Spacelike and timelike Compton scattering at LO and NLO

Lech Szymanowski

National Centre for Nuclear Research (NCBJ), Warsaw

in collaboration with D. Müller, B. Pire , J. Wagner

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Deeply Virtual Compton Scattering: From Observables to GPDs

Ruhr-Universität Bochum, February 10-12, 2014

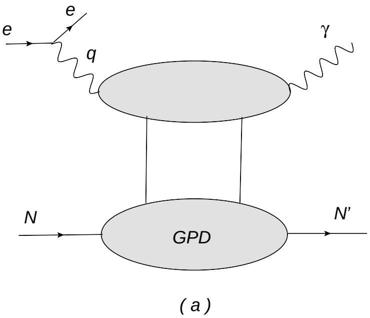


Figure: Deeply Virtual Compton Scattering : $lN \to l'N'\gamma$

Skewness ξ:

$$\xi = -\frac{(P' - P)n}{(P + P')n}$$

TCS

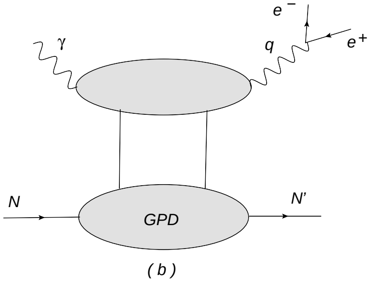


Figure: Timelike Compton Scattering: $\gamma N \to l^+ l^- N'$

Skewness η:

$$\eta = -\frac{(P' - P)n}{(P + P')n}$$

Why TCS?

- GDPs enter factorization theorems for hard exlusive reactions (DVCS, deeply virtual meson production, TCS etc.), in a similar manner as PDFs enter factorization theorem for DIS
- First moment of GPDs enters the Ji's sum rule for the angular momentum carried by partons in the nucleon,
- Deeply Virtual Compton Scattering (DVCS) is a golden channel for GPDs extraction,
- Why TCS: universality of the GPDs, spacelike-timelike crossing and understanding the structure of the NLO corrections,
- Experiments *at low energy*: CLAS 6 GeV $\rightarrow$ CLAS 12 GeV, *at high energy*: COMPASS, RHIC, LHC and AFTER@LHC ?

Coordinates

Berger, Diehl, Pire, 2002

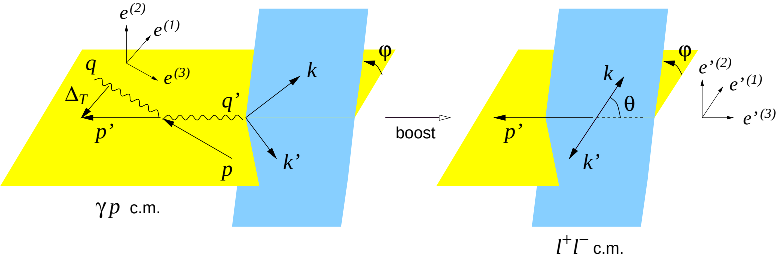


Figure: Kinematical variables and coordinate axes in the γp and $\ell^+\ell^-$ c.m. frames.

The Bethe-Heitler contribution

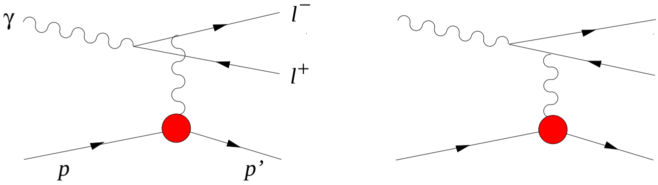


Figure: The Feynman diagrams for the Bethe-Heitler amplitude.

$$\frac{d\sigma_{BH}}{dQ'^2 dt d\cos\theta} \approx 2\alpha^3 \frac{1}{-tQ'^4} \frac{1+\cos^2\theta}{1-\cos^2\theta} \left(F_1(t)^2 - \frac{t}{4M_p^2} F_2(t)^2 \right),$$

