Exploring Crossing and Analyticity of DVCS Amplitude

DVCS Workshop

Bochum, February 12, 2014

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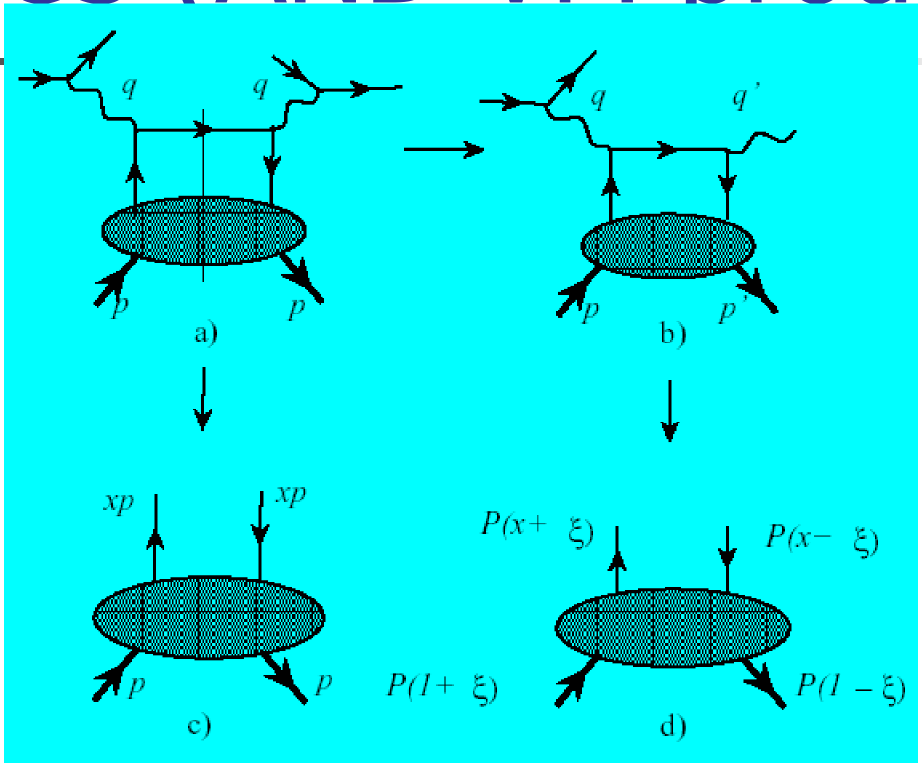
Main Topics

- Analyticity vs QCD factorization
- Crossing, Tomography, Holography
- Analyticity and real-photon limit
- D-term in various processes
- GPDs and (gravitational) formfactors; D-term and inflation

QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – if $1 \leq |x_B|$ polynomial in $1/x_B$

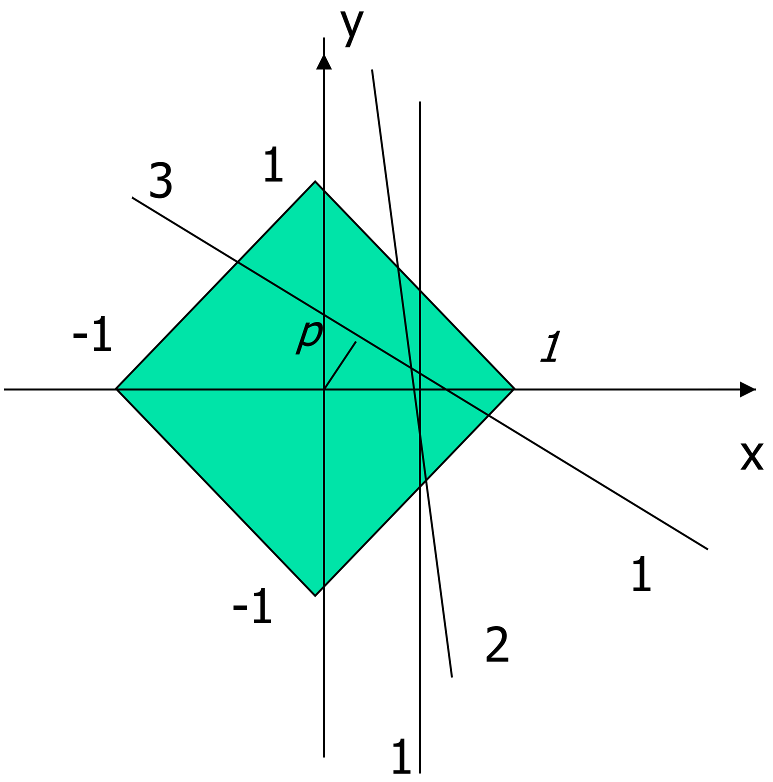
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line- skewness
- Kinematics of DIS: $\xi = 0$
  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

