

DVCS analysis and GPD parametrizations

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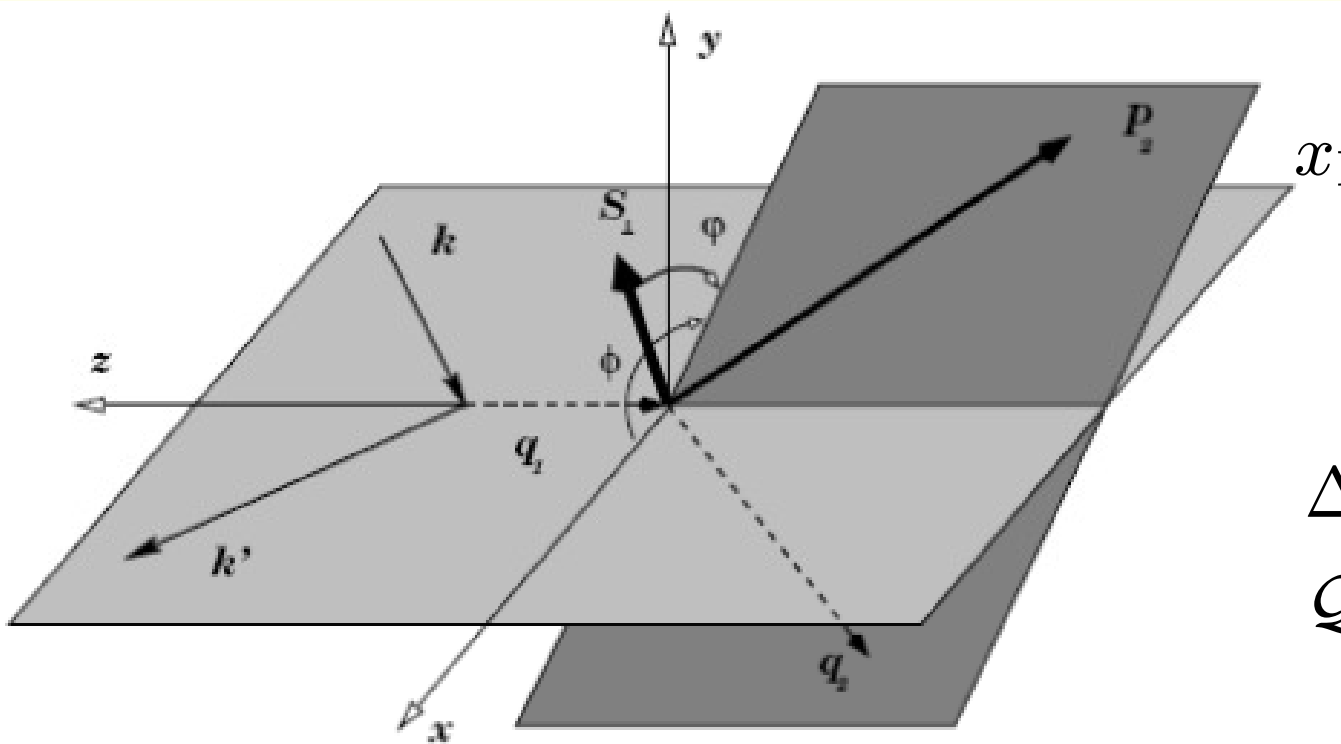
- ❖ ***(D)VCS Compton scattering tensor***
- ❖ ***GPD parametrizations***
- ❖ ***Data description/predictions***
- ❖ ***Conclusions***

Photon leptonproduction $e^\pm N \rightarrow e^\pm N \gamma$

measured by **H1, ZEUS, HERMES, CLAS, HALL A** collaborations

planned at **COMPASS, JLAB@12GeV**, perhaps at ? EIC, ?? LHeC

$$\frac{d\sigma}{dx_{\text{Bj}} dy d|\Delta^2| d\phi d\varphi} = \frac{\alpha^3 x_{\text{Bj}} y}{16 \pi^2 Q^2} \left(1 + \frac{4M^2 x_{\text{Bj}}^2}{Q^2} \right)^{-1/2} \left| \frac{\mathcal{T}}{e^3} \right|^2,$$



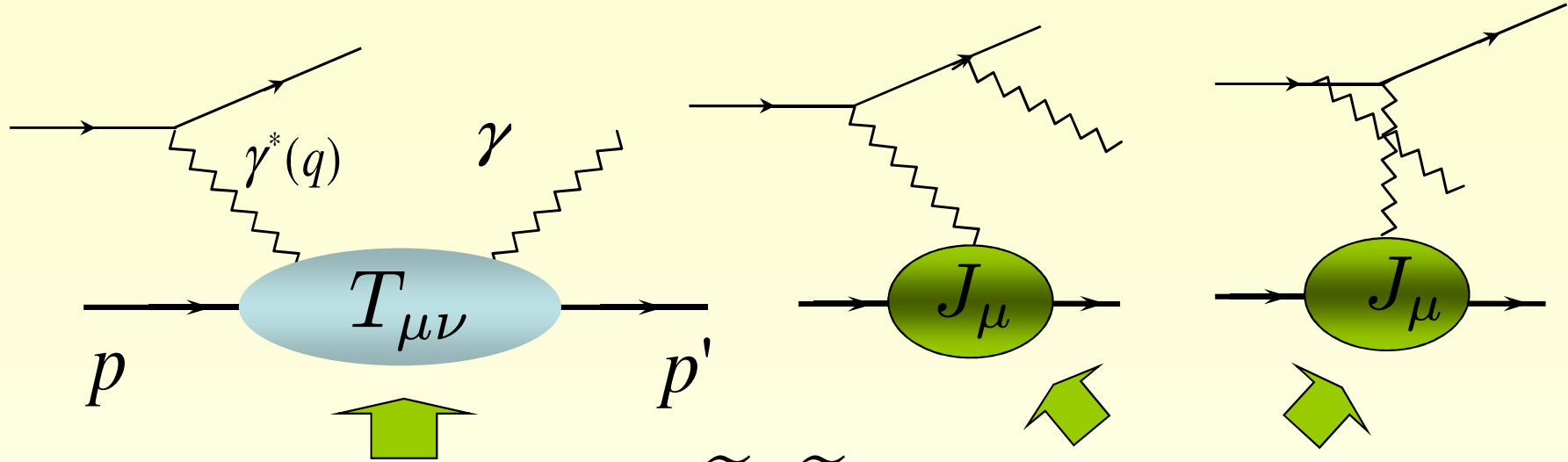
$$x_{\text{Bj}} = \frac{Q^2}{2P_1 \cdot q_1} \approx \frac{2\xi}{1 + \xi},$$

$$y = \frac{P_1 \cdot q_1}{P_1 \cdot k},$$

$$\Delta^2 = t \text{ (fixed, small),}$$

$$Q^2 = -q_1^2 \text{ (> 1 GeV}^2\text{),}$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}}, \dots$ elastic form factors F_1, F_2
(helicity amplitudes)

$$|\mathcal{T}_{\text{BH}}|^2 = \frac{e^6 (1 + \epsilon^2)^{-2}}{x_{\text{Bj}}^2 y^2 \Delta^2 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ c_0^{\text{BH}} + \sum_{n=1}^2 c_n^{\text{BH}} \cos(n\phi) \right\}, \quad \text{exactly known (LO, QED)}$$

$$|\mathcal{T}_{\text{DVCS}}|^2 = \frac{e^6}{y^2 Q^2} \left\{ c_0^{\text{DVCS}} + \sum_{n=1}^2 [c_n^{\text{DVCS}} \cos(n\phi) + s_n^{\text{DVCS}} \sin(n\phi)] \right\}, \quad \begin{array}{l} \text{harmonics} \\ \text{1:1} \\ \text{helicity ampl.} \end{array}$$

$$\mathcal{I} = \frac{\pm e^6}{x_{\text{Bj}} y^3 \Delta^2 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^3 [c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi)] \right\}. \quad \begin{array}{l} \text{harmonics} \\ \text{1:1} \\ \text{helicity ampl.} \end{array}$$

all harmonics are given by twist-2 and -3 GPDs:

$$\begin{aligned} \left\{ \begin{matrix} c_1 \\ s_1 \end{matrix} \right\}^{\mathcal{I}} &\propto \frac{\Delta}{Q} \text{tw-2(GPDs)} + O(1/Q^3), & c_0^{\mathcal{I}} &\propto \frac{\Delta^2}{Q^2} \text{tw-2(GPDs)} + O(1/Q^4), \\ \left\{ \begin{matrix} c_2 \\ s_2 \end{matrix} \right\}^{\mathcal{I}} &\propto \frac{\Delta^2}{Q^2} \text{tw-3(GPDs)} + O(1/Q^4), & \left\{ \begin{matrix} c_3 \\ s_3 \end{matrix} \right\}^{\mathcal{I}} &\propto \frac{\Delta \alpha_s}{Q} (\text{tw-2})^T + O(1/Q^3), \end{aligned}$$

$$c_0^{\text{CS}} \propto (\text{tw-2})^2, \quad \left\{ \begin{matrix} c_1 \\ s_1 \end{matrix} \right\}^{\text{CS}} \propto \frac{\Delta}{Q} (\text{tw-2}) (\text{tw-3}), \quad \left\{ \begin{matrix} c_2 \\ s_2 \end{matrix} \right\}^{\text{CS}} \propto \alpha_s (\text{tw-2}) (\text{tw-2})^{\text{GT}}$$

e.g., n=1 odd harmonic is approximately given by 'CFF' combination

$$\left\{ \begin{matrix} c_{1,\text{unp}}^{\mathcal{I}} \\ s_{1,\text{unp}}^{\mathcal{I}} \end{matrix} \right\} = 8K \left\{ \begin{matrix} -(2-2y+y^2) \\ \lambda y(2-y) \end{matrix} \right\} \left\{ \begin{matrix} \Re \\ \Im \end{matrix} \right\} c_{\text{unp}}^{\mathcal{I}}(\mathcal{F}), \quad c_{\text{unp}}^{\mathcal{I}} = F_1 \mathcal{H} + \frac{x_B}{2-x_B} (F_1 + F_2) \widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2} F_2 \mathcal{E}$$

relations among **harmonics** and (helicity dependent) **CFFs** [Belitsky, DM (10) -- Belitsky, DM, Ji (12)] are not more based on a $1/Q$ expansion:

$$\begin{aligned} s_{1,\text{unp}}^{\mathcal{I}} &= \frac{8\tilde{K}\lambda\sqrt{1-y-\frac{y^2\gamma^2}{4}}(2-y)y}{Q(1+\gamma^2)} \Im \left\{ c_{\text{unp}}^{\mathcal{I}} \left(\left[1 - \frac{\varkappa}{2Q^2} \frac{Q^2+t}{\sqrt{1+\gamma^2}} \right] \mathcal{F}_{++} + \left[1 - \frac{2+\varkappa}{2Q^2} \frac{Q^2+t}{\sqrt{1+\gamma^2}} \right] \mathcal{F}_{-+} + \frac{(Q^2+t)\varkappa_0}{Q^2\sqrt{1+\gamma^2}} \mathcal{F}_{0+} \right) \right. \\ &\quad \left. + \frac{-t(Q^2+t)}{\sqrt{1+\gamma^2}Q^4} \Delta c_{\text{unp}}^{\mathcal{I}} \left(\mathcal{F}_{-+} + \frac{\varkappa}{2} [\mathcal{F}_{++} + \mathcal{F}_{-+}] - \varkappa_0 \mathcal{F}_{0+} \right) \right\}, \end{aligned} \quad (70)$$

new improved **C coefficients** ensure the cancellation of kinematical singularities
relations among CFFs and GPDs are **always** based on a **$1/Q$ expansion**

What is used for the (D)VCS tensor? (helicity amplitudes)

A simple electromagnetic form factor parametrization is accepted:

$$\langle p_2, s_2 | j_\rho(0) | p_1, s_1 \rangle = \bar{u}(p_2, s_2) \left[\gamma_\rho F_1(t) + i \sigma_{\rho\sigma} \frac{\Delta^\sigma}{2M} F_2(t) \right] u(p_1, s_1)$$

What about (DV)CS-tensor parametrizations?

$$T_{\mu\nu} = i \int d^4x e^{\frac{i}{2}(q_1+q_2)\cdot x} \langle p_2 | T \{ j_\mu(x/2) j_\nu(-x/2) \} | p_1 \rangle$$

Prange (1958) [real CS, over-counting amplitudes, Dirac-spinors]

Hearn, Leader (1962) [VCS, Pauli-spinor representation]

Tarrach (1975) [VVCS, kinematical constraints are removed]

DVCS (calculated in terms of GPDs since 1992, various similar parametrizations)

VCS Kroll et al. (1995) in terms of helicity amplitudes [diquark model for nucleon]

VCS Drechsel et al. (1998) [VCS, generalized polarizabilities]

etc.

unique parametrization for the (D)VCS tensor is desired

The so-obtained basis of gauge invariant tensors is expected to be minimal and pole free by construction and is the following one :

$$\mathcal{T}_1 = k \cdot k' T_1 - T_3$$

$$\mathcal{T}_2 = k^2 k'^2 T_1 + k \cdot k' T_2 - \frac{k^2 + k'^2}{2} T_6 + \frac{k^2 - k'^2}{2} T_5$$

$$\mathcal{T}_3 = (\ell \cdot K)^2 T_1 + k \cdot k' T_6 - \ell \cdot K T_7$$

$$\mathcal{T}_4 = \ell \cdot K (k^2 + k'^2) T_1 - \ell \cdot K T_4 - \frac{k^2 + k'^2}{2} T_7 + \frac{k^2 - k'^2}{2} T_8 + k \cdot k' T_9$$

$$\mathcal{T}_5 = -\ell \cdot K (k^2 - k'^2) T_1 + \ell \cdot K T_5 + \frac{k^2 - k'^2}{2} T_7 - \frac{k^2 + k'^2}{2} T_8 + k \cdot k' T_{10}$$

$$\mathcal{T}_6 = \ell \cdot K T_2 - \frac{k^2 + k'^2}{2} T_9 - \frac{k^2 - k'^2}{2} T_{10} - M T_{12} + M \frac{k^2 + k'^2}{4} T_{23} - M \frac{k^2 - k'^2}{4} T_{24} \\ + \frac{k^2 - k'^2}{8} T_{29} - \frac{k^2 + k'^2}{8} T_{30} - \frac{k^2 k'^2}{4} T_{33}$$

$$\mathcal{T}_7 = 8 T_{16} - 4 \ell \cdot K T_{21} + \ell \cdot K T_{34}$$

$$\mathcal{T}_8 = T_{19} + \frac{k^2 - k'^2}{2} T_{22} - \ell \cdot K T_{23} + \frac{k^2 + k'^2}{8} T_{34}$$

$$\mathcal{T}_9 = T_{20} - \frac{k^2 + k'^2}{2} T_{22} + \ell \cdot K T_{24} - \frac{k^2 - k'^2}{8} T_{34}$$

$$\mathcal{T}_{10} = -8 k \cdot k' T_6 + 4 \ell \cdot K T_7 + 4 M k \cdot k' T_{21} - 4 M \ell \cdot K T_{25} - 2 \ell \cdot K T_{32} - 2 k \cdot k' \ell \cdot K T_{33} + M k \cdot k' T_{34}$$

$$\mathcal{T}_{11} = T_{18} - k \cdot k' T_{22} + \ell \cdot K T_{26}$$

$$\mathcal{T}_{12} = \ell \cdot K T_4 - \frac{k^2 - k'^2}{2} T_8 - k \cdot k' T_9 - M T_{14} + M k \cdot k' T_{23} - M \frac{k^2 - k'^2}{2} T_{26} - \frac{k^2 + k'^2}{4} T_{32} - k \cdot k' \frac{k^2 + k'^2}{4} T_{33}$$

$$\mathcal{T}_{13} = \ell \cdot K T_5 - \frac{k^2 + k'^2}{2} T_8 + k \cdot k' T_{10} - M T_{15} + M k \cdot k' T_{24} - M \frac{k^2 + k'^2}{2} T_{26} - \frac{k^2 - k'^2}{4} T_{32} - k \cdot k' \frac{k^2 - k'^2}{4} T_{33}$$

$$\mathcal{T}_{14} = 2 \ell \cdot K T_8 - 2 M k \cdot k' T_{22} + 2 M \ell \cdot K T_{26} - k \cdot k' T_{27} + \ell \cdot K T_{31}$$

$$\mathcal{T}_{15} = -(k^2 - k'^2) T_7 + (k^2 + k'^2) T_8 - 2 k \cdot k' T_{10} - 2 M k \cdot k' T_{24} + M (k^2 - k'^2) T_{25} + M (k^2 + k'^2) T_{26} \\ - k \cdot k' T_{29} + \frac{k^2 + k'^2}{2} T_{31} + \frac{k^2 - k'^2}{2} T_{32}$$

$$\mathcal{T}_{16} = -(k^2 + k'^2) T_2 + (k^2 - k'^2) T_8 + 2 k \cdot k' T_9 - 2 M k \cdot k' T_{23} + M (k^2 + k'^2) T_{25} + M (k^2 - k'^2) T_{26} \\ - k \cdot k' T_{30} + \frac{k^2 - k'^2}{2} T_{31} + \frac{k^2 + k'^2}{2} T_{32}$$

$$\mathcal{T}_{17} = -4 \ell \cdot K T_1 + 2 T_7 + 4 M T_{11} - 2 M T_{25} + T_{32} + k \cdot k' T_{33}$$

$$\mathcal{T}_{18} = 4 T_{17} - 4 \ell \cdot K T_{25} + k \cdot k' T_{34}$$

Tarrach
(1975)

? minimal

Requirements on the parametrization of DVCS tensor

- Lorentz-covariance + gauge invariance + implementing discrete symmetries
- scalar amplitudes (6 for RCS, 12 for (D)VCS, 18 for general parametrization without kinematical constraints)
- simplicity (including simple relation to what is already used)

DVCS tensor parametrizations arise from approximate calculations

- $\chi^\mu \propto n^\mu + \dots$ different choices for light-like vector $n \propto q + a p_1 + b p_2$,
(constructed from in- and out-particle momenta)
- various results differ at twist-2, twist-3, (twist-4) by the order $O(1/Q)$, $O(1/Q^2)$, $O(1/Q^3)$
- DVCS/GPD results are not exact and suffer from breaking of gauge+Lorentz symmetries
- to relate GPDs to observables a convention is needed (if one likes to compare results)

embed GPD findings in a general DVCS tensor parametrization [Belitsky, DM, Kirchner (01)]