For small θ BH contribution becomes very large

The Compton contribution

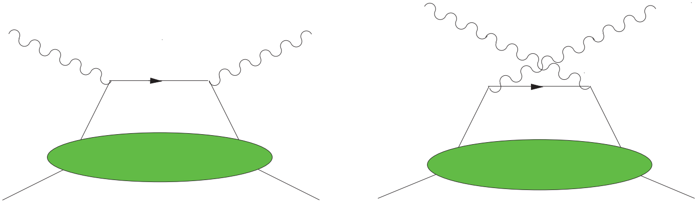


Figure: Handbag diagrams for the Compton process in the scaling limit.

$$\frac{d\sigma_{TCS}}{dQ'^2 d\Omega dt} \approx \frac{\alpha^3}{8\pi} \frac{1}{s^2} \frac{1}{Q'^2} \left(\frac{1+\cos^2\theta}{4} \right) 2(1-\xi^2) \left| \mathcal{H}(\xi,t) \right|^2,$$

$$\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx T(x,\xi,Q') H^q(x,\xi,t),$$

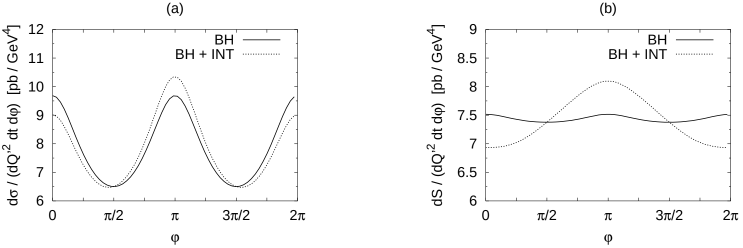
Interference

The interference part of the cross-section for $\gamma p \to \ell^+ \ell^- p$ with unpolarized protons and photons is given at leading order by

$$\frac{d\sigma_{INT}}{dQ'^2\, dt\, d\cos\theta\, d\varphi} \sim \cos\varphi\, \mathrm{Re}\,\mathcal{H}(\xi, t)$$

Linear in GPD's, odd under exchange of the l^+ and l^- momenta $\Rightarrow$ angular distribution of lepton pairs is a good tool to study interference term.

Berger, Diehl, Pire, 2002



B-H dominant for small energies;

Rafayel Paremuzyan PhD thesis

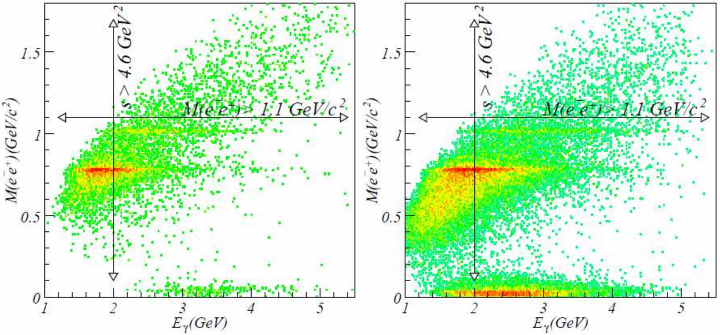


Figure: e^+e^- invariant mass distribution vs quasi-real photon energy. For TCS analysis $M(e^+e^-) > 1.1\,\mathrm{GeV}$ and $s_{\gamma p} > 4.6\,\mathrm{GeV}^2$ regions are chosen. Left graph represents e1-6 data set, right one is from e1f data set.

There is more data from g12 data set, soon to be analyzed. 12 GeV upgrade enables exploration of invariant masses up to $Q^2 = 9\,\mathrm{GeV}^2$ mass.

