Radiative Corrections in Inclusive Processes

Deeply Virtual Compton Scattering: From Observables to GPDs
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A primary goal in DIS:

High-precision measurements of the 'nucleon structure'

➜ measure form factors, structure functions, (generalized) parton distribution functions, ...

- at low Q^2 elastic and quasi-elastic scattering
  ➜ form factors, polarizabilities, ...
- at high Q^2 deep inelastic scattering
  ➜ parton distribution functions, GPDs, GDAs, ...

The interesting physics is encoded in FFs, PDFs, ...
test the dynamics of the strong interaction ➜ QCD precision physics

Lepton scattering: only via electromagnetic and weak interaction

➜ well-controlled and separable perturbative treatment

Measure FFs, PDFs, etc by comparing data with theoretical predictions:

$$\sigma_{\rm exp} = \sigma_{\rm theory}[F_n(x, Q^2, \ldots)]$$

High precision requires knowledge of higher-order corrections

$$\sigma_{\rm theory} = \sigma^{(0)} + \alpha_{\rm em}\sigma^{(1)} + \ldots$$

- Emission of real photons
  experimentally often not distinguished from non-radiative processes:
  soft photons, collinear photons
  ➜ ”radiative corrections”
- Virtual corrections: loop diagrams
  needed to cancel infrared divergences (Bloch-Nordsieck)
- Electroweak effects
  Z-, W-boson exchange ($O(G_F)$)
  and higher-order electroweak corrections ($O(\alpha G_F)$)

Radiative corrections have to be 'removed' to uncover the interesting physics and radiative corrections often considered the uninteresting part

but:

- radiation from the nucleon: DVCS deeply virtual Compton scattering, γ-PDF, is part of the 1-photon radiative corrections
- 2γ exchange (and γZ exchange) is part of the 1-loop photonic corrections: box diagrams involve nuclear form factors

  (corresponding infrared divergences cancel with the interference between photon radiation from the lepton and from the nucleon)

Classical analytical approach: Mo, Tsai
often used in 'private' implementations of experimental collaborations
uses soft-photon approximation, complicated expressions
not precise enough for today's needs

Full Monte-Carlo approach:
HERACLES:
complete electroweak corrections at $O(\alpha)$ (parton model) for NC and CC scattering at HERA, including polarization

Full event generation:
DJANGOH:
universal leptonic corrections at $O(\alpha)$, interface to QCD-based event generation: parton showers, jets, hadronic final state: LEPTO,
includes models for low Q^2 behaviour: elastic tail, SOPHIA for low-mass hadronic final states

Version 4.6.10: http://wwwthep.physik.uni-mainz.de/~hspiesb/djangoh/djangoh.html

J. Kripfganz, H.-J. Möhring, A. Kwiatkowski, G. Schuler, K. Charchula, H.S.

Codes for analytical/numerical calculation of radiative corrections

TERAD by Bardin et al.

HECTOR by Arbuzov, Bardin, Blümlein, Kalinovskaya, Riemann
Semi-analytic, includes leading logs at order $O(\alpha^2)$

POLRAD, ESFRAD and RADGEN, ELRADGEN by
Afanasev, Akushevich, et al.
MC integrator, MC event generator for radiative events

Example for specialized programs: VANDERHAEGHEN ET AL., PRC62 (2000)
$O(\alpha)$ QED corrections to virtual Compton scattering

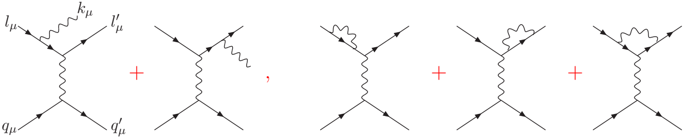
and others: HELIOS, KRONOS, FRANEQ, ...
see Workshop proceedings (Future) Physics at HERA

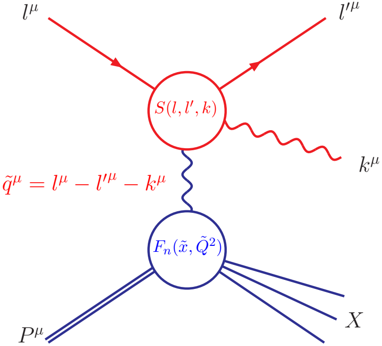
- Radiation from the lepton
  model independent (universal)
- vacuum polarization (boson self energy)
  universal, photon self energy ➜ $\alpha_{em}(Q^2)$
- Radiation from the hadronic initial/final state
  parton model: radiation from quarks
  to be considered as a part of the nucleon structure
- Interference of leptonic and hadronic radiation
  2γ exchange
  new structure
- purely weak corrections

