DVCS Workshop: From Observables to GPDs
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DVCS Radiative Corrections
An Experimentalist's Perspective

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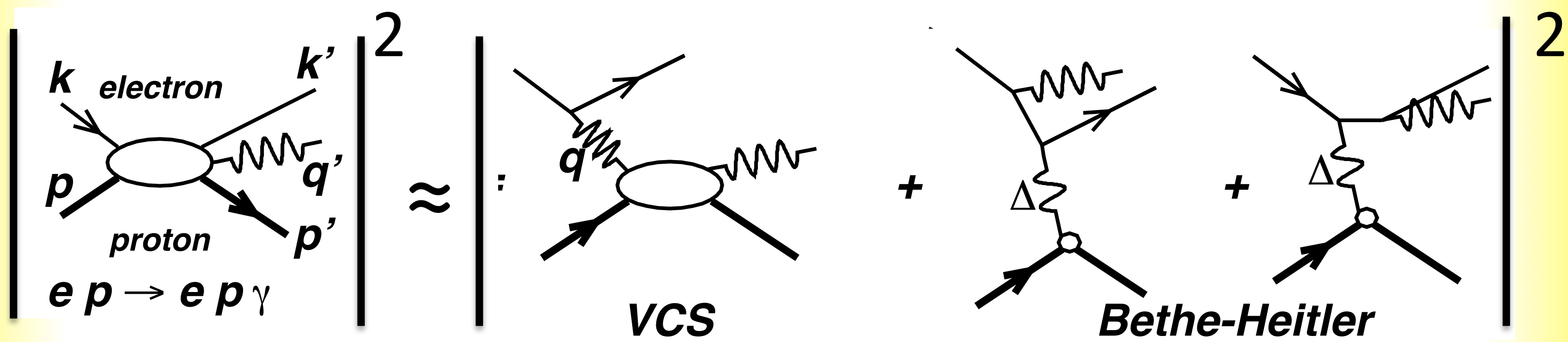
M. Vanderhaeghen, et al., Phys Rev C **62,** 025501 (2000)
I.Akushevich and A. Ilyich, Phys Rev D **85**, 053008 (2012)
A. Afanasev, I. Akushevich, et al, Phys Rev D **66** (2002) (EXCLURAD)

Contrast of Two World Views:

- Theorist:
  - Radiative corrections applied to Lowest Order QED calculation 'Born Approximation'
    ➔ describe the (raw) experimental cross sections.
- Experimentalist:
  - Radiative corrections applied to the data
    ➔ Present to the world (*e.g.* theorists) the dσ that would be true if QED stopped at the Born Approximation.
- Both should not be at work simultaneously on the same data!

The Born Approximation

- dσ*(e,e')* =



- BUT:
  - An accelerating charge **always** radiates
  - The probability of scattering without additional radiation is ***0***.
- Radiative corrections **not** small
  $\sim \alpha \ln(Q^2/m_e^2) \sim 20\%$

