Fitting DVCS at LO and NLO

Krešimir Kumerički

Physics Department
University of Zagreb, Croatia

Based mostly on:

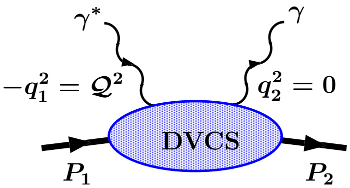
K.K., D. Müller, M. Murray, to appear in Phys. Part. Nucl. (2014), arXiv:1301.1230
K.K., D. Müller, A. Schäfer, JHEP **07** (2011) 073, arXiv:1106.2808
K.K., D. Müller, Nucl. Phys. **B841** (2010) 1-58, arXiv:0904.0458

International Workshop on Deeply Virtual Compton Scattering: *From Observables to GPDs*,
Ruhr-Universität Bochum, February 10-12, 2014

Outline

Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS)



$$P = P_1 + P_2 \,, \qquad t = (P_2 - P_1)^2$$

$$q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

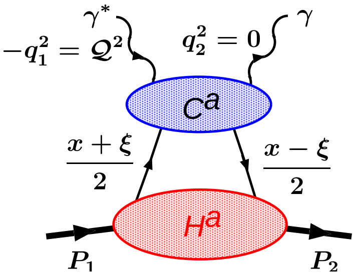
$$\xi = \frac{-q^2}{2P \cdot q} = \frac{x_B}{2 - x_B} \to \text{const}$$

- We work at leading order accuracy where cross-section can be expressed in terms of four Compton form factors (CFFs)

$$\mathcal{F} \in \{\mathcal{H}(\xi, t, \mathcal{Q}^2), \mathcal{E}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2)\}$$

- We use formulas from [Belitsky, Müller, Kirchner '01, Belitsky, Müller '10].

Factorization of DVCS $\longrightarrow$ GPDs



$$P = P_1 + P_2 \,, \qquad t = (P_2 - P_1)^2$$
$$q = (q_1 + q_2)/2$$

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$
$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mathcal{Q}_0^2) \; H^a(x, \eta = \xi, t, \mathcal{Q}_0^2)$$

a=NS,S(Σ,G)

- $H^a(x, \eta, t, \mathcal{Q}_0^2)$ — Generalized parton distribution (GPD)

Dispersion-relation access to GPDs at LO

[Teryaev '05; K.K., Müller and Passek-K. '07, '08; Diehl and Ivanov '07]

- LO perturbative prediction is "handbag" amplitude

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} \int_{-1}^{1} dx \, \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mathcal{Q}^2)$$

- giving access to GPD on the "cross-over" line $\eta = x$

$$\frac{1}{\pi} \mathfrak{Im}\, \mathcal{H}(\xi = x, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} H(x, x, t, \mathcal{Q}^2) - H(-x, x, t, \mathcal{Q}^2)$$

- while dispersion relation connects it to $\mathfrak{Re}\, \mathcal{H}$

$$\mathfrak{Re}\, \mathcal{H}(\xi, t, \mathcal{Q}^2) =$$

$$\frac{1}{\pi} \mathrm{PV} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \mathfrak{Im}\, \mathcal{H}(\xi', t, \mathcal{Q}^2) + \mathcal{C}_{\mathcal{H}}(t, \mathcal{Q}^2)$$

Curse of dimensionality

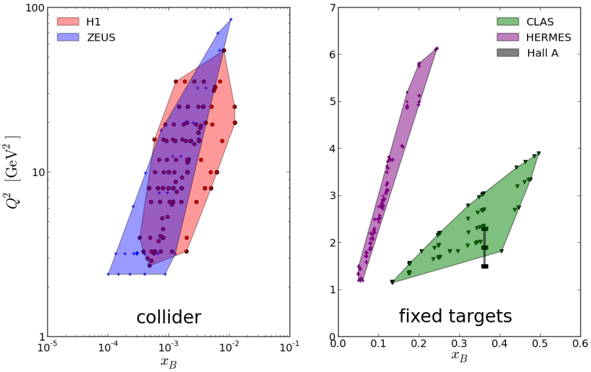
- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*

Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*
- When the dimensionality increases, the volume of the space increases so fast that the available data becomes sparse.
- Analogously, in contrast to *PDFs*(x), it is very difficult to perform truly model independent extraction of *GPDs*(x, ξ, t)
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of *CFFs*(x_B, t)
- (Dependence on additional variable, photon virtuality $\mathcal{Q}^2$, is in principle known — given by evolution equations.)

Fit types

	Fit type	Data used	pQCD order	Target
1.	Local fits	fixed target	LO	CFFs
2.	Global fits	collider/fixed target	((N)N)LO	CFFs/GPDs
3.	Neural nets	fixed target	LO	CFFs



Local fits to HERMES data

- Most complete set of asymmetries is measured in 12 bins:

bin no.	1	2	3	4	5	6	7	8	9	10	11	12
$-t$ [GeV^2]	0.03	0.1	0.2	0.42	0.1	0.1	0.13	0.2	0.08	0.1	0.13	0.19
x_B	0.08	0.1	0.11	0.12	0.05	0.08	0.12	0.2	0.06	0.08	0.11	0.17
Q^2 [GeV^2]	1.9	2.5	2.9	3.5	1.5	2.2	3.1	5.0	1.2	1.9	2.8	4.9

$$A_C \equiv \frac{\mathrm{d}\sigma_{e^+} - \mathrm{d}\sigma_{e^-}}{\mathrm{d}\sigma_{e^+} + \mathrm{d}\sigma_{e^-}} = \frac{\mathcal{A}_{\mathrm{Interference}}(\mathcal{F})}{|\mathcal{A}_{\mathrm{DVCS}}|^2(\mathcal{F}^2) + |\mathcal{A}_{\mathrm{BH}}|^2}$$

$$A_{\mathrm{C}}^{\cos(1\phi)} \propto \left[F_1\, \mathfrak{Re}\,\mathcal{H} - \frac{t}{4M_p^2} F_2\, \mathfrak{Re}\,\mathcal{E} + \frac{x_{\mathrm{B}}}{2}(F_1 + F_2)\, \mathfrak{Re}\,\widetilde{\mathcal{H}} \right]$$

- ...and similarly for other observables $\longrightarrow$ almost linear relation between observables and CFFs
- (up to $|\mathcal{A}_{\mathrm{DVCS}}|^2$ contamination — taken into account)

Mapping

- Inverting these relations gives mapping from the set of observables ...

