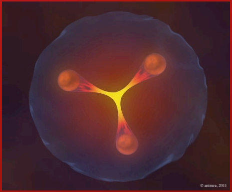
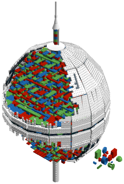
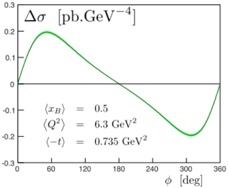
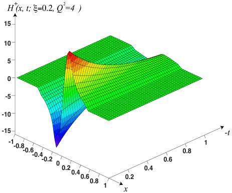


Directions in DVCS analysis: Introduction to discussion







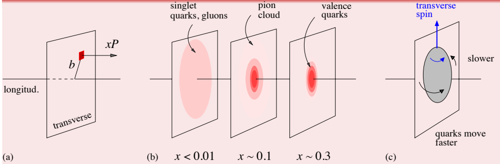
DVCS: From Observables to GPDs | Hervé MOUTARDE

Feb. 11$^{\text{th}}$, 2014

This talk: focus on GPD properties that require extrapolations outside the physical domain.

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in the nucleon.
- Insights on:
  - **Spin** structure,
  - **Energy-momentum** structure.
- **Probabilistic interpretation** of Fourier transform of GPD$(x, \xi = 0, t)$ in **transverse plane**.

Obtain this 3d picture from exclusive measurements. . . ?



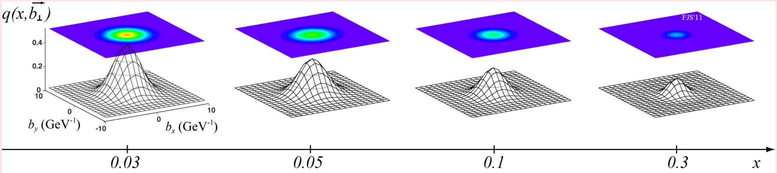
Weiss, AIP Conf. Proc. 1149, 150 (2009)

GPDs and 3D nucleon structure.

This talk: focus on GPD properties that require extrapolations outside the physical domain.

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in the nucleon.
- Insights on:
  - **Spin** structure,
  - **Energy-momentum** structure.
- **Probabilistic interpretation** of Fourier transform of GPD$(x, \xi = 0, t)$ in **transverse plane**.

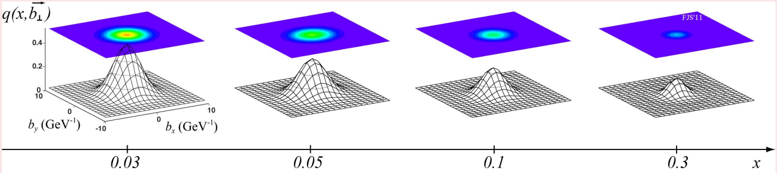
. . . Or how accurate is this picture?



Questions to be answered to devise a fitting strategy:

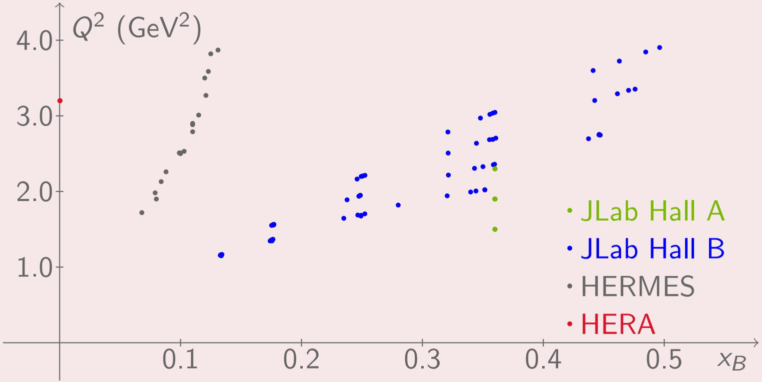
1. Data points and model parameters?
2. Model-independent fitting?
3. Experimental 3D imaging?

Can we propagate uncertainties onto this picture?



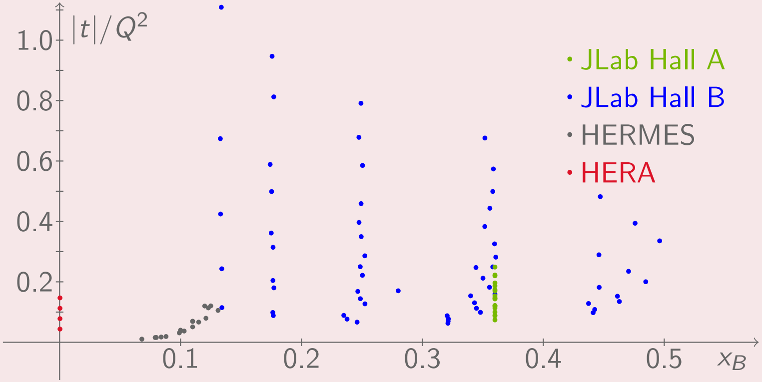
Data points and model parameters?

What is large Q^2 ?



- World data cover **complementary kinematic regions**.

What is large Q^2 ?

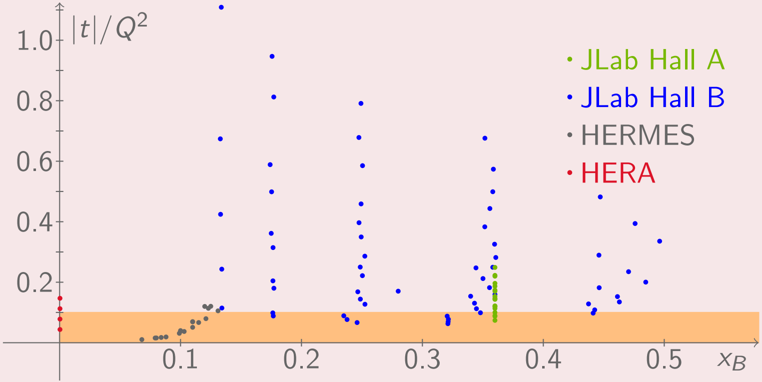


- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.