Virtual Radiative Corrections

Vertex, Self-Energy, Vacuum-Polarization corrections to BH

Vertex, Vacuum-Polarization corrections to VCS

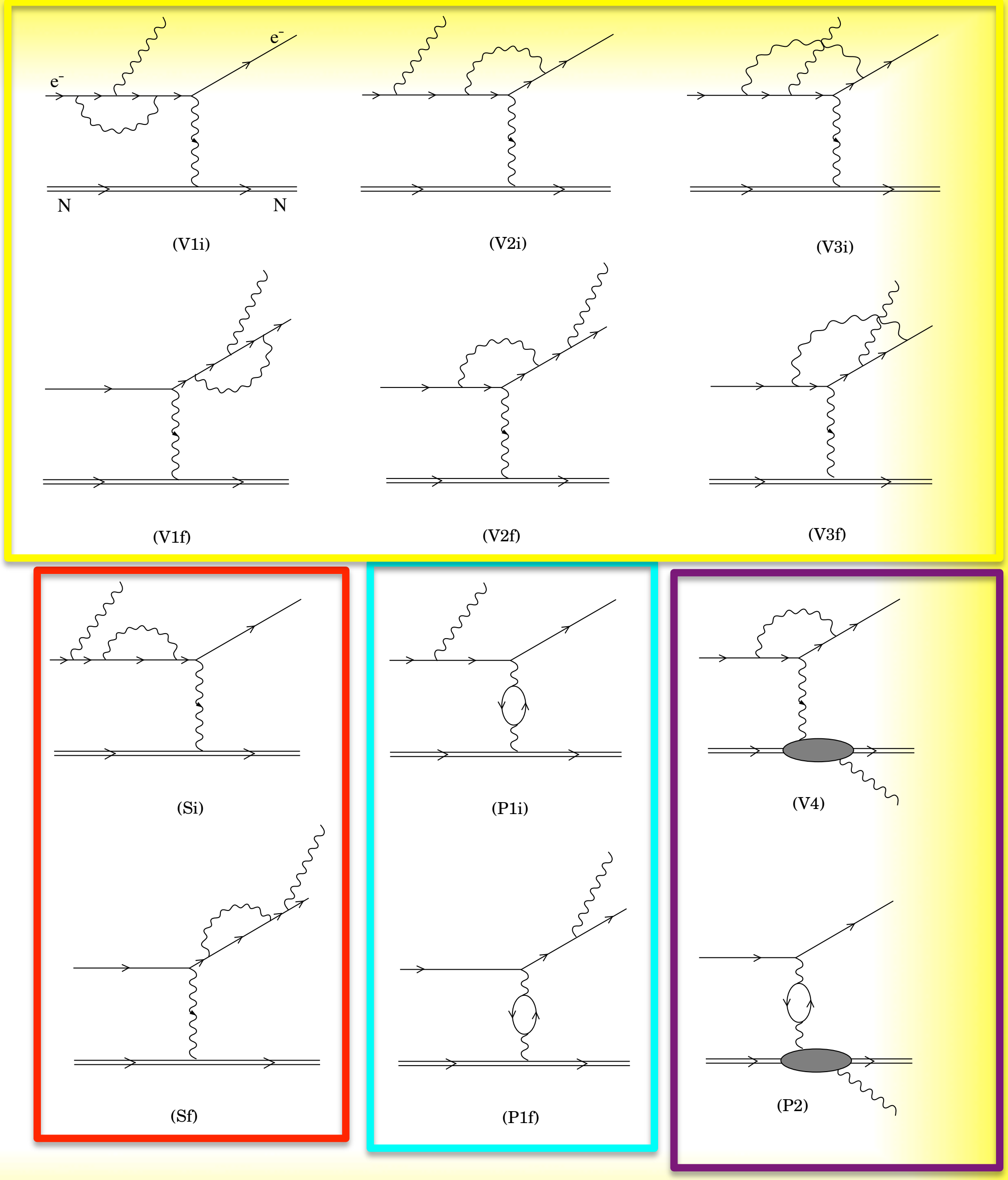


FIG. 2. First order virtual photon radiative corrections to the $ep \rightarrow ep\gamma$ reaction.