$A_{\mathrm{LU,I}}^{\sin(1\phi)}$	$A_{\mathrm{C}}^{\cos(1\phi)}$	$A_{\mathrm{C}}^{\cos(0\phi)}$	$A_{\mathrm{UL,+}}^{\sin(1\phi)}$
$A_{\mathrm{LL,+}}^{\cos(1\phi)}$	$A_{\mathrm{LL,+}}^{\cos(0\phi)}$	$A_{\mathrm{UT,I}}^{\sin(\varphi)\cos(1\phi)}$	$A_{\mathrm{UT,I}}^{\cos(\varphi)\sin(1\phi)}$
$A_{\mathrm{UT,DVCS}}^{\sin(\varphi)\cos(0\phi)}$	$A_{\mathrm{UT,I}}^{\sin(\varphi)\cos(0\phi)}$	$A_{\mathrm{LT,I}}^{\sin(\varphi)\sin(1\phi)}$	$A_{\mathrm{LT,I}}^{\cos(\varphi)\cos(1\phi)}$
$A_{\mathrm{LT,BH+DVCS}}^{\cos(\varphi)\cos(0\phi)}$	$A_{\mathrm{LT,I}}^{\cos(\varphi)\cos(0\phi)}$		

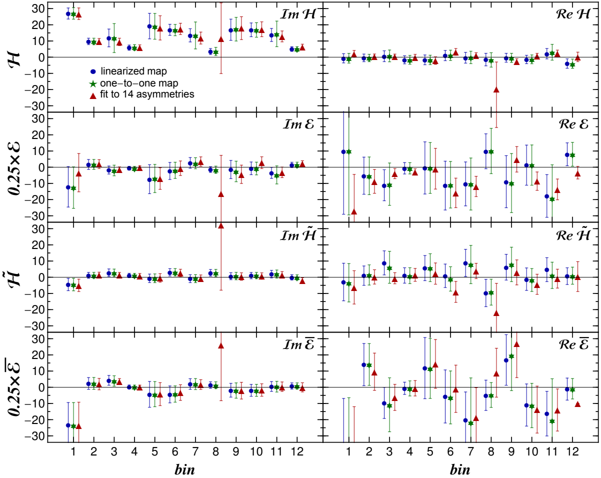
- ... to the set of eight real and imaginary parts of CFFs (sometimes called sub-CFFs or just CFFs) ...

$$\boldsymbol{\mathcal{F}} = \left(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{H}, \mathfrak{Im}\,\mathcal{E}, \mathfrak{Re}\,\mathcal{E}, \mathfrak{Im}\,\widetilde{\mathcal{H}}, \mathfrak{Re}\,\widetilde{\mathcal{H}}, \mathfrak{Im}\,\widetilde{\mathcal{E}}, \mathfrak{Re}\,\widetilde{\mathcal{E}}\right)$$

- ... where error propagation is straightforward.

Mapping — results

- (Here compared also to standard local least-squares fit)



- Only $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\widetilde{\mathcal{H}}$ are reliably constrained.

Method of stepwise regression

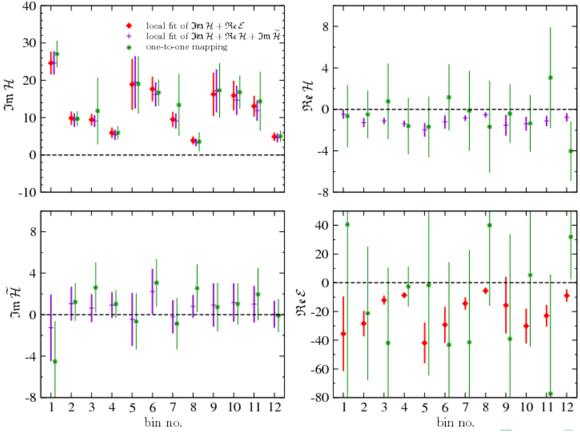
- Constraints from data are too weak to constrain simultaneously all eight $\{\mathfrak{Im}\,\mathcal{F}, \mathfrak{Re}\,\mathcal{F}\}$ CFFs
- Let us take smaller number of CFFs, choosing only those which are reliably extracted. Stepwise regression algorithm:
  1. Perform single-CFF fit with each of 8 CFFs and see which one alone describes data best (it is obviously $\mathfrak{Im}\,\mathcal{H}$).
  2. Combine $\mathfrak{Im}\,\mathcal{H}$ with each of other seven CFFs and see which pair describes data best.
  3. Proceed until there is either no improvement in data description or new CFFs are not extracted with any statistical significance

Method of stepwise regression

- Constraints from data are too weak to constrain simultaneously all eight $\{\mathfrak{Im}\,\mathcal{F}, \mathfrak{Re}\,\mathcal{F}\}$ CFFs
- Let us take smaller number of CFFs, choosing only those which are reliably extracted. Stepwise regression algorithm:
  1. Perform single-CFF fit with each of 8 CFFs and see which one alone describes data best (it is obviously $\mathfrak{Im}\,\mathcal{H}$).
  2. Combine $\mathfrak{Im}\,\mathcal{H}$ with each of other seven CFFs and see which pair describes data best.
  3. Proceed until there is either no improvement in data description or new CFFs are not extracted with any statistical significance
- It turns out that already 2nd step is the final one, and there are two equally good pairs of CFFs:
  1. ($\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$) with $\chi^2/n_{\rm d.o.f.} = 102.3/120$, and
  2. ($\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{E}$) with $\chi^2/n_{\rm d.o.f.} = 103.0/120$.