Kinematics of existing DVCS measurements.

Looking for the Bjorken regime.

What is large Q^2 ?

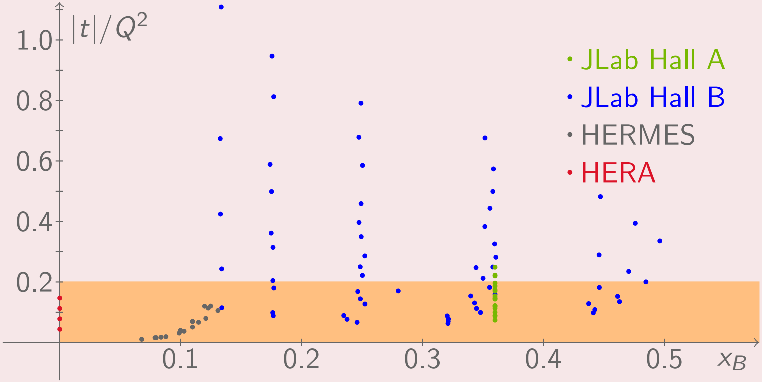


- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- **Higher twists**, finite-t and target mass corrections ?

Kinematics of existing DVCS measurements.

Looking for the Bjorken regime.

What is large Q^2 ?

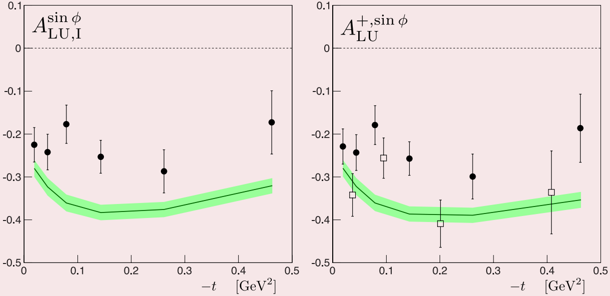


- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- **Higher twists**, finite-t and target mass corrections ?

Observables and GPD combinations.

Apart from HERMES data, most existing measurements are mostly sensitive to the GPD H.

Beam Spin Asymmetry, HERMES



Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

- **Disagreement** between HERMES $A_{\rm LU}$ measurements performed **with and without recoil detector**.
- **Unknown corresponding effect** for other observables.
- Which role for HERMES data in **global fitting**?

- Radyushkin's **Factorized Ansatz** + t-dependence from nucleon **form factor** F_1:

$$H^q(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha\, \delta(\beta+\xi\alpha-x) f^q(\beta,\alpha,t)$$

$$f^q(\beta,\alpha,t) = F_1^q(t) h(\beta) \pi_n(\beta,\alpha)$$

$$\pi_n(\beta,\alpha) = \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{(1-|\beta|)^2-\alpha^2]^n}{(1-|\beta|)^{2n+1}}$$

- Expressions for h and n :

$$h^q_{\rm sea}(\beta) = q_{\rm sea}(|\beta|)\mathrm{sign}(\beta) \qquad n_{\rm sea} = 1$$
$$h^q_{\rm val}(\beta) = q_{\rm val}(\beta)\Theta(\beta) \qquad n_{\rm val} = 1$$

- Add D-term at $z = x/\xi$:

$$D(z) \simeq (1-z^2)\Big(-4.C_1^{3/2}(z) - 1.2C_3^{3/2}(z) - 0.4C_5^{3/2}(z)\Big)$$

Vanderhaeghen *et al.*, Phys. Rev. **D60**, 094017 (1999)

How many parameters to describe GPDs?

Naive counting from a Double Distribution model (2/3).

- Parton Distribution Function:

$$q(x) = Ax^{\eta_1}(1-x)^{\eta_2}(1+\epsilon\sqrt{x}+\gamma x)$$

Martin *et al.*, Eur. Phys. J. **C63**, 189 (2009)
5 parameters per quark flavor

- Kelly parameterization of form factor ($\tau = t/(4M^2)$) :

$$F_1^q(t) = \frac{1+a\tau}{1+b\tau+c\tau^2+d\tau^3}$$

Kelly *et al.*, Phys. Rev. **C70**, 068202 (2004)
4 parameters per quark flavor

- Profile function parameter n :

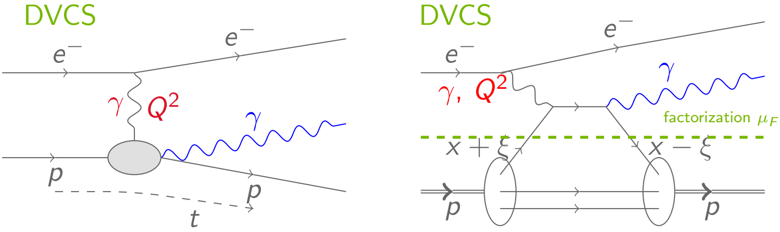
$$\pi_n(\beta,\alpha) = \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{(1-|\beta|)^2-\alpha^2]^n}{(1-|\beta|)^{2n+1}}$$

Mezrag *et al.*, Phys. Rev. **D88**, 014001 (2013)
1 parameters per quark flavor

- Naive counting leads to **9 parameters per quark flavor**!
- Not fully realistic:
  - No **correlations between** x **and** t...
  - ...But generalized form factors computed on the lattice exhibit different t-dependence.

Hägler, Phys. Rept. **490**, 49 (2010)

- Expect $\simeq$ 30 - 40 parameters for u, d, s and g from naive counting, **not considering higher-twist GPDs**.
- Strategy:
  - Find **educated parameterization** (few free parameters) to proceed with **traditional** χ^2**-minimization algorithms**.
  - Use **uneducated parameterization** (lot of free parameters) but proceed with **alternative fitting procedures** (neural networks? etc.)



- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) = \int_{-1}^{+1} dx\, H_q^+(x,\xi,\mu_F)\ T_q\left(x,\xi,\alpha_S(\mu_F),\frac{Q}{\mu_F}\right) + \int_{-1}^{+1} dx\, H_g(x,\xi,\mu_F)\ T_g\left(x,\xi,\alpha_S(\mu_F),\frac{Q}{\mu_F}\right)$$

Belistky and Müller, Phys. Lett. **B417**, 129 (1998)

Pire *et al*, Phys. Rev. **D83**, 034009 (2011)

The cross-over line.

Existence of a relation between $Re\mathcal{H}(\xi)$ and $H(x, \xi = x)$.

- Write dispersion relation **at fixed** t **and** Q^2:

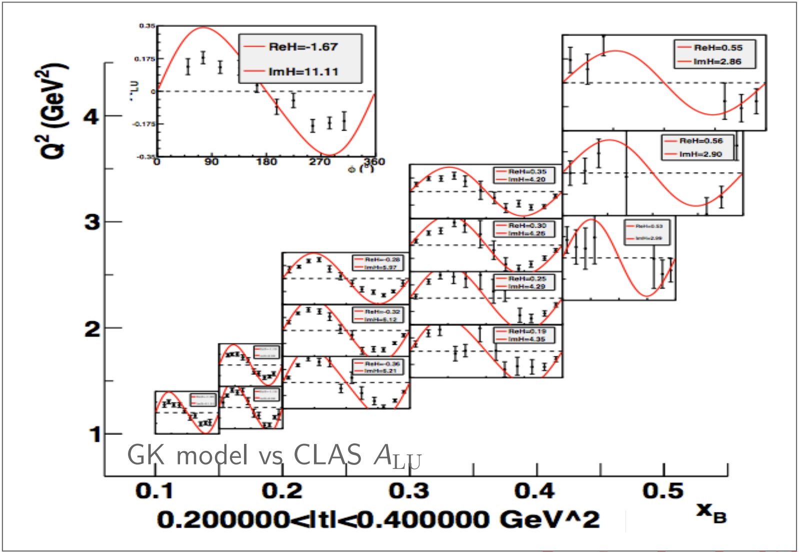
$$Re\mathcal{H}(\xi, t) = \Delta(t) + \frac{2}{\pi}\mathcal{P}\int_0^1 \frac{\mathrm{d}x}{x} \frac{Im\mathcal{H}(x, t)}{\left(\frac{\xi^2}{x^2} - 1\right)}$$

- Use LO relation $Im\mathcal{H}(x, t) = \pi\big(H(x, x, t) - H(-x, x, t)\big)$.
- Up to the D-term form factor $\Delta(t)$, all the information accessible **at LO and fixed** Q^2 is contained on the cross-over line.

Teryaev, hep-ph/0510031
Anikin and Teryaev, Phys. Rev. **D76**, 056007 (2007)
Diehl and Ivanov, Eur. Phys. J. **C52**, 919 (2007)

Dispersion relations and actual data.

Too few kinematic bins to provide model-independent constraints?



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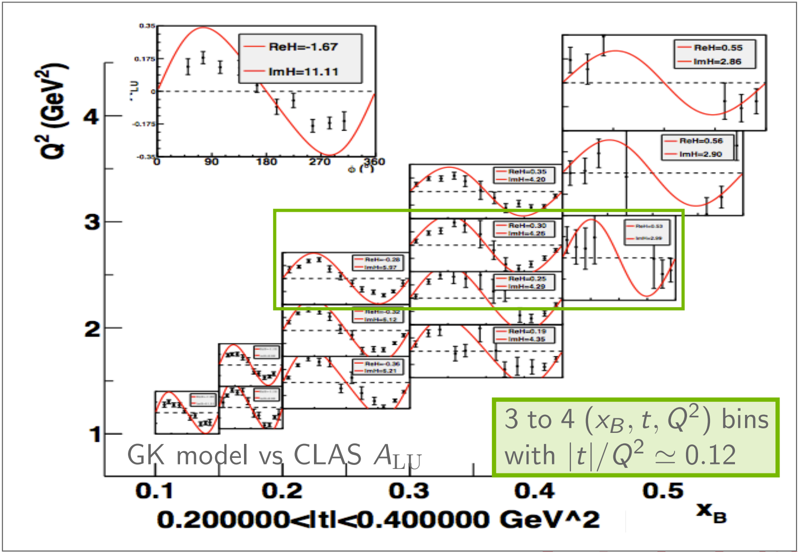

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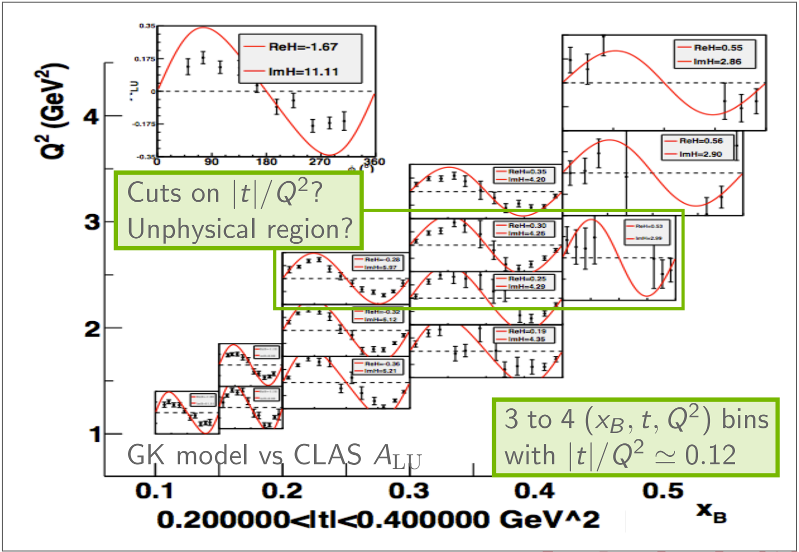


Model-independent fitting?