Note: for NC-scattering, straightforward separation
Rule: respect gauge invariance
IR divergences: need to combine real and virtual radiation

Feynman diagrams for leptonic radiation at $O(\alpha)$ (NC)

for *eq* scattering:





radiative leptonic tensor $S_{\mu\nu}(l, l', k)$ is

- gauge invariant
- infrared finite
- universal (1γ exchange)

(includes Born + loops: $\delta^{(4)}(k^\mu)$)

- Shifted kinematics: $q^\mu \to q^\mu - k^\mu$

Observed cross section: convolution of true cross section $\otimes$ radiator function

$$\mathrm{d}\sigma^{\mathrm{obs}}(p,q) = \int \frac{d^3k}{2k^0} R(l,l',k)\,\mathrm{d}\sigma^{\mathrm{true}}(p,q-k)$$

or, for the structure functions:

$$F_n^{\mathrm{obs}}(x,Q^2) = \int \mathrm{d}\tilde{x}\mathrm{d}\tilde{Q}^2 R_n(x,Q^2;\tilde{x},\tilde{Q}^2) F_n^{\mathrm{true}}(\tilde{x},\tilde{Q}^2)$$

Can be extended to include higher-order effects: multi-photon emission, soft-photon exponentiation, e^+e^--pair creation,
R_n known analytically to second order, $O(\alpha^2)$

In turn: determination of the true F_n = unfolding, may be ill-defined
Difficult to treat radiative and detector effects separately (acceptance cuts, efficiencies, ...)

$$\mathrm{d}\sigma^{\mathrm{obs}}(p,q) = \int \frac{d^3k}{2k^0} R(l,l',k)\,\mathrm{d}\sigma^{\mathrm{true}}(p,q-k)$$

Note: shifted kinematics: $q \to q-k$, e.g.,

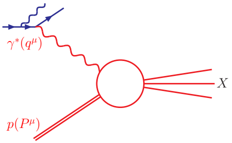
$$Q^2 = -(l-l')^2 \to \tilde{Q}^2 = -(l-l'-k)^2$$

- leptonic variables: measure E and θ of scattered lepton ➜ x and Q^2
- hadronic variables: measure E, θ from hadronic final state ➜ $\tilde{x}$ and $\tilde{Q}^2$
- mixed variables: combine information from leptonic and hadronic final state

Radiative tail ➜ expect strong dependence on experimental prescriptions for measuring kinematic variables

(Note: $\tilde{Q}^2 \ll Q^2$ possible: $\tilde{Q}^2_{\mathrm{min}} = \frac{x^2}{1-x} M_N^2$)

➜ radiative tail, Compton peak back to photoproduction
➜ γ-PDF



➜ need full Monte-Carlo treatment

Using partial fractioning, write: $R(l, l', k) = \frac{I}{k \cdot l} + \frac{F}{k \cdot l'} + \dots$

- initial state radiation, $k \cdot l$ small for $\sphericalangle(\mathrm{e_{in}}, \gamma) \to 0$
- final state radiation, $k \cdot l'$ small for $\sphericalangle(\mathrm{e_{out}}, \gamma) \to 0$

narrow peaks, width $\simeq \sqrt{m_e/E_e}$: collinear or mass singularities

upon angular integration: large logarithm $\propto \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \simeq 10\,\%$

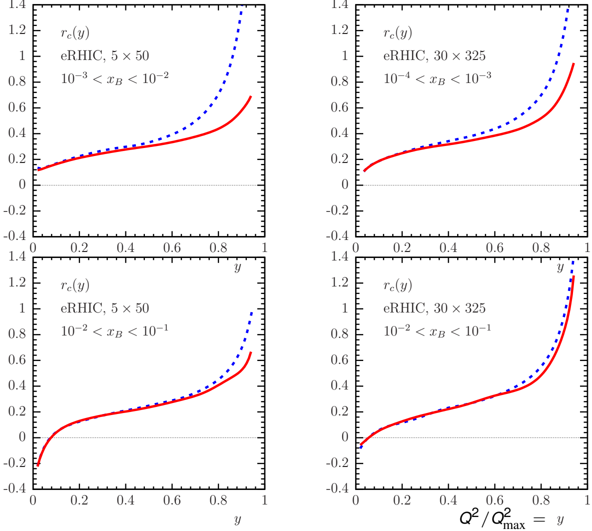
Note: $E^2_{\gamma,\mathrm{max}} \propto Q^2 \frac{1-x}{x}$

➜ large corrections at large Q^2, large y and at small x
➜ Radiation suppressed at small Q^2 and at large x,
large negative corrections from uncancelled virtual contributions

➜ Separation is unique only for the large logarithms !

