Description of hard exclusive processes within the SCET framework

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Hard exclusive processes: theory

QCD Factorization

if QCD factorization holds then one can compute systematically logarithmic corrections improving a description (systematic approach, model independent analysis)

$$A(Q^2, \Lambda^2) \simeq \frac{1}{Q^{2n}} T(\alpha_s(Q^2), \ln Q^2 / \Lambda^2) * \Phi_{\text{coll}}(\Lambda) + \mathcal{O}(1/Q^2)$$

form factors

$$\gamma^* \gamma \to \pi, \eta, \dots$$

 $\gamma^* \pi \to \pi$

DV production

$$\gamma^* p \to p\gamma$$

 $\gamma^*_L p \to p + h, \ h = \pi, \rho, \dots$

Large angle scattering

 $\gamma\gamma \to \pi\pi, \, K\bar{K}, \dots$

Hard exclusive processes: theory

QCD Factorization

if QCD factorization holds then one can compute systematically logarithmic corrections improving a description (systematic approach, model independent analysis)

Problems

collinear factorization

 $\begin{array}{lll} \mbox{collinear factorization} & F_2^N(Q^2) \\ \mbox{does not work for} & & F_2^*(\pi,\rho,...)p \end{array} \Rightarrow \begin{array}{ll} F_2/F_1 & \mbox{`difficult''} \\ \mbox{observables} & & \sigma_T/\sigma_L \end{array}$

asymptotic results applicable for a very³ LARGE Q^2 especially critical for small leading-order amplitudes $\sim \alpha_s(Q^2)$

 $F_{\pi}(Q^2) \sim \alpha_s(Q^2)/Q^2 \qquad \gamma_L^* p \to (\pi, \rho, ...) p$

FF F_1 at large- Q^2 hard and soft spectator contributions





hard-spectator scattering

soft-spectator scattering

hard- and soft-spectator contributions have the same power and large rapidity log's from the soft-collinear overlap Fadin, Milshtein 1981,82

Soft spectator scattering might be important in processes with baryons

WACS
$$\gamma p \to \gamma p$$
 $\gamma p \to (\pi, \rho, ...)p$

Questions

Can we extend the factorization framework and to develop a description of the configurations with the soft & collinear modes? (systematic approach)

Can one obtain a reliable theoretical description which has predictive power?

Soft spectator scattering in the EFT framework

1. Factorize of the hard modes: $p_h^2 \sim Q^2 \gg \Lambda^2$ (hard subprocess)

$$F^{(s)}(Q^2, Q\Lambda, \Lambda^2) \simeq H(Q^2) * f(Q, \Lambda) + \mathcal{O}(1/Q)$$

 $QCD \rightarrow Effective field theory which includes collinear and soft particles$

$$f({\it Q},\Lambda) = \langle out | O | in
angle_{
m EFT}$$
 well defined in EFT

- The UV-behavior of the operators describing $f(Q, \Lambda)$ matches the IR-behavior of the QCD diagrams describing $H(Q^2)$
- There is a systematic power counting in the EFT framework

Dynamical modes in the EFT framework



hard-collinear

$$p_{hc} \sim (Q, \sqrt{\Lambda Q}, \Lambda)$$

Soft Collinear Effective Theory

description of the soft-spectator contribution involves 3 different scales

 $p = (p_+, p_\perp, p_-) \qquad \text{QCD}$

 $p_h \sim (Q, Q, Q)$ hard $p_h^2 \sim Q^2 \sim \mu_h^2$

 $p_{hc} \sim (Q, \sqrt{\Lambda Q}, \Lambda)$ hard-collinear

 $p_{hc}^2 \sim Q\Lambda \sim \mu_{hc}^2$

 $p_c \sim ({m Q}, \Lambda, \Lambda^2/{m Q})$ collinear

 $p_c^2 \sim p_s^2 \sim \Lambda^2 \sim \mu_s^2$

 $p_s \sim (\Lambda, \Lambda, \Lambda)$ soft

 $\begin{array}{c} \text{Soft-spectator} \\ \text{amplitude} \end{array} F($

$$F(\mu_h^2 \sim Q^2, \, \mu_{hc}^2 \sim Q\Lambda, \, \mu_s^2 \sim \Lambda^2)$$

Soft Collinear Effective Theory

description of a hard-spectator contribution involves only 2 scales

- $p = (p_+, p_\perp, p_-)$ QCD
 - $p_h \sim (Q, Q, Q)$ hard



 $p_h^2 \sim Q^2 \sim \mu_h^2$

$$p_c \sim (Q, \Lambda, \Lambda^2/Q)$$
 collinear $p_c^2 \sim p_s^2 \sim \Lambda^2 \sim \mu_s^2$