- this does not solve the ambiguity problem in GPD calculations (see Volodya's talk)
- provides the basis to discuss the physics case of (D)VCS measurements

$$\mathcal{T}_{ab}^{\text{VCS}}(\phi) = (-1)^{a-1} \varepsilon_2^{\mu*}(b) T_{\mu\nu} \varepsilon_1^\nu(a)$$

$$\mathcal{T}_{ab}^{\text{VCS}} = \mathcal{V}(\mathcal{F}_{ab}) - b \mathcal{A}(\mathcal{F}_{ab}) \quad \text{for } a \in \{0, +, -\}, b \in \{+, -\} \quad \text{parameterization of (DV)CS helicity amplitudes}$$

$$\mathcal{V}(\mathcal{F}_{ab}) = \bar{u}_2 \left(\not{q} \mathcal{H}_{ab} + i \sigma_{\alpha\beta} \frac{m^\alpha \Delta^\beta}{2M} \mathcal{E}_{ab} \right) u_1$$

$$\mathcal{A}(\mathcal{F}_{ab}) = \bar{u}_2 \left(\not{q} \gamma_5 \tilde{\mathcal{H}}_{ab} + \gamma_5 \frac{m \cdot \Delta}{2M} \tilde{\mathcal{E}}_{ab} \right) u_1, \quad m^\mu = \frac{q_1^\mu + q_2^\mu}{(p_1 + p_2) \cdot (q_1 + q_2)}$$

$$\begin{aligned} T_{\mu\nu} = & -\tilde{g}_{\mu\nu} \frac{q \cdot V_T}{p \cdot q} + i \tilde{\varepsilon}_{\mu\nu} \frac{q \cdot A_T}{p \cdot q} + \left(q_{2\mu} - \frac{q_2^2}{p \cdot q} p_\mu \right) \left(q_{1\nu} - \frac{q_1^2}{p \cdot q} p_\nu \right) \frac{q \cdot V_L}{p \cdot q} \\ & + \left(q_{1\nu} - \frac{q_1^2}{p \cdot q} p_\nu \right) \left(g_{\mu\rho} - \frac{p_\mu q_{2\rho}}{p \cdot q} \right) \left[\frac{V_{\text{LT}}^\rho}{p \cdot q} + \frac{i \epsilon^{\rho qp\sigma}}{p \cdot q} \frac{A_{\text{LT}}^\sigma}{p \cdot q} \right] \\ & + \left(q_{2\mu} - \frac{q_2^2}{p \cdot q} p_\mu \right) \left(g_{\nu\rho} - \frac{p_\nu q_{1\rho}}{p \cdot q} \right) \left[\frac{V_{\text{TL}}^\rho}{p \cdot q} + \frac{i \epsilon^{\rho qp\sigma}}{p \cdot q} \frac{A_{\text{TL}}^\sigma}{p \cdot q} \right] \\ & + \left(g_\mu{}^\rho - \frac{p_\mu q_2^\rho}{p \cdot q} \right) \left(g_\nu{}^\sigma - \frac{p_\nu q_1^\sigma}{p \cdot q} \right) \left[\frac{\Delta_\rho \Delta_\sigma + \tilde{\Delta}_\rho^\perp \tilde{\Delta}_\sigma^\perp}{2M^2} \frac{q \cdot V_{\text{TT}}}{p \cdot q} + \frac{\Delta_\rho \tilde{\Delta}_\sigma^\perp + \tilde{\Delta}_\rho^\perp \Delta_\sigma}{2M^2} \frac{q \cdot A_{\text{TT}}}{p \cdot q} \right] \end{aligned}$$

(one) parameterization of (DV)CS tensor
equivalent to *Tarrach's* one

relations of CFFs to helicity dependent CFFs are easily calculated:

$$\begin{aligned} \mathcal{F}_{+b} = & \left[\frac{1 + b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}} + \frac{(1-x_B)x_B^2(4M^2-t)(1+\frac{t}{Q^2})}{Q^2\sqrt{1+\epsilon^2}(2-x_B+\frac{x_B t}{Q^2})^2} \right] \mathcal{F}_T \\ & + \frac{1 - b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}} \frac{\tilde{K}^2}{M^2(2-x_B+\frac{x_B t}{Q^2})^2} \mathcal{F}_{\text{TT}} + \frac{2x_B \tilde{K}^2}{Q^2\sqrt{1+\epsilon^2}(2-x_B+\frac{x_B t}{Q^2})^2} \mathcal{F}_{\text{LT}} \end{aligned}$$

$$\mathcal{F}_{0+} = \frac{\sqrt{2} \tilde{K}}{\sqrt{1+\epsilon^2} Q (2-x_B+\frac{x_B t}{Q^2})} \left\{ \left[1 + \frac{2x_B^2(4M^2-t)}{Q^2(2-x_B+\frac{x_B t}{Q^2})} \right] \mathcal{F}_{\text{LT}} \right.$$

$$\left. + x_B \left[1 + \frac{2x_B(4M^2-t)}{Q^2(2-x_B+\frac{x_B t}{Q^2})} \right] \mathcal{F}_T + x_B \left[2 - \frac{4M^2-t}{M^2(2-x_B+\frac{x_B t}{Q^2})} \right] \mathcal{F}_{\text{TT}} \right\}$$

usable for
DVCS - RCS,
extendable to
timelike (D)VCS,
double(D)VCS or DIS

What is used to connect GPDs to DVCS?

Vanderhaeghen Guidal Guichon (VGG) code

- numerical squaring, well defined wrt. convention (includes twist-three, no transversity)
- implemented GPD model is often called VGG – it is GPV [Goeke, Polyakov, Vanderhaeghen (01), based on Radyushkin ansatz]
- model is often not specified if confronted with data

Guchion (Vanderhaeghen) code used by Moutarde & Sabatie

- unpublished, I am not able to figure out what goes into the code (users can not tell me)
- users claims: the code is `exact' and uniquely separates leptonic and hadronic parts, which is simply wrong

BM(K)J versions

- analytic squaring with (BMK) and without (BMJ) approximation
- approximate versions went into `private codes' and Monte Carlos (Freund & McDermott (MILOU), Guzey&Teckentrup, HERMES, JLAB, COMPASS)
- we (KM) use (apart from transverse target) BM11 contained in BMJ12 (will be upgraded to 12 CFFs, needed for kinematical twist-4 corrections)

➤ numerical comparison on precision level is only possible with VGG

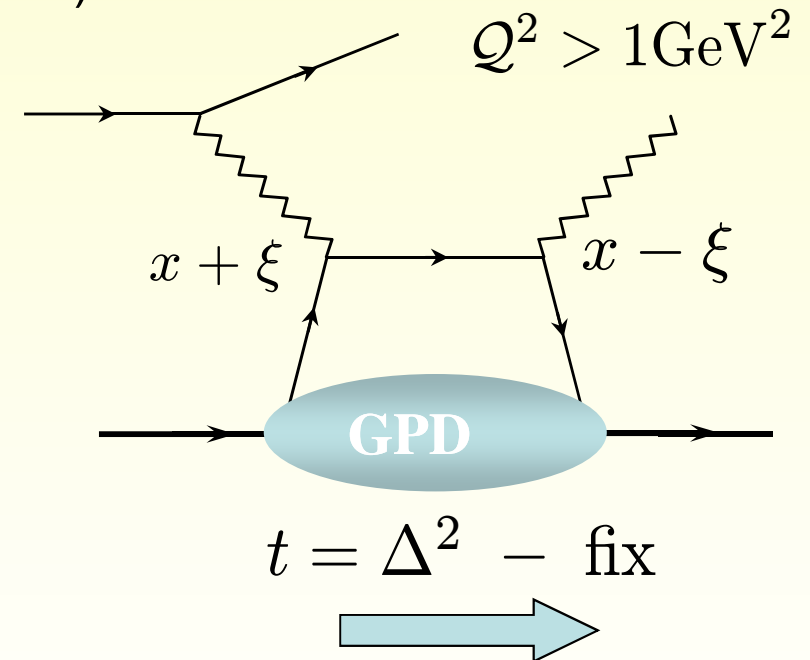
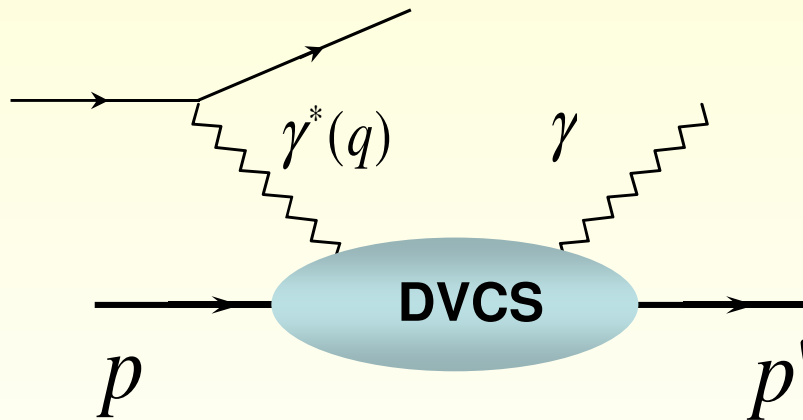
➤ practically, results agree often very well since we use similar light-like vector n this has not to be the case, [see Volodya's talk](#)

GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,

e.g., hard electroproduction of photons (DVCS)

[DM et. al (91/94)
Radyushkin (96)
Ji (96)]



$$\mathcal{F}(\xi, Q^2, t) = \int_{-1}^1 dx \, C(x, \xi, \alpha_s(\mu), Q/\mu) F(x, \xi, t, \mu) + O\left(\frac{1}{Q^2}\right)$$

CFF

hard scattering part

GPD

higher twist

Compton form factor

perturbation theory
(our conventions/microscope)

universal
(conventional)

depends on
approximation

observable

Field theoretical GPD definition

GPDs are defined as matrix elements of **renormalized light-ray** operators:

DM, Robaschik, Geyer,
Dittes, Hořejší (94)

$$F(x, \eta, \Delta^2, \mu^2) = \int_{-\infty}^{\infty} d\kappa e^{i\kappa x n \cdot P} \langle P_2 | \mathcal{RT} : \phi(-\kappa n) [(-\kappa n), (\kappa n)] \phi(\kappa n) : | P_1 \rangle, n^2 = 0$$

momentum fraction x , skewness $\eta = \frac{n \cdot \Delta}{n \cdot P}$ $\Delta = P_2 - P_1$ $P = P_1 + P_2$ $\Delta^2 \equiv t$

For a nucleon target we have four chiral even twist-two GPDs:

$$\begin{aligned} \bar{\psi}_i \gamma_+ \psi_i &\Rightarrow i_q^V = \bar{U}(P_2, S_2) \gamma_+ U(P_1, S_1) H_i + \bar{U}(P_2, S_2) \frac{i\sigma_{+\nu} \Delta^\nu}{2M} U(P_1, S_1) E_i \\ \bar{\psi}_i \gamma_+ \gamma_5 \psi_i &\Rightarrow i_q^A = \bar{U}(P_2, S_2) \gamma_+ \gamma_5 U(P_1, S_1) \tilde{H}_i + \bar{U}(P_2, S_2) \frac{\gamma_5 \Delta_+}{2M} U(P_1, S_1) \tilde{E}_i \end{aligned}$$

shorthands:

chiral even GPDs: $F = \{H, E, \tilde{H}, \tilde{E}\}$

& CFFs: $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}}\}$

chiral odd GPDs: $F_T = \{H_T, E_T, \tilde{H}_T, \tilde{E}_T\}$

$\mathcal{F}_T = \{\mathcal{H}_T, \mathcal{E}_T, \tilde{\mathcal{H}}_T, \tilde{\mathcal{E}}_T\}$

GPD properties (from definition)

- polynomiality arises from Lorentz covariance
(but GPDs are not Lorentz invariant or covariant)

$$\int_{-1}^1 dx x^n F(x, \eta, t) = \text{polynom of order } n \text{ or } n + 1 \text{ in } \eta$$

- symmetric in η (time reversal invariance+hermiticity)
- satisfied within double distribution representation (GPD duality)

$$F(x, \eta, t) = \int_{-1}^1 dy \int_{-1+|y|}^{1-|y|} dz \delta(x - y - z\eta) [f(y, z, t) + x\Delta f(y, z, t)]$$

- lowest moment: partonic form factor – related to observables
- first moment: expectation value of energy-momentum tensor
- reduction to parton densities (PDFs)

$$q(x) = \lim_{\Delta \rightarrow 0} H(x, \eta, t), \quad \Delta q(x) = \lim_{\Delta \rightarrow 0} \tilde{H}(x, \eta, t)$$

- positivity constraints (requirement on GPD and scheme) [Pobylitsa(00,02)]
are **only** automatically satisfied in the LCWF **overlap representation**

GPD representations

- x-(momentum fraction) representation (mostly indirectly used)
- double distribution representation (used in models: GPV, BMK, GK,...)
- conformal partial wave expansion, starting point for smearing [**Radyushkin (97)**; **Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)**]
Shuvaev transformation [**A. Shuvaev (99)**, **J. Noritzsch (00)**]
'dual' param. [**M. Polyakov, A. Shuvaev (02)**; **M. Polyakov (07)**, **Semenov-Tian-Shansky (07)**]
Mellin-Barnes representation [**DM, Schaefer (05)**; **Kirch, Manashov, Schäfer (05)**; ...]
- LC-wave function overlap representation (not used in phenomenology)

toy GPD:

$$F(x, \eta) = \theta\left(\frac{\eta + x}{1 - x}\right) \frac{7(1 + \eta)}{8\eta^2} \left(\frac{x + \eta}{1 + \eta}\right)^{\frac{3}{2}} \left[\frac{1}{2} \frac{1 - x}{1 + \eta} + 1 - \frac{x}{\eta} \right] + \{\eta \rightarrow -\eta\}$$

$$F(x, 0) = \frac{35}{32} \frac{(1 - x)^3}{\sqrt{x}}$$

$$F(\xi, \xi) = \frac{7}{4(1 + \xi)} \sqrt{\frac{1 + \xi}{2\xi} \frac{1 - \xi}{1 + \xi}} \Leftrightarrow \frac{7}{4} \frac{1 - X}{\sqrt{X}}, \quad X = \frac{2\xi}{1 + \xi}$$

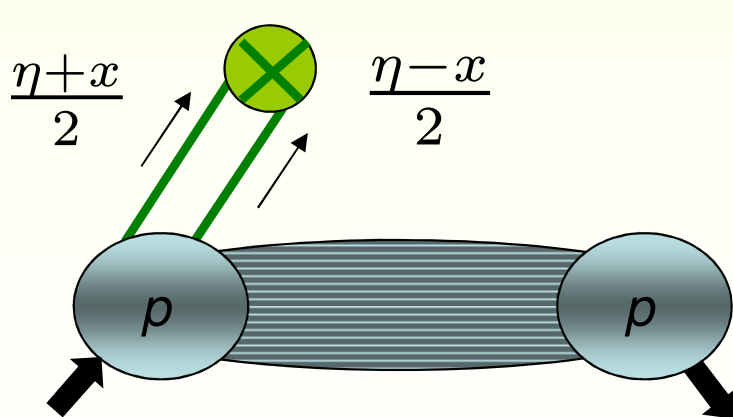
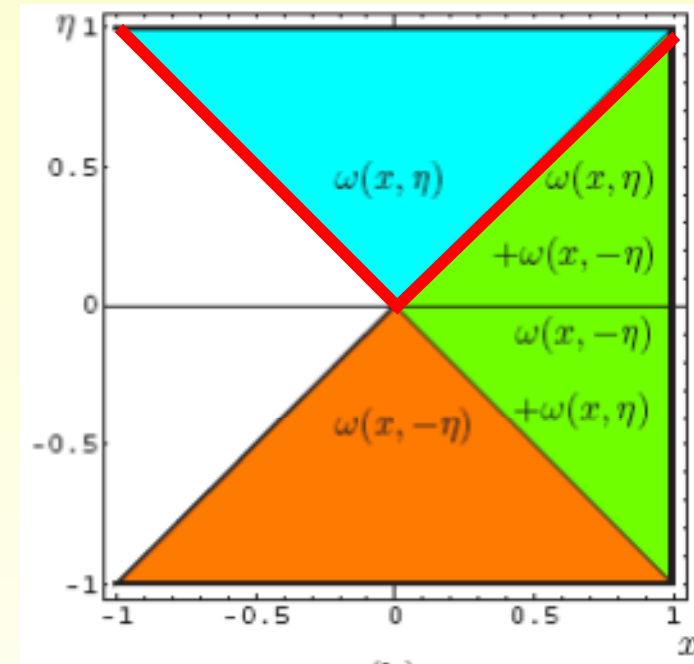
A partonic duality interpretation

quark GPD (anti-quark $x \rightarrow -x$):

$$F(x, \eta, t) = \theta(-\eta \leq x \leq 1) \omega(x, \eta, t) + \theta(\eta \leq x \leq 1) \omega(x, -\eta, t)$$

$$\omega(x, \eta, t) = \frac{1}{\eta} \int_0^{\frac{x+\eta}{1+\eta}} dy (a + bx) f(y, (x-y)/\eta, t)$$

dual interpretation on partonic level:

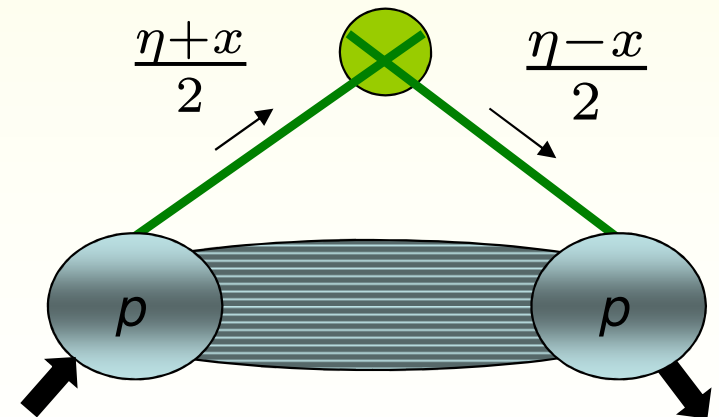


central region - $\eta < x < \eta$
mesonic exchange in t -channel

support extension
is unique [DM et al. 92]



ambiguous (D -term)
[DM, A. Schäfer (05)
KMP-K (07)]



outer region $\eta < x$
partonic exchange in s -channel

Double distribution (DD) representations

- general DD representation might be quoted as

$$F(x, \eta, t) = \int_0^1 dy \int_{-1+y}^{1-y} dz \delta(x - y - z\eta) [f(y, z, t) + x\Delta f(y, z, t)]$$

$$\Delta f(y, z, t) = 0 \quad \text{for signature-odd and} \quad F \in \{H + E, \tilde{H}\}$$

DD can be converted into $f + \Delta f + \eta z \Delta f$ with $f, \Delta f$ symmetric in z

- if representation is fixed, DDs are obtained by Radon transform
- alternatively one may use, e.g., [Polyakov, Weiss] D -term

[Belitsky,
DM et al.;
Teryaev (01)]

$$H^{q(+)}(x, \eta, t) = \int_0^1 dy \int_{-1+y}^{1-y} dz \delta(x - y - z\eta) f'(y, z, t) + \theta(|x| \leq |\eta|) \text{sign}(\eta) D(x/\eta)$$

with $f' = f + w \otimes \Delta f$ and $D(x) = \lim_{\eta \rightarrow \infty} H^{q(+)}(x\eta, \eta, t)$

