Kinematic power corrections in off-forward hard processes

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based on

V. Braun, A. Manashov: PRL 107 (2011) 202001;
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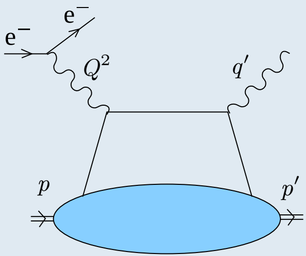
Hard exclusive processes involve off-forward matrix elements

DVCS: $\gamma^* N(p) \to \gamma N(p')$ **D. Müller, X. Ji, A. Radyushkin**



Operator Product Expansion

$$\mathcal{A}_{\mu\nu} \sim \langle p'|\,T\left\{j^{em}_\mu(x) j^{em}_\nu(0)\right\}|p\rangle \simeq \sum_N C^N_{\mu\nu}(x^2,\mu^2)\,\langle p'|\mathcal{O}_N(\mu^2)|p\rangle$$

Kinematic variables: hadron mass m^2 momentum transfer $t=(P-P')^2$

How to calculate effects $\sim m^2/Q^2$ and t/Q^2?

current conservation

DVCS Amplitude

$$\mathcal{A}_{\mu\nu}(p,q,q') = i\int d^4x\, e^{-i(z_1 q - z_2 q')}\langle p'|T\{j_\mu(z_1 x) j_\nu(z_2 x)\}|p\rangle$$

$z_1 - z_2 = 1.$

translation invariance: $\mathcal{A}$ **depends only on the difference** $z_1 - z_2$

current conservation:

$$q^\mu \mathcal{A}_{\mu\nu}(p,q,q') = (q')^\nu \mathcal{A}_{\mu\nu}(p,q,q') = 0$$

only valid in the sum of all twists but not for each twist separately

$$T\{j_\mu(z_1 x) j_\nu(z_2 x)\} = T^{t=2}_{\mu\nu}(z_1,z_2) + T^{t=3}_{\mu\nu}(z_1,z_2) + T^{t=4}_{\mu\nu}(z_1,z_2) + \dots$$

twist-3

Anikin, Teryaev; Belitsky, Müller; Kivel, Polyakov, Schäfer, Teryaev, 2001

Kinematic power corrections

OPE = expansion over multiplicatively renormalized operators

$$T\{j_\mu^{em}(x) j_\nu^{em}(0)\} \quad = \quad \sum_N C_{\mu\nu}^N(x)\, \mathcal{O}_N(0) \; + \; \text{higher twists}$$

$\mathcal{O}_N$ – twist two operators: $\quad (\mathcal{O}_N^{\mu_1\cdots\mu_{N+1}} = \bar{q}\gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_{N+1}} q)$

Higher twist operators related to $\mathcal{O}_N$:

- twist-3: $\quad \partial_{[\mu}\, \mathcal{O}_{\mu_1]\ldots\mu_{N+1}}$
- twist-4: $\quad \partial_\mu \partial^\mu \mathcal{O}_{\mu_1\ldots\mu_{N+1}}, \qquad \partial^\mu \mathcal{O}_{\mu\ldots\mu_{N+1}}$

"Kinematical power corrections" = taking into account contributions of these operators to the OPE only

General structure

$$T\{j(x)j(0)\} = \sum_N a_N \mathcal{O}_N + \sum_N \left(b_N \partial^2 \mathcal{O}_N + c_N (\partial \mathcal{O})_N \right) + \text{all others}$$

all others $\neq$ all quark-gluon operators

To determine the coefficients a_N, b_N in the leading order in α_s it is sufficient to take matrix elements over free quarks. But it does not work for the coefficients c_N.

matrix elements of $(\partial\mathcal{O})_N \equiv \partial^\mu \mathcal{O}_{\mu\mu_1\ldots\mu_N}$ over free quarks vanish

S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto, Phys. Lett. B38, 333 (1972):

$(\partial\mathcal{O})_N$ can be expressed in terms of quark-gluon operators

$N = 1$ **example**

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}gF_{\nu\mu}\gamma^\mu q\,, \qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]$$