Theory vs experiment

R.Paremuzyan and V.Guzey:

$$R = \frac{\int d\phi \ \cos\phi \ d\sigma}{\int \ d\phi \ d\sigma}$$

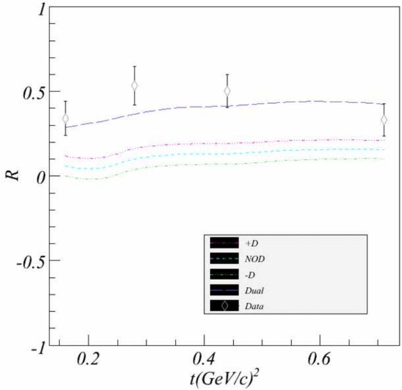


Figure: Theoretical prediction of the ratio R for various GPDs models.

Motivation for NLO

Why do we need NLO corrections to TCS:

- gluons enter at NLO,
- DIS versus Drell-Yan: big K-factors $\log \frac{-Q^2}{\mu_F^2} \to \log \frac{Q^2}{\mu_F^2} \pm i\pi$,
- reliability of the results, factorization scale dependence,

X.D.Ji and J. Osborne, Phys. Rev. D58 (1998)
A. Belitsky, D. Mueller, Phys. Lett. B 417 (1998)
Belitsky, Mueller, Niedermeier, Schafer, Phys.Lett.B474 ,2000.
Pire, L.Sz., Wagner, Phys.Rev.D83, 2011.

General Compton Scattering:

$$\gamma^*(q_{in})N \to \gamma^*(q_{out})N'$$

- DVCS: $q_{in}^2 < 0\,, \quad q_{out}^2 = 0$
- TCS: $q_{in}^2 = 0\,, \quad q_{out}^2 > 0$
- DDVCS: $q_{in}^2 < 0\,, \quad q_{out}^2 > 0$

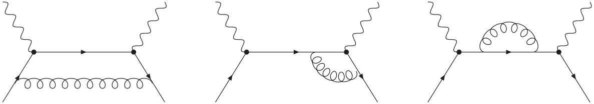
Amplitude:

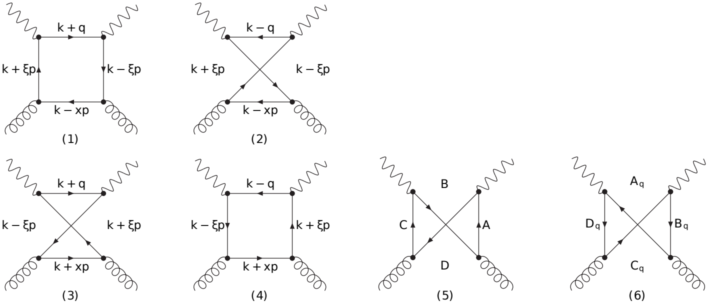
$$\mathcal{A}^{\mu\nu} = -g_T^{\mu\nu} \int_{-1}^{1} dx \left[\sum_q^{n_F} T^q(x) F^q(x) + T^g(x) F^g(x) \right] + i\epsilon_T^{\mu\nu} \int_{-1}^{1} dx \left[\sum_q^{n_F} \widetilde{T}^q(x) \widetilde{F}^q(x) + \widetilde{T}^g(x) \widetilde{F}^g(x) \right] ,$$

$$\mathcal{H}(\xi, t) \sim \int_{-1}^{1} dx T(x, \xi, Q') F(x, \xi, t),$$

Compton Form Factor

Diagrams




k + q
k + ξp
k − ξp
k − xp
(1)
k − q
k + ξp
k − ξp
k − xp
(2)
k + q
k − ξp
k + ξp
k + xp
(3)
k − q
k − ξp
k + ξp
k + xp
(4)
B
C
A
D
(5)
A q
D q
B q
C q
(6)

Method of calculation

X.D.Ji and J. Osborne, Phys. Rev. D58 (1998)

- algebraic reduction of number of propagators
- dimensional regularization
- after renormalization only infrared divergences (IR) remain as poles $\frac{1}{\epsilon}$
- IR divergences removed with the help of evolution equation