Stepwise regression — results

- Scenario 1: Fit of $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\widetilde{\mathcal{H}}$. $\chi^2/n_{\rm d.o.f.} = 148.8/144$. (In good agreement with [Guidal '10])
- Scenario 2: Fit of $\mathfrak{Im}\,\mathcal{H}$ and $\mathfrak{Re}\,\mathcal{E}$. $\chi^2/n_{\rm d.o.f.} = 134.2/144$.



Modelling GPDs in moment space

- Instead of considering momentum fraction dependence $H(x, \ldots)$
- ...it is convenient to make a transform into complementary space of conformal moments j:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^j \, C_j^{3/2}(x/\eta) \, H^q(x, \eta, \ldots)$$

- They are analogous to Mellin moments in DIS: $x^j \to C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials

Advantages of conformal moments

1. The evolution equations are most simple: There is **no mixing** among moments at LO, and in special ($\overline{CS}$) scheme not even at NLO
2. Powerful analytic methods of **complex j** plane are available (similar to complex angular momentum of Regge theory)
3. Stable and fast **computer code** for evolution and fitting
4. Moments are equal to matrix elements of **local** operators and are thus directly accessible on the **lattice**

- GeParD computer code implements conformal moment GPD models and QCD evolution up to NLO ($\overline{\mathrm{MS}}$ and $\overline{\mathrm{CS}}$) and NNLO ($\overline{\mathrm{CS}}$ scheme), as well as DVCS and DVMP coefficient functions and observables. For fitting we use either χ^2 minimization (MINUIT) or neural networks (PYBRAIN).

l-PW model — only leading SO(3) partial wave

- cf. [Dieter's talk]

$$\mathbf{H}_j(\xi, t, \mu_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(t) \, \mathrm{B}\big(1 + j - \alpha_\Sigma(0), 8\big) \\ N'_\mathrm{G} F_\mathrm{G}(t) \, \mathrm{B}\big(1 + j - \alpha_\mathrm{G}(0), 6\big) \end{pmatrix}$$

$\alpha_a(t) = \alpha_a(0) + 0.15t$
$$F_a(t) = \frac{j+1-\alpha(0)}{j+1-\alpha(t)} \left(1 - \frac{t}{M_0^{a\,2}}\right)^{-p_a}$$

...corresponding in forward case to PDFs of form

$$\Sigma(x) = N'_\Sigma \, x^{-\alpha_\Sigma(0)} \, (1-x)^7 \; ; \qquad G(x) = N'_\mathrm{G} \, x^{-\alpha_\mathrm{G}(0)} \, (1-x)^5$$

- $M_0^G = \sqrt{0.7}\,\mathrm{GeV}$ is fixed by the J/ψ production data
- Free parameter (for DVCS): M_0^Σ

For small ξ (small x_{Bj}) valence quarks are less important $\Rightarrow \Sigma \approx$ sea

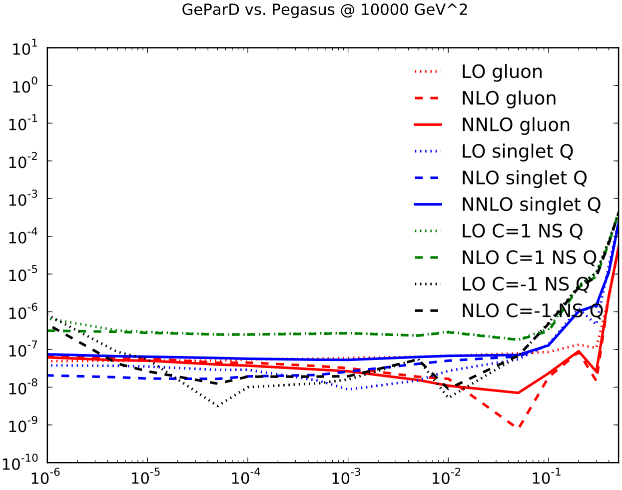
Inclusion of subleading PW — flexible models

$$\mathbf{H}_j(\eta,t) = \underbrace{\begin{pmatrix} N'_{\rm sea}\, F_{\rm sea}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm sea}(0),8\big) \\ N'_{\rm G}\, F_{\rm G}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm G}(0),6\big) \end{pmatrix}}_{\text{skewness } r\approx 1.6 \text{ (too large)}} + \underbrace{\begin{pmatrix} s_{\rm sea} \\ s_{\rm G} \end{pmatrix} \begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}}_{<0 \atop \text{negative skewness}}$$

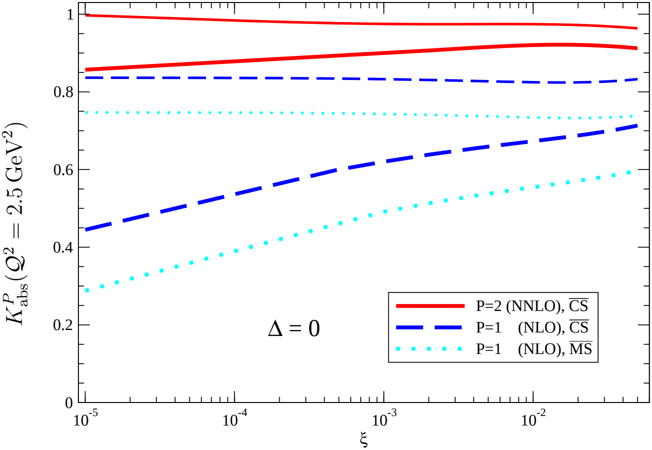
- nl-PW — addition of second PW needed for good fits
- two new parameters: $s^{(2)}_{\rm sea}$ and $s^{(2)}_{\rm G}$
- nnl-PW — addition of third PW (doesn't improve fits much but makes possible positive gluon GPDs at small $\mathcal{Q}^2$) — $s^{(4)}_{\rm sea}$ and $s^{(4)}_{\rm G}$

Checking the evolution code

- We checked agreement in the forward limit with QCD PEGASUS code for PDF evolution [Vogt '04]