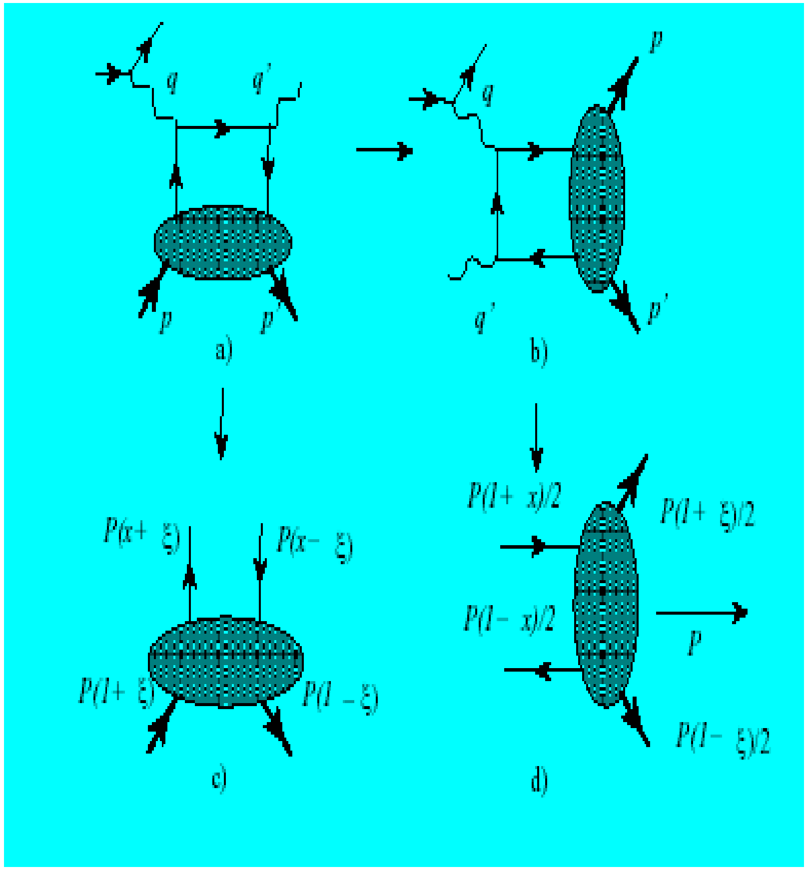


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in *x* weighted with x^n - are polynomials in $1/\xi$ of power *n+1*
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Radon Tomography

- Require 2 channels (calls for universal description of GPDs and GDAs)
- Performed (Gabdrakhmanov,OT'12) for photon (Pire, Szymanowski, Wallon, Friot, El Beiyad) GPDs/GDAs

$$F_{1D}(\beta,\alpha) = [2(1-|\beta|-|\alpha|) - 1 + \delta(\alpha)]sgn(\beta), \qquad D_1(\alpha) = (|\alpha|-1)(2|\alpha|+1)sgn(\alpha)$$

- Realistic case – very difficult numerically
- Limited angle tomography – in progress

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity ("dynamical") -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