Twist expansion

Twist-4 contributions to $T_{\mu\nu}$

$$iT\Big\{j_\mu(z_1x)j_\nu(z_2x)\Big\} = T^{(a)}_{\mu\nu}(z_1,z_2) + T^{(b)}_{\mu\nu}(z_1,z_2) + \ldots$$

Balitsky, Braun, 89

$$T^{(a)}_{\mu\nu}(z_1,z_2) = -\frac{1}{2\pi^2x^4z_{12}^3}\,\bar q(z_1x)\gamma_\mu \not{x}\gamma_\nu\, Q^2 q(z_2x) + \Big(\mu\leftrightarrow\nu,\ z_1\leftrightarrow z_2\Big)\,,$$

$$T^{(b)}_{\mu\nu}(z_1,z_2) = -\frac{g}{8\pi^2x^2z_{12}}\int_0^1 du\, x^\alpha\, \bar q(z_1)Q^2\gamma_\mu\Big[\widetilde F_{\alpha\beta}(z^u_{21})\gamma_5 + i(2u-1)F_{\alpha\beta}(z^u_{21})\Big]\gamma^\beta\gamma_\nu\, q(z_2)$$
$$+\Big(\mu\leftrightarrow\nu,\ z_1\leftrightarrow z_2\Big)\,,$$

Twist separation:

Spinor formalism $x_\mu\to x_{\alpha\dot\alpha}=x_\mu\sigma^\mu_{\alpha\dot\alpha}$, $T_{\mu\nu}\to T_{\alpha\beta\dot\alpha\dot\beta}$, **etc**

$$T_{\mu\nu} = -\frac{1}{\pi^2x^4}\Big\{x^\rho\Big[S_{\mu\rho\nu\sigma}\mathbb{V}^\sigma + i\epsilon_{\mu\nu\rho\sigma}\mathbb{A}^\sigma\Big] + x^2\Big[(x_\mu\partial_\nu + x_\nu\partial_\mu)\mathbb{X} + (x_\mu\partial_\nu - x_\nu\partial_\mu)\mathbb{Y}\Big]\Big\}\,,$$

The expansion of functions $\mathbb{V}_\sigma$ and $\mathbb{A}_\sigma$ starts from twist two, wheareas $\mathbb{X}$ and $\mathbb{Y}$ are already twist-four.

Twist expansion

Examples

$$\mathbb{V}_\mu^{t=2}(z_1,z_2) = \frac{1}{2}\partial_\mu \int_0^1 du\, \mathcal{O}_+^{t=2}(uz_1,uz_2) - (z_1\leftrightarrow z_2)\,,$$

$$\begin{aligned}\mathbb{V}_\mu^{t=4}(z_1,z_2) = x^2\partial_\mu \int_0^1 \frac{du}{u^2}\Big\{ & u^2(1-u^2+u^2\ln u)\, z_1 z_2\, \partial^2 O_+^{t=2}(z_1 u, z_2 u) \\ & - \Big[(1-u^2)\Big(z_2\partial_{z_2} - \frac{z_1}{z_{12}}\Big) + (1-u^2+u^2\ln u)\, z_2 \partial^2_{z_2} z_{12}\Big] R(uz_1,uz_2) \\ & + \Big[(1-u^2)\Big(z_1\partial_{z_1} - \frac{z_2}{z_{21}}\Big) + (1-u^2+u^2\ln u)\, z_1 \partial^2_{z_1} z_{21}\Big] \bar{R}(uz_1,uz_2) \\ & + x^\mu\Big(\ldots\Big)\Big\} - (z_1\leftrightarrow z_2),\end{aligned}$$

$$O_+^{t=2}(z_1,z_2) = \Big(\bar{q}(z_1x)\gamma_+ q(z_2x)\Big)^{t=2}$$

$$R(z_1,z_2) = ig\int_{z_2}^{z_1} dw\,(w-z_2)\,Q(z_1,w,z_2)\,,\qquad \bar{R}(z_1,z_2) = ig\int_{z_2}^{z_1} dw\,(z_1-w)\,\bar{Q}(z_1,w,z_2)\,.$$