Local fits

Take each kinematic bin independantly of the others.
Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ... as independent parameters.

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

Neural networks

Exploratory stage for GPDs.

Local fits

Take each kinematic bin independantly of the others.
Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ... as independent parameters.

M. Guidal, Eur. Phys. J. **A39**, 5 (2009)

- **Almost model-independent**: relies on twist-2 dominance assumption and assume bounds for the fitting domain.
- Interpretation of **uncertainties** on extracted quantities? Contributions from measurements uncertainties, correlations between CFFs and fitting domain boundaries.
- Interpretation of **extracted quantities**? *e.g.* mixing of quark and gluon GPDs due to NLO effects.
- **Oscillations** between different (x_B, t, Q^2) bins may happen.
- **Extrapolation** problem left open.

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

where $a = \mathcal{O}(Q^{-1})$, $b = \mathcal{O}(Q^{-4})$, $c = \mathcal{O}(Q^{-1})$, $d = \mathcal{O}(Q^{-2})$, $e = \mathcal{O}(Q^{-5})$.

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem (8 fit parameters : real and imaginary parts of 4 CFFs $\mathcal{H}$, $\mathcal{E}$, $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$).

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs** (or **add** $\simeq$ 5-10 % **systematic uncertainty**).

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c \cos\phi + d \cos 2\phi + e \cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs**.
- Add physical input? **Dispersion relations**, etc.

Kumericki *et al.* , arXiv:1301.1230

Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Kumericki, Nucl. Phys. **B841**, 1 (2010)

- **Model-dependent** approach.
- Allows the **implementation of theoretical constraints** on GPDs or CFFs.
- Guideline for **extrapolation** outside the physical domain.
- Compromise between number of parameters and number of described GPDs (flavor dependence, higher-twists, . . .)?
- Impact on the **choice of a fitting strategy**?

Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

Moutarde, Phys. Rev. **D79**, 094021 (2009)

- Avoid unphysical oscillations between different (x_B, t, Q^2) bins by comparing to a **global fit by a smooth function**:

$$H^+ = 2\sum_{n=0}^{N}\sum_{l=0}^{n+1} B_{nl}(t)\theta(|x| < \xi)\left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^{(3/2)}\left(\frac{x}{\xi}\right) P_{2l}\left(\frac{x}{\xi}\right)$$

- Number of fit parameters describing the B_{nl} coefficients **increases with** N^2... Extension to other GPDs seems difficult.
- **Extrapolation** problem left open.

Neural networks

Exploratory stage for GPDs.

Kumericki *et al.* , JHEP **1107**, 073 (2011)

- Already used for PDF fits.
- **Almost model-independent**: neural network description, twist-2, H-dominance?
- Good agreement between model fit and neural network fit in the fitting domain.
- **More reliable uncertainties** in extrapolations?
- **Overtraining** as a generic feature of (too) flexible models.

Summary of first extractions.

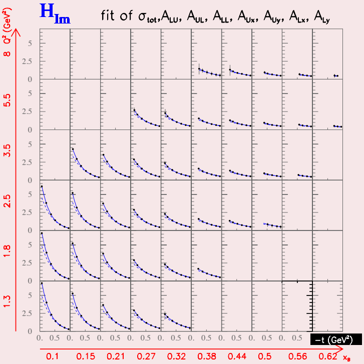
Feasibility of twist-2 analysis of existing data.

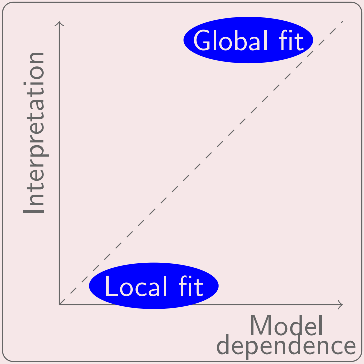
- **Dominance** of twist 2 and **validity** of a GPD analysis of DVCS data.
- $Im\mathcal{H}$ **best determined**. Large uncertainties on $Re\mathcal{H}$.
- However sizable **higher twist contamination** for DVCS measurements.
- Already some indications about the **invalidity** of the H-dominance hypothesis with **unpolarized data**.

JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ – $\mathcal{O}(10\ \%)$ statistical accuracy.

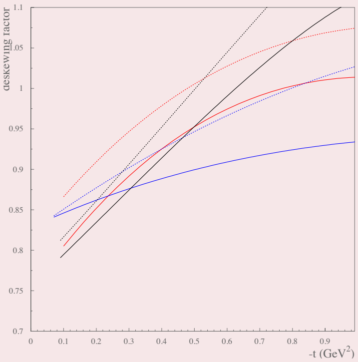
CLAS 12 pseudo-data (M. Guidal and H. Avakian)

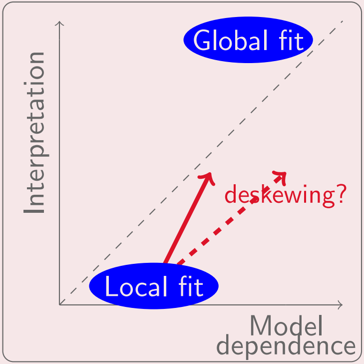




Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

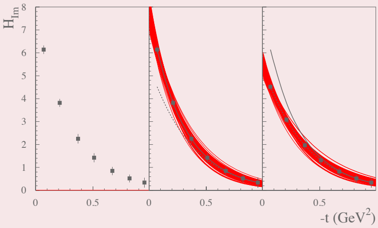
Model-estimate of $H(\xi, 0, t)/H(\xi, \xi, t)$