(N)NLO corrections

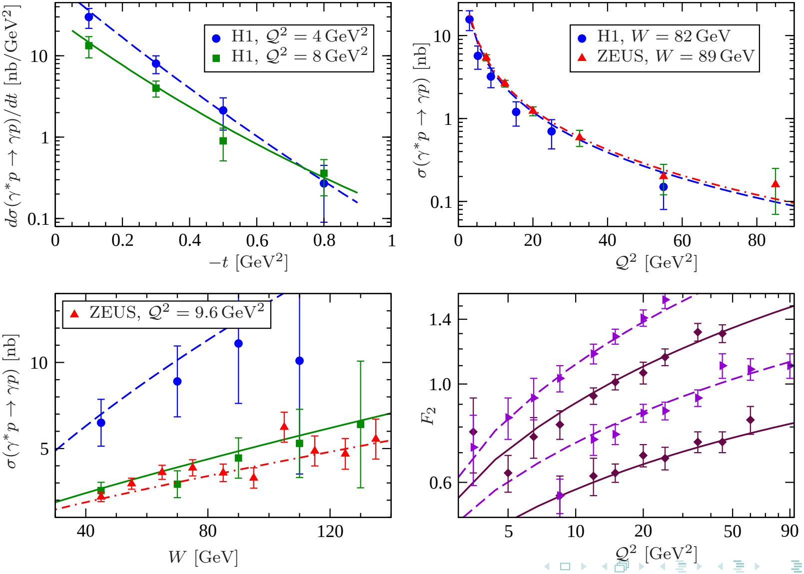


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0) = \alpha_\Sigma(0) + 0.05$

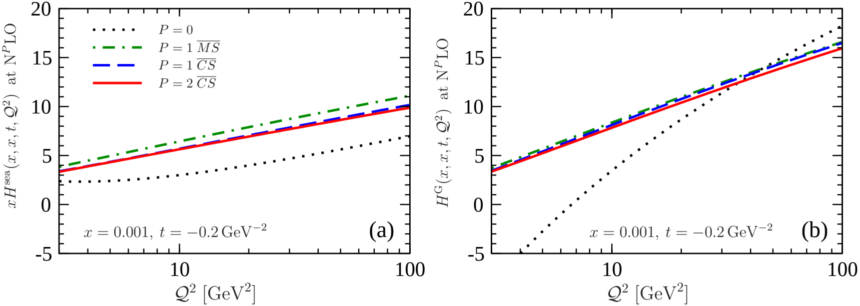
Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0) = \alpha_\Sigma(0) - 0.02$

$$K^P_{\rm abs} \equiv \left| \frac{\mathcal{H}^{(P)}}{\mathcal{H}^{(P-1)}} \right|$$

Example of fit result



Resulting small-x $H(x, x, t)$



- P=0: LO; P=1: NLO; P=2: NNLO
- The whole procedure is extended to meson production [Müller, Lautenschlager, Passek-Kumerički, Schäfer '13]

Extending global analysis to fixed target data

- **Hybrid models** at LO (1st just for *unpolarized* target)
- Sea quarks and gluons modelled like just described (conformal moments + SO(3) partial wave expansion + $\mathcal{Q}^2$ evolution).
- Valence quarks model (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t) = n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- Fixed: n (from PDFs), $\alpha(t)$ (eff. Regge), p (counting rules)

$$\alpha^{\rm val}(t) = 0.43 + 0.85\,t/\mathrm{GeV}^2 \quad (\rho,\,\omega)$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations

$$\mathfrak{Re}\,\mathcal{H}(\xi,t) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)\mathfrak{Im}\,\mathcal{H}(\xi',t) - \frac{C}{\left(1-\frac{t}{M_C{}^2}\right)^2}$$

- Typical set of free parameters:

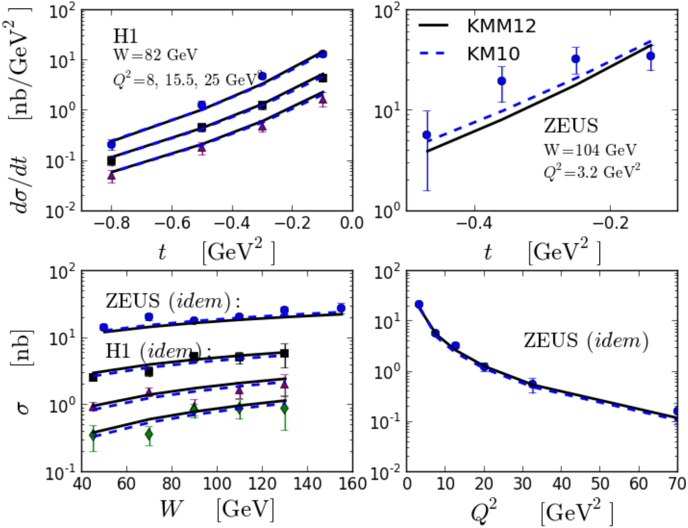
$M_0^{\rm sea}$, $s_{\rm sea}^{(2,4)}$, $s_G^{(2,4)}$	sea* quarks and gluons H
$r^{\rm val}$, $M^{\rm val}$, $b^{\rm val}$	valence H
$\tilde{r}^{\rm val}$, $\tilde{M}^{\rm val}$, $\tilde{b}^{\rm val}$	valence $\widetilde{H}$
C, M_C	subtraction constant (H, E)
r_π, M_π	"pion pole" $\widetilde{E}$

- Global fit to 175 data points turns out fine:

`KM10` model: $\chi^2/d.o.f. = 135.9/160$.

*$s_{\rm sea,G}$ = strengths of subleading partial waves. LO evolution is included.