- ambiguity: DDs are generalized functions, i.e., it is allowed to add $\delta^{(n)}(y)$ [or D -, F -, ...-terms], one might end up with double counting
- Is D -term an integral part of a GPD or not? (answer depends on believes) ¹⁵

Uses of DDs in phenomenology

Radyushkin's double distribution ansatz (RDDA) is employed
(original DD + D-term for $H, E + \pi$ -pole for $\check{E}$)

GPV (VGG code), BMK, GK:

$$f(y, z, t) = F_f(t) y^{-\alpha' t} \frac{q_f(y)}{1-y} \frac{\Gamma\left(\frac{3}{2} + b\right) \left(1 - \frac{z^2}{(1-y)^2}\right)^b}{\sqrt{\pi} \Gamma(1+b)}$$

in form factor modeling:

$$\int_{-1+x}^{1-x} dz f(x, z, t) = q_f(x) \exp\{t g_f(x)\}$$

NOTE:

- GPV & BMK (I give it up 2005) $\alpha'=0$, quark angular momentum mostly fixed
- VGG code now (form factor sum rule can be violated, J_u/J_d issue)
- GK uses not Diehl-Kroll ansatz from form factor fits; only J_{sea} is a free parameter
profile parameter b is fixed (integer value)

NLO PDFs are refitted with integer β , evolution is **not** GPD evolution

- RDDA is so **rigid** that it is a **holographic** model [Kumericki, DM (10)]
($F(x, x, t)$ and $F(x, 0, t)$ allow to restore the whole GPD)

large- x & small- x behavior are tied:

$$\frac{F(\xi, \xi, t)}{F(\xi, 0, t)} \underset{\xi \rightarrow 1}{=} \frac{2^b \Gamma\left(\frac{3}{2} + b\right) \Gamma(1+b-\alpha(t)) \Gamma(\beta-b)}{\sqrt{\pi} \Gamma(1+b) \Gamma(1-\alpha(t)+\beta)} \frac{(1-\xi)^b}{(1-\xi)^\beta}$$

$$\frac{F(\xi, \xi, t)}{F(\xi, 0, t)} \underset{\xi \rightarrow 0}{=} \frac{\Gamma\left(\frac{3}{2} + b\right) \Gamma(1+b-\alpha(t))}{\Gamma\left(1+b-\frac{\alpha(t)}{2}\right) \Gamma\left(\frac{3}{2} + b - \frac{\alpha(t)}{2}\right)}$$

Conformal partial wave expansion of GPDs

- a GPD can be expanded with respect to conformal partial waves of the collinear conformal group $SO(2,1)$ (similar to $SO(3)$ expansion)

- expansion in terms of discrete conformal spin $j+2$ for $\eta > 1, |x/\eta| \leq 1$

$$F(x, \eta, t) = \sum_{j=0}^{\infty} (-1)^j p_j(x, \eta) F_j(\eta, t) \quad \mathbf{z=x/\eta} \longleftrightarrow \mathbf{j+2}$$

- conformal moments (partial wave amplitudes) are polynomials:

$$F_j(x, \eta) = \frac{\Gamma(3/2)\Gamma(1+j)}{2^j\Gamma(3/2+j)} \int_{-1}^1 dx \eta^{j+1} C_j^{3/2} \left(\frac{x}{\eta} \right) F(x, \eta, t)$$

- conformal partial waves ensure the polynomiality condition:

$$p_j(x, \eta) = \frac{\Gamma(5/2+j)}{j!\Gamma(1/2)\Gamma(2+j)} \frac{d^j}{dx^j} \int_{-1}^1 du (1-u^2)^{j+1} \delta(x-u\eta)$$

- ✓ **crossing symmetry** allows for a more convenient representation
(technicality, e.g., Sommerfeld-Watson transform, numerous failures in the literature)

- ✓ PWs evolve autonomously  trivial implementation of LO evolution
NLO done by perturbative expansion

Summing up conformal PWs

- GPD support is a consequence of Poincaré covariance (polynomiality)

$$H_j(\eta, t, \mu^2) = \int_{-1}^1 dx c_j(x, \eta) H(x, \eta, t, \mu^2), \quad c_j(x, \eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

$$\mu \frac{d}{d\mu} H_j(\eta, t, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \gamma_j^{(0)} H_j(\eta, t, \mu^2)$$

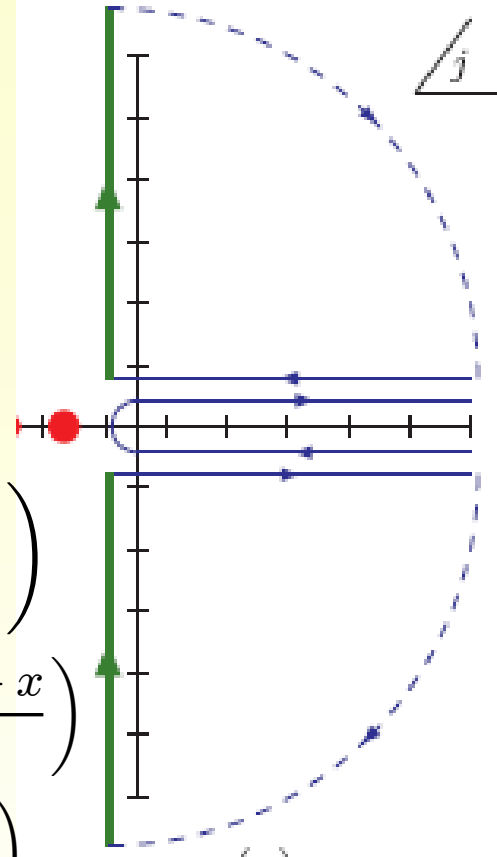
- inverse relation is given as series of (mathematical) generalized distributions:

$$H(x, \eta, t) = \sum_{j=0}^{\infty} (-1)^j p_j(x, \eta) H_j(\eta, t), \quad p_j(x, \eta) \propto \theta(|x| \leq \eta) \frac{\eta^2 - x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:

- smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- dual parameterization [M. Polyakov, A. Shuvaev (02), Polyakov (07), Semenov-Tian-Shansky]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- **Mellin-Barnes integral** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

Sommerfeld-Watson transform



- ✓ rewrite sum as an integral around the real axis:

$$F(x, \eta, \Delta^2) = \frac{1}{2i} \oint_{(0)}^{(\infty)} dj \frac{1}{\sin(\pi j)} p_j(x, \eta) F_j(\eta, \Delta^2)$$

- ✓ find appropriate analytic continuation of p_j and F_j
(Carlson's theorem)

$$p_j(x, \eta) = \theta(\eta - |x|) \eta^{-j-1} \mathcal{P}_j \left(\frac{x}{\eta} \right) + \theta(x - \eta) \eta^{-j-1} \mathcal{Q}_j \left(\frac{x}{\eta} \right)$$

$$\mathcal{P}_j(x) = \frac{2^{j+1} \Gamma(5/2 + j)}{\Gamma(1/2) \Gamma(1 + j)} (1 + x) {}_2F_1 \left(\begin{matrix} -j - 1, j + 2 \\ 2 \end{matrix} \middle| \frac{1 + x}{2} \right)$$

$$\mathcal{Q}_j(x) = -\frac{\sin(\pi j)}{\pi} x^{-j-1} {}_2F_1 \left(\begin{matrix} (j + 1)/2, (j + 2)/2 \\ 5/2 + j \end{matrix} \middle| \frac{1}{x^2} \right)$$

- ✓ change integration path so that singularities remain on the l.h.s.

$$F(x, \eta, \Delta^2) = \frac{i}{2} \int_{c-i\infty}^{c+i\infty} dj \frac{1}{\sin(\pi j)} p_j(x, \eta) F_j(\eta, \Delta^2)$$

- ✓ **NOTE:** continuation of GPD conformal moments has not worked out numerically
RDDA with integer b , β , like GK model, can be transformed in Mellin space

Implementing constraints

- form factor and PDF constraints can be trivially implemented, e.g., (also Lattice constraints could be treated in this way, if they are considered as reliable)

$$F_j(\eta = 0, t) = q_{F,j} F_F(t|M_j), \quad q_{F,j} = \int_0^1 dx x^j q_F(x), \quad F_F(t|M_{j=0}) = F_F(t)$$

- flexible skewness dependence can be implemented, e.g., by SO(3) PW expansion (Wigner matrices) [‘dual’ model]

[Polyakov (99),
Lebed, Ji (00),
Diehl(03),...]

$$F_j(t, \eta) = \sum_J f_j^J(t) \eta^{j(+1)-J} \hat{d}_J^F(\eta), \quad \hat{d}_J^F(\eta = 0) = 1$$

- all PWs contribute in the small- ξ approximation of CFFs
- taking leading PW yields the Shuvaev claim (tying GPDs to PDFs at small- ξ)
- two PWs can be used to mimic the RDDA
- three PWs can be used to control normalization and evolution flow at small- ξ
- to have flexibility at large- ξ one must resum, i.e., in fitting one should replace Wigner matrices by some effective functions

see Kirill's talk

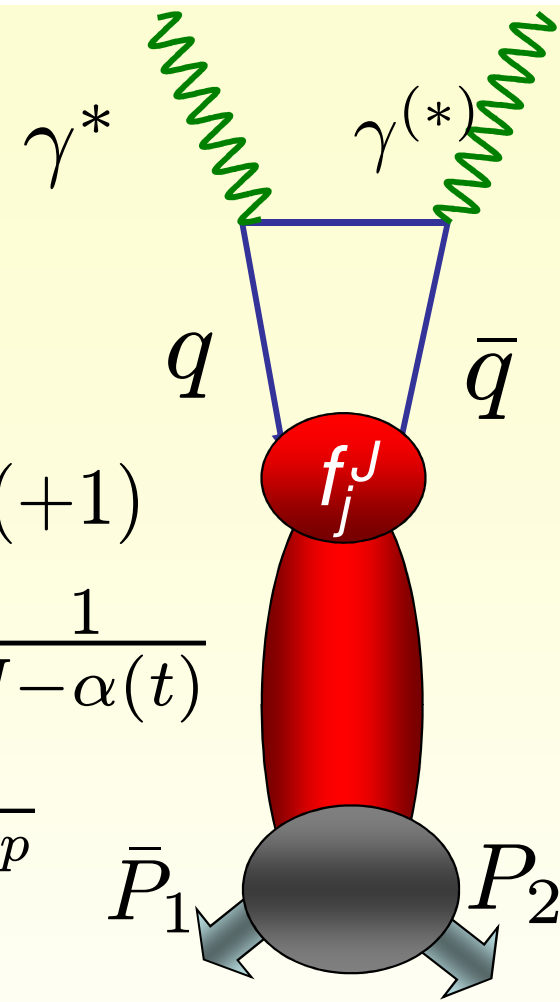
GPD ansatz from t -channel view

- ❖ at short distance a quark/anti-quark state is produced, labeled by **conformal spin** $j+2$
- ❖ they form an intermediate mesonic state with total angular momentum **J** strength of **coupling** is
- ❖ mesons propagate with
- ❖ decaying into nucleon anti-nucleon pair with given angular momentum J , described by an **impact form factor**

$$f_j^J, J \leq j (+1)$$

$$\frac{1}{m^2(J) - t} \propto \frac{1}{J - \alpha(t)}$$

$$\frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p}$$



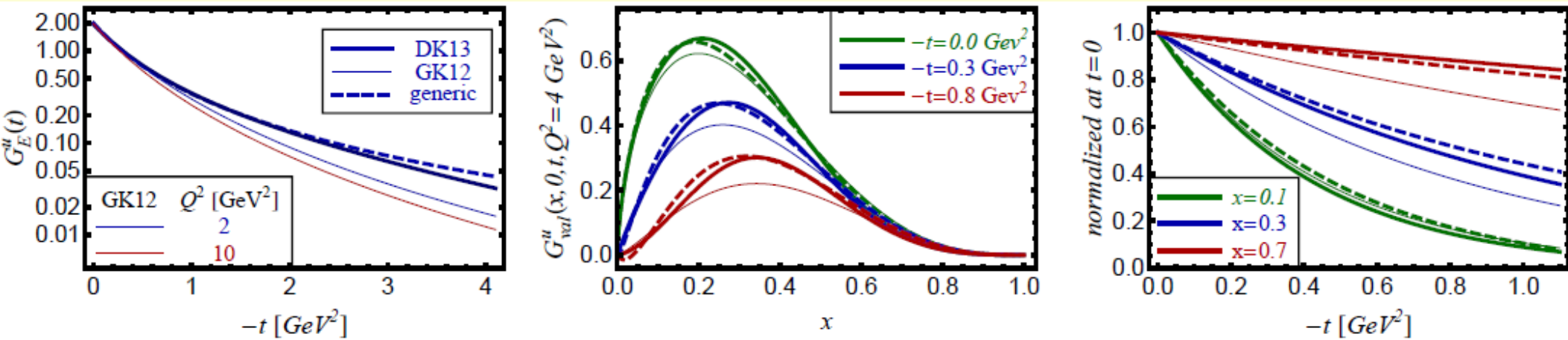
- (conformal) GPD moments expanded in Wigner's rotation matrices

$$F_j(t, \eta) = \sum_J^{j(+1)} \frac{f_j^J}{J - \alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p} \eta^{j(+1)-J} \hat{d}_J^F(\eta), \quad \hat{d}_J^F(\eta = 0) = 1$$

[Polyakov (99), Lebed, Ji (00), Diehl(03),...]

- labeling by t -channel quantum numbers J^{PC}
- so-called D-term arises from 0^{++} , (f^0 or σ) 2^{++} , 4^{++} , ..., has even $J=j+1$ (or $j = -1$ in DR) pole ($J (=0)$ has multiple meanings [KMP-K(07&08)])
- usable for large x (employing effective rotation matrices)

Comments on skewless GPD modeling

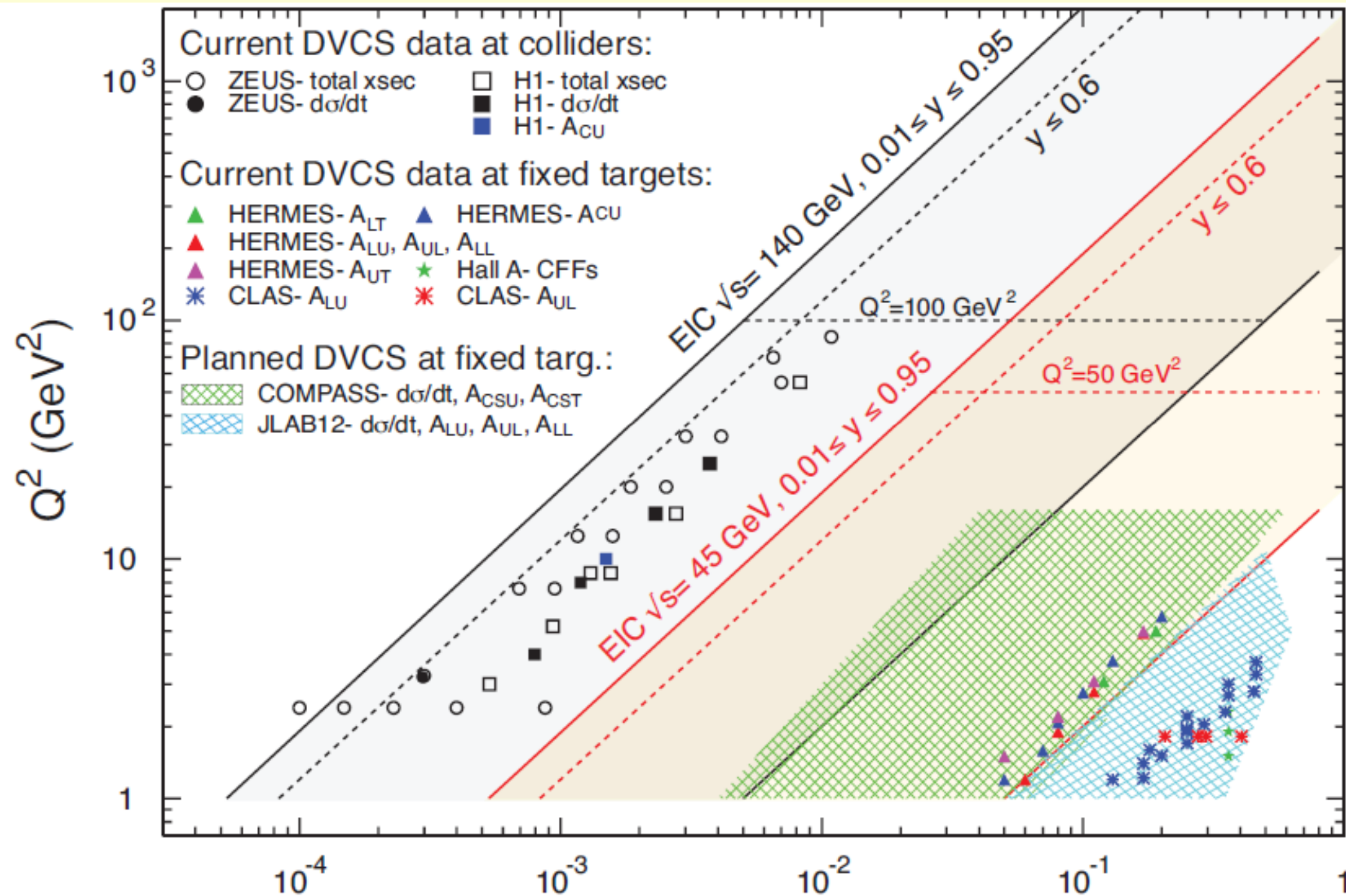


- GK12 uses as in VGG a Regge inspired ansatz for valence quarks
 - not the ansatz as in Guidal et. al or Diehl-Kroll form factor fits
 - induces only a slight violation of form factor sum rules
 - $J_u^{(-)} / J_d^{(-)}$ values fixed (strong J_u / J_d variations are senseless)!
- generic modeling agrees with DK13, having a different functional form

$$F_j^{q^{(-)}}(x, t) = q_F(x) e^{tB(x)} \quad G_j^{u^{(-)}}(t) = \frac{2}{\left(1 + \frac{t}{M_j^2}\right)^2} \frac{\Gamma(1+j-\alpha)\Gamma(2-\alpha+\beta)}{\Gamma(1-\alpha)\Gamma(2+j-\alpha+\beta)}, \quad M_j^2 = \frac{\alpha'}{1+j-\alpha}$$

- How strongly influences the PDF parameterization the t -dependence?
- Do η - and t -dependencies factorize (as commonly assumed)? (properly not)

DVCS world data set



Can one 'measure' GPDs?

