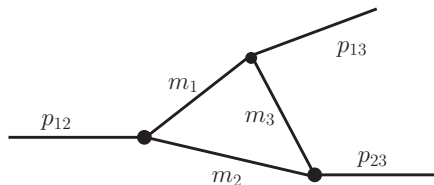
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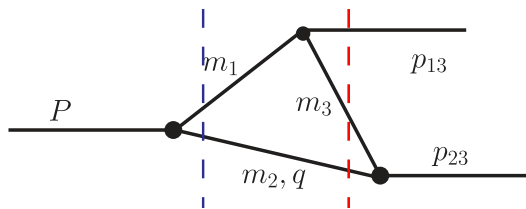
- **Triangle singularity**: leading Landau singularity for a triangle diagram, **anomalous threshold**
studied extensively in 1960s
- Solutions of Landau equation for a triangle diagram: Landau (1959)

$$1 + 2 y_{12} y_{23} y_{13} = y_{12}^2 + y_{23}^2 + y_{13}^2, \quad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_{ij}^2}{2 m_i m_j}$$

quadratic equation of y_{ij} , always **two solutions**

- Do they affect the physical amplitude?

Some details (I)



Consider the scalar three-point loop integral

$$I = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{[(P - q)^2 - m_1^2 + i\epsilon] (q^2 - m_2^2 + i\epsilon) [(p_{23} - q)^2 - m_3^2 + i\epsilon]}$$

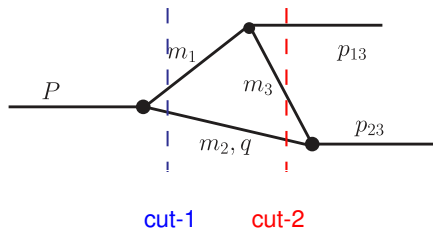
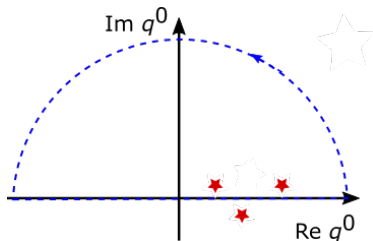
Rewriting a propagator into two poles:

$$\frac{1}{q^2 - m_2^2 + i\epsilon} = \frac{1}{(q^0 - \omega_2 + i\epsilon)(q^0 + \omega_2 - i\epsilon)} \quad \text{with} \quad \omega_2 = \sqrt{m_2^2 + \vec{q}^2}$$

focus on the positive-energy poles

$$I \simeq \frac{i}{8m_1 m_2 m_3} \int \frac{dq^0 d^3 \vec{q}}{(2\pi)^4} \frac{1}{(P^0 - q^0 - \omega_1 + i\epsilon)(q^0 - \omega_2 + i\epsilon)(p_{23}^0 - q^0 - \omega_3 + i\epsilon)}$$

Some details (II)



Contour integral over $q^0 \Rightarrow$

$$\begin{aligned}
 I &\propto \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{1}{[P^0 - \omega_1(q) - \omega_2(q) + i\epsilon][p_{23}^0 - \omega_2(q) - \omega_3(\vec{p}_{23} - \vec{q}) + i\epsilon]} \\
 &\propto \int_0^\infty dq \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\epsilon} f(q)
 \end{aligned}$$

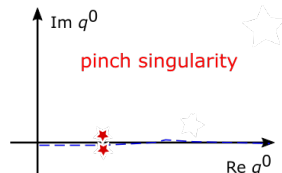
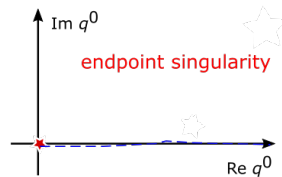
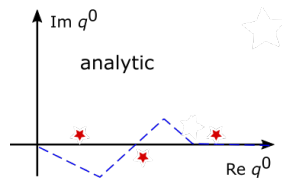
The second cut:

$$f(q) = \int_{-1}^1 dz \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2} + p_{23}^2 - 2p_{23}qz + i\epsilon}$$

Some details (III)