 $p_s \sim (A, A, A)$ soft

 $\begin{array}{ll} \mbox{Hard-spectator} \\ \mbox{amplitude} \end{array} \quad F(\mu_h^2\sim Q^2, \mu_{hc}^2\sim Q\Lambda, \ \mu_s^2\sim \Lambda^2) \end{array}$

Soft spectator scattering in the SCET framework

1. Factorize of the hard modes: $p_h^2 \sim Q^2 \gg \Lambda^2$ (hard subprocess)

 $\mathsf{QCD} \to \mathsf{SCET-I} \qquad F^{(s)}(Q^2, Q\Lambda, \Lambda^2) \simeq H(Q^2) * f(Q\Lambda, \Lambda^2)$





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 $f(Q\Lambda,\Lambda^2) = \langle out | O | in \rangle_{\rm SCET}$

well defined in a field theory

ightarrow moderate values of Q²: $~Q\Lambda \sim m_N^2~~$ hard-collinear scale is not large

$$Q^2 = 4 - 25 \text{GeV}^2$$

 $\Lambda \simeq 0.3 \text{GeV}$
 $Q\Lambda \simeq 0.6 - 1.5 \text{GeV}^2$

Soft spectator scattering in the SCET framework

2. Factorization of hard-collinear modes

$$p_{hc}^2 \sim Q\Lambda \gg m_N^2$$

 $SCET-I \rightarrow SCET-II = collinear + soft$

 $f(Q\Lambda, \Lambda^2) \simeq J_{hc}(Q\Lambda) * S[p_s] * \phi_N[p_c]$

 $p_{hc} \sim (Q, \sqrt{\Lambda Q}, \Lambda)$

hard-collinear subprocess

- provides an estimate of power of 1/Q at large Q
- allows one to establish an overlap between the hard- and soft-spectator contributions

Soft spectator contribution

NK, Vanderhaeghen PRD,2010 $Q^2 \gg Q \Lambda \sim m_N^2$





Soft-Collinear Effective Theory Form Factor

$$p \simeq Q \frac{\bar{n}}{2}$$
 $p' \simeq Q \frac{\bar{n}}{2}$
 $\langle p' | \bar{\chi}_n \gamma_{\perp}^{\mu} \chi_{\bar{n}} + \bar{\chi}_{\bar{n}} \gamma_{\perp}^{\mu} \chi_n | p \rangle_{\text{SCET}} = \bar{N}(p') \gamma_{\perp}^{\mu} N(p) f_1(\Lambda Q, \mu_F)$
quark antiquark
active quark $\chi_{\bar{n}} = P \exp \left\{ ig \int_{-\infty}^{0} ds \, n \cdot A_{hc}(sn) \right\} \frac{1}{4} \bar{n} \psi_{hc}(0)$

The scale dependence μ_F is defined by the renormalization of the SCET operator

U

$$\mu \frac{d}{d\mu} f_1(\mu) = \left(-\frac{\alpha_s}{\pi} \ln \frac{Q^2}{\mu^2} + \frac{3\alpha_s}{2\pi}\right) f_1(\mu)$$

RG can be used to evolve μ_F from Q to $\mu_{hc} \sim \sqrt{Q}\Lambda$

Sudakov Logs provides enhancement in TL kinematics (observed)

The overlap of the soft and collinear regions: the end-point singularities in f_1 and hard-spectator contributions

Hard or soft dominance?









Can the soft-overlap mechanism provide the large contribution? How it behaves with respect to Q^2 ?

Phenomenology

- Large numerical effect for moderate values of Q
- Soft-overlap contribution is subleading in 1/Q²

LC wave functions Isgur, Smith 1984 QCD sum rules

Nesterenko, Radyushkin 1982,83 Braun et al, '00, '02, '06, '13 GPD or handbag model Radyushkin 1998 Kroll et al, 2002, '05, '10 WACS/annihilation $\gamma p \rightarrow \gamma p \quad \gamma \gamma \rightarrow p \bar{p}$

Wide Angle Compton Scattering in SL region $s\sim -t\sim -u\sim Q\gg \Lambda^2$

WACS amplitude is described by 6 independent scalar amplitudes: Babusci et al, 1998