- **CFF** given as **GPD convolution**:

$$\mathcal{F}(\xi, t, Q^2) \stackrel{\text{LO}}{=} \int_{-1}^1 dx \left(\frac{1}{\xi - x - i\epsilon} \mp \frac{1}{\xi + x - i\epsilon} \right) F(x, \eta = \xi, t, Q^2)$$

$$\stackrel{\text{LO}}{=} i\pi F^\pm(x = \xi, \eta = \xi, t, Q^2) + \text{PV} \int_0^1 dx \frac{2x}{\xi^2 - x^2} F^\pm(x, \eta = \xi, t, Q^2)$$

- $F(x, x, t, Q^2)$ viewed as **"spectral function"** (s-channel cut):

$$F^\pm(x, x, t, Q^2) \equiv F(x, x, t, Q^2) \mp F(-x, x, t, Q^2) \stackrel{\text{LO}}{=} \frac{1}{\pi} \Im \mathcal{F}(\xi = x, t, Q^2)$$

- **CFFs** satisfy **'dispersion relations'**
(not the physical ones, threshold ξ_0 set to 1)

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

$$\Rightarrow \Re \mathcal{F}(\xi, t, Q^2) = \frac{1}{\pi} \text{PV} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \Im \mathcal{F}(\xi', t, Q^2) + \mathcal{C}(t, Q^2)$$

[Terayev (05)]

$\Rightarrow$ **access** to the **GPD** on the **cross-over line** $\eta = x$ (at LO)

access to the subtraction constant (for H, E related to 'D-term')

Status of theory

[Belitsky, DM (97);
Mankiewicz et. al (97);
Ji, Osborne (97/98);
Pire, Szymanowski, Wagner
(11); DM, Pire, Szymanowski,
Wagner]

DM, T. Lautschlager,
K. Passek-Kumericki.
A. Schaefer (13)

- ✓ **twist-two** coefficient functions at **next-to-leading** order

NLO effects are well understood generically

large- ξ : logarithmical enhancement

valence region: weak evolution implies moderate effects

small- ξ : model dependence

- ✓ anomalous dimensions and evolution kernels at **next-to-leading** order

evolution effects can be called moderate, except for H/E at small- ξ

NLO analyses have to include NLO evolution

[Belitsky, DM (98)
+ Freund (01)]

- ✓ gluon transversity at **next-to-leading** order [Belitsky, DM (00)]

- ✓ **next-to-next-to-leading** order in a specific conformal subtraction scheme

NLO $\rightarrow$ NNLO corrections can be called moderate w.r.t. LO $\rightarrow$ NLO

[DM (06);
KMP-K,
Schaefer 06]

- ✓ **twist-three** including quark-gluon-quark correlation at LO

- ✓ partially, **twist-three** sector at **next-to-leading** order

[Kivel, Mankiewicz (03)]

[Anikin, Teryaev, Pire (00);
Polyakov et. al (00),
Belitsky DM (00); Kivel et. al,
Weiss, Radyushkin (00)]

- ? 'target mass corrections' (not understood) [Belitsky DM (01)]

- ✓ kinematical twist-four corrections [Braun, Manashov (11)]

DVCS phenomenology is often done at LO, DVMP/DVCS NLO fits just started

Strategies to analyze DVCS data

(ad hoc) modeling: VGG code [Goeke et. al (01) based on Radyushkin's DDA]
BMK model [Belitsky, DM, Kirchner (01) based on RDDA]
'aligned jet' model [Freund, McDermott, Strikman (02)]
Goloskokov/Kroll (05) based on RDDA (pinned down by DVMP)
'dual' model [Polyakov, Shuvaev 02; Guzey, Teckentrup 06; Polyakov 07]
" -- " [KMP-K (07) in MBs-representation]
polynomials [Belitsky et al. (98), Liuti et. al (07), Moutarde (09)]

dynamical models: not applied [Radyushkin et.al (02); Tiburzi et.al (04); Hwang DM (07)]...
(respecting Lorentz symmetry)

flexible models: any representation by including *unconstrained* degrees of freedom
(for fits) KMP-K (07/08) for H1/ZEUS in MBs-integral-representation

CFFs (real and imaginary parts) and GPD fits/predictions

- i. CFF extraction with formulae (local) [BMK (01), HALL-A (06)] and [KK, DM, Murray]
least square fits (local) [Guidal, Moutarde (08...)]
neural networks – a start up [KMS (11)]
- ii. 'dispersion integral' fits [KMP-K (08), KM (08...)]
- iii. flexible GPD modeling [KM (08...)]
- vi. model comparisons VGG code, however also BMK01 (up to 2005)
& predictions Goloskokov/Kroll (07) model based on RDDA²⁶

Asking for CFFs (physics case)

- CFFs are defined for the whole kinematical region [Belitsky, DM, Ji (12)]
- contain (generalized) polarizabilities
- their access requires a complete measurement

toy example DVCS off a scalar target

[KK, DM, Murray (13)]

- for the first step we use s-channel helicity conservation hypothesis (neglecting twist-three and transversity associated CFFs)

- linearized set of equations (approximately valid)

$$A_{\text{LU,I}}^{\sin(1\phi)} \approx N c_{\mathfrak{Jm}}^{-1} \mathcal{H}^{\mathfrak{Jm}} \quad \text{and} \quad A_{\text{C}}^{\cos(1\phi)} \approx N c_{\mathfrak{Re}}^{-1} \mathcal{H}^{\mathfrak{Re}}$$

- normalization N is bilinear in CFFs

$$0 \lesssim N(\mathbf{A}) \approx \frac{1}{1 + \frac{k}{4} |\mathcal{H}|^2} \approx \frac{\int_{-\pi}^{\pi} d\phi \mathcal{P}_1(\phi) \mathcal{P}_2(\phi) d\sigma_{\text{BH}}(\phi)}{\int_{-\pi}^{\pi} d\phi \mathcal{P}_1(\phi) \mathcal{P}_2(\phi) [d\sigma_{\text{BH}}(\phi) + d\sigma_{\text{DVCS}}(\phi)]} \lesssim 1$$

- cubic equation for N with two non-trivial solutions

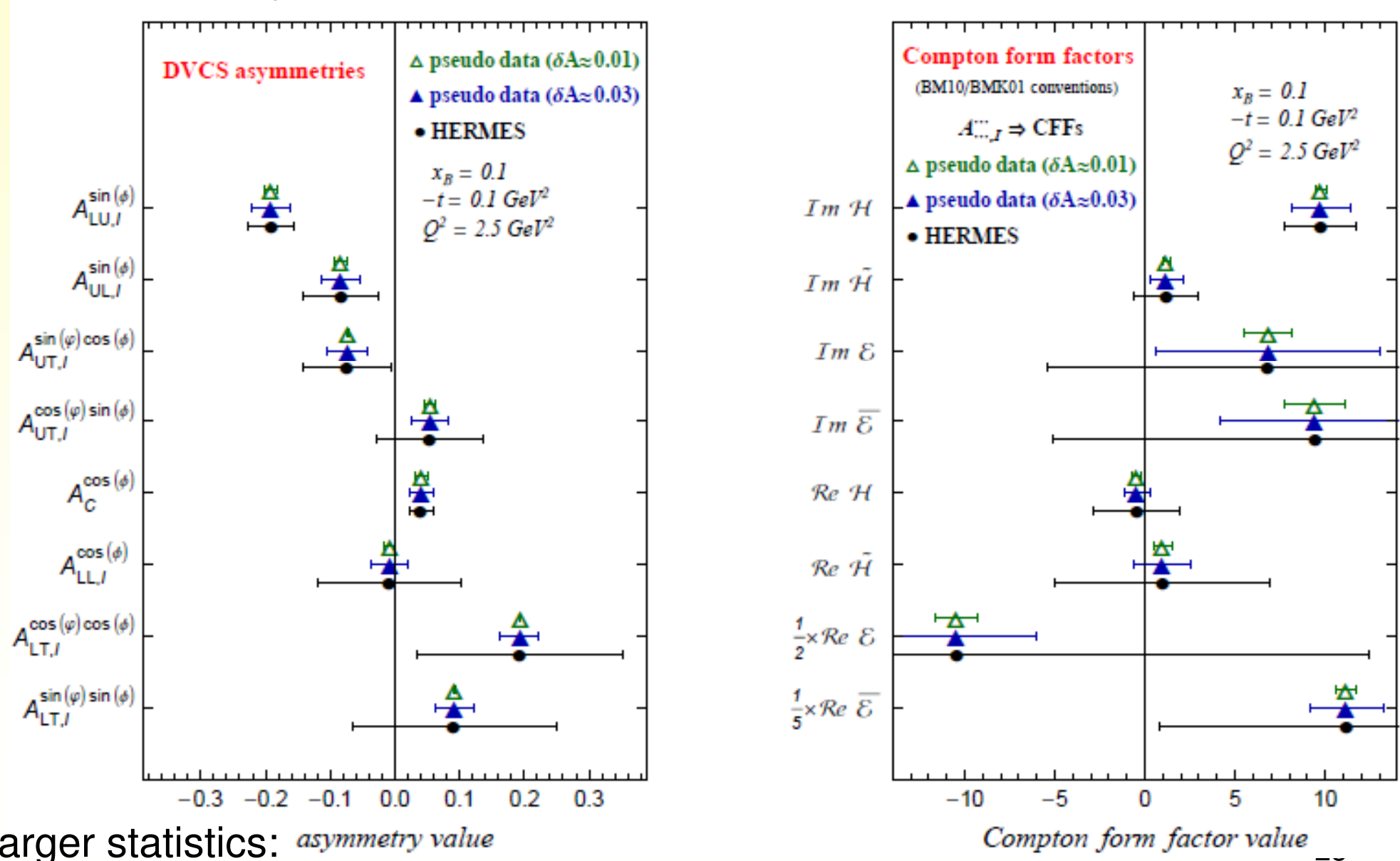
$$N(\mathbf{A}) \approx \frac{1}{2} \left(1 \pm \sqrt{1 - k c_{\mathfrak{Jm}}^2 \left(A_{\text{LU,I}}^{\sin(1\phi)} \right)^2 - k c_{\mathfrak{Re}}^2 \left(A_{\text{C}}^{\cos(1\phi)} \right)^2} \right)$$

+ BH regime
- DVCS regime

- standard error propagation

NOTE: there is no need to **linearize**, we do mapping numerically

- a complete measurement allows in principle to pin down all CFFs KK, DM, Murray (13)
- missing information in incomplete measurements can be filled with noise
(Michel's philosophy: use noise together with hypotheses and model constraints, our results are compatible)



- larger statistics: *asymmetry value*
 some CFF $\mathcal{E}$ constraint might have been obtained by HERMES

A simple valence quarks GPD model

- model of GPD $H(x, x, t)$ within DD motivated ansatz at $Q^2 = 2 \text{ GeV}^2$

fixed:

PDF normalization eff. Reage pole large t -counting rules

$$H(x, x, t) = \frac{n r 2^\alpha}{1+x} \left(\frac{2x}{1+x} \right)^{-\alpha(t)} \left(\frac{1-x}{1+x} \right)^b \frac{1}{\left(1 - \frac{1-x}{1+x} \frac{t}{M^2} \right)^p}.$$

free parameters: r -ratio at small x large x -behavior p -pole mass

- unpolarized valence quarks : asking for r, b, M parameters

$$n = 1.0, \quad \alpha(t) = 0.43 + 0.85t/\text{GeV}^2, \quad p = 1$$

- flexible parameterization of subtraction constant
(so-called D-term convoluted with hard amplitude)

$$\mathcal{D}(t) = \frac{-C}{(1-t/M_c^2)^2}$$

- analogous ansatz for polarized quark GPD + pion-pole contribution
- no $E(x, x, t)$ nor $\hat{E}(x, x, t)$ is set up
- KM...> 2010 hybrid models GPD evolution for sea /gluon + DR for valence

KM10 fits to DVCS off unpolarized proton

- a hybrid model: three effective SO(3) PWs for sea quarks/gluons
dispersion relations for valence
still E GPD is neglected (only D-term)
still $\hat{E}$ GPD only flexible pion pole contribution
- asking for GPD H and 'D-term' ($\hat{H}$ is considered as effective d.o.f.)

leading order, including evolution for sea quarks/ gluons
quark twist-two dominance hypothesis within CFF convention [BM10]

- data selection (taking moments of azimuthal angle harmonics)

KM10a: neglecting HALL-A data

KM10b: forming ratios of moments

KM10: original HALL-A data

neglecting large $-t$ BSA CLAS data

15 parameter fit, e.g.,
including all HALL-A data

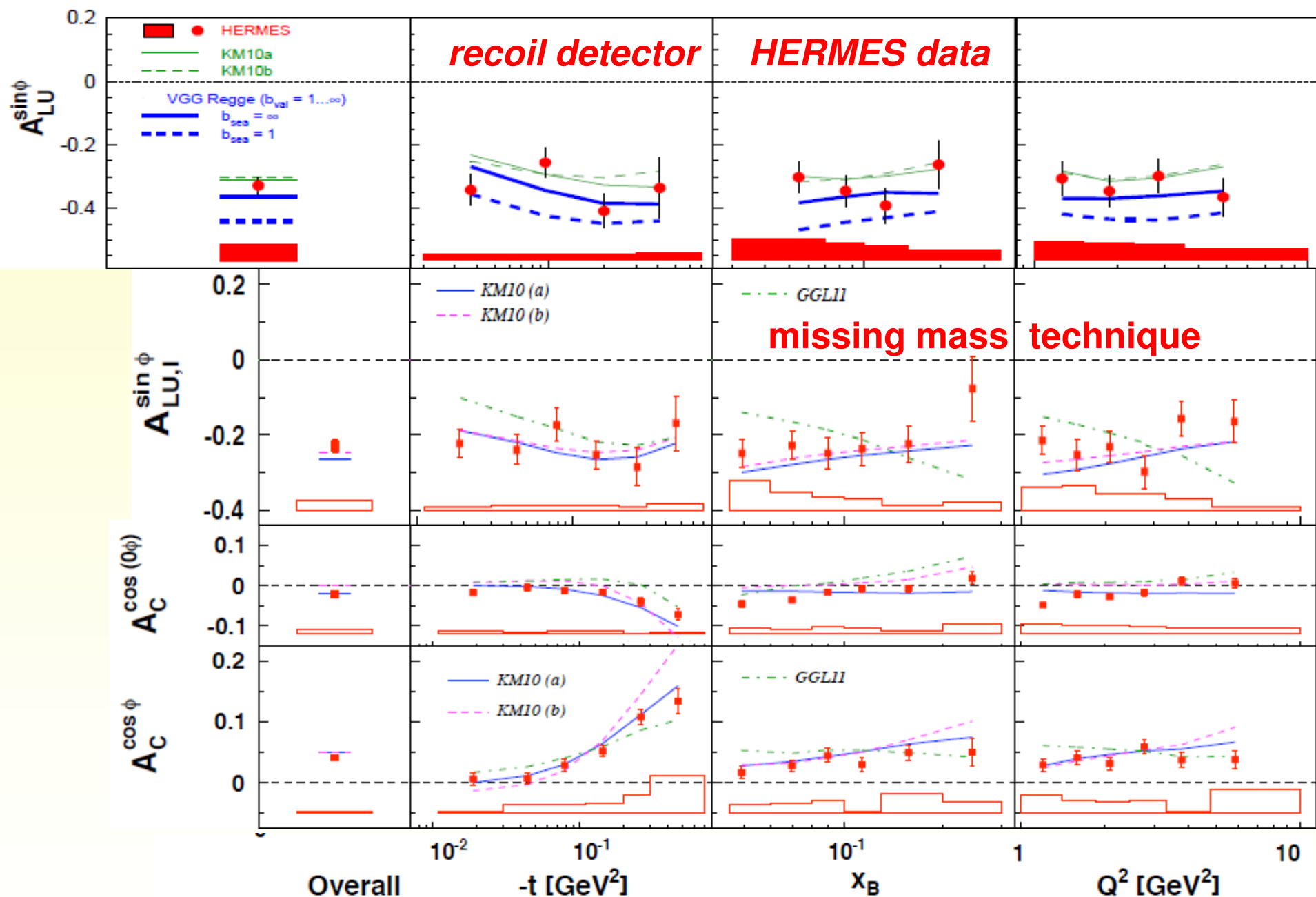
175 data points

$\chi^2/\text{d.o.f.} = 132/165$

```
-----  
MO2S = 0.51 +- 0.02  
SECS = 0.28 +- 0.02  
SECG = -2.79 +- 0.12  
THIS = -0.13 +- 0.01  
THIG = 0.90 +- 0.05  
    Mv = 4.00 +- 3.33 (edge)  
    rv = 0.62 +- 0.06  
    bv = 0.40 +- 0.67  
    C = 8.78 +- 0.98  
    MC = 0.97 +- 0.11  
tMv = 0.88 +- 0.24  
trv = 7.76 +- 1.39  
tbv = 2.05 +- 0.40  
rpi = 3.54 +- 1.77  
Mpi = 0.73 +- 0.37  
-----
```

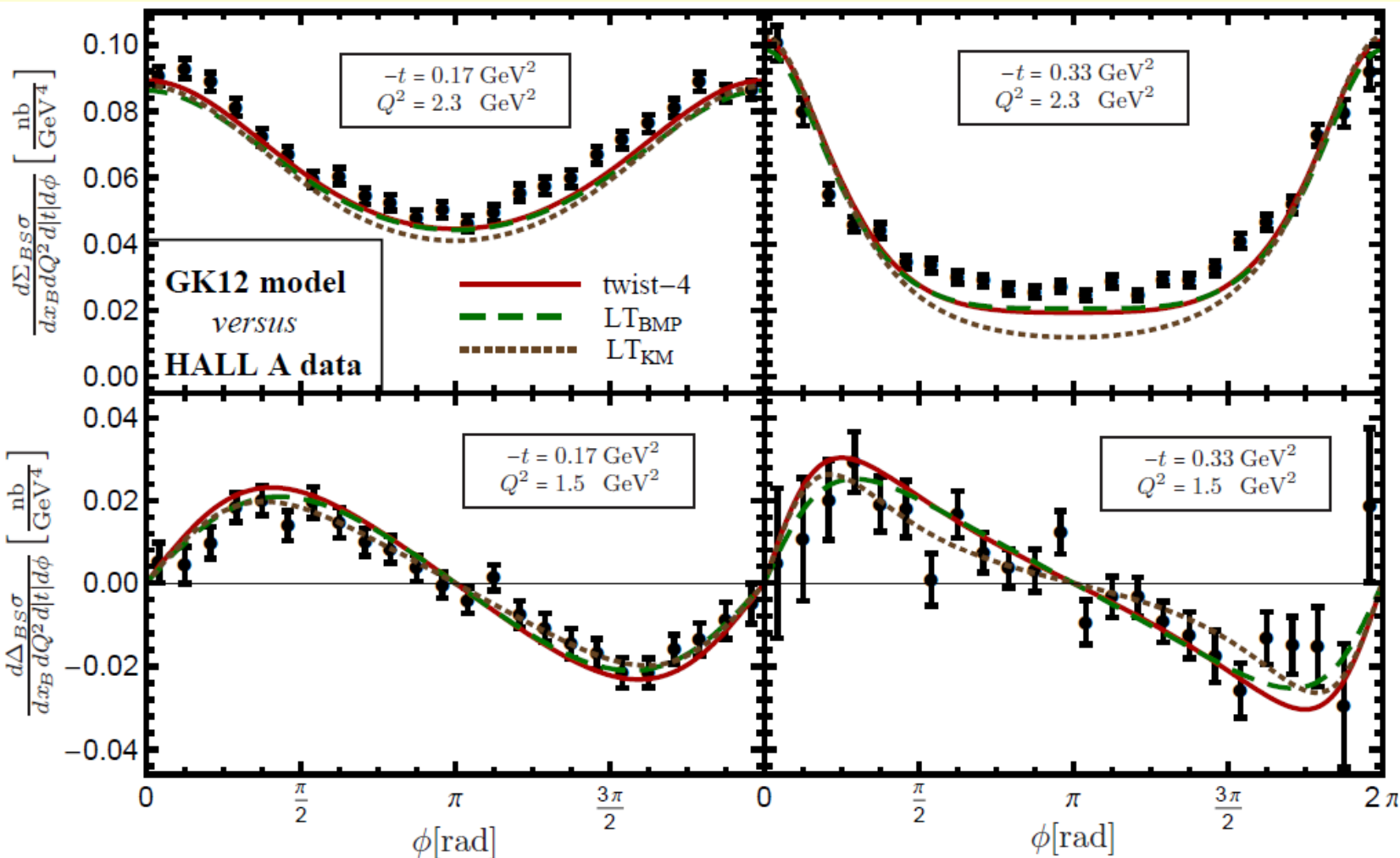
30

- results are given as **xs.exe** on <http://calculon.phy.hr/qpd/>



- recoil detector data are compatible with missing mass technique ones
- fits produce curves where data are scattered around
- recoil data: RDDA is not so much disfavored as it was before the case

How to understand Hall A data?