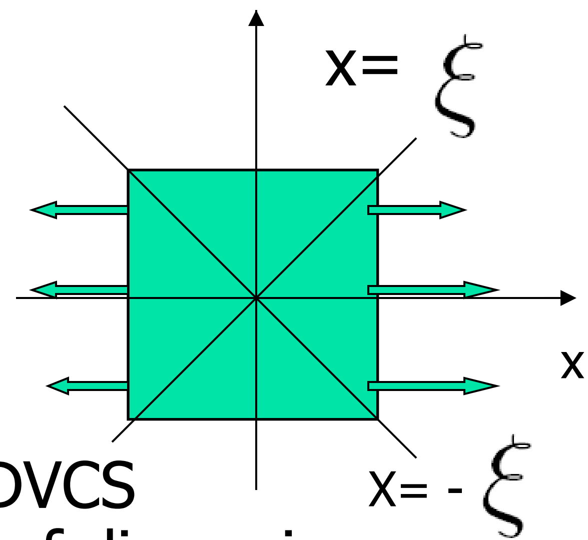
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta \mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- ERBL → "GDA" region
- Strategy (now adopted) of GPD's
  studies: start at diagonals
  (through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Holographic property - IV

- Follows directly from DD -> preserved by (LO) evolution; NLO –Diehl, D.Ivanov’07
- Asymptotic GPD -> Pure real DVCS Amplitude (=subtraction term) growing like ξ^{-2}
- Direct consequence of finite asymptotic value of the quark momentum fraction

Holography vs NLO

- 3D -> 2D generally speaking is lost
- Depends on factorization scheme
- Special role of scheme preserving the coefficient function (DVCS scheme – D. Mueller, K. Kumericky)- but leads to complex GPDs for TCS
- Nucleon from the point of view of information ~ (momentum space!) black hole – 3D information encoded in 2D
- Measuring procedure (in particular, factorization scheme) dependent – c.f. " Black hole complementarity"

Angular distribution in hadron pairs production

- Back to GDA region
- Moments of H(x,x) - define the coefficients of powers of cosine!– $1/\xi$
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Continuation of D-term from t to s channel – dispersion relation in t (Pasquini, Vanderhaegen)

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\,\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\,\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\,\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\mathrm{CQM}}(2)=\Delta^n_{\mathrm{CQM}}(2)\approx 4.4, \qquad \Delta^p_{\mathrm{latt}}\approx\Delta^n_{\mathrm{latt}}\approx 1.1$$

- Numerically close to Thomson term for REAL proton (but NOT neutron) Compton Scattering!
- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!
- Stability of subtraction against NPQCD? twist -3 – related to twist 2 (Moiseeva, Polyakov'08) Large -> small Q (talk of M. Strikman)?

HT summation and analyticity(OT'13)

- For HT series $c_i = \langle f(x)\, x^i \rangle$ - moments of HT "density"- geometric series rather than exponent: $\Sigma\, c_i\, (- M^2 / Q^2) = \langle M^2\, f(x)/(x M^2 + Q^2) \rangle$
- Supported by analyticity in Q^2
- DIS -> good description of JLab data on BjSR at all Q^2
- Similar representation for subtraction constant $D(Q^2, t) = D_0(t) + \langle Q^2\, d(x)/(x M^2 + Q^2) \rangle$

2-photon scattering

- Real photons limit

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2 - \nu^2)} + \Delta$$

- v=(s-u)/4M -> (t-u)/4M

- Scattering at 90^0 in c.m. is defined by subtractiion constant
- Dominance of Thomson term (better for proton-antiproton – sum of charges squared argument)

Is D-term independent?

- Fast enough decrease at large energy ->

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2 - \nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$

$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$

$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

“D – term” 30 years before…

- Cf Brodsky, Close, Gunion’72
- D-term – a sort of renormalization constant
- Recover through special regularization procedure (D. Mueller,K. Semeneov-Tyan-Shansky)?
- Cf mass-shell (‘physical”) and MS renormalizations