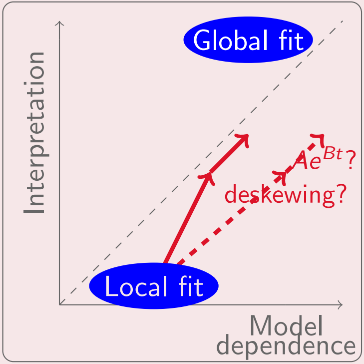




Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

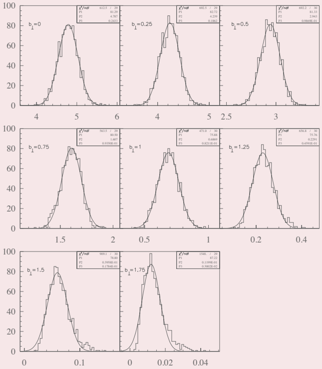
Extracted $Im\mathcal{H}$ as function of t and Ae^{Bt} fit

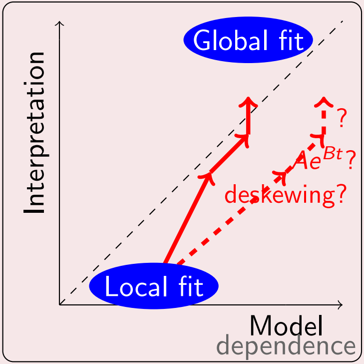




Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

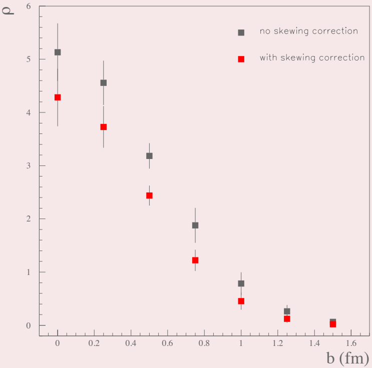
2D Fourier transform of fit function (error propagation)

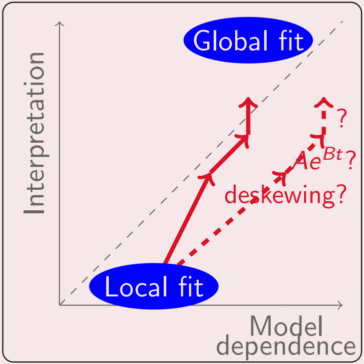




Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

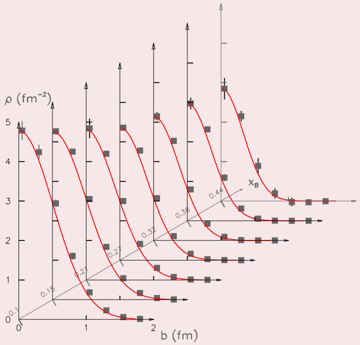
$b_\perp$-dependence of spatial density

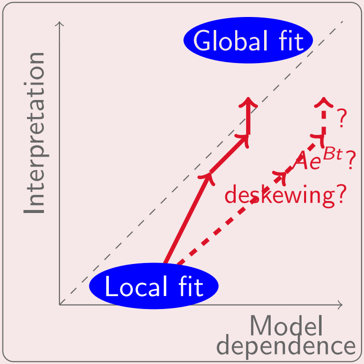




Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

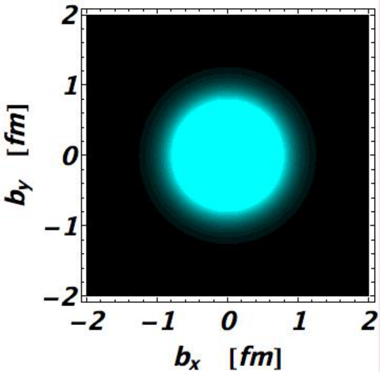
Spatial density as function of x_B

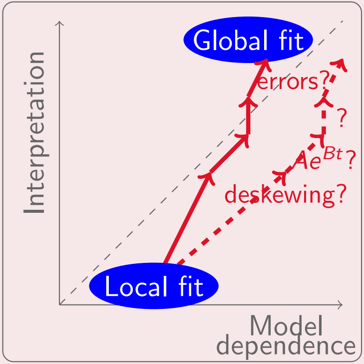




Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

Contour plot of spatial charge density





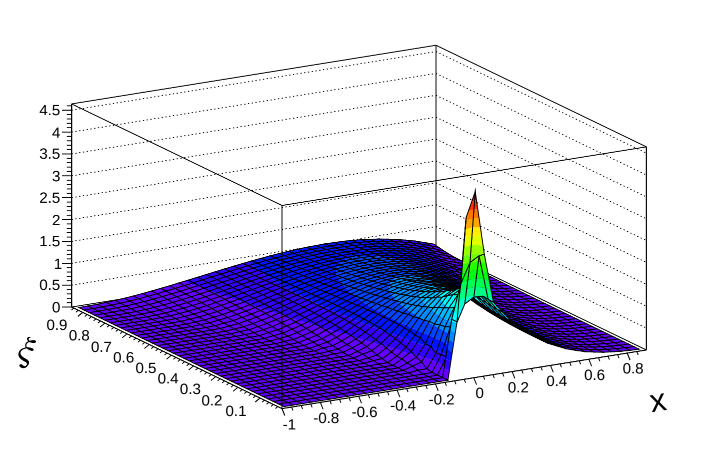
Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

Experimental 3D imaging?

From principles to actual data.

Direct experimental access to a restricted domain.

GPD H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2.