[Braun, Manashov, Pirnay, DM (14)]

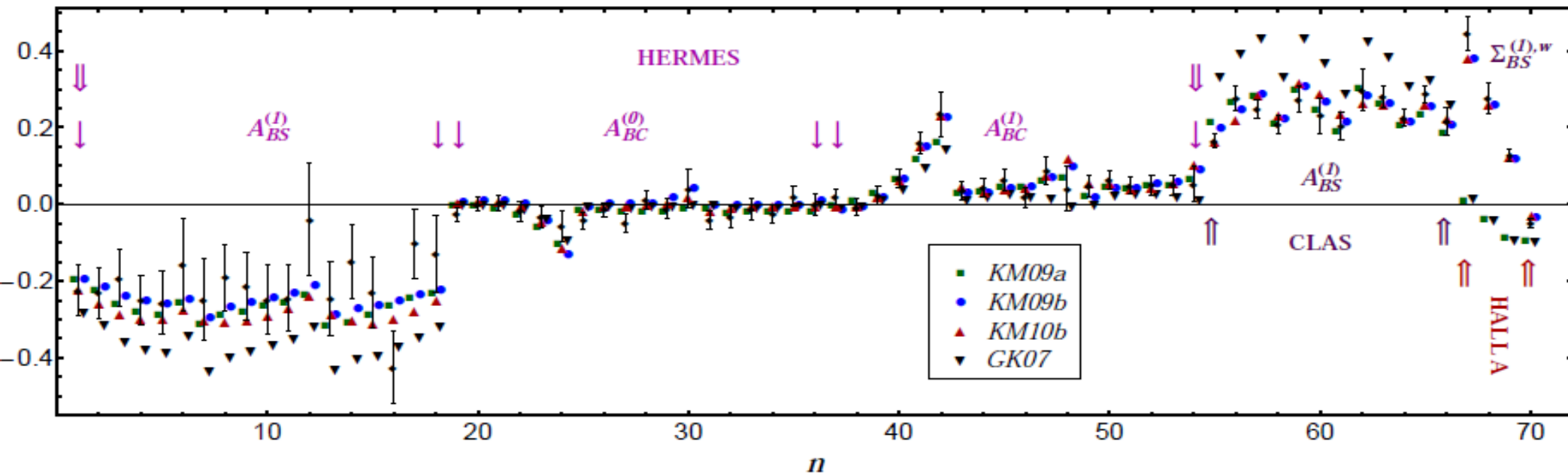
GK12 model evaluated with KM and BMP prescription

including kinematical corrections

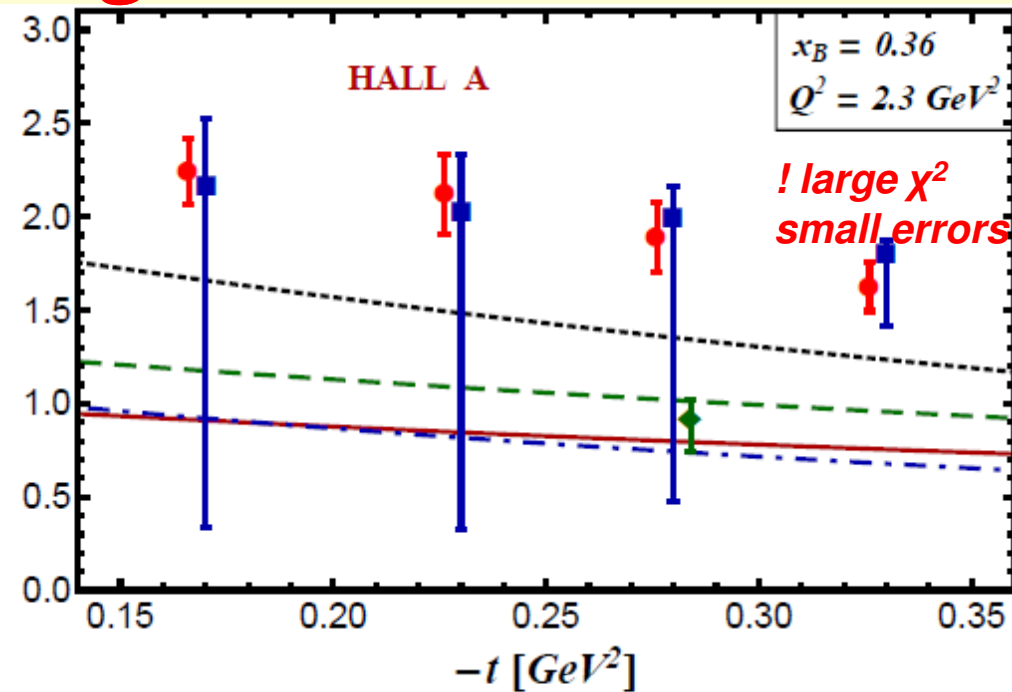
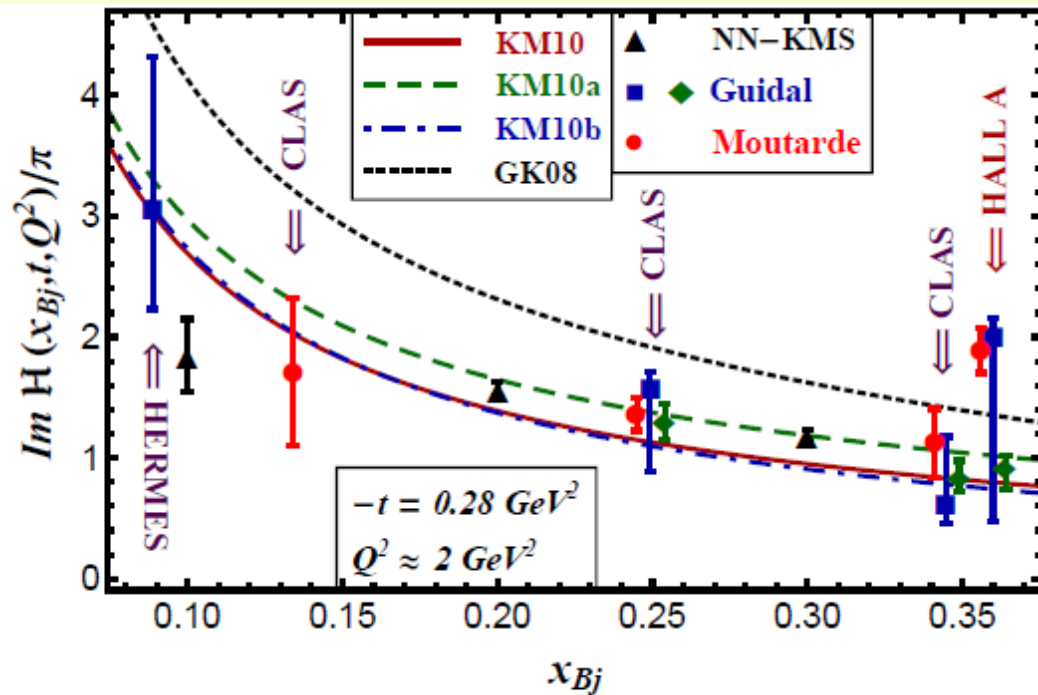
- standard models can not explain HALL A data
- wrong understanding on CFF hierarchy? – inclusion of higher-twist?
- exclusivity issue in all other fixed target data?
- Is (QED) correction procedure understood?
- naive understanding of 'power corrections' [VGG (99)] is misleading

Fixed target DVCS data

- HERMES(02-12) 12x34 asymmetries (+few bins) $0.05 \leq \langle x_B \rangle \leq 0.2$, $\langle |t| \rangle \leq 0.6 \text{ GeV}^2$
 $[\sin(\varphi), \dots, \cos(3\varphi)]$, $\langle Q^2 \rangle \approx 2.5 \text{ GeV}^2$
two kinds of electrons, all polarization options
- HERMES(12) A_{LU} with recoil detector
 (compatible with old data, differences in GPD interpretation)
- CLAS(07) 12x12 $[A_{LU}(\varphi)]$ $0.14 \leq \langle x_B \rangle \leq 0.35$, $\langle |t| \rangle \leq 0.3 \text{ GeV}^2$
 40x12 $[A_{LU}(\varphi)]$ (large $|t|$ or bad sta.) $\langle Q^2 \rangle \approx 1.8 \text{ GeV}^2$
 (06,08) A_{UL} and A_{LU}
- HALL A(06) 12x24 $[\Delta\sigma(\varphi)]$ $\langle x_B \rangle = 0.36$, $\langle |t| \rangle \leq 0.33 \text{ GeV}^2$
 3x24 $[\sigma(\varphi)]$ $\langle Q^2 \rangle \approx 1.8 \text{ GeV}^2$



KM... versus CFF fits & large- x “model” fit



GUIDAL

twist-two dominance hypothesis

7 parameter fit to all harmonics of unpolarized cross section

propagated errors + “theoretical” error estimate

GUIDAL

same + longitudinal TSA

Moutarde

H dominance hypothesis within a smeared polynomial expansion

propagated errors + “theoretical” error estimate

NN

neural network within H dominance hypothesis

green (blue) [red] curves (KM10...) without (with) HALL A data (ratios)

GK08

black curve GPDs (based on RDDA) obtained from handbag approach to DVMP

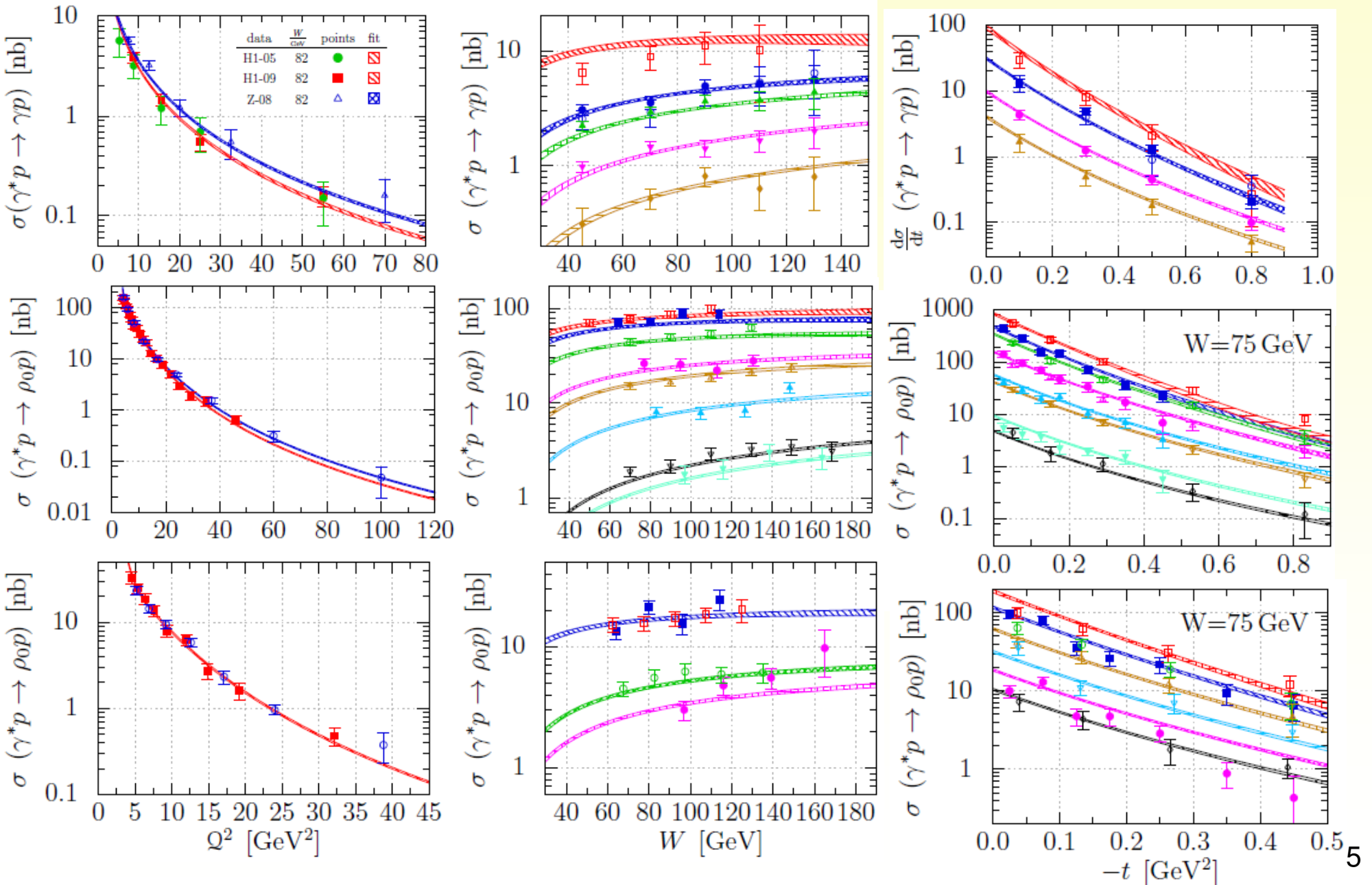
- reasonable agreement for HERMES and CLAS kinematics
- large x -region and real part remains unsettled

DIS+DVCS+DVMP phenomenology at small- x_B (H1,ZEUS)

works somehow without DIS at LO

[T. Lautenschlager, DM, A. Schäfer (13)]

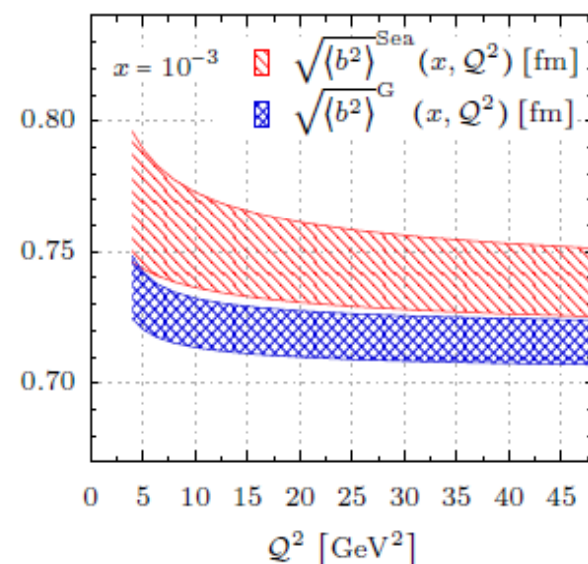
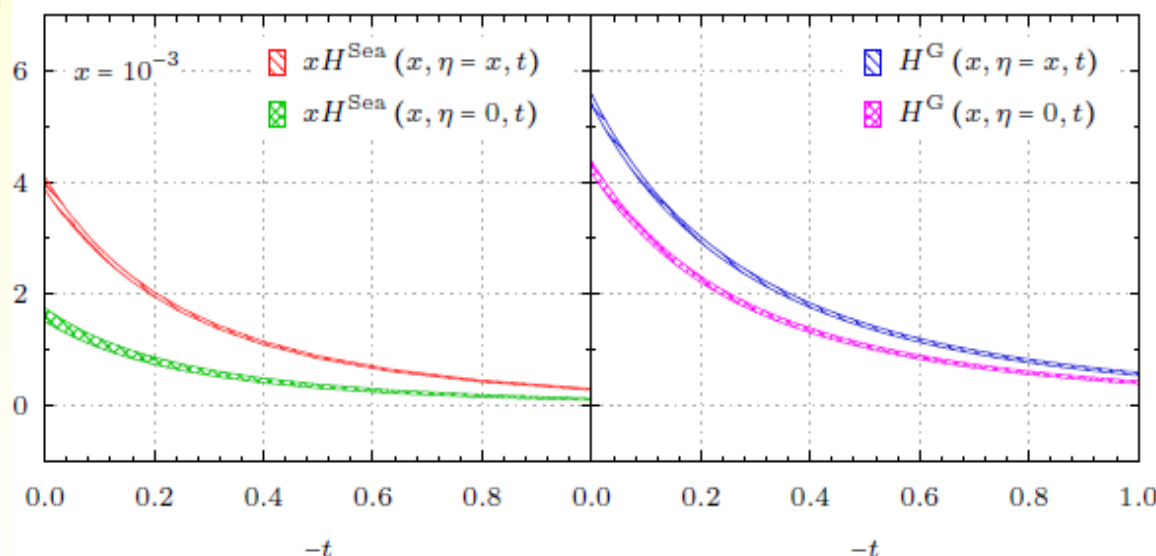
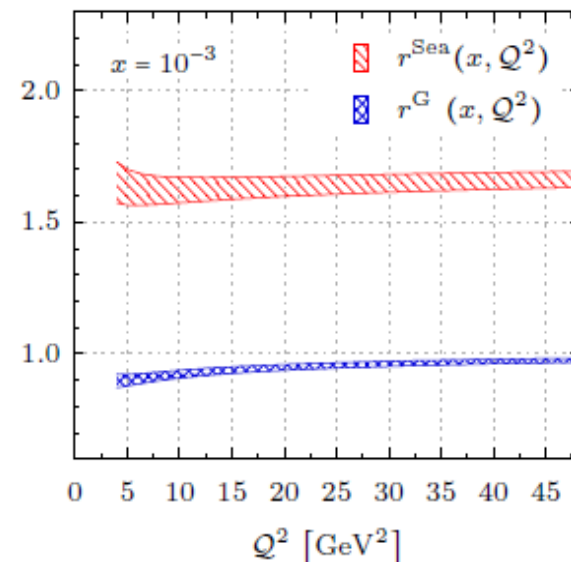
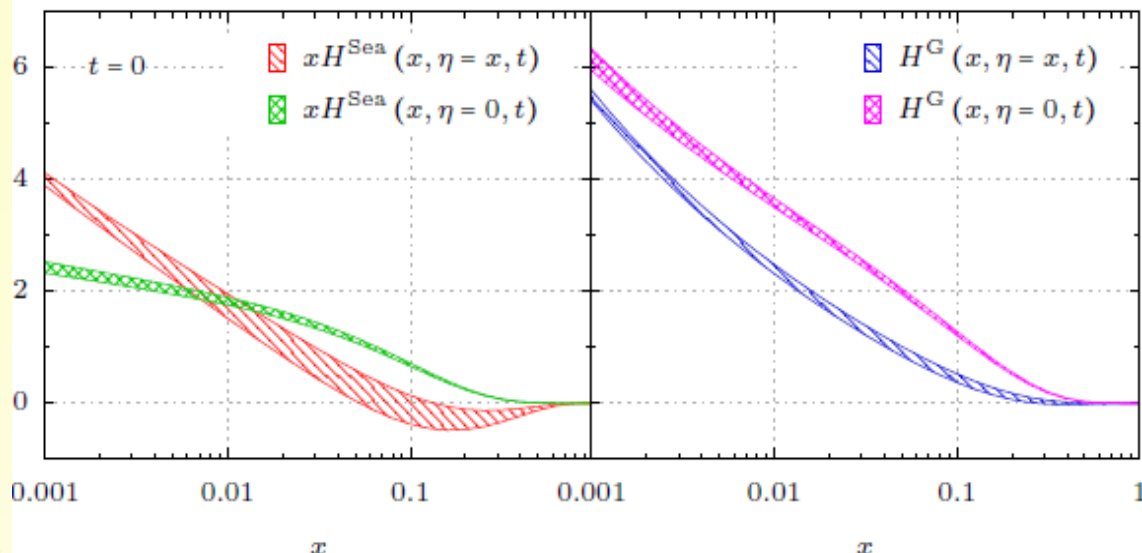
works at NLO ($Q^2 > 4 \text{ GeV}^2$), done with Bayes theorem (probability distribution function)