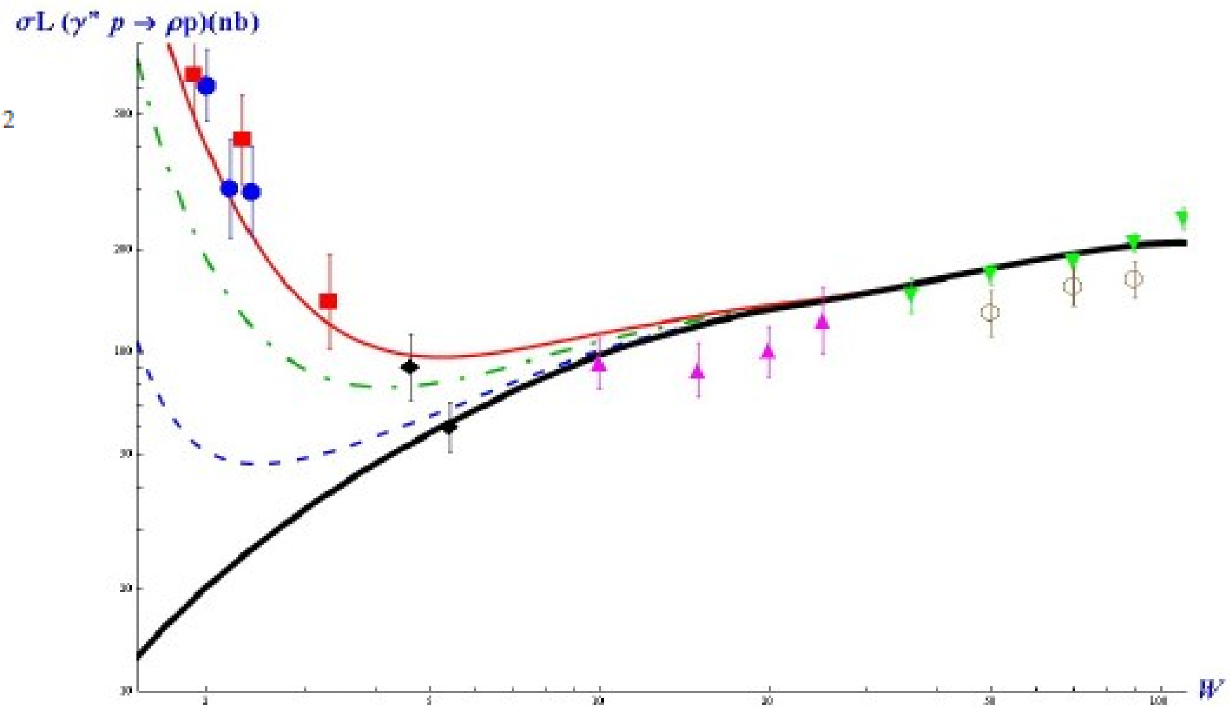
Subtraction in exclusive electroproduction (absent in Brodsky et al. approach)

- May qualitatively explain the low energy enhancement (Gabdrakhmanov,OT'12)
- GK model

$$\sigma(W) \approx \sigma_0(W) \left| \frac{A_{Collinear}(W) + a \cdot \Delta}{A_{Collinear}(W)} \right|^2$$

- a=3,4.8
- Large x

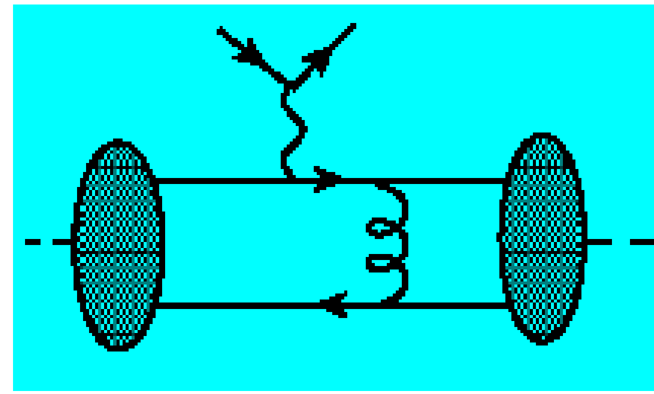
$$H(x,x) < const \sqrt{H(x,0)}$$



Applications of analyticity: complicated hard reactions

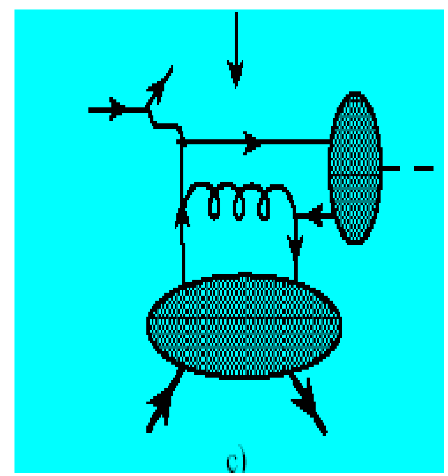
- Starting from (Pion) form factor- 2 DA's