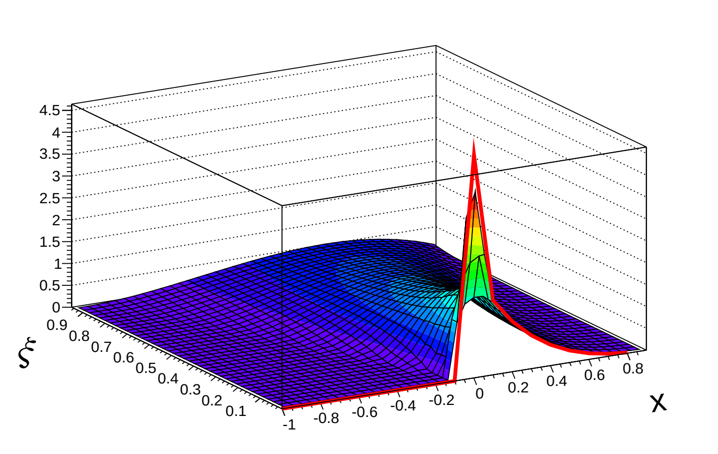


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

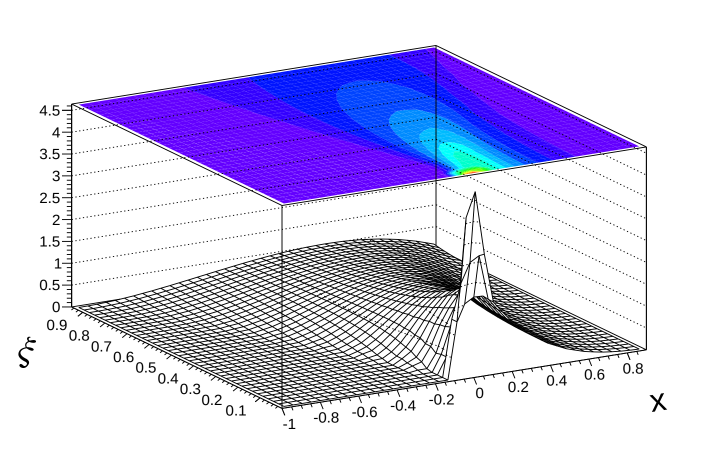
Direct experimental access to a restricted domain.

Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.



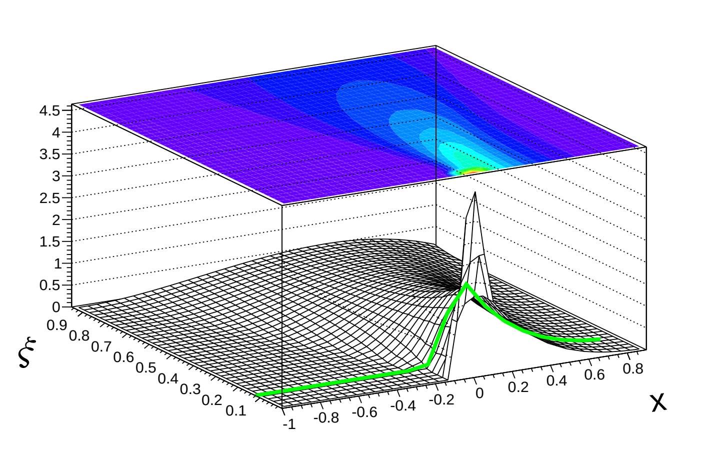
GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

What is the physical region?



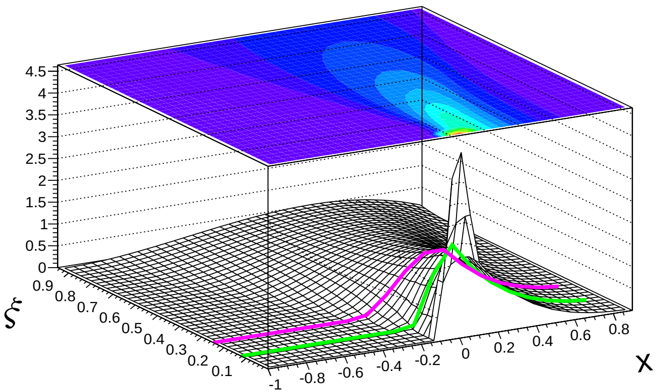
GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

$\xi_{\min}$ from finite beam energy.



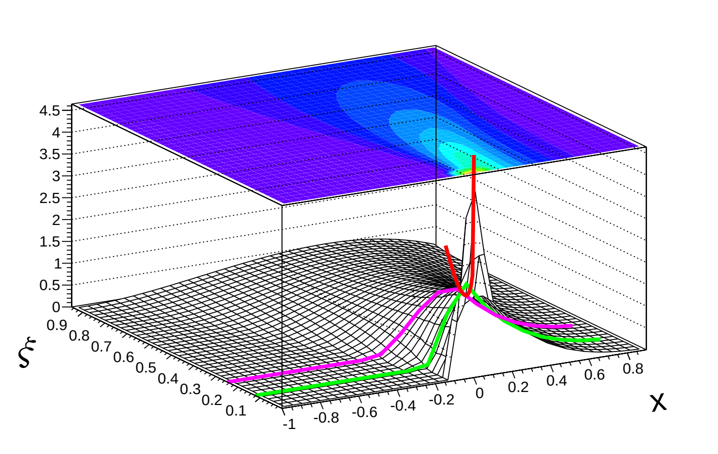
GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

$\xi_{\rm max}$ from kinematic constraint on 4-momentum transfer.



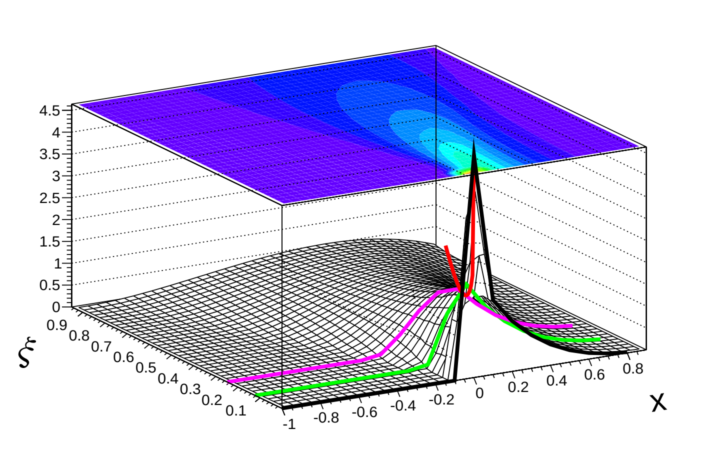
GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

The cross-over line $x = \xi$.