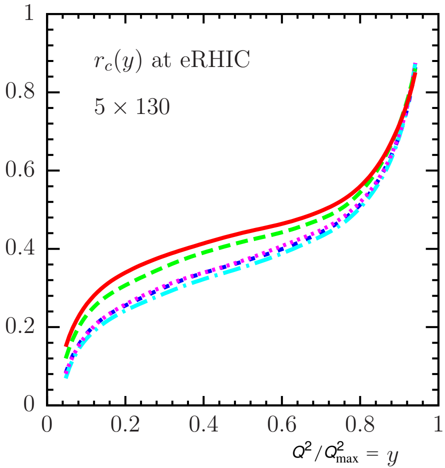
$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\rm Born}(y) - 1$, influence of a cut on $W_{had} > 1.4$ GeV

$e^- p$ at 5×50 GeV^2 $\quad$ $e^- p$ at 30×325 GeV^2



need detector to provide information about the hadronic final state

$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\mathrm{Born}}(y) - 1$
for e^-N at 5×130 GeV^2, $10^{-3} \leq x_{Bj} \leq 10^{-2}$,

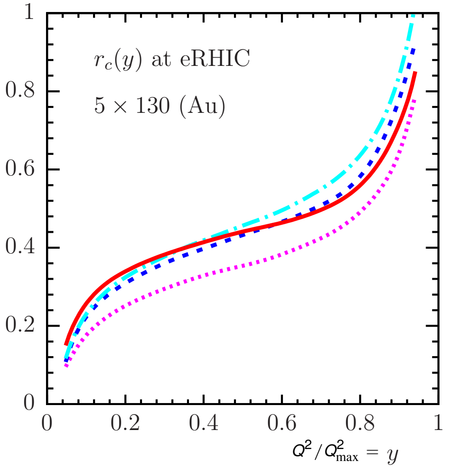


different targets

- full red: Au, CTEQ61M with EPS09
- dashed green: Fe, CTEQ61M with EPS09
- dotted magenta: He, CTEQ61M with EPS09
- dash-dotted light-blue: Au, CTEQ61M, *p*PDF
- dashed blue: *p*, CTEQ61M

➜ Radiative corrections depend on structure functions

$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\mathrm{Born}}(y) - 1$
for e^-Au at 5×130 GeV^2, $10^{-3} \leq x_{Bj} \leq 10^{-2}$,



different nuclear PDFs

- full red: CTEQ61M with EPS09
- dash-dotted light-blue: CTEQ61M with EPS08
- dashed blue: CTEQ61M with EKS98
- dotted magenta: HKN

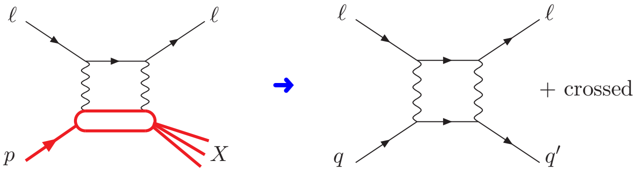
PDFs do not cancel in the ratio !

(deep) inelastic *ep*:

factorized into

PDFs $\otimes$ 2γ-box

for *eq* scattering

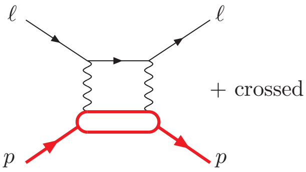


Need interference of radiation from the lepton and the hadron to obtain IR-finite result (well-known for DIS)

elastic *ep*:

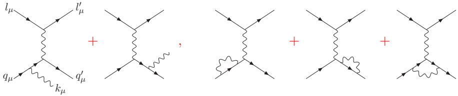
• assume dominance of a few intermediate states: *p* + resonances