Virtual Radiative Corrections

Intrinsically different for BH & VCS

Self-Energy, Unique to BH

Vacuum-Polarization

BH: $1/[1+ \Pi(t)]$

VCS: $1/[1+ \Pi(Q^2)]$

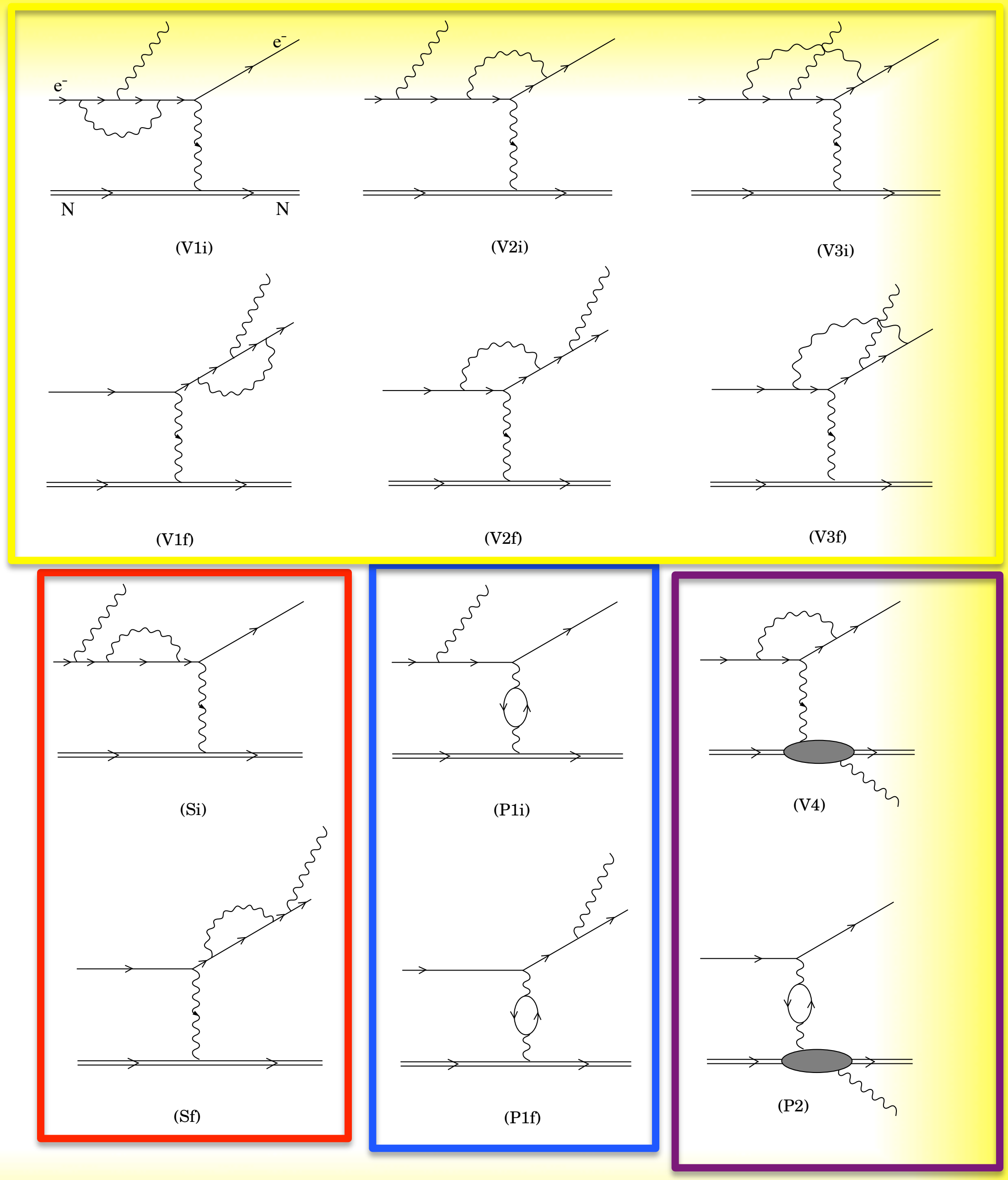


FIG. 2. First order virtual photon radiative corrections to the $ep \to ep\gamma$ reaction.