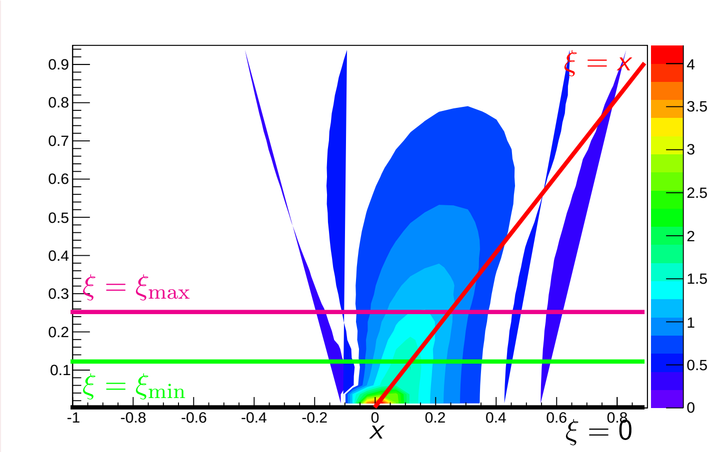
GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

The black curve is what is needed for transverse plane imaging!



GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

Density plot of H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2



GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

Nucleon charge radius.

A training ground for the extrapolation problem.

- Traditional definition of the proton charge radius $\langle r_E^2 \rangle$.

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E}{dq^2} \right|_{q^2=0} .$$

- What is **measured** is $G_E(q^2)$ with $q^2 \neq 0$. To obtain the charge radius, one need to derivate the data, and extrapolate it to vanishing q^2.
- Taylor expand G_E:

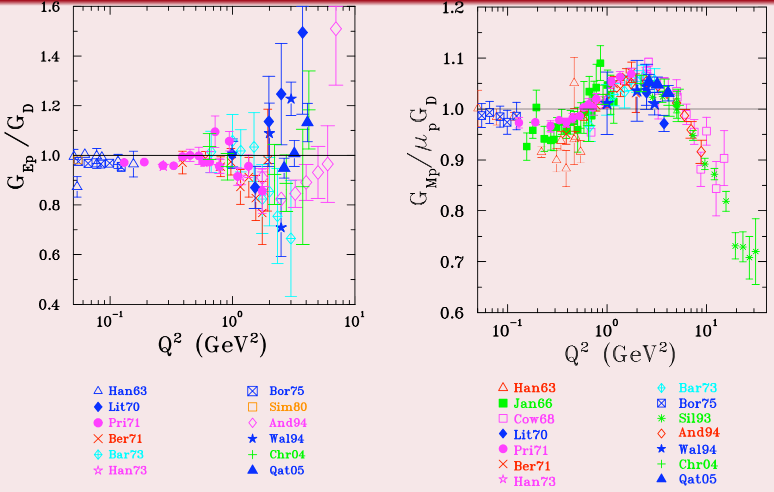
$$G_E(q^2) = 1 - q^2 \langle r_E^2 \rangle /6 + q^4 \langle r_E^4 \rangle /120 - \ldots$$

- **Higher moments** are increasing with order, hence giving a **large contribution** to $G_E(q^2)$.
- No reason for the $\langle r_E^2 \rangle$ term to dominate! *e.g.* compute $\langle r^{2n} \rangle / \langle r^2 \rangle^n$ for an exponential charge density.

Nucleon charge radius.

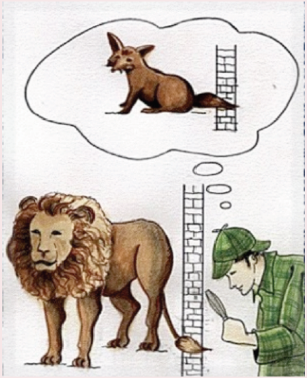
A training ground for the extrapolation problem.

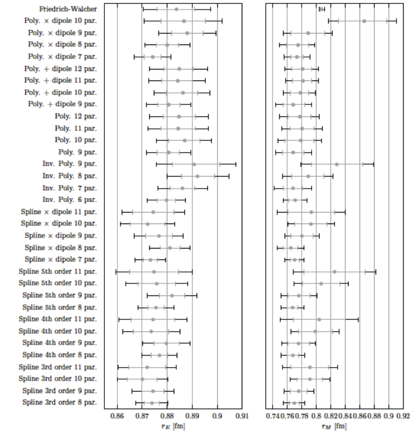
Measurements



Perdrisat *et al.* , Prog. Part. Nucl. Phys. **59**, 694 (2007)

Extrapolations. . .





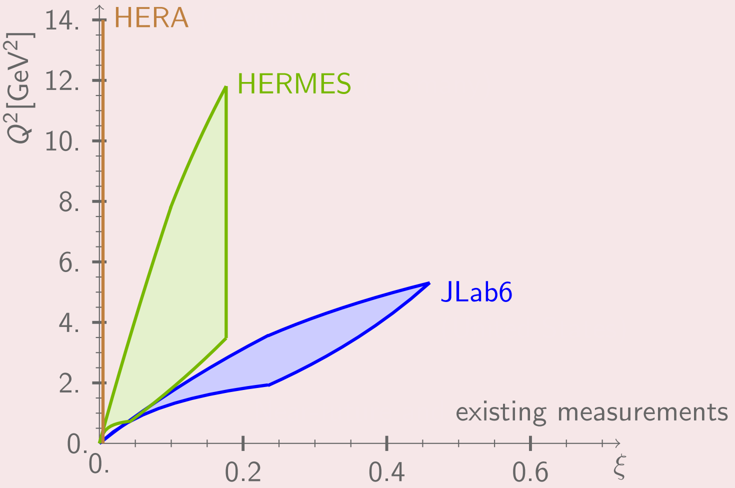
Bernauer *et al.* (A1 Coll.), arXiv:1307.6227

Conclusions

Need for global fits of world experimental data.