$\Longrightarrow$ factorization scale μ_F appears

Structure of scattering amplitude:

$$\begin{aligned}\mathcal{A}^{\mu\nu} &= -g_T^{\mu\nu}\int_{-1}^{1} dx \left[\sum_q^{n_F} T^q(x)F^q(x) + T^g(x)F^g(x)\right] \\ &+ i\epsilon_T^{\mu\nu}\int_{-1}^{1} dx \left[\sum_q^{n_F} \widetilde{T}^q(x)\widetilde{F}^q(x) + \widetilde{T}^g(x)\widetilde{F}^g(x)\right],\end{aligned}$$

$$\begin{aligned}T^q(x) &= \left[C_0^q(x) + C_1^q(x) + \ln\left(\frac{Q^2}{\mu_F^2}\right)\cdot C_{coll}^q(x)\right] - (x \to -x)\,, \\ T^g(x) &= \left[C_1^g(x) + \ln\left(\frac{Q^2}{\mu_F^2}\right)\cdot C_{coll}^g(x)\right] + (x \to -x)\,, \\ \widetilde{T}^q(x) &= \left[\widetilde{C}_0^q(x) + \widetilde{C}_1^q(x) + \ln\left(\frac{Q^2}{\mu_F^2}\right)\cdot \widetilde{C}_{coll}^q(x)\right] + (x \to -x)\,, \\ \widetilde{T}^g(x) &= \left[\widetilde{C}_1^g(x) + \ln\left(\frac{Q^2}{\mu_F^2}\right)\cdot \widetilde{C}_{coll}^g(x)\right] - (x \to -x)\,.\end{aligned}$$

DVCS:
Quark coefficient functions:

$$
\begin{aligned}
C_0^q(x,\xi) &= -e_q^2 \frac{1}{x+\xi-i\varepsilon}, \\
C_1^q(x,\xi) &= \frac{e_q^2 \alpha_S C_F}{4\pi} \frac{1}{x+\xi-i\varepsilon}\left[9 - 3\frac{x+\xi}{x-\xi}\log(\frac{x+\xi}{2\xi}-i\varepsilon) - \log^2(\frac{x+\xi}{2\xi}-i\varepsilon)\right], \\
C_{coll}^q(x,\xi) &= \frac{e_q^2 \alpha_S C_F}{4\pi} \frac{1}{x+\xi-i\varepsilon}\left[-3 - 2\log(\frac{x+\xi}{2\xi}-i\varepsilon)\right], \\
\widetilde{C}_0^q(x,\xi) &= -e_q^2 \frac{1}{x+\xi-i\varepsilon}, \\
\widetilde{C}_1^q(x,\xi) &= \frac{e_q^2 \alpha_S C_F}{4\pi} \frac{1}{x+\xi-i\varepsilon}\left[9 - \frac{x+\xi}{x-\xi}\log(\frac{x+\xi}{2\xi}-i\varepsilon) - \log^2(\frac{x+\xi}{2\xi}-i\varepsilon)\right], \\
\widetilde{C}_{coll}^q(x,\xi) &= \frac{e_q^2 \alpha_S C_F}{4\pi} \frac{1}{x+\xi-i\varepsilon}\left[-3 - 2\log(\frac{x+\xi}{2\xi}-i\varepsilon)\right],
\end{aligned}
$$

where $C_F = (N_c^2-1)/(2N_c)$

DVCS
Gluon coefficient functions:

$$
\begin{aligned}
C_1^g(x,\xi) &= \frac{\Sigma e_q^2 \alpha_S T_F}{4\pi} \frac{1}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \times \\
&\quad \left[2\frac{x+3\xi}{x-\xi} \log\left(\frac{x+\xi}{2\xi} - i\varepsilon\right) - \frac{x+\xi}{x-\xi} \log^2\left(\frac{x+\xi}{2\xi} - i\varepsilon\right) \right], \\
C_{coll}^g(x,\xi) &= \frac{\Sigma e_q^2 \alpha_S T_F}{4\pi} \frac{2}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \left[-\frac{x+\xi}{x-\xi} \log\left(\frac{x+\xi}{2\xi} - i\varepsilon\right) \right], \\
\widetilde{C}_1^g(x,\xi) &= \frac{\Sigma e_q^2 \alpha_S T_F}{4\pi} \frac{1}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \times \\
&\quad \left[-2\frac{3x+\xi}{x-\xi} \log\left(\frac{x+\xi}{2\xi} - i\varepsilon\right) + \frac{x+\xi}{x-\xi} \log^2\left(\frac{x+\xi}{2\xi} - i\varepsilon\right) \right], \\
\widetilde{C}_{coll}^g(x,\xi) &= \frac{\Sigma e_q^2 \alpha_S T_F}{4\pi} \frac{2}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \left[\frac{x+\xi}{x-\xi} \log\left(\frac{x+\xi}{2\xi} - i\varepsilon\right) \right],
\end{aligned}
$$

where $T_F = \frac{1}{2}$

D. Mueller, B. Pire, L. Sz. and J. Wagner, Phys. Rev. D 86 (2012)

The relation between the coefficient functions for NLO DVCS and NLO TCS:

$$
\begin{aligned}
{}^{TCS}T^q(x,\xi_T) &= +\left[{}^{DVCS}T^q + i\pi C^q_{coll}\right]^* (x,\xi_S)|_{\xi=\eta} \\
{}^{TCS}T^g(x,\xi_T) &= +\left[{}^{DVCS}T^g + i\pi C^g_{coll}\right]^* (x,\xi_S)|_{\xi=\eta} \,.
\end{aligned}
$$

for ξ symmetric CFF

$$
\begin{aligned}
{}^{TCS}\tilde{T}^q(x,\xi_T) &= -\left[{}^{DVCS}\tilde{T}^q + i\pi \tilde{C}^q_{coll}\right]^* (x,\xi_S)|_{\xi=\eta} \\
{}^{TCS}\tilde{T}^g(x,\xi_T) &= -\left[{}^{DVCS}\tilde{T}^g + i\pi \tilde{C}^g_{coll}\right]^* (x,\xi_S)|_{\xi=\eta} \,.
\end{aligned}
$$

for ξ antisymmetric CFF

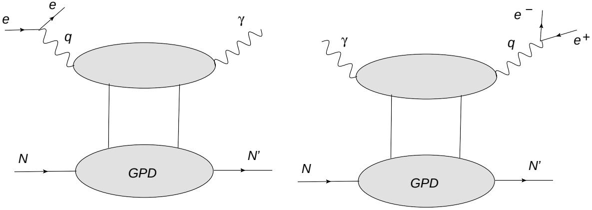
$$\xi_S = \xi - i\epsilon \qquad\qquad \xi_T = \xi_S^*|_{\xi=\eta} = \eta + i\epsilon$$

$$e(k_1)N(p) \to e'(k_2)\gamma(q_{out})N'(p')$$

with a spacelike $q_{in}^2 = (k_1 - k_2)^2 = -\mathcal{Q}^2$

$$\gamma(q_{in})N(p) \to l^+(k^+)l^-(k^-)N'(p')$$

with a timelike $q_{out}^2 = (k^+ + k^-)^2 = +\mathcal{Q}^2$.



(a)   (b)

the scaling variable ξ and skewness $\eta > 0$:

$$\xi = -\frac{q_{out}^2 + q_{in}^2}{q_{out}^2 - q_{in}^2}\eta\,, \qquad \eta = \frac{q_{out}^2 - q_{in}^2}{(p+p')\cdot(q_{in}+q_{out})}\,.$$

$\xi = +\eta > 0$ in DVCS and $\xi = -\eta < 0$ in TCS

• the rule:

$$\mathcal{F}(\xi=\eta,t,\mathcal{Q}^2)\overset{\textsf{SL}\to\textsf{TL}}{\Longrightarrow}\mathcal{F}(\xi=-\eta,t,-\mathcal{Q}^2)\,,$$

where the c.o.m. energy square $s=(p+q_{in})^2$ might differ

• DVCS: physical sheet

$$\xi=\frac{Q^2}{2s+Q^2}\ \to\ \xi_S=\frac{Q^2}{2(s+i\,\epsilon)+Q^2}=\xi-i\,\epsilon$$