Real Radiative Corrections

BH

Factorize in soft limit

VCS

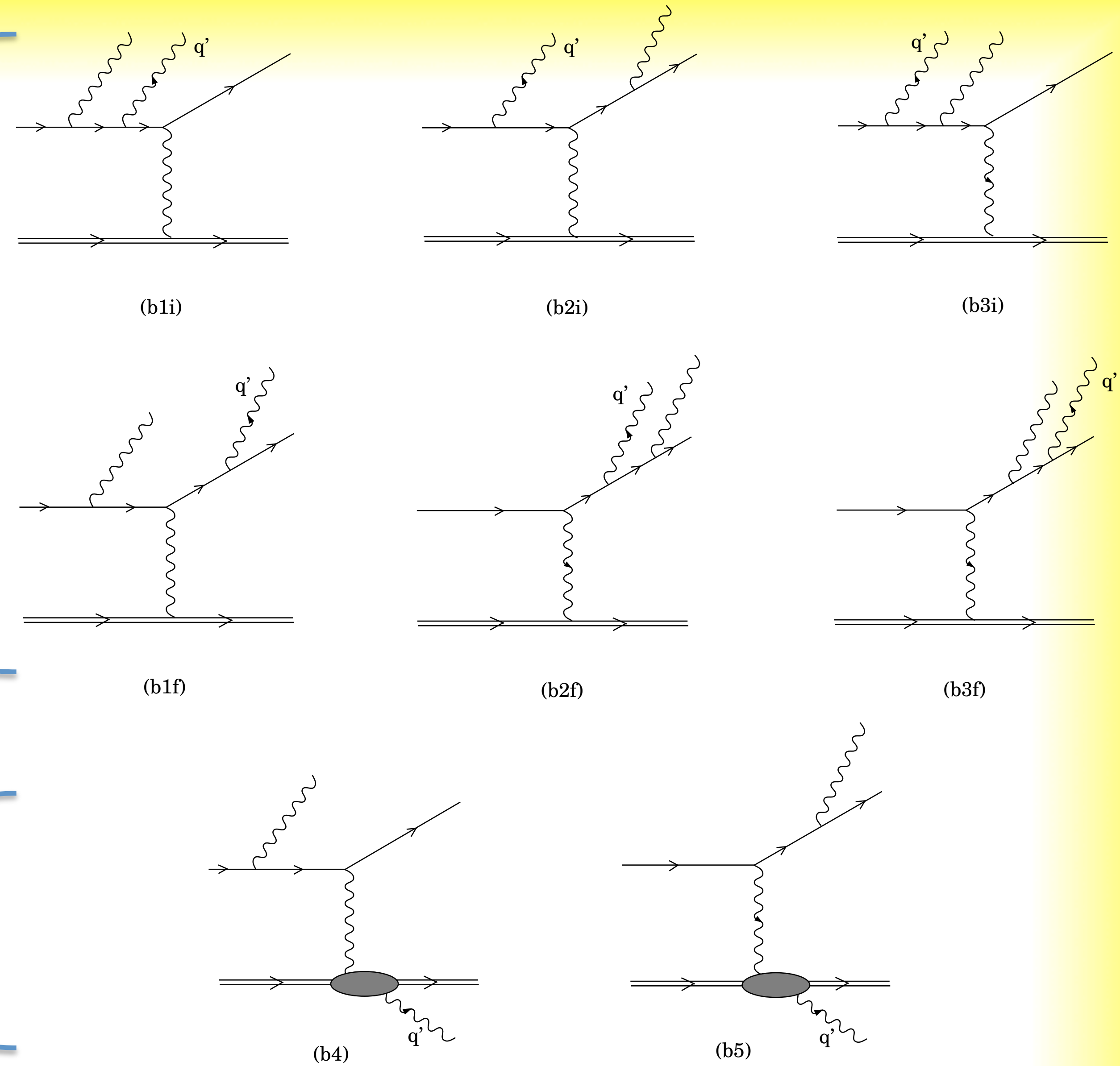


FIG. 3. First order soft photon emission contributions to the $ep \to ep\gamma$ reaction.

Integral of Soft Bremsstrahlung

- M. Vanderhaeghen *et al*:
  - Neglect non-factorized part of real radiative correction?
  - Integrate $d^3\boldsymbol{l}$ in frame `*S*´: $\mathbf{P}' + \boldsymbol{l} = \mathbf{q} + \mathbf{P} - \mathbf{q}' = \boldsymbol{0}$

$$d\sigma\Big|_{\text{Real Soft}} = d\sigma\Big|_{\text{`Born}'}\left[1 + \alpha\int^{\Delta M_X^2} \frac{d^3 l}{(2\pi)^2 l}\left(\frac{k'^{\mu}}{k'\cdot l} - \frac{k^{\mu}}{k\cdot l}\right)\left(\frac{k'_{\mu}}{k'\cdot l} - \frac{k_{\mu}}{k\cdot l}\right)\right]$$

$$= d\sigma\Big|_{\text{`Born}'}\left[1 + \delta_R(\Delta M_X^2) + \text{divergent integral cancelled by virtual terms}\right]$$

Finite part of Real Radiative Correction

$$\delta_R = \frac{\alpha}{\pi}\left\{\ln\left(\frac{(\Delta E_S)^2}{\tilde{E}_e\tilde{E}'_e}\right)\left[\ln\frac{Q^2}{m_e^2}-1\right]-\frac{1}{2}\ln^2\frac{\tilde{E}_e}{\tilde{E}'_e}+\frac{1}{2}\ln^2\frac{Q^2}{m_e^2}-\frac{\pi^2}{3}+\mathrm{DiLog}\left(\cos^2\frac{\tilde{\theta}_e}{2}\right)\right\}$$