Two-step fit (CFF, then GPDs)? Extrapolations? Uncertainties?

Kinematic reach of existing or near-future DVCS measurements



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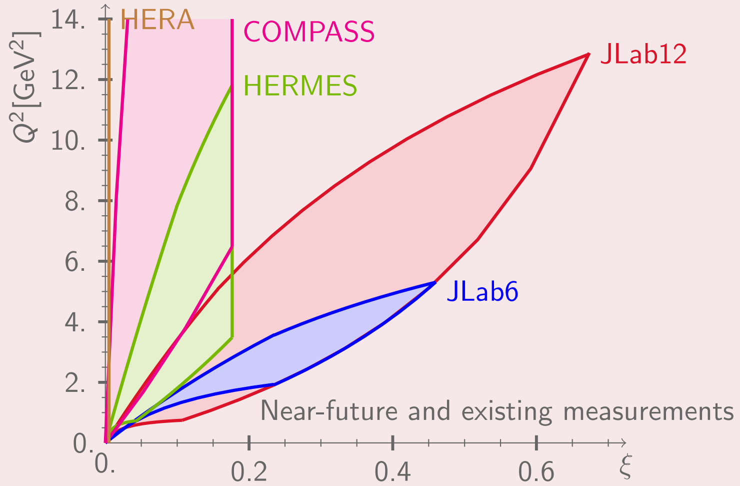

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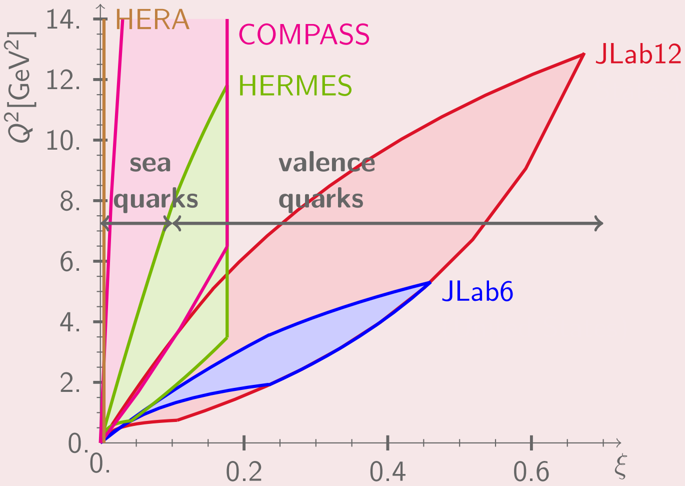

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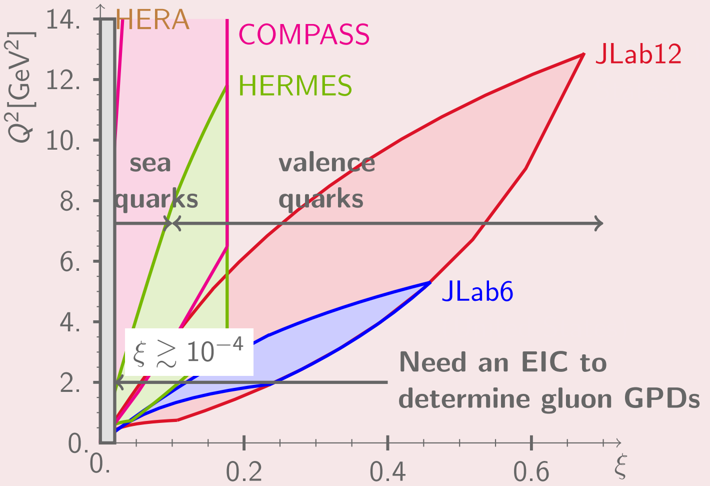

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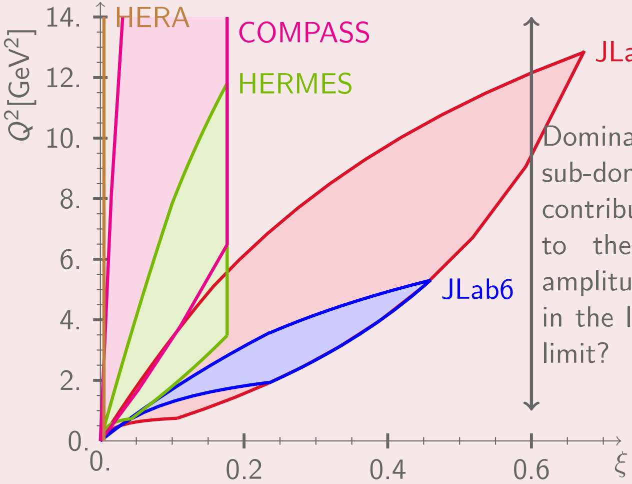

Dominant and sub-dominant contributions to the DVCS amplitude in the large Q^2 limit?

Conclusions and prospects.

Rome wasn't built in a day.

- Reminder: PDFs fits have been performed by **more groups** for a **longer time**.
- **Encouraging results** have been obtained in the last five years in fitting DVCS data.
- In progress: inclusion of DVMP data in fits.
- Today it is not clear that existing strategies will be able to handle **very precise** data on a **large kinematic domain**.
- **All approaches should be explored**, each with its own advantages and drawbacks.
- Global fits seem **unavoidable** at some point (direct GPD fit? Two-step fit, CFF, then GPDs? Extrapolations?).
- Experimental 3D imaging is far more complicated than PDF or charge radius fitting, but **possible in principle**.

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