 $\begin{array}{c} & T_{2,4,6} \Leftrightarrow M_{h,h}^{\lambda,\lambda'} & \text{helicity conserving} \\ & & \\ \end{array} \\ \end{array}$

 $Q o \infty$ $T_{2,4,6} \sim 1/Q^4$ $T_{1,3,5} \sim 1/Q^5$ 20

NK, Vanderhaeghen 2012, 2013

 $T_i(s,t) = C_i(s,t) \mathcal{F}_1(t) + \Psi * H_i(s,t) * \Psi \qquad i = 2,4,6$



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NK, Vanderhaeghen 2012, 2013

 $T_i(s,t) = C_i(s,t) \mathcal{F}_1(t) + \Psi * H_i(s,t) * \Psi \qquad i = 2,4,6$

 $\langle p' | \bar{\chi}_n \gamma_\perp \chi_{\bar{n}} - \bar{\chi}_{\bar{n}} \gamma_\perp \chi_n | p \rangle_{SCET} = \bar{N}(p') \gamma_\perp N(p) \mathcal{F}_1(t)$ $p' \simeq Q \frac{n}{2} \qquad p \simeq Q \frac{\bar{n}}{2}$

WACS phenomenology $s \sim -t \sim -u \sim Q \gg \Lambda^2$

NK, M. Vanderhaeghen 2012, 2013

$$T_i(s,t) = C_i(s,t) \mathcal{F}_1(t) + \Psi * H_i(s,t) * \Psi$$
 $i = 2,4,6$
 $T_i(s,t) \approx 0$ $i = 1,3,5$

 $\mathcal{F}_1(t), \Psi$ unknown nonperturbative functions

 $C_i(s,t) \sim \mathcal{O}(1)$ $H_i(s,t) \sim \mathcal{O}(\alpha_s^2)$

$$T_i(s,t) = C_i(s,t) \mathcal{F}_1(t) + \Psi * H_i(s,t) * \Psi$$

regular = singular + singular \Rightarrow each term must be regularized NK, 2012

use the universality: one SCET FF \mathcal{F}_1 defines the all three amplitudes following features: $\mathcal{F}_1(t)$ does not depend on s

 $T_2(s,t) = C_2(s,t) \, \mathcal{F}_1(t) + \Psi * H_2(s,t) * \Psi$ using the simple structure

 $\Rightarrow \mathcal{F}_1(t) = \mathcal{R}(s,t) - \Psi * H_2(s,t) * \Psi/C_2(s,t)$

regular ratio

regular = regular +

•
$$\mathcal{R}(s,t) = rac{T_2(s,t)}{C_2(s,t)}$$
 $\mu_F^2 = -t$

 $T_{2}(s,t) = C_{2}(s,t)\mathcal{R}(s,t)$ $T_{i}(s',t) = C_{i}(s',t)\mathcal{R}(s,t) + \Psi * \left\{ H_{i}(s',t) - C_{i}(s',t)\frac{H_{2}(s,t)}{C_{2}(s,t)} \right\} * \Psi$ i = 4,6 $s' \neq s!$

regular each term is regular!



Brooks, Dixon, 2000



The hard-spectator contribution predicts the cross section at least an order of magnitude below the data

> Vanderhaeghen et al, 1997 Brooks, Dixon, 2000, Thomson et al, 2006

Data: Cornell, Shupe et al, PRD 1979

Wide Angle Compton Scattering & FF

$$\frac{d\sigma^{\gamma p \to \gamma p}}{dt} = \frac{f_N^4 \alpha_s^4}{s^6} A(\theta)$$
$$F_1(Q^2) = \frac{f_N^2 \alpha_s^2}{Q^4} I_N$$

$$\frac{s^6 d\sigma^{\gamma p \to \gamma p}/dt}{[Q^4 F_1]^2} = A(\theta)/I_N$$

Data JLab, Hall A, 2007 Thomson et al, 2006



 $Q^4 F_1(Q^2) \approx 1 \text{GeV}^4$ $Q^2 = 7 - 15 \text{GeV}^2$



the ratio

$$\mathcal{R}(s,t) = \frac{T_2(s,t)}{C_2(s,t)}$$

$$\iota_F^2 = -t$$

$$\mathcal{R}(s,t) = \frac{T_2(s,t)}{C_2(s,t)} \simeq \frac{T_4(s',t)}{C_4(s',t)} \simeq \frac{T_6(s'',t)}{C_6(s'',t)} \simeq \mathcal{R}(t)$$

this can be checked experimentally

$$\frac{d\sigma}{dt} \simeq \frac{\pi \alpha^2}{s^2} |\mathcal{R}(s,t)|^2 (-su) \left(\frac{1}{2} |C_2(s,t)|^2 + \frac{1}{2} |C_4(s,t)|^2 + |C_6(s,t)|^2\right)$$