Relation between singularities of **integrand and integral**

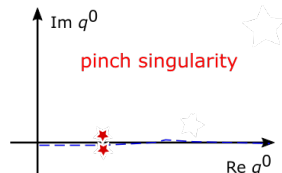
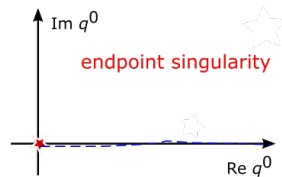
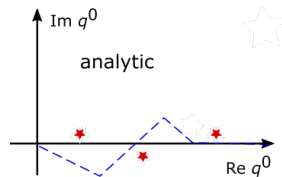
- singularity of integrand does **not necessarily** give a singularity of integral:
integral contour can be deformed to avoid the singularity
- Two cases that a singularity cannot be avoided:
 - ↳ endpoint singularity
 - ↳ pinch singularity



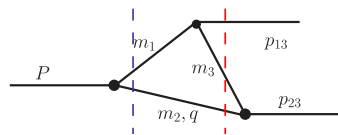
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Some details (IV)



$$I \propto \int_0^\infty dq \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\epsilon} f(q)$$

$$f(q) = \int_{-1}^1 dz \frac{1}{A(q, z)} \equiv \int_{-1}^1 dz \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\epsilon}$$

Singularities of the **integrand of I** in the rest frame of initial particle:

- 1st cut: $M - \omega_1(l) - \omega_2(l) + i\epsilon = 0 \Rightarrow$

$$q_{\text{on}\pm} \equiv \pm \left(\frac{1}{2M} \sqrt{\lambda(M^2, m_1^2, m_2^2)} + i\epsilon \right)$$
- 2nd cut: $A(q, \pm 1) = 0 \Rightarrow$ endpoint singularities of $f(q)$

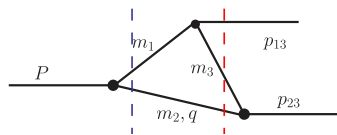
$$z = +1: \quad q_{a+} = \gamma(\beta E_2^* + p_2^*) + i\epsilon, \quad q_{a-} = \gamma(\beta E_2^* - p_2^*) - i\epsilon,$$

$$z = -1: \quad q_{b+} = \gamma(-\beta E_2^* + p_2^*) + i\epsilon, \quad q_{b-} = -\gamma(\beta E_2^* + p_2^*) - i\epsilon$$

$$\beta = |\vec{p}_{23}|/E_{23}, \quad \gamma = 1/\sqrt{1 - \beta^2} = E_{23}/m_{23}$$

$E_2^*(p_2^*)$: energy (momentum) of particle-2 in the cmf of the (2,3) system

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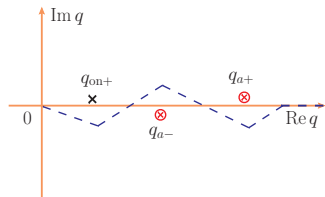
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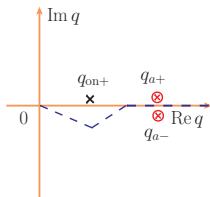
Some details (V)

All singularities of the integrand:

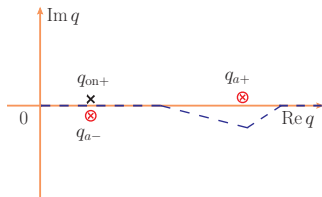
$$\begin{aligned}
 q_{0n+}, & \quad q_{a+} = \gamma (\beta E_2^* + p_2^*) + i \epsilon, & \quad q_{a-} = \gamma (\beta E_2^* - p_2^*) - i \epsilon, \\
 q_{0n-} < 0, & \quad q_{b-} = -q_{a+} < 0 \text{ (for } \epsilon = 0), & \quad q_{b+} = -q_{a-},
 \end{aligned}$$



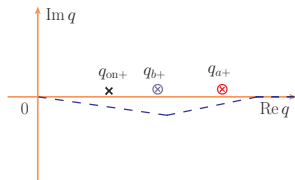
(a)



(b)



(c)