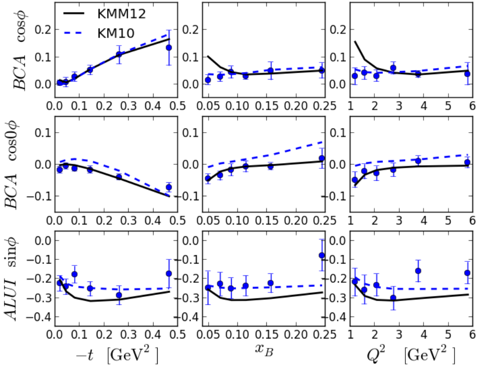
H1 (2007), ZEUS (2008)



HERMES (2008)

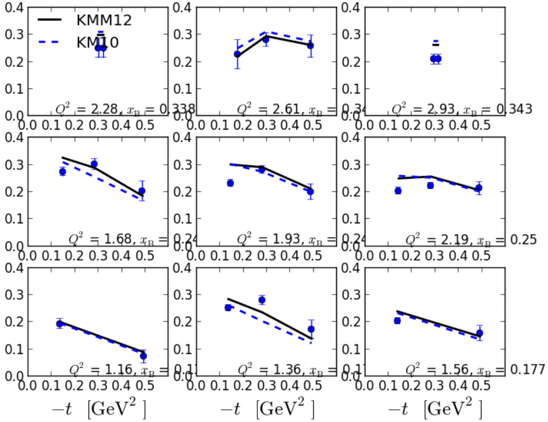
$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi} \cos\phi \sim \Re\mathfrak{e}\, \mathcal{H}$$

$$BSA \equiv \frac{d\sigma_{e\uparrow} - d\sigma_{e\downarrow}}{d\sigma_{e\uparrow} + d\sigma_{e\downarrow}} \sim A_{LU}^{\sin 1\phi} \sin\phi \sim \Im\mathfrak{m}\, \mathcal{H}$$



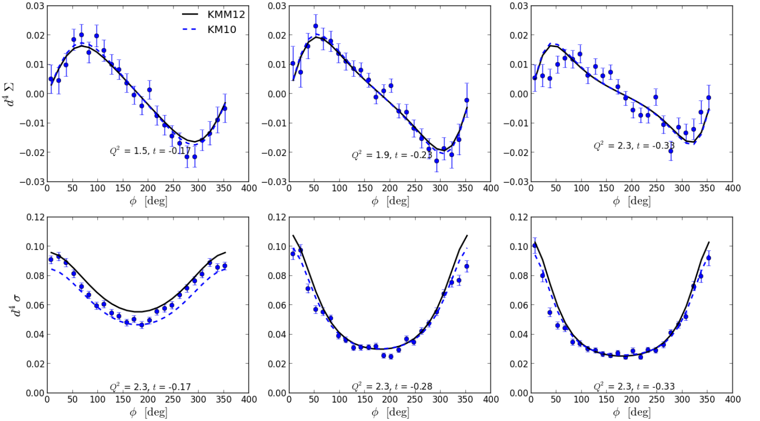
CLAS (2007)

- BSA. (Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.)



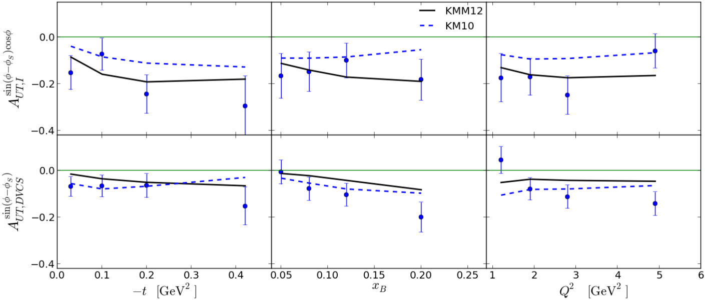
Hall A (2006)

- Fit to unpolarized cross section $d\sigma/(dx_B dQ^2 dt d\phi) \quad \sim \mathfrak{Re}\,\mathcal{H}$
- KM10 fit needs unusually large $\mathfrak{Re}\,\tilde{\mathcal{H}}$.



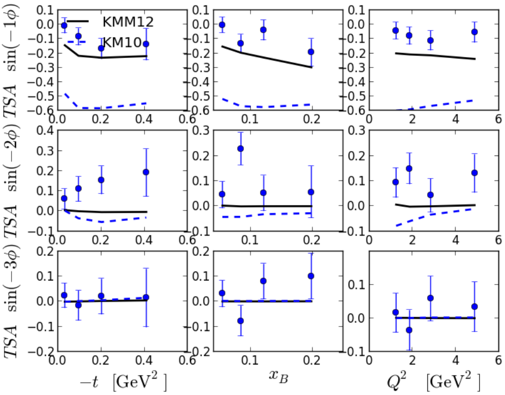
Including data with polarized target

- KMM12: $\chi^2/n_{\rm d.o.f.} = 124.1/80$, strictly speaking not a good fit, but best what we have at the moment

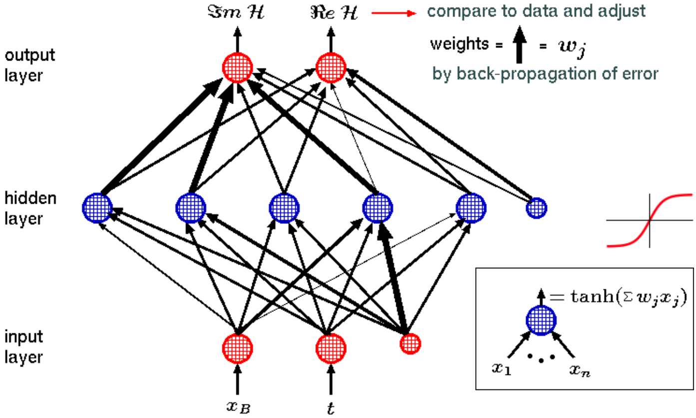


Polarized target (II)

- Surprisingly large $\sin(2\phi)$ harmonic of A_{UL} cannot be described within this leading twist framework