To the leading order accuracy

$$C_i = C_i^{\rm lo} + \frac{\alpha_s}{4\pi} C_F \ C_i^{\rm nlo} + \dots$$

$$\frac{d\sigma}{dt} \simeq \frac{2\pi\alpha^2}{s^2} \left| \mathcal{R}(s,t) \right|^2 \left(\frac{s}{-u} + \frac{-u}{s} \right) \right|_{m=0} = \frac{d\sigma_0^{\mathrm{KN}}}{dt} |\mathcal{R}(s,t)|^2$$

$$|\mathcal{R}(s,t)| \approx \sqrt{\frac{d\sigma^{\exp}/dt}{d\sigma_{0}^{\mathrm{KN}}/dt}} \left(1 - \frac{1}{2} \frac{\alpha_{s}}{4\pi} C_{F} \frac{C_{2}^{\mathrm{\tiny LO}} \mathrm{Re}\left[C_{2}^{\mathrm{\tiny NLO}} - C_{4}^{\mathrm{\tiny NLO}}\right] + C_{6}^{\mathrm{\tiny LO}} \mathrm{Re}\left[C_{6}^{\mathrm{\tiny NLO}}\right]}{|C_{2}^{\mathrm{\tiny LO}}|^{2} + |C_{6}^{\mathrm{\tiny LO}}|^{2}}\right)$$

used data: JLab/Hall-A, 2007



NK, Vanderhaeghen 2013

all power corrections m/Q are neglected

with NLO corrections & kinematical power corrections

$$\bar{\mathcal{R}}| = \sqrt{\frac{d\sigma^{\exp}}{dt}} : \sqrt{\frac{\pi\alpha^2}{(s-m^2)^2}} \left((s-m^2)(m^2-u)\frac{1}{2}(|\bar{C}_2|^2 + |\bar{C}_4|^2) + (m^4-su)|\bar{C}_6|^2 \right)$$

massless approximation $C_i(s,t)|_{m=0} = C_i(s,\cos\theta)$

$$\bar{C}_i(s,t) = C_i\left(s,\cos\theta = 1 + \frac{2ts}{(s-m^2)^2}\right) = C_i(s,\cos\theta)|_{m=0} + \mathcal{O}(m/s).$$





empirical fit:

$$|\mathcal{R}(s,t)| = \left(\frac{\Lambda^2}{-t}\right)^c$$

	$\Lambda, { m GeV}$	α	$\chi^2/d.o.f$
$ \mathcal{R} , \mathrm{NLO}$	0.95 ± 0.02	1.67 ± 0.05	2.7
$ \bar{\mathcal{R}} , \mathrm{LO}$	1.0 ± 0.02	1.88 ± 0.05	1.1
$ \bar{\mathcal{R}} , \mathrm{NLO}$	0.98 ± 0.02	1.80 ± 0.05	1.25

TPE factorization within the SCET framework

Basic idea is to construct expansion with respect to large scale 1/Q in the large-angle scattering domain $s\sim -t\sim -u\gg \Lambda^2$

NK, Vanderhaeghen, 2012

large values of ${\mathcal E}$



WACS phenomenology: longitudinal polarization KLL

$$K_{\rm LL} = \frac{\sigma_{\parallel}^R - \sigma_{\parallel}^L}{\sigma_{\parallel}^R + \sigma_{\parallel}^L} = \frac{s^2 - u^2}{s^2 + u^2} + \frac{\alpha_s}{\pi} C_F K_{\rm LL}^{\rm NLO}$$

JLAB, 2004 s=6.9GeV² t=-4GeV² u=-0.84GeV²

 \Rightarrow Does not depend on s & \mathcal{R}

data: JLab/Hall-A, 2004

NK, Vanderhaeghen, to appear with the kinematical power corr's



J.J. Thomson



Theoretical model "pudding model" 1904



Gold foil experiment, 1909







H.Geiger



E.Marsden





DVCS at large momentum transfer

DVCS $Q^2 \sim s \gg -t \gg \Lambda^2$ $-t \sim 3 - 10 \ {
m GeV}^2$ moderate values