fixed:
meson DA
flavor content

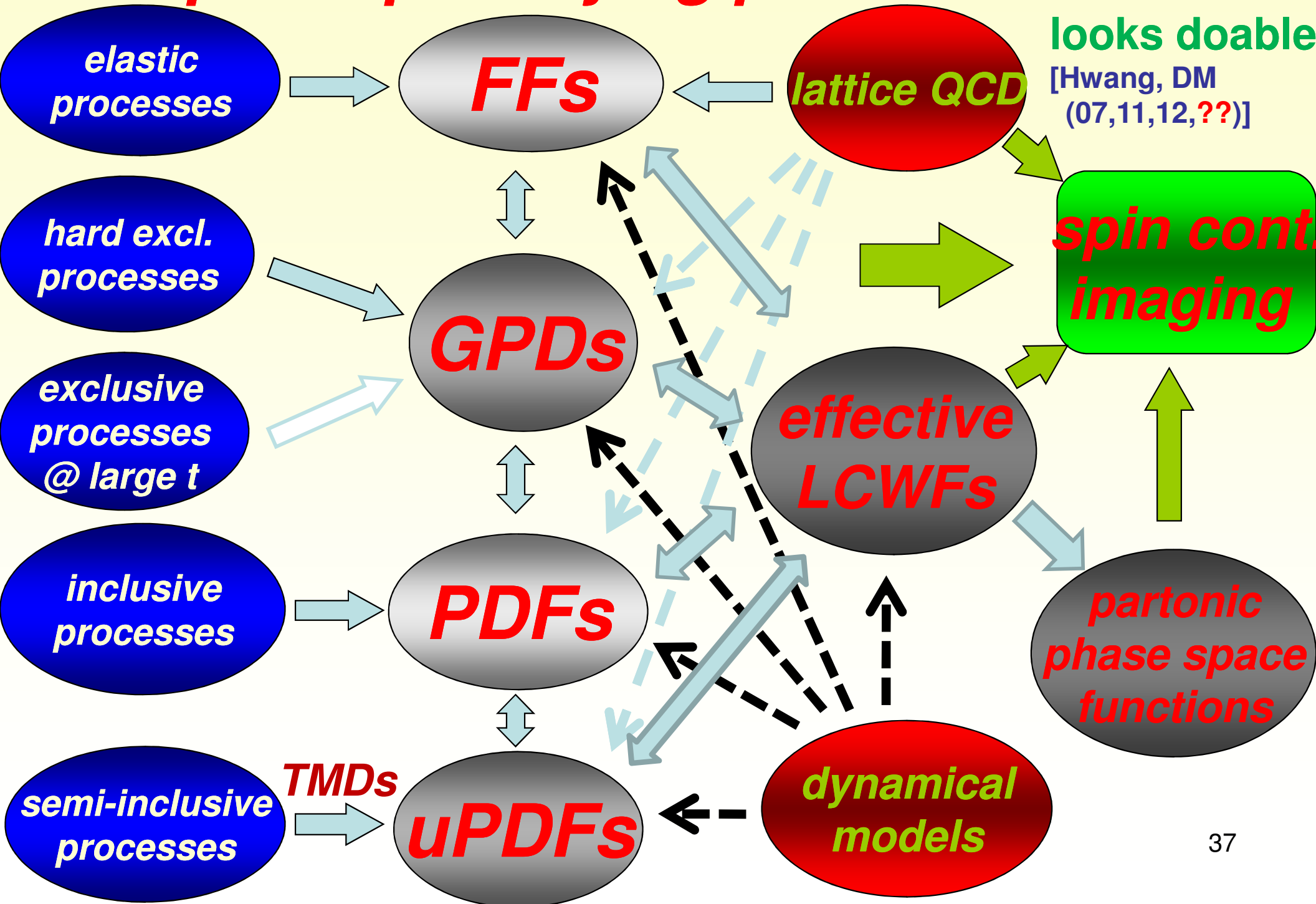
errors might
be perhaps
larger

entirely model
dependency
for $x > 10^{-2}$



- going from LO to NLO increases the skewness ratios (known since 'ever', [\[KMP-K\(07\)\]](#))
- gluons are more centralized as sea quarks (expected from DVCS & J/ψ interpretation)
- cross-talk of skewness and t -dependency has been addressed by pdf
- NLO GPDs look rather compatible to Goloskokov/Kroll and Martin et. al finding
- there is also DVCS beam charge and perhaps **beam spin** data are coming up

Prospect: quantifying partonic content



Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons (to some extent done at small x_B)
- to address the spin content of the nucleon (not possible at present in pheno.)
- providing a bridge to LCWFs & non-perturbative methods (e.g., lattice)
- modeling in terms of effective LCWFs is doable (require efforts)

first decade of hard exclusive leptonproduction measurements

- CFFs have their own interest, bridging low and high virtuality regimes
- should be straightforward to improve global (flexible) model fits to DVCS
- DVCS and DVMP data are describable in global fits at small x
- moving on: to NLO, kinematical twist, full GPD models, DVCS+DVMP+...
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena: handle on GPD E & 3D

need :

tools/technology for global NLO QCD fits (inclusive + exclusive)
theory development (desired but not urgent needed for phenomenology)

back ups

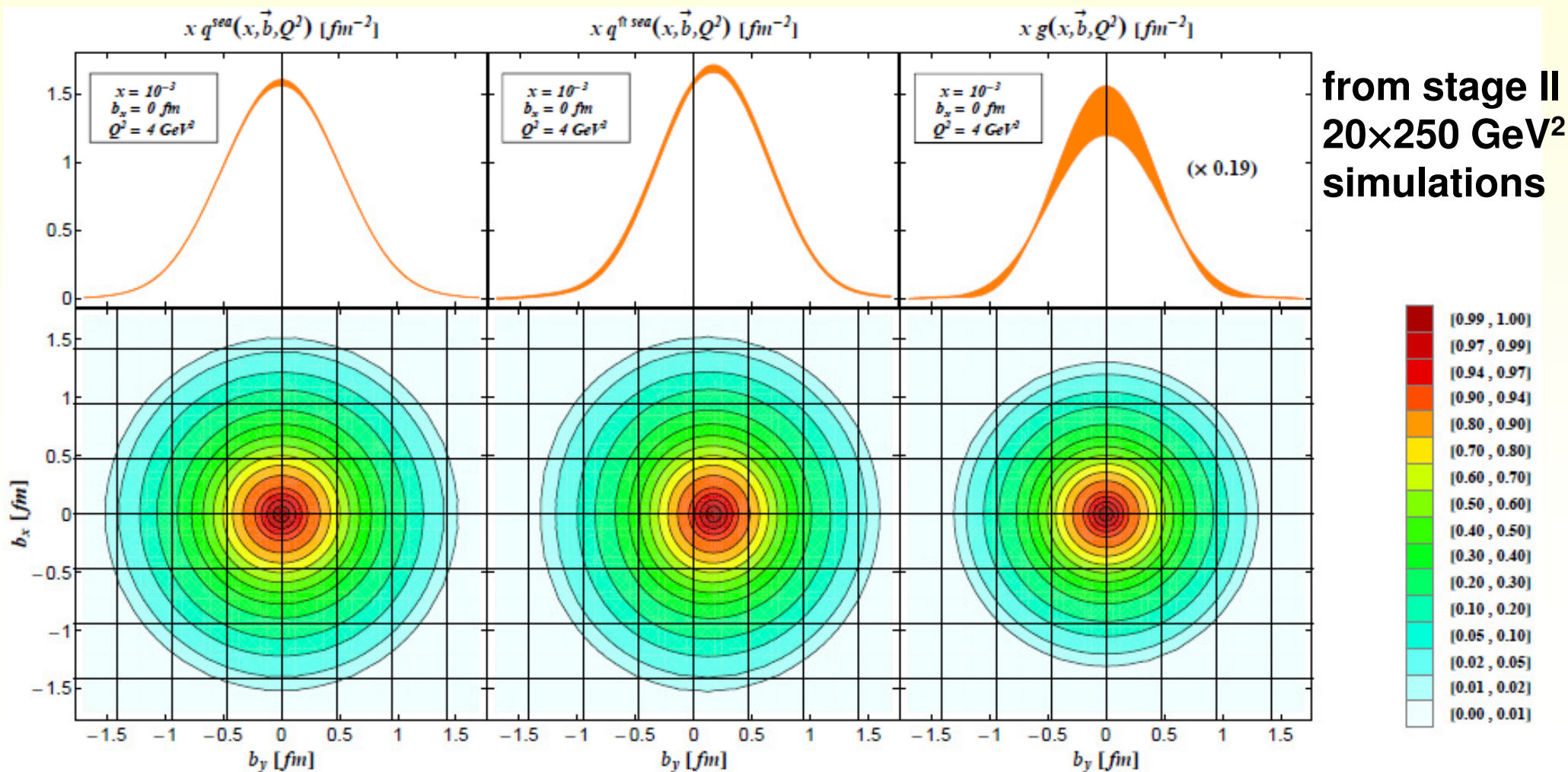
GPD phenomenology lessons: first decade

- qualitatively GPD formalism works in DVCS (from the start up)
- first look: no serious problems in DVMP (apart from ? about very large x_B data) also supported by hand-bag model description of Goloskokov/Kroll
- description of present DVCS data is reached/feasible with flexible models for unpolarized target– but GPD understanding induces tension among data large unidentified contribution called $\hat{H}$ is disfavored by polarized target data
- many uncertainties: exclusivity, correction procedure, assumptions
- HERMES gave proof of principle that one can go for a complete measurement partonic interpretation:
 - RDDA (GVP01, BMK01, VGG code in its many versions, GK07, ...) a bit disfavored at LO can not reach a $\chi^2/dof \sim 1...1.6$ (its like $\chi^2/nop \sim 5...10$) should work at NLO **[Freund, McDermott (02)]**
 - GPD H is dominant (? 15% accuracy), tomography at small- x_B
 - GPD $\hat{H}$ is constrained
 - no access to GPD E from present data, pion pole model for $\hat{E}$ is disfavored
 - D-term related subtraction constant comes out negative (& sizable)
Goke et. al model prediction (perhaps fit result might be not stable)

The Future

- ✓ Compass
- ✓ JLAB@12 GeV
- ? ENC@GSI
- ? LHeC@CERN
- ? EIC@BNL or EIC@JLAB

Aschenauer, Firzo
KK, DM (13)



Impact of EIC data to extract GPD H

two simulations from S. Fazio for DVCS cross section ~ 650 data points

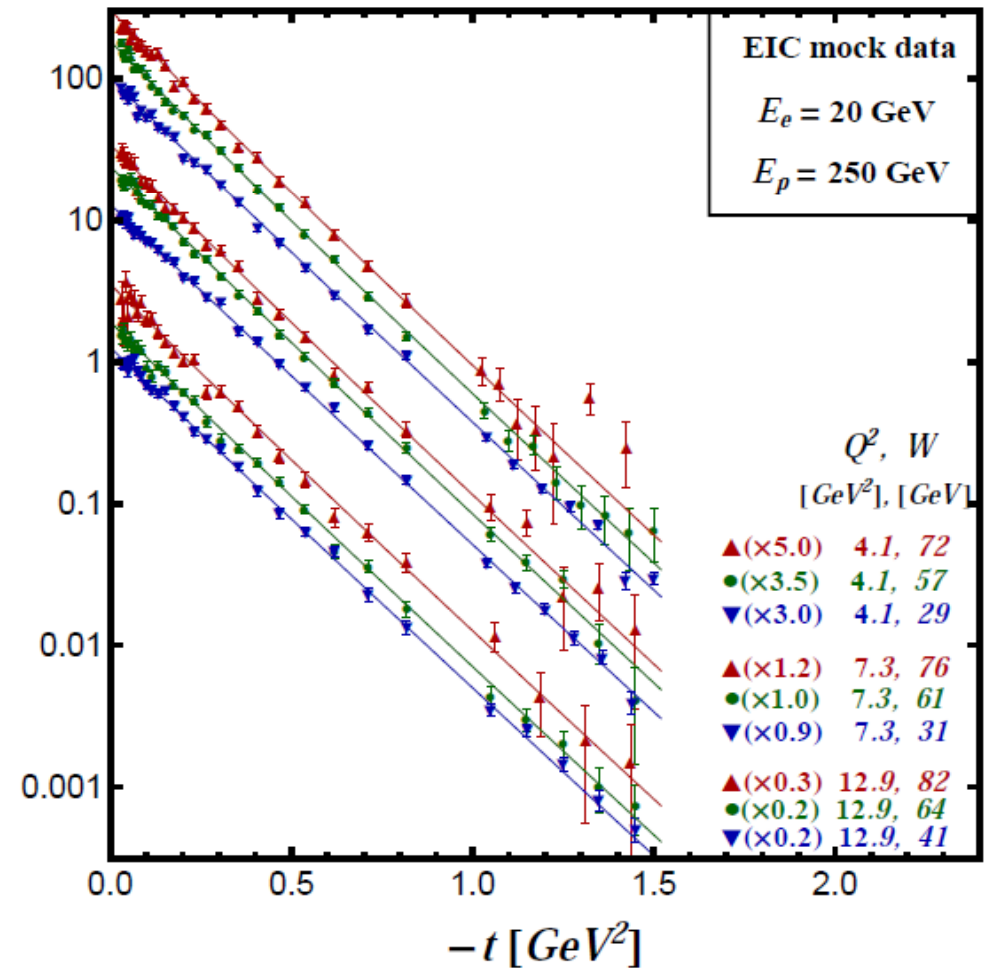
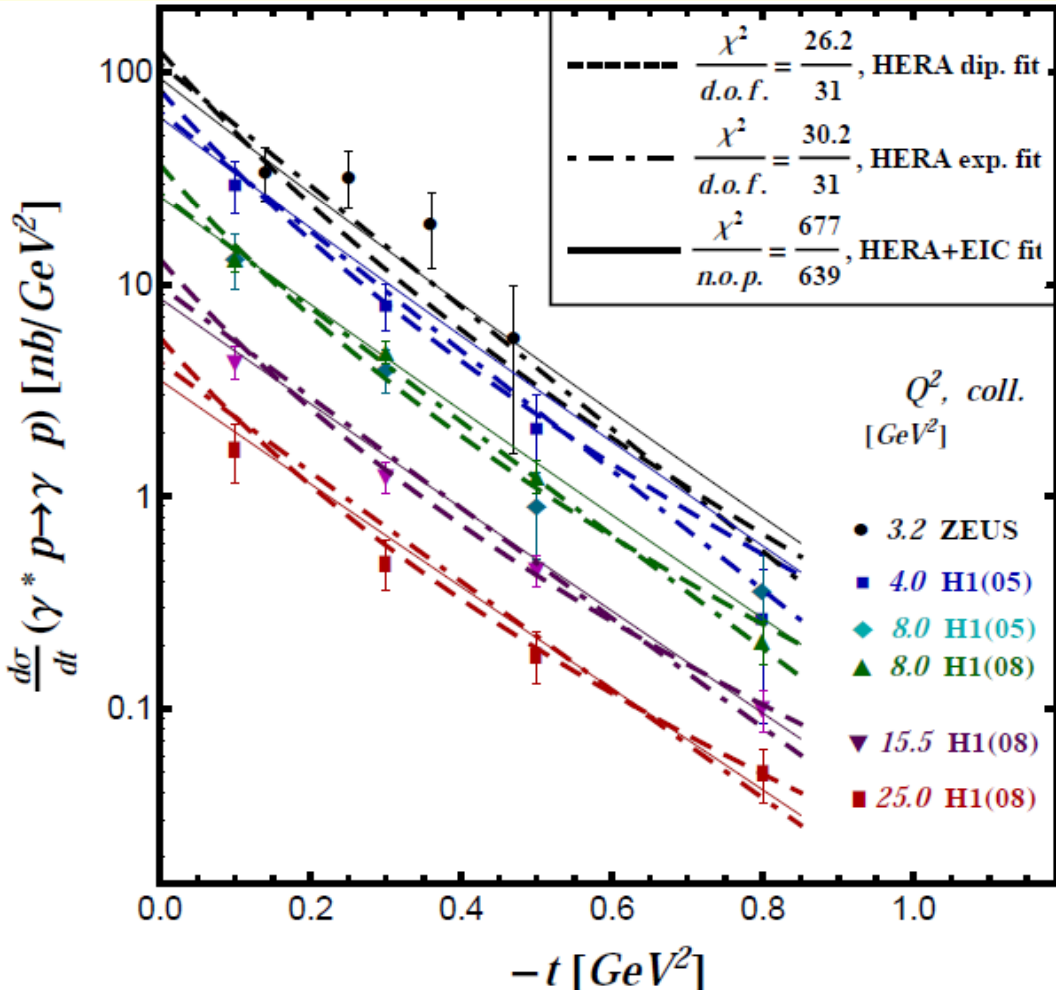
$-t < \sim 0.8 \text{ GeV}^2$ for $\sim 10/\text{fb}$