- with

$$\Delta E_S = \frac{M_X^2 - M^2}{\sqrt{M_X^2}}$$

$$\tilde{E}_e \approx \frac{k\cdot(P+q-q')}{\sqrt{M_X^2}} = \frac{M}{\sqrt{M_X^2}}\left(E_e^{\mathrm{Lab}} - \frac{Q^2}{2M} - \frac{k\cdot q'}{M}\right)$$

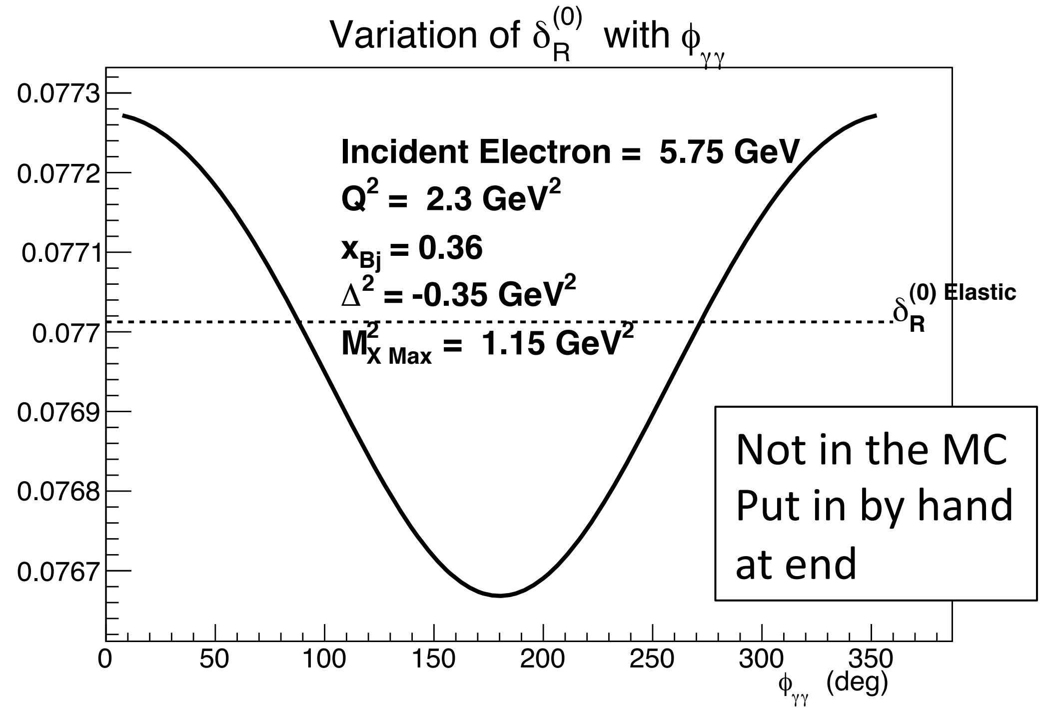
$$\tilde{E}_e' \approx \frac{k'\cdot(P+q-q')}{\sqrt{M_X^2}} = \frac{M}{\sqrt{M_X^2}}\left(E_e^{'\mathrm{Lab}} + \frac{Q^2}{2M} - \frac{k'\cdot q'}{M}\right)$$

$$\sin^2\frac{\tilde{\theta}_e}{2} = \left[\frac{\left(E_e E'_e\right)^{\mathrm{Lab}}}{\left(\tilde{E}_e\tilde{E}_e\right)_S}\right]\sin^2\frac{\theta_e^{\mathrm{Lab}}}{2}$$