Twist expansion

Spinor notation:

$$Q(z_1, w, z_2) = \bar{q}_+^L(z_1 n) f_{++}(wn) q_-^L(z_2 n)$$
$$\bar{Q}(z_1, w, z_2) = \bar{q}_-^L(z_1 n) \bar{f}_{++}(wn) q_+^L(z_2 n)$$

Q, $\bar{Q}$ **nonquasipartonic operator of twist**-4

$$f_{\alpha\beta} = \frac{i}{4}(\sigma_{\mu\nu} F^{\mu\nu})_{\alpha\beta} \qquad \bar{f}_{\dot\alpha\dot\beta} = \frac{i}{4}(\bar{\sigma}_{\mu\nu} F^{\mu\nu})_{\dot\alpha\dot\beta}$$

$$n^2 = \tilde{n}^2 = 0 \qquad n_{\alpha\dot\alpha} = n_\mu \sigma^\mu_{\alpha\dot\alpha} = \lambda_\alpha \bar{\lambda}_{\dot\alpha} \qquad \tilde{n}_{\alpha\dot\alpha} = \mu_\alpha \bar{\mu}_{\dot\alpha} \quad q_+^L = q_\alpha^L \lambda^\alpha \quad q_-^L = q_\alpha^L \mu^\alpha$$

Vector notation

$$(n\tilde{n})\bar{q}_L(z_1)\Big[F_{+\mu}(w) + i\widetilde{F}_{+\mu}(w)\Big]\gamma^\mu q_L(z_2) \sim Q(z_1, w, z_2)\,,$$

How to isolate the contribution of $(\partial O)_N$ to Q?

Nonlocal& local operators

$\mathcal{O}_{Nq}$ – **local multiplicatively renormalized quark-gluon operators:**

$$Q(z_1,w,z_2) = \sum_{Nq} \Psi_{Nq}(z_1,w,z_2)\,\mathcal{O}_{Nq} = $$
$$= \sum_{N} \Psi_{N}(z_1,w,z_2)(\partial O)_N + {\sum_{Nq}}' \Psi_{Nq}(z_1,w,z_2)\,\mathcal{O}_{Nq}$$

If the function $\Psi_N(z_1,w,z_2)$ **is known then**

$$R(z_1,z_2) = ig\sum_N (\partial O)_N \left(\int_{z_2}^{z_1} dw\,(w-z_2)\Psi_N(z_1,w,z_2)\right) + \ldots$$

How to determine Ψ_N **??**

Nonlocal & local operators

RG-equation

$$\Big(\mu\partial_\mu + \beta(g)\partial_g + \mathbf{H}\Big) Q(z_1, w, z_2) = 0 \qquad \Longleftrightarrow \qquad \Big(\mu\partial_\mu + \beta(g)\partial_g + \gamma_{Nq}\Big) \mathcal{O}_{Nq} = 0$$

$\mathbf{H}$ is an integral operator (Hamiltonian). It is known in the explicit form, V. Braun, J. Rohrwild, A.M, 2010

$$\mathbf{H}\Psi_{Nq}(z_1, w, z_2) = \gamma_{Nq}\Psi_{Nq}(z_1, w, z_2)\,.$$

Conformal symmetry $\mapsto$ There exists a scalar product such that $\mathbf{H} = \mathbf{H}^\dagger$

$$\mathcal{O}_{Nq} = \langle \Psi_{Nq} | Q \rangle \Longrightarrow (\partial O)_N = \langle \Psi_N | Q \rangle$$