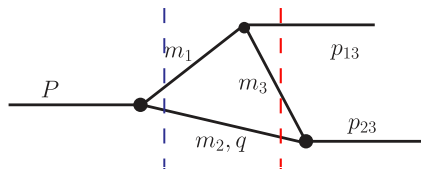


2-body threshold
singularity at
 $m_{23} = m_2 + m_3$

triangle singularity at

$$q_{0n+} = q_{a-}$$

Some details (VI)



Rewrite $q_{a-} = p_2 - i\epsilon$, $p_2 \equiv \gamma(\beta E_2^* - p_2^*)$

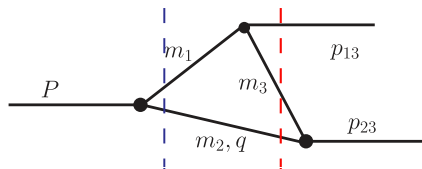
Kinematics for $p_2 > 0$, which is relevant to triangle singularity:

- $p_3 = \gamma(\beta E_3^* + p_2^*) > 0 \Rightarrow$
particles 2 and 3 move in the same direction in the rest frame of initial particle
- velocities in the rest frame of the initial particle: $v_3 > \beta > v_2$

$$v_2 = \beta \frac{E_2^* - p_2^*/\beta}{E_2^* - \beta p_2^*} < \beta, \quad v_3 = \beta \frac{E_3^* + p_2^*/\beta}{E_3^* + \beta p_2^*} > \beta$$

particle 3 moves faster than particle 2 in the rest frame of initial particle

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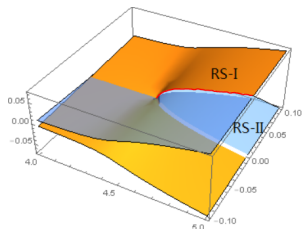
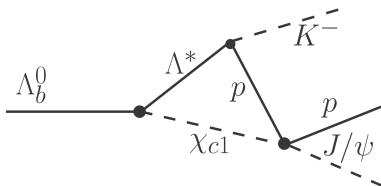
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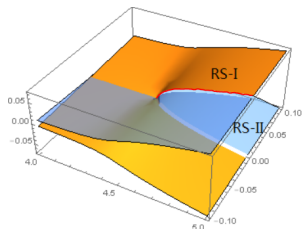
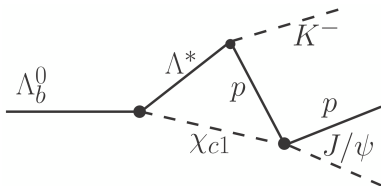
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Coleman-Norton theorem



- Coleman–Norton theorem: S. Coleman and R. E. Norton, Nuovo Cim. 38 (1965) 438
The singularity is on the **physical boundary** if and only if the diagram can be interpreted as a classical process in space-time.
 - ☞ **physical boundary**: upper edge (lower edge) of the unitary cut in the first (second) Riemann sheet
- Translation:
 - ☞ all three intermediate states can go **on shell**
 - ☞ $\vec{p} \parallel \vec{p}_{\chi_{c1}}$, the proton can catch up with the χ_{c1} to rescatter

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Analysis of the kinematics

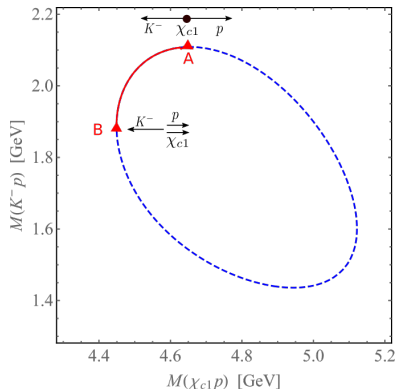
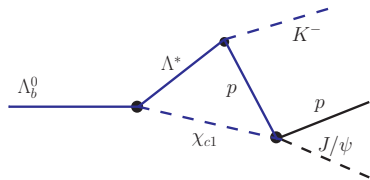
Dalitz plot for $\Lambda_b \rightarrow \chi_{c1} \Lambda^* \rightarrow \chi_{c1} p \bar{K}$:

Starting from a large Λ^* mass, in Λ_b rest frame

- when $M_{\Lambda^*} > M_{\Lambda_b} - M_{\chi_{c1}}$, cannot go on-shell
- at point **A**, $M_{\Lambda^*} = M_{\Lambda_b} - M_{\chi_{c1}}$, χ_{c1} is at rest
- at point **B**, proton and χ_{c1} has the same velocity, $p \chi_{c1}$ threshold
- **between A and B**, $\vec{p}_p \parallel \vec{p}_{\chi_{c1}}$ and proton moves **faster** than χ_{c1}

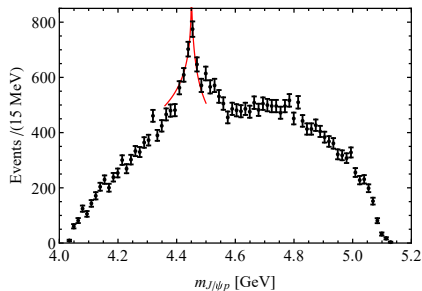
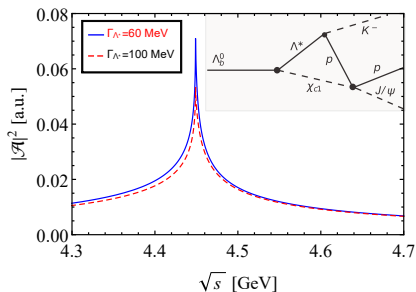
$$M_{K^- p, A} = M_{\Lambda_b} - M_{\chi_{c1}},$$

$$M_{K^- p, B} = \sqrt{\frac{M_{\Lambda_b}^2 M_p + M_K^2 M_{\chi_{c1}}}{M_{\chi_{c1}} + M_p}} - M_{\chi_{c1}} M_p$$



Triangle singularity for $P_c(4450)$

- At point B, the triangle singularity is **exactly at the $\chi_{c1}p$ threshold, 4.449 GeV**, requiring $M_{\Lambda^*} = 1.89$ GeV
- Coincidentally, **four-star baryon $\Lambda(1890)$** : $J^P = 3/2^+$, $\Gamma : 60 - 200$ MeV
- triangle loop with **S -wave $\chi_{c1}p$** : $J^P = \frac{1}{2}^+, \frac{3}{2}^+$



- impossible to produce a narrow peak for $\chi_{c1}\bar{p}$ in other partial waves

More comments

Strength of the triangle singularity is determined by

- couplings:

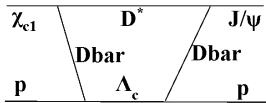
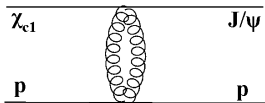
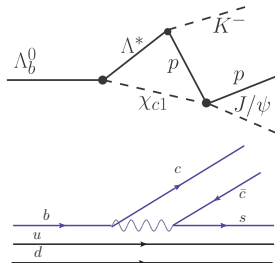
- $\Lambda_b \rightarrow \Lambda^* \chi_{c1}$ is from $b \rightarrow c\bar{c}s$, not measured, but should have a sizeable branching fraction:

$$\text{Br}(B^+ \rightarrow J/\psi K^+) \simeq 1 \times 10^{-3},$$

$$\text{Br}(B^+ \rightarrow \chi_{c1} K^+) \simeq 0.5 \times 10^{-3}$$

- $\Lambda^*(1890) \rightarrow N \bar{K}$: largest branching fraction, Br= 20 – 35%

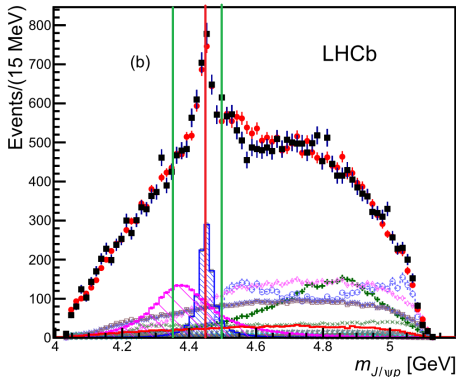
- $\chi_{c1} p \rightarrow J/\psi p$: OZI suppressed, $\mathcal{O}(1/N_c)$ [recall: OZI suppressed meson-meson scattering: $\mathcal{O}(1/N_c^2)$]



lattice QCD predicts possible $c\bar{c}$ -nucleus bound states at $M_\pi = 805$ MeV

More triangle singularities?