• TCS: physical sheet

$$\xi_S\ \overset{SL\to TL}{\to}\ -\eta-i\,\epsilon=-\xi_T\ ,\qquad \xi_T=\xi_S^*|_{\xi=\eta}$$

• example:

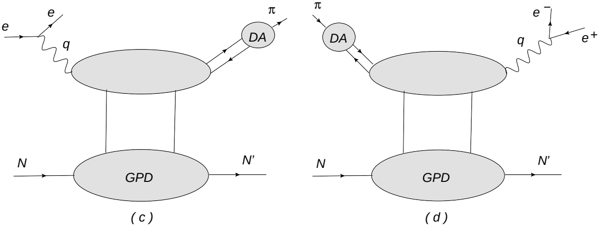
$$\frac{1}{\xi-i\epsilon\mp x}\overset{\textsf{SL}\to\textsf{TL}}{\Longrightarrow}\frac{1}{-\eta-i\epsilon\mp x}=\frac{-1}{\xi_T\pm x}=-\left(\frac{1}{\xi_S\pm x}\right)^*|_{\xi=\eta}\,.$$

$${}^{S}C(x,\xi_S)\overset{\textsf{SL}\to\textsf{TL}}{\Longrightarrow}{}^{T}C(x,\xi_T)=\mp{}^{S}C^*(-x,\xi_S)\,,$$

• $s\ +\cdots=-Q^2\Longrightarrow(s+i\epsilon)\ +\cdots=-(Q^2-i\epsilon)$

$$\ln\frac{\mathcal{Q}^2}{\mu^2}\overset{\textsf{SL}\to\textsf{TL}}{\Longrightarrow}\ln\frac{\mathcal{Q}^2}{\mu^2}-i\pi\,.$$

$\gamma_L^* \, p \, \to \, \pi^+ \, n$ versus $\pi^- \, p \to \, \gamma_L^* \, n$



$$\widetilde{\mathcal{F}} \propto \frac{1}{\mathcal{Q}} \int_0^1 du \int_{-1}^1 dx \, \widetilde{F}^{ud}(x,\xi,t) \, {}^S T_{ud}(u,x,\xi) \varphi_\pi(u) \, .$$

$${}^S T_{ud} = \left[e_u C(u,x,\xi_S) - e_d C(u,-x,\xi_S) \right] ,$$

$$C \stackrel{\rm NLO}{=} \alpha_s(\mu^2) C_0 + \frac{\alpha_s^2(\mu^2)}{2\pi} \left[C_{div} \ln \frac{\mathcal{Q}^2}{\mu^2} + C_1 \right] ,$$

$$C_0(u,x,\xi) = \frac{1}{u(\xi - x)} , \qquad C_{div} = \frac{-\beta_0}{2} C_0 + C_{coll}^F + C_{coll}^\varphi \, .$$

DVMP

hard ^{T}CF to LO accuracy:

$$ {}^{T}T_{du}(u, x, \xi_T) = \left[e_u C_0(u, x, \xi_T) - e_d C_0(u, -x, \xi_T) \right]. $$

Timelike vs spacelike NLO relation for DVMP:

$$ {}^{T}C(u, x, \xi_T) \overset{\mathrm{NLO}}{=} - \left[C^* - i\pi \frac{\alpha_s}{2\pi} C^*_{div} \right] (u, -x, \xi_S). $$

Summary:

• we studied the NLO corrections to DVCS, TCS and DDVCS from both quarks and gluons

• we established simple crossing relation between NLO CF's for DVCS and TCS and between two crossed DVMP processes

• phenomenological/experimental studies:

• talk by Jakub Wagner

H. Moutarde, B. Pire, F. Sabatie, L. Sz and J. Wagner, Phys. Rev. D **87** (2013) 054029

• talk by Rafayel Paremuzyan