Conformal symmetry and $SU(1,1)$ scalar product

collinear conformal transformations $SL(2,\mathbb{R}) \Leftrightarrow SU(1,1)$

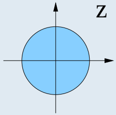
$$x_\mu = z n_\mu\,, \qquad z \in \mathbb{R} \to z' = \frac{az+b}{cz+d}\,, \qquad \Leftrightarrow \qquad z \in \mathbb{C} \to z' = \frac{az+b}{\bar{b}z+\bar{a}}$$

representations are labeled by conformal spin

$$\varphi(z) \to T^j\varphi(z) = \frac{1}{(\bar{b}z+\bar{a})^{2j}}\varphi\left(\frac{az+b}{\bar{b}z+\bar{a}}\right)$$

This is a unitary transformation with respect to the following scalar product:

$$\langle\phi,\psi\rangle_j = \frac{2j-1}{\pi}\int_{|z|<1} d^2z\,(1-|z|^2)^{2j-2}\bar{\phi}(z)\psi(z) \equiv \int_{|z|<1}\mathcal{D}_j z\,\bar{\phi}(z)\psi(z)\,, \qquad ||\phi||^2 = \langle\phi,\phi\rangle$$



similar for several variables

$$\langle\phi,\psi\rangle_{j_1,j_2} = \int_{|z_1|<1}\mathcal{D}_{j_1}z_1\int_{|z_2|<1}\mathcal{D}_{j_2}z_2\,\bar{\phi}(z_1,z_2)\psi(z_1,z_2)$$

$$\langle\phi,\psi\rangle_{j_1,j_2,j_3} = \int_{|z_1|<1}\mathcal{D}_{j_1}z_1\int_{|z_2|<1}\mathcal{D}_{j_2}z_2\int_{|z_3|<1}\mathcal{D}_{j_3}z_3\,\bar{\phi}(z_1,z_2,z_3)\psi(z_1,z_2,z_3)$$

Generators

- **Generators of infinitesimal $SU(1,1)$ transformations**

$$S_+ = z^2\partial_z + 2jz\,, \qquad S_0 = z\partial_z + j\,, \qquad S_- = -\partial_z$$

satisfy the usual $SL(2)$ algebra

$$[S_+, S_-] = 2S_0\,, \qquad [S_0, S_\pm] = \pm S_\pm$$

- **For products of fields, e.g. $\phi(z_1)\phi(z_2)\ldots$**

$$S_+^{(j_1,j_2)} = S_+^{(j_1)} + S_+^{(j_2)} = z_1^2\partial_{z_1} + z_2^2\partial_{z_2} + 2j_1 z_1 + +2j_2 z_2\,,$$

$$S_+^{(j_1,j_2,j_3)} = z_1^2\partial_{z_1} + z_2^2\partial_{z_2} + z_3^2\partial_{z_3} + 2j_1 z_1 + +2j_2 z_2 + 2j_3 z_3\,,$$

- **Hermiticity properties**

$$S_0^\dagger = S_0 \qquad (S_+)^\dagger = -S_- \qquad \langle\phi, S\psi\rangle = \langle S^\dagger\phi, \psi\rangle$$

- **reproducing kernel (unit operator)**

$$\phi(z) = \int_{|w|<1} \mathcal{D}_j w\, \mathcal{K}_j(z,\bar{w})\phi(w)\,, \qquad \mathcal{K}_j(z,\bar{w}) = \frac{1}{(1-z\bar{w})^{2j}}$$

Divergency of conformal operator

We want to calculate $(\partial\mathcal{O})_N$

- **Light-ray operators**

$$O_+(z_1,z_2) = \bar{q}(z_1 n)\,\gamma_+\, q(z_2 n)$$

- **(Local) conformal operators (standard representation)**

$$\mathcal{O}_N(x) \quad = \quad (-\partial_+)^N \bar{q}(x)\gamma_+ C_N^{3/2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) q(x)$$