$1 \text{ GeV}^2 < -t < 2 \text{ GeV}^2$ for $\sim 100/\text{fb}$ (cut: $-t < 1.5 \text{ GeV}^2$, $4 \text{ GeV}^2 < Q^2$ to ensure $-t < Q^2$)

pseudo data are re-generated with GeParD

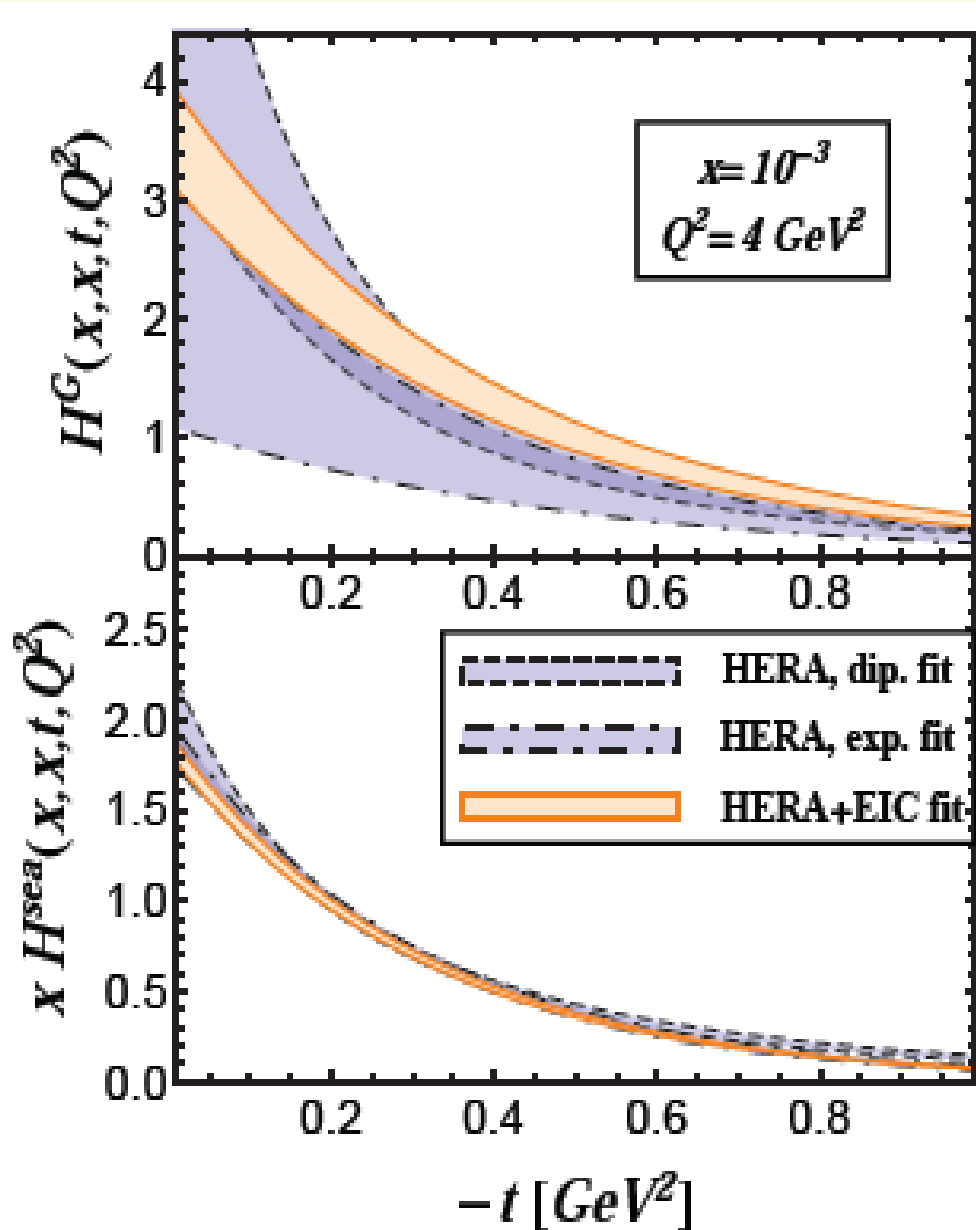
statistical errors rescaled

5% systematical errors added in quadrature, 3% Bethe-Heitler uncertainty



Imaging (probabilistic interpretation)

$$q(x, \vec{b}, \mu^2) = \frac{1}{4\pi} \int_0^\infty d|t| J_0(|\vec{b}| \sqrt{|t|}) H(x, \eta = 0, t, \mu^2)$$

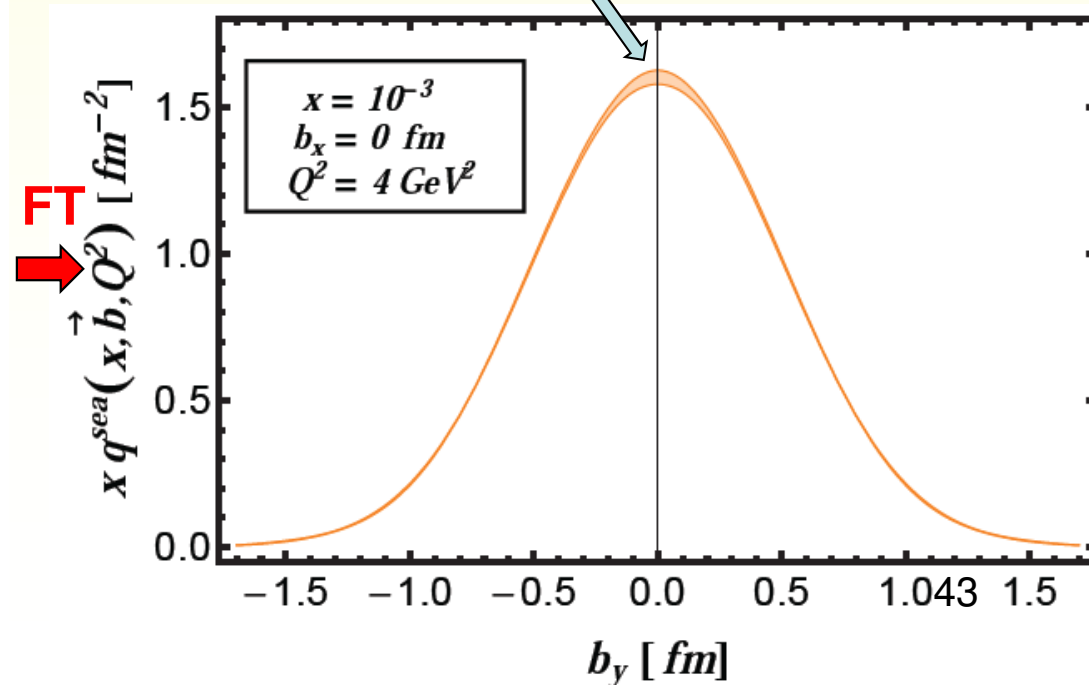


skewness effect vanishes ($s_2, s_4 \rightarrow 0$)

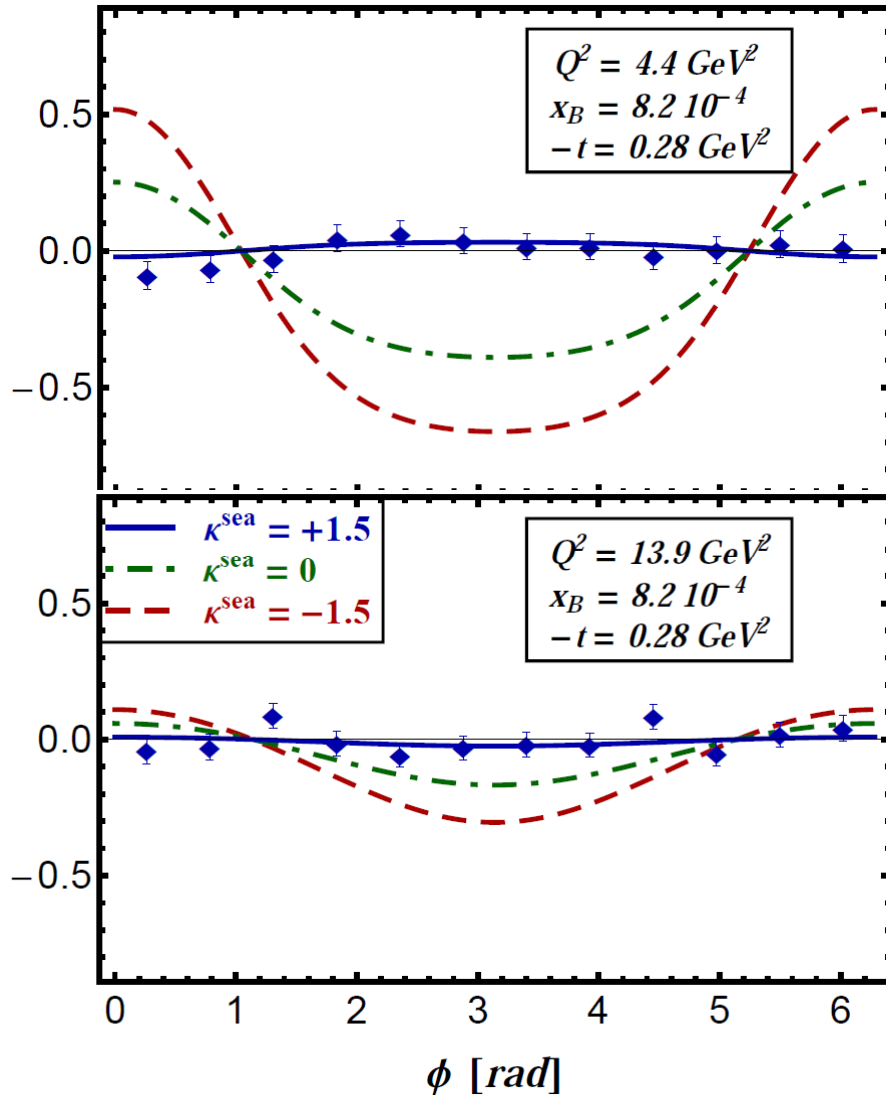
- reduce fit uncertainties
- increase model uncertainties

extrapolation errors for $-t \rightarrow 0$
(large b uncertainties – small effect)

extrapolation errors into $-t > 1.5 GeV^2$
(small b uncertainties)



Single transverse target spin asymmetry



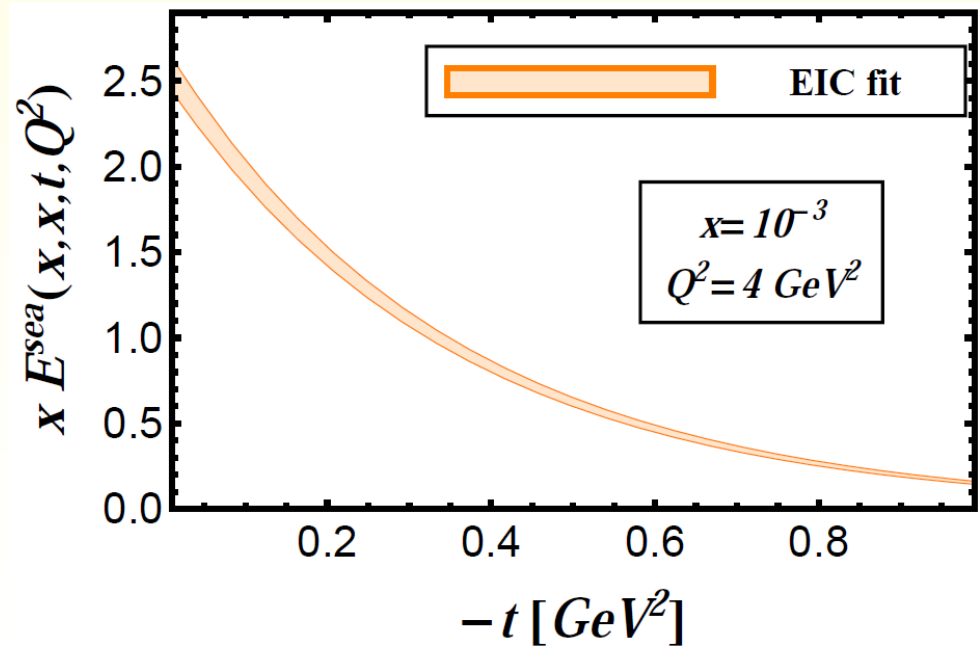
20x250 2x5/fb mock data

(~1200 data points with statistical errors
+ 5% systematics at cross section level)

flexible GPD model for E^{sea} and E^G

normalization (and t -dependency) of E^{sea}
is reasonable constraint

E^G is essentially unconstrained

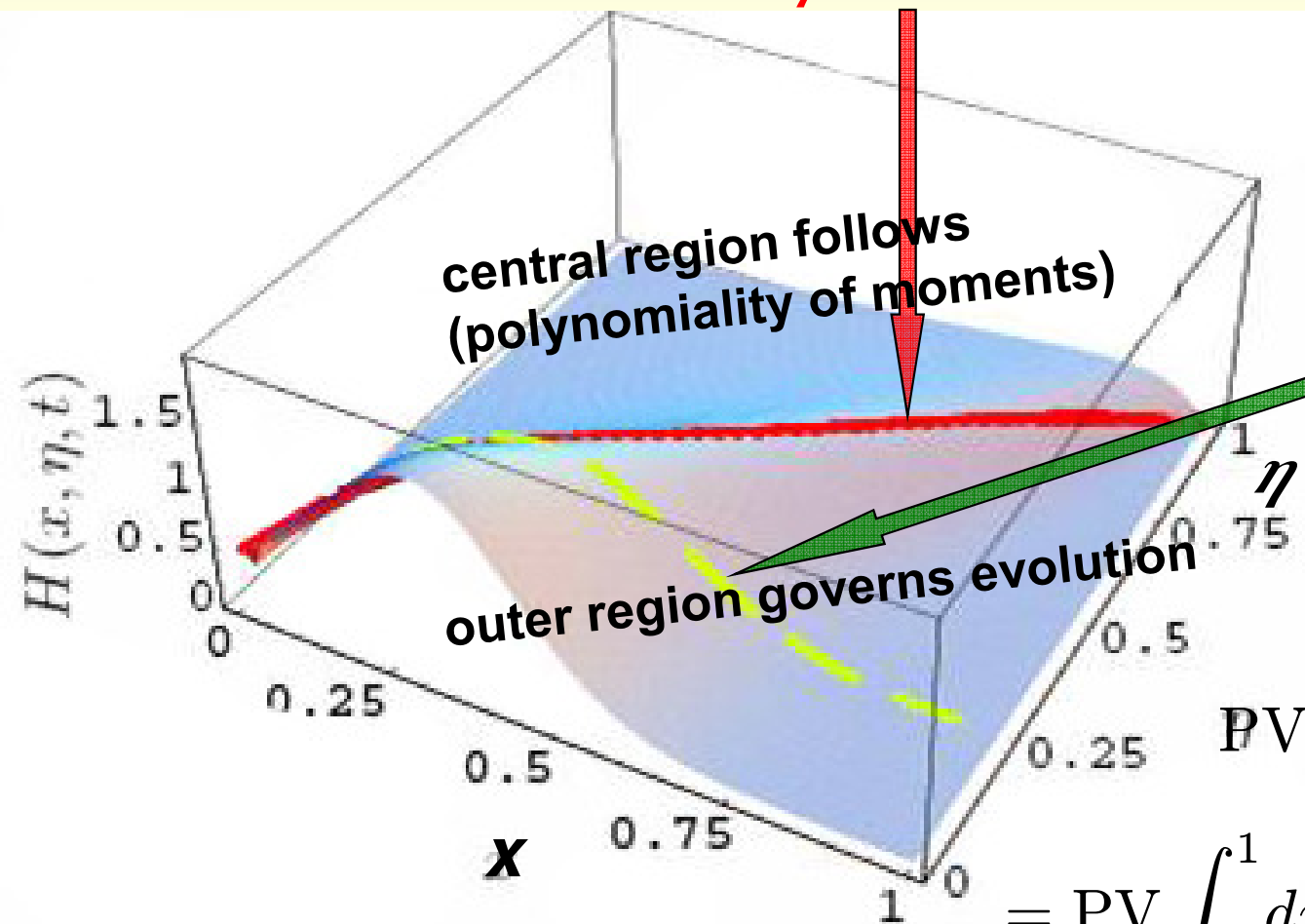


Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} F(x, x, t, \mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) F(y, x, \mu^2)$$

GPD at $\eta = x$ is 'measurable' (LO)