• assume factorization into GPDs $\otimes$ partonic scattering



Dedicated precision measurements to determine 2γ contributions: lepton charge asymmetry (*Re*) and lepton polarization asymmetry (*Im*)

At large Q^2: DIS, parton model
emission of photons
like emission of gluons



infrared divergences (soft photons / gluons) cancel with loops, collinear emission gives rise to corrections $\frac{\alpha}{2\pi} \log m_q^2$, but quark masses are ill-defined
➜ factorize and absorb collinear divergences into parton distribution functions

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

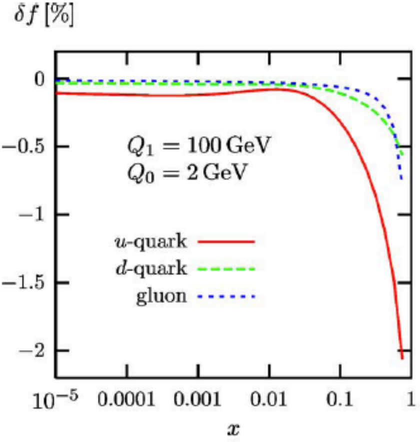
$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x) = \sum_f d\hat{\sigma}_f \hat{q}_f(x, Q^2)$$

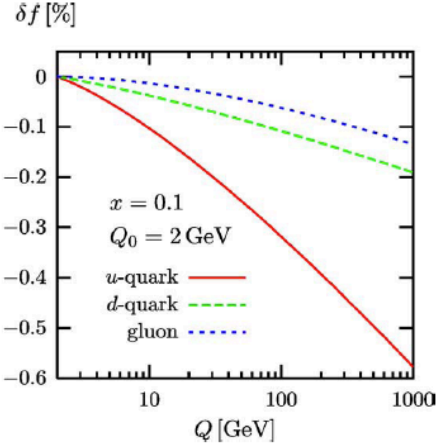
renormalized parton distribution functions

$$\hat{q}_f(x, Q^2) = (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

➜ modified scaling violations

well-known in QCD, $\overline{\text{MS}}$ factorization different charges of *u*- and *d*-quarks ➜ isospin-violating effect





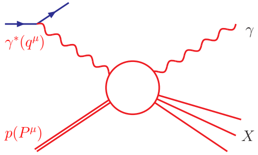
implemented in MRST2004, NNPDF3 HS; Roth, Weinzierl, PLB590

relevant for precision predictions, e.g. *W* production at the LHC

Different point of view: $\gamma^* p \to \gamma + X$
not as part of radiative corrections but with observed photon

➜ DVCS: deeply virtual Compton scattering
$\gamma^* N \to \gamma N'$ at large s, small t
one of the cleanest tools to determine
GPDs, generalized parton distribution functions

➜ at large t: direct photons in perturbative QCD



Leptonic bremsstrahlung (Bethe-Heitler process) is large background

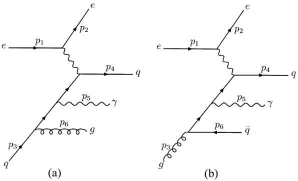
Precision measurements require $O(\alpha)$ corrections to $\gamma^* p \to \gamma + X$
➜ i.e. $\gamma^* p \to \gamma + \gamma + X$ and one-loop corrections to $\gamma^* p \to \gamma + X$

see Igor Akushevich: BHRADGEN, PRD85
and Marc Vanderhaeghen et al., 2000 for DVCS
also Haas, Makarenko, 2011: rad. corr. for luminosity measurement with $ep \to ep\gamma$

More efforts required to meet the needs of high-precision measurements:

- Multi-photon effects: radiation and loops
  ➜ IR/soft photon exponentiation and radiator functions at $O(\alpha^2)$
- Hadronic radiation: radiation from quarks
  subtraction and modified parton showers
  including $q \to q + \gamma$
  mixed QED+QCD corrections

  lepton-hadron interference and ...
- $\gamma\gamma$ and γZ box



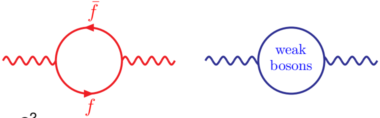
- Simulation of the hadronic final state for scattering off heavy nuclei
  ➜ interface to dedicated programs, e.g. DPMJET
- Monte Carlo simulation for DVCS ➜ include 1-loop + 1-photon and 2-photon radiation

Many results available, but often not implemented in Monte Carlo simulation programs

- High precision needs careful treatment of radiative corrections
- Closely related to experimental conditions, need full Monte Carlo treatment including simulation of hadronic final states

More Material ...