- **Alternatively:**

$$\mathcal{O}_N = \rho_N \Big\langle (z_1 - z_2)^N, O_+(z_1,z_2)\Big\rangle_{11} = \rho_N \iint\limits_{|z_i|<1} \mathcal{D}_1 z_1 \mathcal{D}_1 z_2\, \bar{z}_{12}^N\, O_+(z_1,z_2)$$

$$\boldsymbol{\delta_K}\mathcal{O}_N \sim \Big\langle z_{12}^N, \boldsymbol{S_+} O_+(z_1,z_2)\Big\rangle = -\Big\langle \boldsymbol{S_-} z_{12}^N, O_+(z_1,z_2)\Big\rangle = 0$$

$$(\partial\mathcal{O})_N^{\mathbf{free}} \sim \Big\langle z_{12}^N, (\boldsymbol{\partial O_+})(z_1,z_2)\Big\rangle = \Big\langle z_{12}^N, \boldsymbol{S_+}\partial_\nu \bar{q}(z_1)\partial^\nu q(z_2)\ldots\Big\rangle = 0$$

Divergency of conformal operator

In interacting theory

$$(\partial\mathcal{O})_N = \left\langle z_{12}^N, \int_{z_1}^{z_2} dw\, f(z_1, z_2, w)\; Q(z_1, w, z_2)\right\rangle$$

Trick:

$$Q(z_1, w, z_2) = \left\langle \mathcal{K}(z_1, \zeta_1)\, \mathcal{K}(w, \zeta_2)\, \mathcal{K}(z_2, \zeta_3),\, Q(\zeta_1, \zeta_2, \zeta_3)\right\rangle$$

$$(\partial\mathcal{O})_N = \left\langle \Psi_N(\zeta_1, \zeta_2, \zeta_3)\,,\, Q(\zeta_1, \zeta_2, \zeta_3)\right\rangle$$

with

$$\Psi_N(\zeta_1, \zeta_2, \zeta_3) = \left\langle z_{12}^N, \int_{z_1}^{z_2} dw\, f(z_1, z_2, w)\, \mathcal{K}(z_1, \zeta_1)\, \mathcal{K}(w, \zeta_2)\, \mathcal{K}(z_2, \zeta_3)\right\rangle$$

$$\sim \int_0^1 d\alpha\, \bar{\alpha} \int_0^1 d\beta\, \left(\beta + \frac{1}{N+1}\alpha\right) (\zeta_{12}^\alpha - \zeta_{32}^\beta)^{N-1}$$

$\zeta_{12}^\alpha = (1-\alpha)\zeta_1 + \alpha\zeta_2$.

$$||\Psi_N||^2 \sim \gamma_N \sim \psi(N+3) + \psi(N+1) - 2\psi(1)$$

$$R(z_1,z_2) = z_{12}\int_{z_2}^{z_1}\frac{dw}{z_{12}}\int_{z_2}^{w}\frac{dw'}{z_{12}}\frac{w'-z_2}{z_1-w'}\left[\frac{1}{2}S_+\mathcal{O}_1(w,w') - (S_0-1)\mathcal{O}_2(w,w')\right],$$

$$\bar{R}(z_1,z_2) = z_{12}\int_{z_2}^{z_1}\frac{dw}{z_{12}}\int_{z_2}^{w}\frac{dw'}{z_{12}}\frac{z_1-w}{w-z_2}\left[\frac{1}{2}S_+\mathcal{O}_1(w,w') - (S_0-1)\mathcal{O}_2(w,w')\right].$$

$$\mathcal{O}_1(w,w') = \left[i\mathbf{P}^\mu,\left[i\mathbf{P}_\mu,\mathcal{O}_+^{t=2}(w,w')\right]\right], \qquad \mathcal{O}_2(w,w') = \left[i\mathbf{P}^\mu,\frac{\partial}{\partial x^\mu}\mathcal{O}_+^{t=2}(w,w')\right].$$

$$T_{\mu\nu} = -\frac{1}{\pi^2 x^4}\left\{x^\alpha\left[S_{\mu\alpha\nu\beta}\mathbb{V}^\beta + i\epsilon_{\mu\nu\alpha\beta}\mathbb{A}^\beta\right] + x^2\left[(x_\mu\partial_\nu + x_\nu\partial_\mu)\mathbb{X} + (x_\mu\partial_\nu - x_\nu\partial_\mu)\mathbb{Y}\right]\right\},$$

Translation invariance and current conservation up to twist-5.