- $\chi_{c0,c1,c2} p \rightarrow J/\psi p$ are related through heavy quark spin symmetry
 - Weak decay $b \rightarrow c\bar{c}s$, $V - A \xrightarrow{\text{Fierz}} \bar{c}\gamma^\mu(1 - \gamma_5)c$
 - ☞ $\Lambda_b \rightarrow \Lambda^* J/\psi$ and $\Lambda_b \rightarrow \Lambda^* \chi_{c1}$ are easy
 - ☞ for χ_{c2} : strongly suppressed, χ_{c0} : also suppressed; for $B^+ \rightarrow \chi_{cJ} K^+$
 $\text{Br}_1 \simeq 5 \times 10^{-4} > \text{Br}_0 \simeq 1.5 \times 10^{-4} \gg \text{Br}_2 \simeq 1.1 \times 10^{-5}$
- ⇒ no obvious peak around the $\chi_{c0} p$ or $\chi_{c2} p$ threshold



- Two coincidences for the LHCb $P_c(4450)$ structure:
 - ☞ located exactly at the $\chi_{c1} p$ threshold
 - ☞ four-star $\Lambda(1890)$ makes a triangle singularity exactly at the same position
- To control the strength, we need:
 - ☞ $\text{Br}(\Lambda_b \rightarrow \Lambda^*(1890)\chi_{c1}) \leftarrow \text{LHCb}$
 - ☞ $\chi_{c1} p \rightarrow J/\psi p$, might get information from lattice QCD
- More measurements are necessary to reveal the nature of the $P_c(4450)$
 - ☞ J^P unambiguously
 - ☞ $\Lambda_b \rightarrow \chi_{c1} p K$
 - ☞ searching for $P_c(4450)$ in processes with a different kinematics

THANK YOU FOR YOUR ATTENTION!

Backup slides

- Some recent work using **triangle singularity** to explain (part of) peak structures [$\eta(1405/1475)$, $a_1(1420)$, ...]:

J. J. Wu, X. H. Liu, Q. Zhao and B. S. Zou, PRL108(2012)081803;

X. G. Wu, J. J. Wu, Q. Zhao and B. S. Zou, PRD87(2013)014023(2013);

Q. Wang, C. Hanhart and Q. Zhao, PLB725(2013)106;

M. Mikhasenko, B. Ketzner and A. Sarantsev, PRD91(2015)094015;

N. N. Achasov, A. A. Kozhevnikov and G. N. Shestakov, PRD92(2015)036003;

X. H. Liu, M. Oka and Q. Zhao, PLB753(2016)297;

A. P. Szczepaniak, PLB747(2015)410; PLB757(2016)61;

F. Aceti, L. R. Dai and E. Oset, arXiv:1606.06893 [hep-ph];

.....

- Very old knowledge from 1960s:

Classical books:

R. J. Eden, P. V. Landshoff, D. I. Olive and
J. C. Polkinghorne, *The Analytic S-Matrix*
Cambridge University Press, 1966.

T.-S. Chang, *Introduction to Dispersion Relation*
(2 volumes, in Chinese, written in 1965),
Science Press, Beijing 1980, 1983.

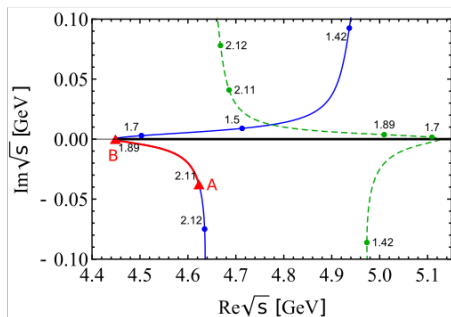
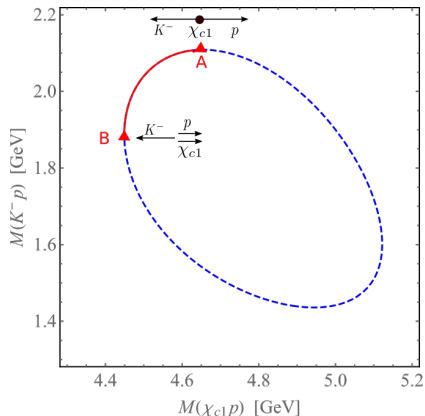
Recent lecture notes by one of the key players:

I. J. R. Aitchison, arXiv:1507.02697 [hep-ph].
*Unitarity, Analyticity and Crossing Symmetry in Two-
and Three-hadron Final State Interactions.*



Tsung-Sui Chang
(1915–1969)

Trajectories of triangle singularities in complex energy plane



numbers: assumed masses for Λ^*

blue: proton and χ_{c1} are parallel, in the 2nd Riemann sheet

green: proton and χ_{c1} are anti-parallel

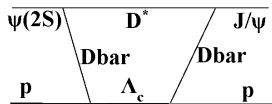
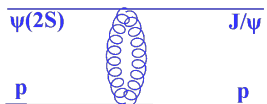
$$M_{\Lambda_b} = 5.62 \text{ GeV}, M_{\chi_{c1}} = 3.51 \text{ GeV}, \quad \sqrt{s} \equiv M(\chi_{c1}p)$$

$$M_{K^-p,A} = M_{\Lambda_b} - M_{\chi_{c1}}, \quad M_{K^-p,B} = \sqrt{\frac{M_{\Lambda_b}^2 M_p + M_K^2 M_{\chi_{c1}}}{M_{\chi_{c1}} + M_p} - M_{\chi_{c1}} M_p}$$

More triangle singularities?

Considering other possible $c\bar{c}-\Lambda^*$ combinations:

- $h_c, \eta_c(1S, 2S)$: spin-singlet
 $\Rightarrow h_c[\eta_c(1S, 2S)]p \rightarrow J/\psi p$ breaks **heavy quark spin symmetry**, suppressed relative to $\chi_{c1}p \rightarrow J/\psi p$
- J/ψ :
 $J/\psi p \rightarrow J/\psi p$: elastic, no peak will show up (due to Schmid theorem)
- $\psi(2S)$: radial excitation different from J/ψ



in comparison with $\chi_{c1}p \rightarrow J/\psi p$

left: **strongly suppressed**; right: might be slightly suppressed, not very clear

For possible triangle singularities for $\Lambda_b \rightarrow J/\psi p K$, the $\chi_{c1}-\Lambda^*(1890)$ seems the **most prominent** one among all $c\bar{c}-\Lambda^*$ combinations

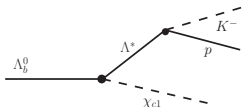
How to distinguish triangle-singularity from genuine resonance?

- Schmid theorem:

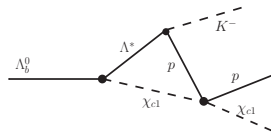
C. Schmid, Phys. Rev. 154 (1967) 1363

see also, A. V. Anisovich, V. V. Anisovich, Phys. Lett. B 345 (1995) 321

Triangle singularity **cannot** produce an additional peak in the invariant mass distribution of the **elastic channel** when neglecting inelasticity



(a)



(b)

Nearby the effective singularity: $\mathcal{A}_{(a)+(b)}(s) \sim e^{2i\delta_{\chi_{c1}p}(s)} \mathcal{A}_{(a)}(s)$

here $\delta_{\chi_{c1}p}$ is the elastic $\chi_{c1}p$ scattering phase shift

- corrections from **coupled channels**

A. Szczepaniak, PLB757(2016)61

How to distinguish triangle-singularity from genuine resonance?

- Method-1: measuring the process $\Lambda_b^0 \rightarrow \chi_{c1} p K^-$
 - ☞ if a narrow near-threshold peak in $\chi_{c1} p \Rightarrow$ a real exotic resonance
- Method-2: processes (such as photoproduction) with a different kinematics
 - Q. Wang, X.-H. Liu, Q. Zhao, PRD92(2015)034022;
 - V. Kubarovsky, M. Voloshin, PRD92(2015)031502;
 - M. Karliner, J. L. Rosner, PLB752(2015)329; ...