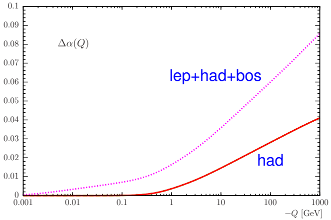
Self energy diagrams of the exchanged boson (γ and Z)



$\propto \log \frac{Q^2}{m_f^2} \rightarrow O(10\,\%)$ small, $O(1\,\%)$

Photon self energy
= vacuum polarization,
absorbed in the running
fine structure constant:

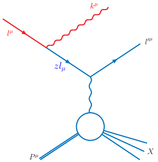
$$\alpha \rightarrow \alpha(Q^2) = \frac{\alpha}{1 - \Pi_\gamma(Q^2)}$$



Z-boson self energy: a small correction if written in terms of:

$$\frac{\alpha}{s_W^2 c_W^2} \rightarrow \frac{M_Z^2 G_\mu \sqrt{2}}{\pi} \frac{1 - \Delta r}{1 - \Pi_Z(Q^2)}$$

(with s_W^2, c_W^2: sin and cos of the weak mixing angle; G_μ the muon decay constant; Δr one-loop corrections to the muon decay: renormalization)



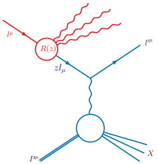
e.g., for initial-state radiation:
assume $k^\mu = (1-z)l^\mu$
➜ Radiator function

$$R_{\mathrm{ISR}} = \frac{\alpha}{2\pi}\frac{1+z^2}{1-z}\log\frac{Q^2}{m_e^2}$$

$(+\delta(1-z)$ from loops $\rightarrow$ $+$-distribution $1/(1-z)_+)$

$$\mathrm{d}\sigma_{\mathrm{ISR}} = \int\frac{\mathrm{d}z}{z}R_{\mathrm{ISR}}(z)\,\mathrm{d}\sigma_{\mathrm{Born}}(l^\mu \rightarrow zl^\mu)$$

(similar for final-state radiation)



Can be extended to include multi-photon emission:

$$R^{(2)}_{\mathrm{ISR}}(z) = \int_z^1\frac{\mathrm{d}z'}{z'}R^{(1)}_{\mathrm{ISR}}(z')R^{(1)}_{\mathrm{ISR}}(z/z') + \ldots$$

Solution of evolution equations like DGLAP
Known at $O(\alpha^2)$ (complete) and partially at $O(\alpha^3)$

Corrections due to soft photons are universal

sum of real and virtual contributions: $\delta^{\rm IR}$ (finite and gauge invariant)

$$1 + \delta^{\rm tot} = 1 + \delta^{\rm IR} + \delta^{\rm fin} \rightarrow \exp(\delta^{\rm IR})(1 + \delta^{\rm fin})$$

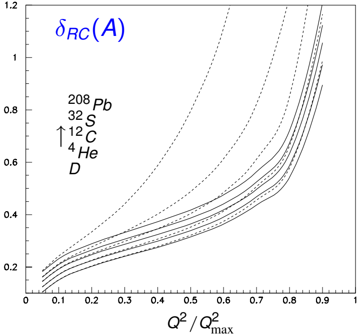
$\delta^{\rm IR}$ contains $\log(E_\gamma^{\max})$ and $L_e = \log(m_e^2/Q^2)$:

$$1 + \frac{\alpha}{2\pi}(L_e - 1)\ln\frac{E_\gamma^{\max}}{E_e} + \ldots \rightarrow \left(\frac{E_\gamma^{\max}}{E_e}\right)^{\frac{\alpha}{2\pi}(L_e - 1)}(1 + \ldots)$$

(in the $\gamma^* p$ cms: $E_\gamma^{\max} = \frac{1}{2}\sqrt{y(1-x)S}$, i.e. important at low y and large x)

Yennie, Frautschi, Suura, 1961

Contribution from elastic and quasi-elastic tails
(scattering off the nucleus or off individual nucleons)



Full lines: inelastic contribution
dashed lines: fully inclusive corrections, incl. elastic and quasi-elastic tails

➜ cut on mass of hadronic final state: $W_A^2 = Q^2(1-x)/x + M_A^2$, $p_{T,\mathrm{had}}$, $E - p_z$