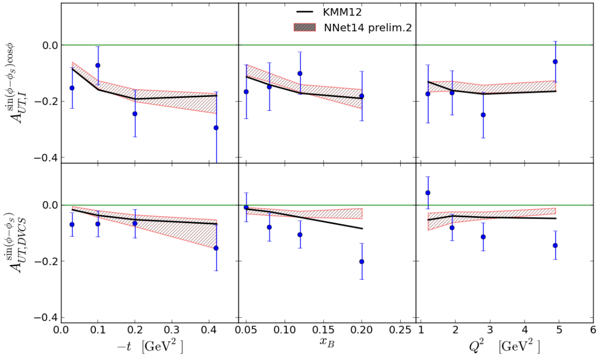
Multilayer perceptron



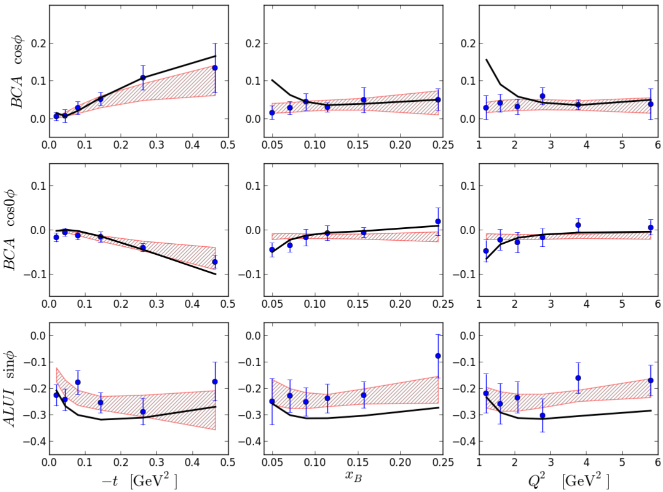
- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias

Preliminary neural Net HERMES fit

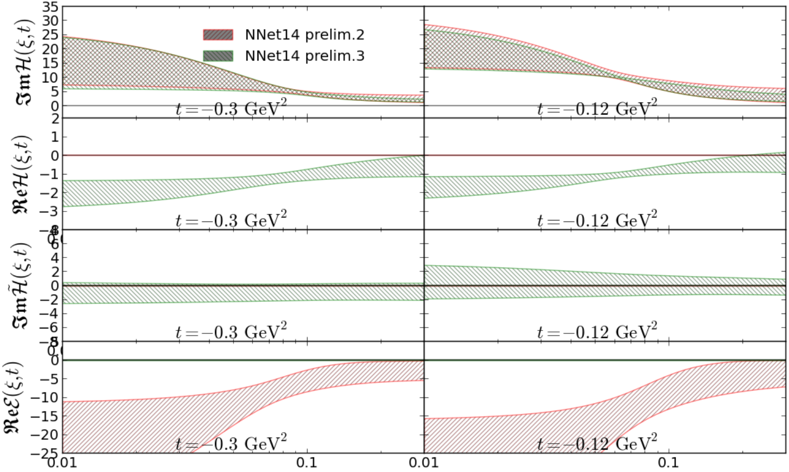
- Fit to all HERMES DVCS data with two types of neural nets
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{H}, \mathfrak{Im}\,\tilde{\mathcal{H}})$: $\chi^2/n_{\rm pts} = 135.4/144$
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{E})$: $\chi^2/n_{\rm pts} = 120.2/144$



Neural Net HERMES fit - BSA/BCA

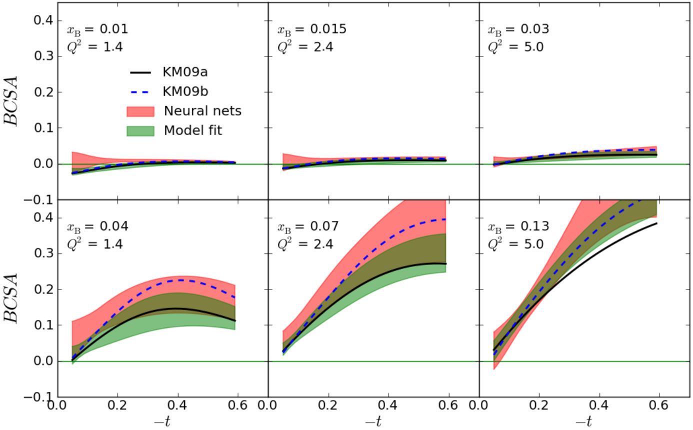


Neural Net HERMES fit - CFFs

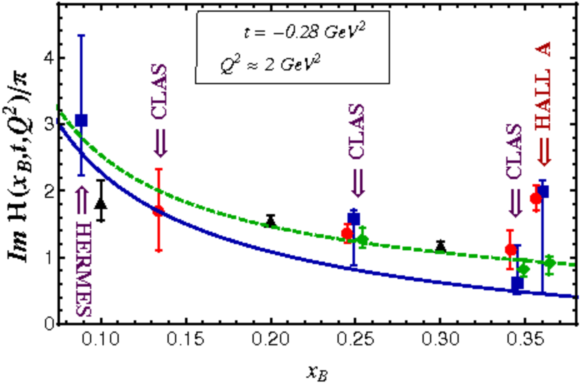


Prediction for COMPASS II BCSA

$$BCSA = \frac{d\sigma_{\mu^{\downarrow +}} - d\sigma_{\mu^{\uparrow -}}}{d\sigma_{\mu^{\downarrow +}} + d\sigma_{\mu^{\uparrow -}}} \qquad (E_\mu = 160\,\mathrm{GeV})$$



Comparison to others



[Guidal '08, Guidal and Moutarde '09], seven CFF fit (blue squares), [Guidal '10] $\mathcal{H}$, $\tilde{\mathcal{H}}$ CFF fit (green diamonds), [Moutarde '09] H GPD fit (red circles)

New directions

- Improved global LO fits with all unpolarized and polarized proton data.
- Adding deeply virtual meson production and going NLO [Müller, Lautenschlager, Schäfer '13]
- Including higher twists
- Global neural network fits