1. Factorize the largest virtualities



$$\mathcal{H}(\xi, Q^2, t) \simeq \sum_q e_q^2 \int dx H^q(x, \xi, t, \mu^2 = |t|) \int dx' U(x, \xi, x', \ln[Q^2/|t|]) \left\{ \frac{1}{\xi - x' - i\varepsilon} - \frac{1}{\xi + x' - i\varepsilon} \right\}$$

GPD evolution hard kernel

GPDs at large momentum transfer

 2^{nd} step: factorize modes with the virtualities $\sim -t$



$$H^{q}(x,\xi,t) = C_{h}(|t|) \left\{ f_{1}^{q}(|t|)\delta(1-x) - \delta(1+x) f_{1}^{\bar{q}}(|t|) \right\} + \Psi * T_{H}^{q}(x,\xi,t) * \Psi$$

$$\int_{-1}^{1} dx H^{q}(x,\xi,t) = \underbrace{C_{h}(|t|) \left\{ f_{1}^{q}(|t|) - f_{1}^{\bar{q}}(|t|) \right\} + \Psi * T_{H}^{q}(t) * \Psi}_{H}$$

 $F_1^q(|t|)$

GPDs at large momentum transfer

$$H^{q}(x,\xi,t) = C_{h}(|t|) \left\{ f_{1}^{q}(|t|)\delta(1-x) - \delta(1+x) f_{1}^{\bar{q}}(|t|) \right\} + \Psi * T_{H}^{q}(x,\xi,t) * \Psi$$

use the factorization formulas for em FFs and WACS

$$H^{q}(x,\xi,t) = F_{1}^{q}(|t|)q_{0}^{val}(x) + \mathcal{R}^{q}(t)q_{0}^{s}(x)$$

+ $\Psi * \left[T_H^q - q_0^{val}(x) T_F^q / C_1(t) - q_0^s(x) T_2^q(s,t) / C_2(s,t) \right] * \Psi.$

$$q_0^{val}(x) = \frac{1}{2} \left\{ \delta(1-x) + \delta(1+x) \right\}$$
$$q_0^s(x) = \frac{1}{2} \left\{ \delta(1-x) - \delta(1+x) \right\}$$

DV Compton FFs at large momentum transfer

If the soft term dominates then

$$H^q(x,\xi,t) \simeq F_1^q(|t|)q_0^{val}(x) + \mathcal{R}^q(t)q_0^s(x)$$
$$H^q(x,\xi,t) \simeq \tilde{H}^q(x,\xi,t) \qquad F_1^q(-t) \simeq g_A^q(-t)$$

Compton FFs $Q
ightarrow \infty$ Q^2/s is fixed, $-t/Q^2$ is small, $-t \gg \Lambda^2$

$$\begin{aligned} \mathcal{H}(\boldsymbol{\xi},\boldsymbol{Q}^{2},t) &\simeq -\mathcal{R}(t) \frac{2\boldsymbol{\xi}}{1-\boldsymbol{\xi}^{2}} \left\{ 1 + \mathcal{O}(\boldsymbol{\alpha}_{s}(\boldsymbol{Q}^{2})) + \mathcal{O}(\boldsymbol{\alpha}_{s}(-t)) \right\} \\ \tilde{\mathcal{H}}(\boldsymbol{\xi},\boldsymbol{Q}^{2},t) &\simeq -e_{u}F_{1}(-t) \frac{1}{1-\boldsymbol{\xi}^{2}} \left\{ 1 + \mathcal{O}(\boldsymbol{\alpha}_{s}(\boldsymbol{Q}^{2})) + \mathcal{O}(\boldsymbol{\alpha}_{s}(-t)) \right\} \end{aligned}$$

$$e_u^2 F_1^u(t) + e_d^2 F_1^d(t) \approx e_u^2 F_1^u(t) \approx e_u F_1(t)$$

Conclusions

- The Q² behavior of the soft spectator contribution can not be computed from pQCD but can be described by universal matrix elements within SCET factorization framework
 - There are indications that the soft spectator contribution is large or even dominant for moderate values of Q² : $Q\Lambda \sim m_N^2$
- Luckily the soft-overlap configurations are suppressed for DVCS!
- SCET factorization framework can be used in order to estimate the behavior of GPDs at large -t
- Factorization for the processes with the helicity flip amplitudes in DV kinematics

$$\gamma^*_{\perp} p \to (\pi, \rho, ...) p \implies \sigma_T / \sigma_L$$

in progress ...