$$F \sim \left(\int \frac{\phi(x)}{1-x}\right)^2$$

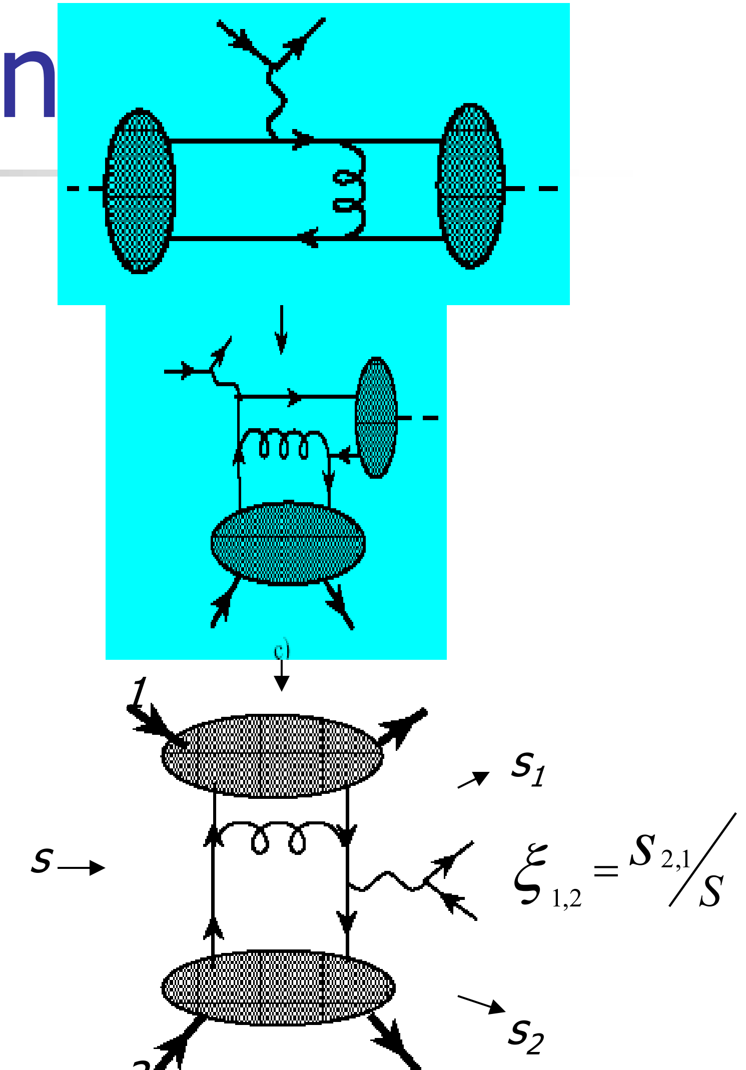


- 1 DA -> GPD :Exclusive mesons production (Frankfurt, Strikman): analytic continuation =factorization + D-subtraction

$$M \sim \int \frac{\phi(x)}{1-x} \int dx \frac{H(x,\xi)}{x-\xi+i\varepsilon}$$



Next step: 2 DA's -> 2 GPD's- Double Diffraction



- Exclusive double diffractive DY process
- Analytic continuation:

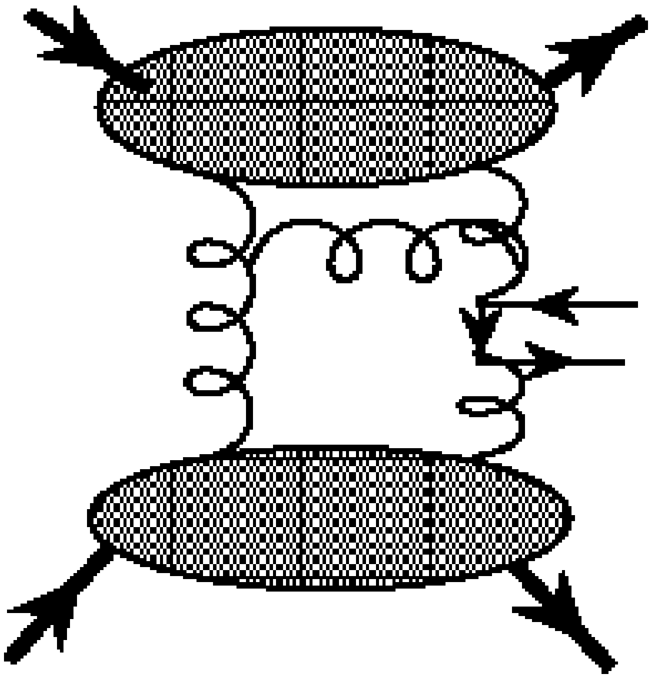
$$M \sim \int dx \frac{H(x,\xi_1)}{x-\xi_1 \pm i\varepsilon}\int dy \frac{H(y,\xi_2)}{y-\xi_2 \mp}$$

- DIFFERS from direct calculation – NO factorization in physical region

$$M \sim \int dx \int dy \frac{H(x,\xi_1)H(y,\xi_2)}{(x-\xi_1)(y-\xi_2)+i\varepsilon}$$

Double Diffraction: gluons

- One or both GPDs may be gluonic
- Complementary description of LHC DD (Higgs, Quarkonia, dijets)



Double Diffraction: properties and problems

- Holographic equation: DR contains double and single (linear in D-term) dispersion integrals as well as subtraction (quadratic in D-term)
- Analytic continuation in relation to

various cuts is still unclear. Possible cancellation of cuts – real amplitude?

D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF – 1st moment of D-term

$$\langle P + q/2|T^{\mu\nu}|P - q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Moment of D-term – positive
- Vacuum – Cosmological Constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$

- 2D effective CC – negative in scattering, positive in annihilation

$$\Lambda = C(q^2)q^2$$

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe

1-st moments - EM, 2-nd - Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe ainteraction with both classical and TeV gravity

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q) \qquad M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu \qquad \sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q) \qquad M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT’07)
- Anomalous gravitomagnetic moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Gravitomagnetism

- Gravitomagnetic field – action on spin – ½ from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM $\vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$

Equivalence principle for moving particles

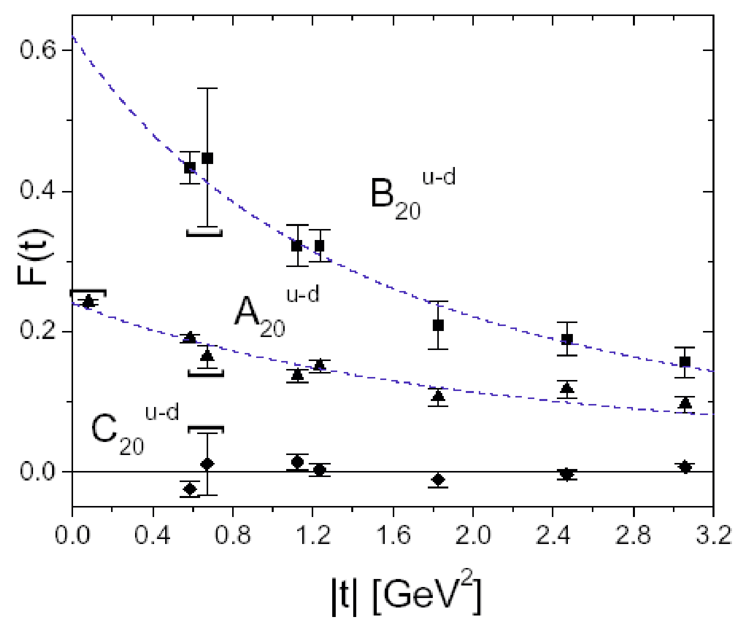
- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