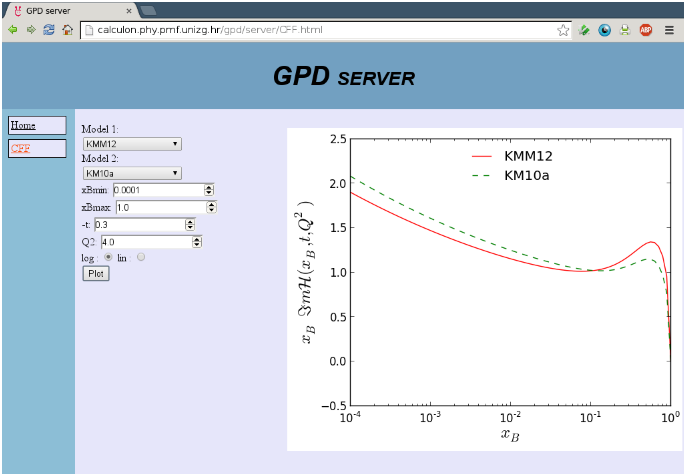
KM models are available at WWW

- http://calculon.phy.hr/gpd/ — binary code for cross sections

```
% xs.exe

  xs.exe  ModelID  Charge  Polarization  Ee  Ep  xB  Q2  t  phi

returns cross section (in nb) for scattering of lepton of energy Ee
on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
   0 debug, always returns 42,
   1 KM09a - arXiv:0904.0458 fit without Hall A,
   2 KM09b - arXiv:0904.0458 fit with Hall A,
   3 KM10  - preliminary hybrid fit with LO sea evolution, from Trento presentation,
   4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
   5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi  -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938  0.111 3. -0.3 0

  0.18584386497251
```

GPD page and server

- Durham-like CFF/GPD server page



- Do we need "Les Houches Accord" CFF/GPD interface?

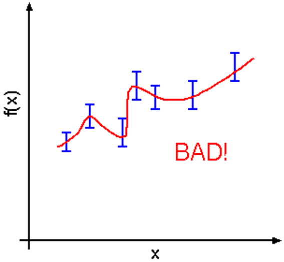
The End!

Function fitting by a neural net

- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).

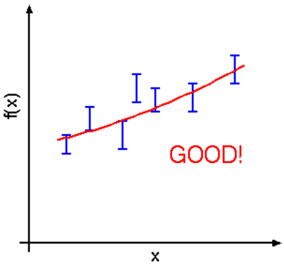
Function fitting by a neural net

- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).
- With simple training of neural nets to data there is a danger of overfitting (a.k.a. overtraining)



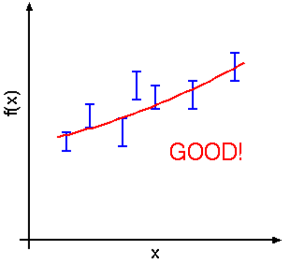
Function fitting by a neural net

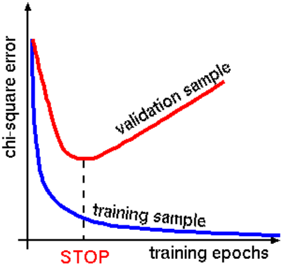
- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).
- With simple training of neural nets to data there is a danger of overfitting (a.k.a. overtraining)



Function fitting by a neural net

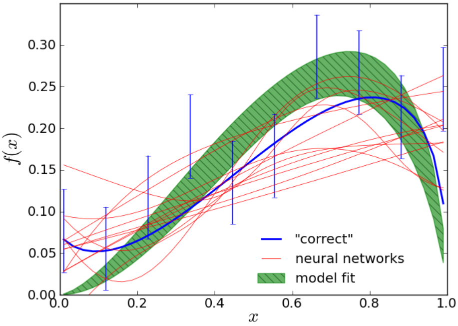
- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).
- With simple training of neural nets to data there is a danger of overfitting (a.k.a. overtraining)
- Solution: Divide data (randomly) into two sets: *training* sample and *validation sample*. Stop training when error of validation sample starts increasing.





Toy fitting example

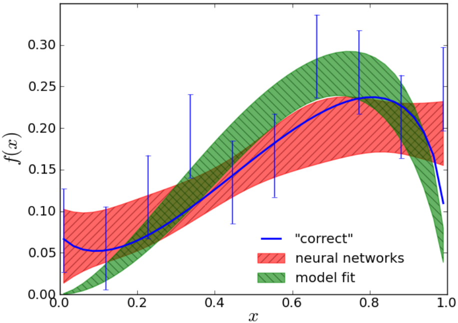
- Fit to data generated according to function (which we pretend not to know).



- Fit with
  1. Standard Minuit fit with ansatz $f(x) = x^a(1-x)^b$
  2. Neural network fit

Toy fitting example

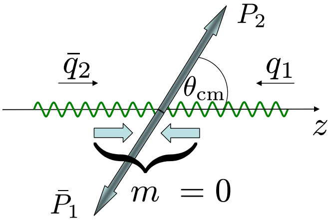
- Fit to data generated according to function (which we pretend not to know).



- Fit with
  1. Standard Minuit fit with ansatz $f(x) = x^a(1-x)^b$
  2. Neural network fit

Modelling conformal moments of GPDs (I)

- How to model η-dependence of GPD's $H_j(\eta, t)$?
- Idea: consider crossed t-channel process $\gamma^* \gamma \to p\bar{p}$

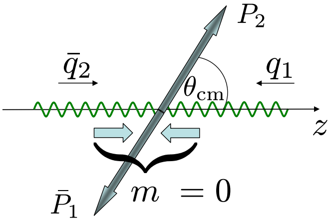


When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

Modelling conformal moments of GPDs (I)

- How to model η-dependence of GPD's $H_j(\eta, t)$?
- Idea: consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$



When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

- ...and dependence on $\theta_{\rm cm}$ in t-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta, \ldots) = \mathcal{H}^{(t)}(\cos\theta_{\rm cm} = -\frac{1}{\eta}, \ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos\theta)$$

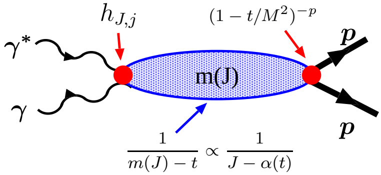
- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
  $\nu = 0, \pm 1$ — depending on hadron helicities

Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$ leads to

$$H_j(\eta, t) = \eta^{j+1}\, H_j^{(t)}(\cos\theta = -\frac{1}{\eta}, s^{(t)} = t)$$

- and t-channel partial waves are modelled as:



$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J - \alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p}\, \eta^{j+1-J} d^J_{0,\nu}$$

- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

Fit results - LO

- For consistency, we don't take standard PDFs, but fit GPDs to DIS data. This determines $N_{\rm sea}$, N_G, $\alpha_{\rm sea}(0)$ and $\alpha_G(0)$, leaving only $M_0^{\rm sea}$, $s_{\rm sea}$ and s_G for DVCS data

- χ^2 values:

model	α_s	χ^2/d.o.f. DIS	χ^2/d.o.f. DVCS	χ^2_t/n.o.p.	χ^2_W/n.o.p.	$\chi^2_{Q^2}$/n.o.p.
l, dipole	LO	49.7/82	**280./100**	**181./56**	**63.6/29**	**36.2/16**
l, exp.	LO	49.7/82	**316./100**	**192./56**	**79./29**	**44.9/16**
nl, dipole	LO	49.7/82	95.9/98	53.2/56	27./29	15.8/16
nl, exp.	LO	49.7/82	97.9/98	49.1/56	31.2/29	17.7/16
Σ, dipole	LO	49.7/82	101./98	57.7/56	27.4/29	16./16
Σ, exp.	LO	49.7/82	102./98	51./56	32.3/29	18.6/16
l, dipole	LO	**321./182**		**189./56**	**51.1/29**	**27.9/16**

- Parameter values:

model	α_s	$N^{\rm sea}$	$\alpha^{\rm sea}(0)$	$(M^{\rm sea})^2$ [GeV^2]	$s^{\rm sea}$	$\alpha^G(0)$	s^G	$B^{\rm sea}$ [GeV^{-2}]	$b^{\rm eff}$ [GeV^{-2}]	BCA
l, dipole	LO	**0.152**	**1.158**	**0.062**		**1.247**		**33.**	**5.7**	**0.19**
l, exp.	LO	**0.152**	**1.158**			**1.247**		**29.**	**5.1**	**0.23**
nl, dipole	LO	0.152	1.158	0.48	-0.15	1.247	-0.81	4.8	5.5	0.13
nl, exp.	LO	0.152	1.158		-0.18	1.247	-0.86	3.1	5.8	0.14
Σ, dipole	LO	0.152	1.158	0.42	-11.	1.247	-32.	5.4	5.5	0.14
Σ, exp.	LO	0.152	1.158		-13.	1.247	-34.	3.1	5.8	0.15

(boldface numbers = bad fits)

Fit results - NLO

- χ^2 values:

model	α_s	χ^2/d.o.f DIS	χ^2/d.o.f DVCS	χ^2_t/n.o.p	χ^2_W/n.o.p	$\chi^2_{Q^2}$/n.o.p
l	NLO($\overline{\mathrm{MS}}$)	71.6/82	**148./100**	**77.6/56**	**36.8/29**	**33.9/16**
l	NLO($\overline{\mathrm{CS}}$)	71.6/82	105./100	62.9/56	25.1/29	17./16
nl	NLO($\overline{\mathrm{MS}}$)	71.6/82	102./98	60.2/56	23.9/29	17.5/16
nl	NLO($\overline{\mathrm{CS}}$)	71.6/82	104./98	61.4/56	24.9/29	18.1/16
Σ	NLO($\overline{\mathrm{MS}}$)	71.6/82	101./98	60./56	23.9/29	17.5/16
Σ	NLO($\overline{\mathrm{CS}}$)	71.6/82	104./98	61.5/56	24.9/29	18.1/16

- Parameter values:

model	α_s	N^{sea}	$\alpha^{\mathrm{sea}}(0)$	$(M^{\mathrm{sea}})^2$	s^{sea}	$\alpha^{\mathrm{G}}(0)$	s^{G}	B^{sea}	b^{eff}	BCA
l	NLO($\overline{\mathrm{MS}}$)	**0.168**	**1.128**	**0.71**		**1.099**		**3.5**	**5.0**	**0.10**
l	NLO($\overline{\mathrm{CS}}$)	0.168	1.128	0.57		1.099		4.2	5.7	0.09
nl	NLO($\overline{\mathrm{MS}}$)	0.168	1.128	0.59	0.04	1.099	0.02	4.0	5.6	0.09
nl	NLO($\overline{\mathrm{CS}}$)	0.168	1.128	0.58	-0.01	1.099	-0.01	4.1	5.6	0.09
Σ	NLO($\overline{\mathrm{MS}}$)	0.168	1.128	0.60	3.10	1.099	1.10	4.0	5.7	0.09
Σ	NLO($\overline{\mathrm{CS}}$)	0.168	1.128	0.58	-0.42	1.099	-0.58	4.1	5.6	0.09

(boldface numbers = bad fits)

- $s^{\mathrm{sea,G}}$ small $\longrightarrow$ skewness ratio $r \sim 1.5$

Parameter values

	KMM12	KM10
Mv	0.951 +- 0.282	4.00 +- 3.33
rv	1.121 +- 0.099	0.62 +- 0.06
bv	0.400 +- 0.000	0.40 +- 0.67
C	1.003 +- 0.565	8.78 +- 0.98
MC	2.080 +- 3.754	0.97 +- 0.11
tMv	3.523 +- 13.17	0.88 +- 0.24
trv	1.302 +- 0.206	7.76 +- 1.39
tbv	0.400 +- 0.001	2.05 +- 0.40
rpi	3.837 +- 0.141	3.54 +- 1.77
Mpi	4.000 +- 0.036	0.73 +- 0.37
M02S	0.462 +- 0.032	0.51 +- 0.02
SECS	0.313 +- 0.039	0.28 +- 0.02
THIS	-0.138 +- 0.012	-0.13 +- 0.01
SECG	-2.771 +- 0.228	-2.79 +- 0.12
THIG	0.945 +- 0.107	0.90 +- 0.05