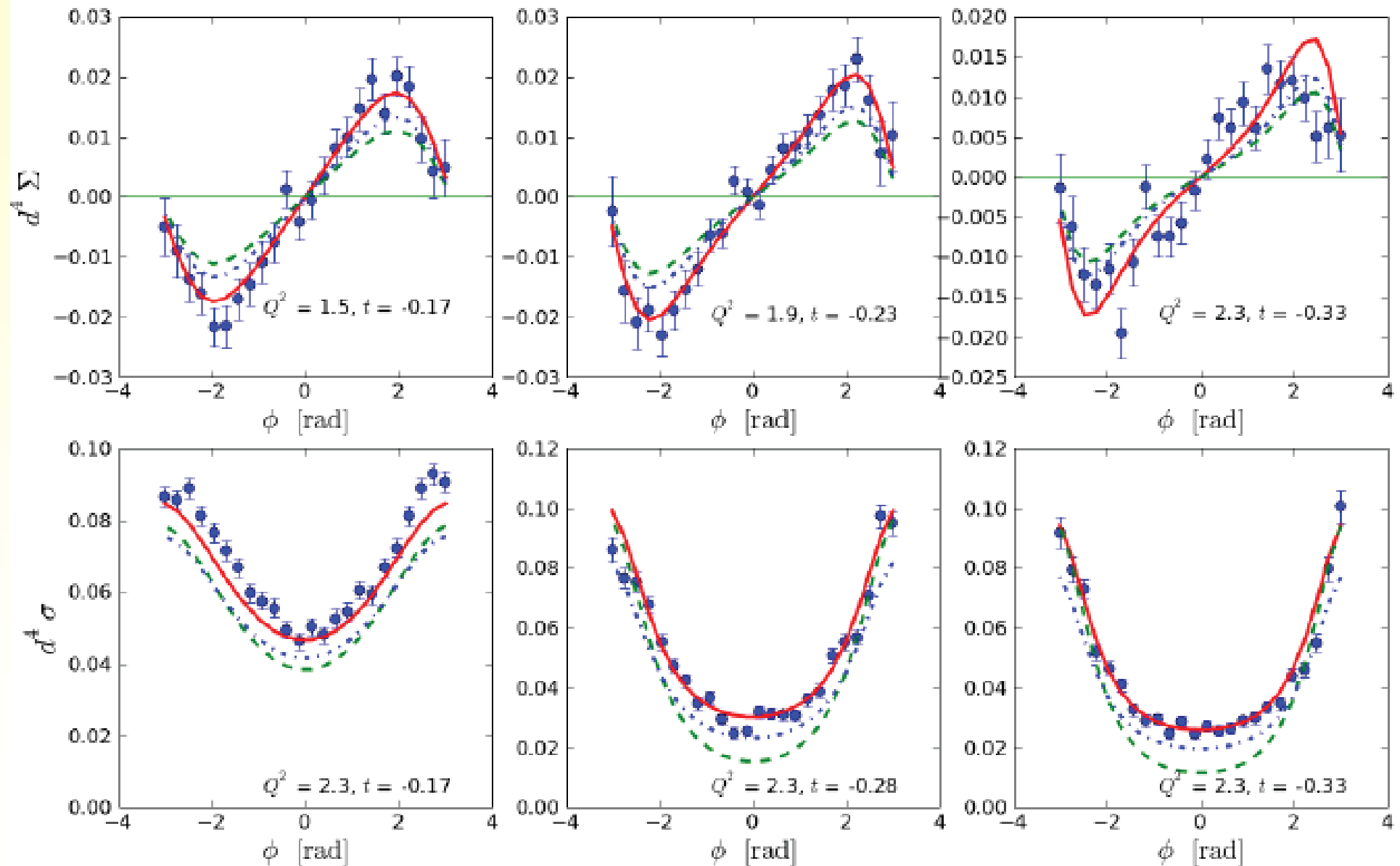


**net contribution of
outer + central region is
governed by a sum rule:**

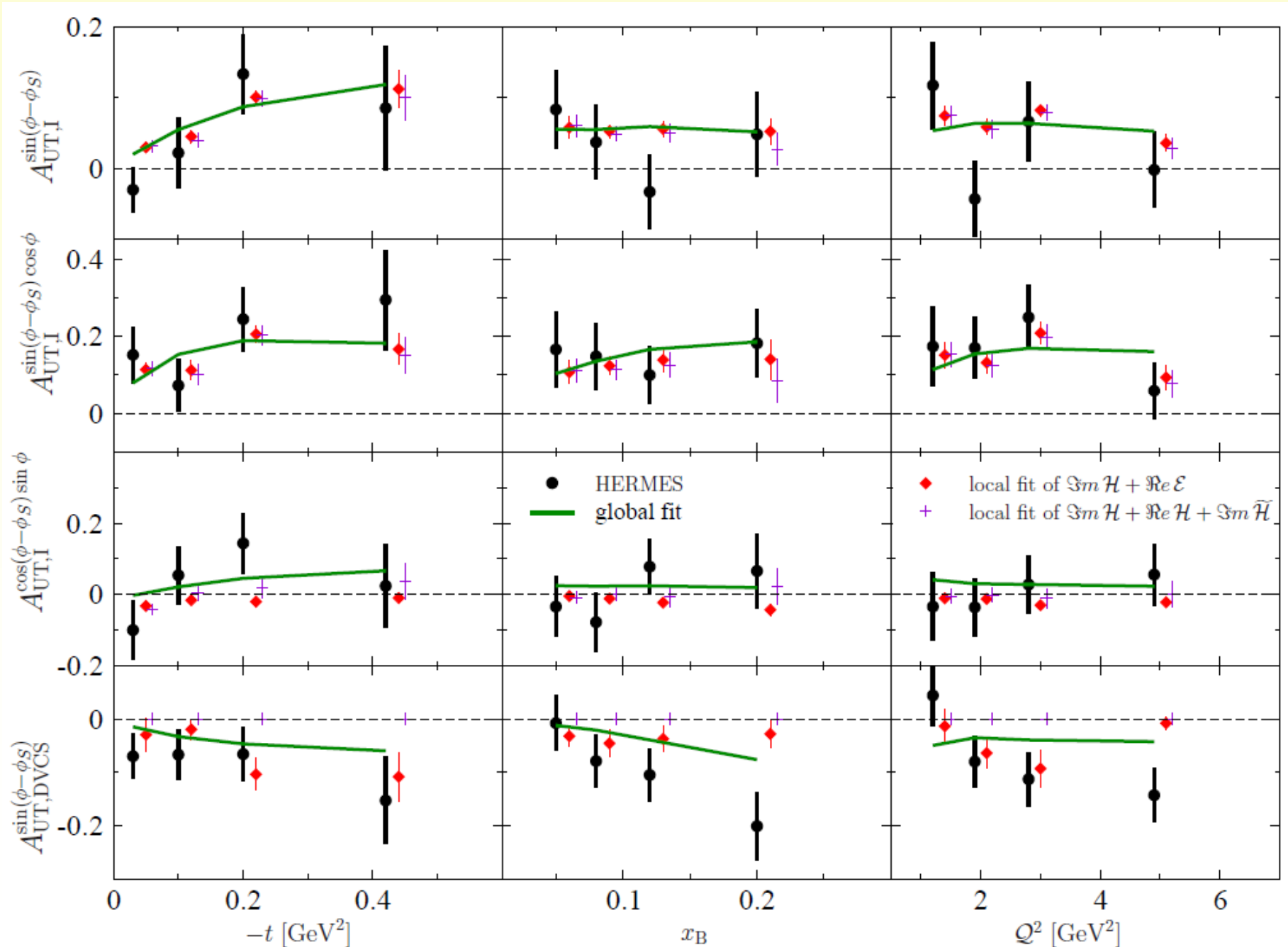
$$\text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} F^+(x, \eta, t) = \text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} F^+(x, x, t) + \frac{1}{45} C(t)$$

HALL A ϕ -dependence

- ϕ -dependence is described (if we fit to it)

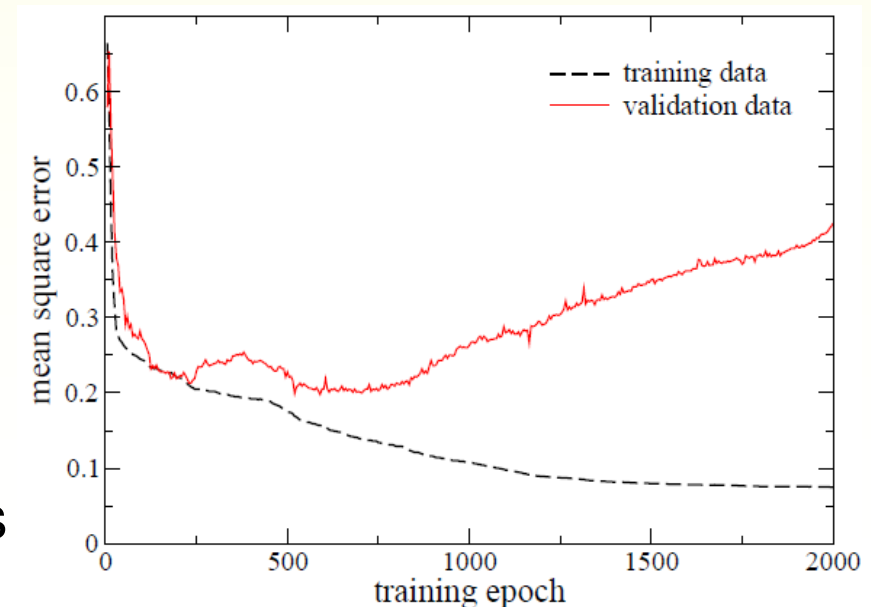
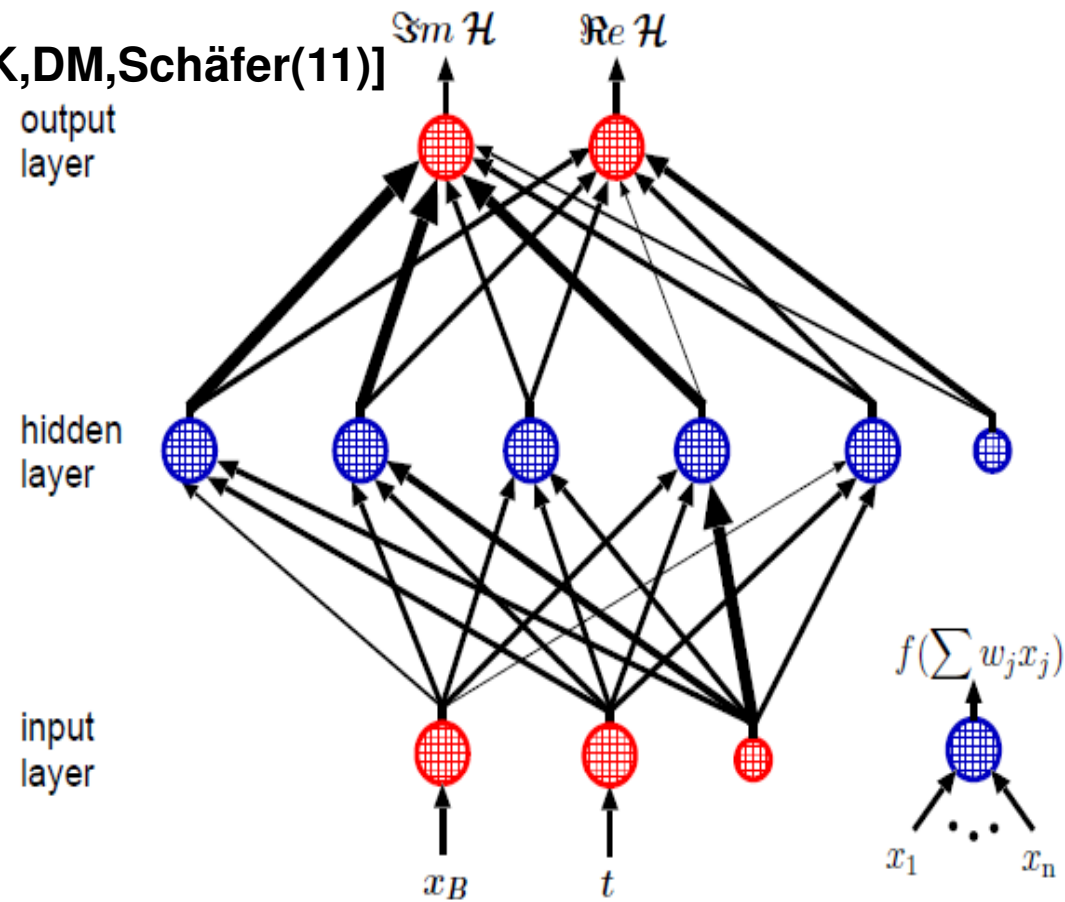


- KMM12 (KM10 type model) includes polarized target DVCS data (global fit to most of data , $\chi^2/d.o.f \approx 1.6$ - best what is there at present e.g., transverse polarized HERMES asymmetries looks as)



Neural Networks [KK,DM,Schäfer(11)]

- kinematical values are represented by the input layer
- propagated through the network, where weights are set randomly
- random values for $Im\mathcal{H}$ and $Re\mathcal{H}$
- calculation of χ^2
- backwards propagation (PyBrain)
- adjusting weights so that error decreases
- repeat procedure
- taking next kinematical point

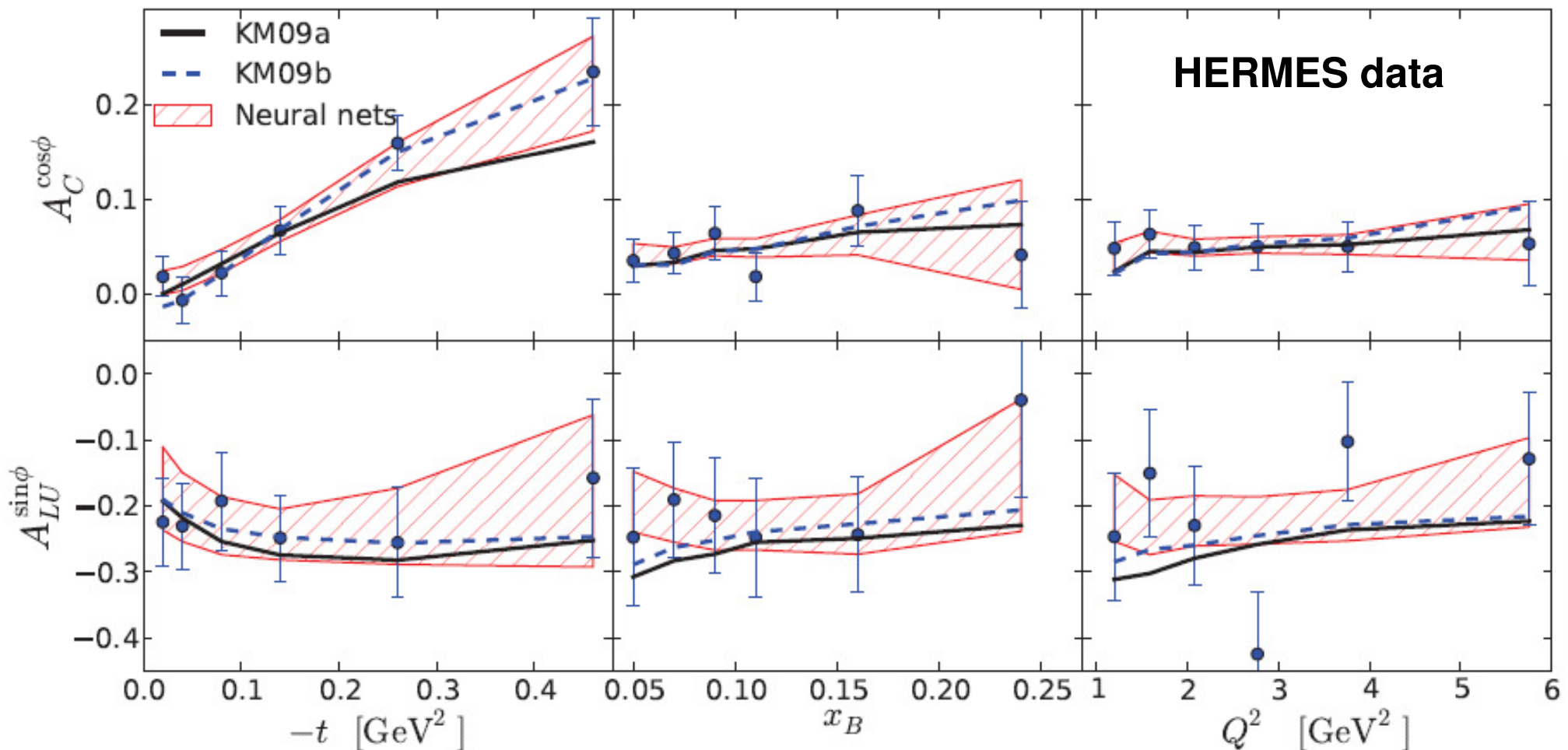


Monte Carlo procedure to propagate errors, i.e., generating a replica data set

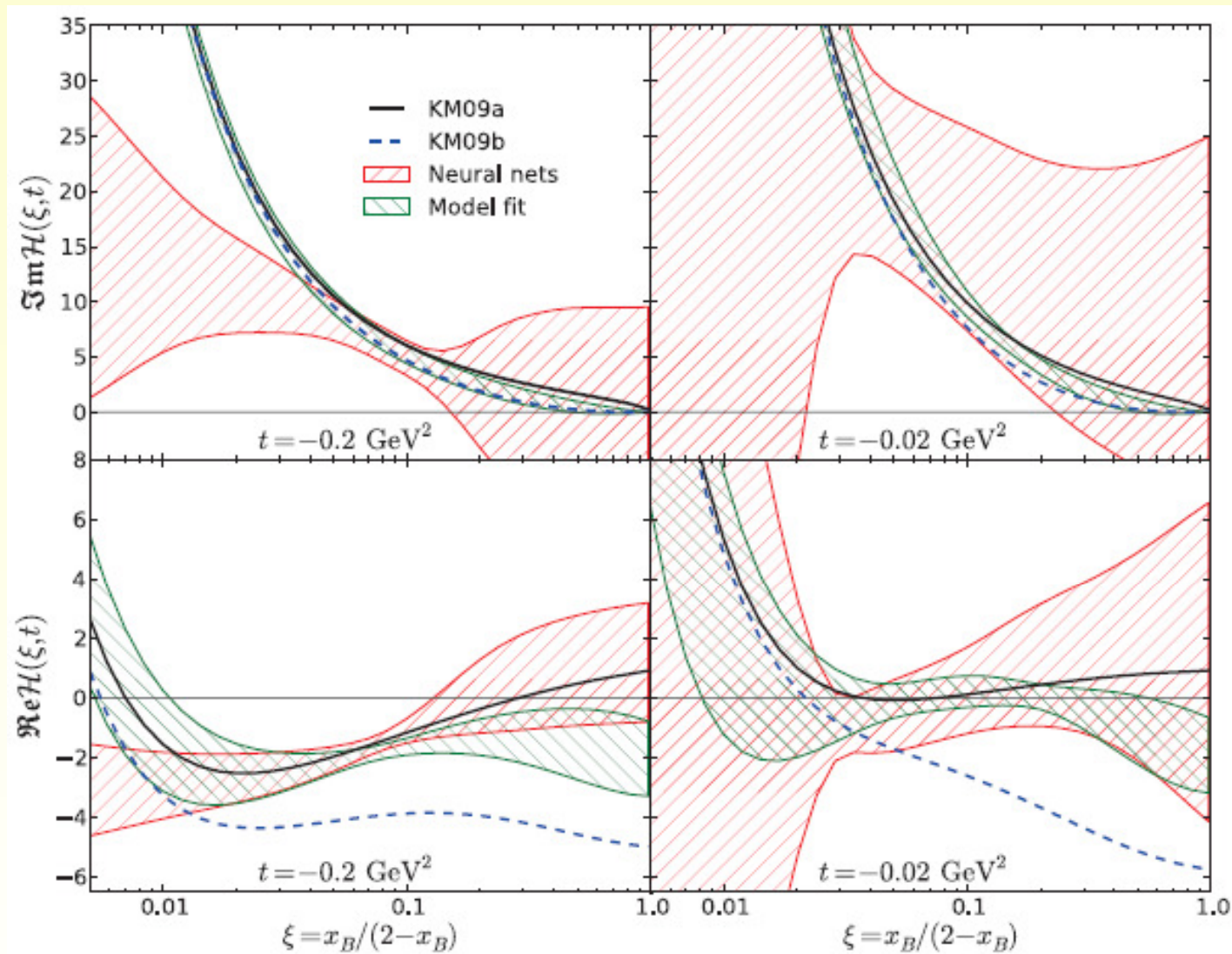
avoiding over fitting (fitting to noise), dividing data set, taking a control example if error increases after decreasing – one stops

A first use of neural network fits

(ideal) tool for error propagation and quantifying model uncertainties
used to access real and imaginary part of $\mathcal{H}$ CFF from HERMES
results are compatible to model, CFF fits, and mapping



Model prediction versus unbiased error propagation



- model fits and neural networks are complimentary
- meaning of error bands should be properly understood
- error propagation is practically an art (full information is not given)⁵⁰

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