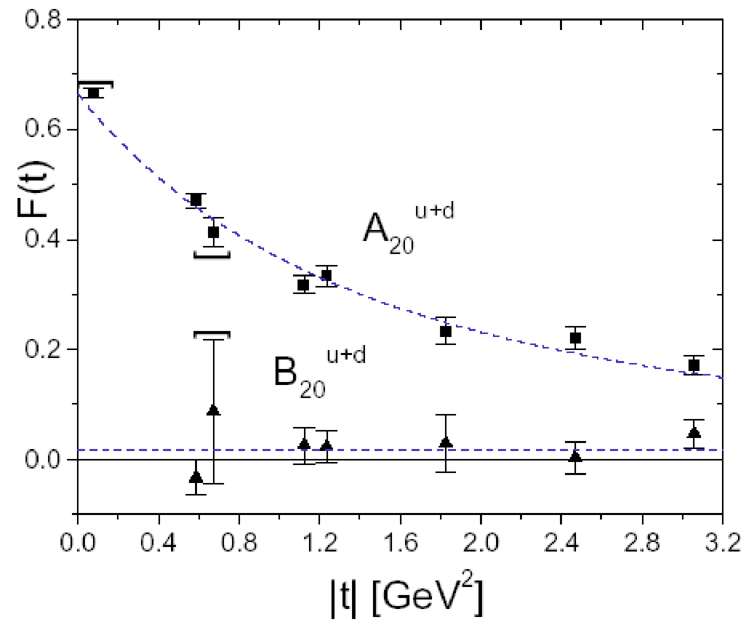
$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, O.T.)

Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- May lead to gravity-resistant (also in BH) confinement
- Supported by smallness of E (isoscalar AMM)
- Polyakov Vanderhaeghen: dual model with E=0

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho’s AMM gives g close to 2.

- Maybe because of similarity of moments
- $g-2=\langle E(x)\rangle$; $B=\langle xE(x)\rangle$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP:

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

EEP and Sivers function

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_{T}(x) \square$

- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_{T}(x) = \sum_{q,G} \int dx x E(x) = 0$$

EEP and Sivers function for deuteron

- ExEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^\rho (J^\alpha_{5S} x^\nu) = 0.$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P | J^\alpha_{5S} | P+q \rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P | J^\gamma_{5S} | P+q \rangle$$

$$\langle P, S | J^5_\mu(0) | P+q, S \rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$

CONCLUSIONS

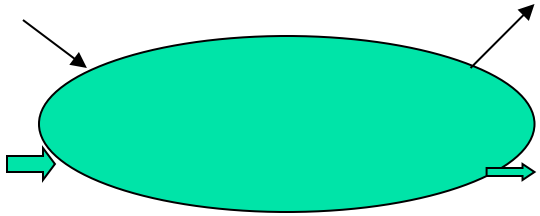
- Crossing analogs of GPD -> GDA
- Tomography: global fits (modeling) for GPD/GDA desirable. Limited angle tomography under investigation
- Analyticity for DVCS – holographic property of GPD's: special role of $x = \pm\xi$ sections 3D -> 2D and subtraction due to D-term
- D-term – link between hard/soft?
- Importance of GPDs – relation to gravity via Gravitational formfactors. Unique way to probe coupling to quarks and gluons seperately. D-term – analogy between inflation and annihilation

Radon (OT’01) and Abel (Moiseeva, Polyakov’08) Transforms: even vs odd-dimensional spaces

- Even (integrals over lines in plane): integral (global) inversion formula
- Odd (integrals over planes in space) – differential (local) inversion formula – Huygens principle
- Triple distributions – THREE pions production (Pire, OT’01) or (deuteron) Decay PD. Relation to nuclei breakup in studies of SRC?!

Relating GPDs and TMDs

- GTMD ~ qH amplitude s = - k_T^2



- Duality – Veneziano-like expression

$$\mathcal{A}(s,t) \sim \frac{\Gamma(1-\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}; \qquad \alpha_s = \alpha_0 + \alpha' s$$

- x-moments to have dipole expressions?