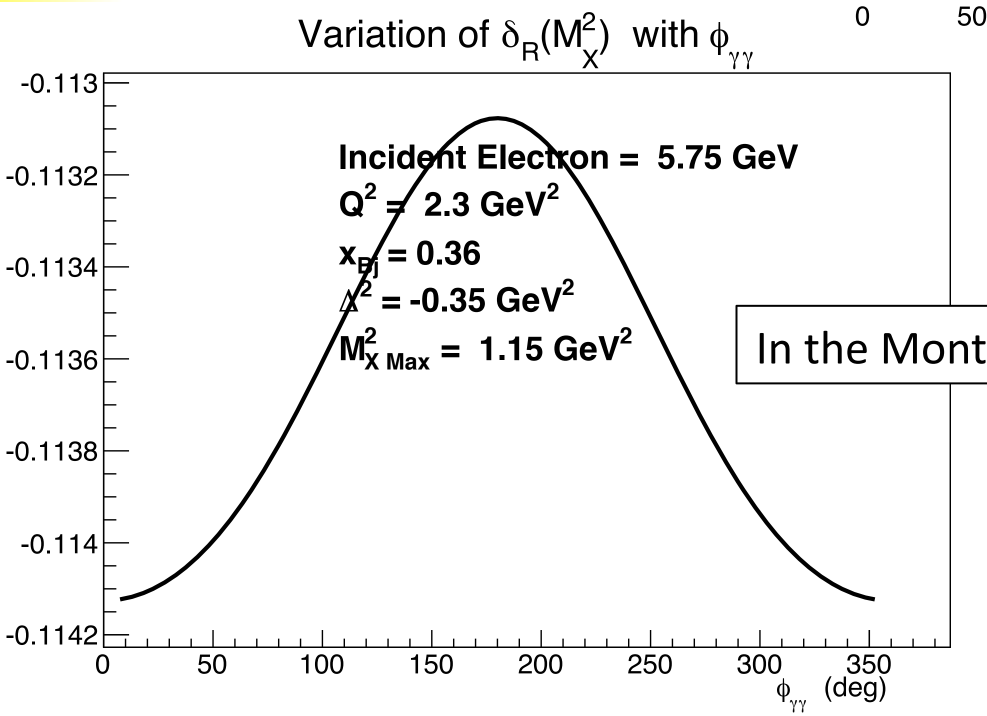
- All variables are external:
- At detector, not vertex
- Soft approx.: $Q^2_{\mathrm{vertex}} \approx Q^2_{\mathrm{detected}}$

$\phi_{\gamma\gamma}$ dependence of δ_R

(very small)



Not in the MC
Put in by hand
at end



In the Monte-Carlo

E00-110
Kin-3
Variations
< ±0.005

Exponentiation

$$(1+\delta_R) \Rightarrow e^{+\delta_R(M_X^2)} = \left(\frac{\Delta E_S}{\tilde{E}_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]} \left(\frac{\Delta E_S}{\tilde{E}'_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]} e^{\delta_R^{(0)}}$$

- “Exact” (soft-radiation) result
  - No peaking approximation
- $\delta_R^{(0)}$ depends weakly on M_X^2 cutoff,
  - set $M_X^2 \rightarrow M^2$ in definition of $\delta_R^{(0)}$

$$\delta_R^{(0)} = \frac{\alpha}{\pi}\left\{-\frac{1}{2}\ln^2\frac{\tilde{E}_e}{\tilde{E}'_e} + \frac{1}{2}\ln^2\frac{Q^2}{m_e^2} - \frac{\pi^2}{3} + \mathrm{DiLog}\left(\cos^2\frac{\tilde{\theta}_e}{2}\right)\right\}$$

- Consistency requires exponentiation of virtual radiative corrections also.

Peaking Approximation

- Exact: Generate soft photon in 4π phase space
- Peaking: Generate soft photons only parallel to **k** or **k'**

$$\left[\left(\frac{\Delta E_S}{\tilde{E}_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]}\left(\frac{\Delta E_S}{\tilde{E}'_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]}\right]_{Q^2_{Exp}} = ? \left[\left(\frac{\Delta E_{\mathrm{Inc}}}{E_{e,\mathrm{Inc}}}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]}\left(\frac{\Delta E_{\mathrm{Scatt}}}{E_{e,\mathrm{Scatt}}}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]}\right]_{Q^2_{Vertex}}$$

Monte-Carlo Implementation of Peaking Approximation

- For specific **k**, Q^2_{Vertex}, and x_{Vertex}
  - Uniform deviate r_i in [0,1]
  - Energy loss on incident electron: $\Delta E_i = k\, r_i^{1/\delta}$
- Calculate $\mathbf{k}'_{\mathrm{vertex}}$
  - Uniform deviate r_s in [0,1]
  - Energy loss on incident electron: $\Delta E_s = k\, r_s^{1/\delta}$

$$\delta = \frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2} - 1\right]$$

- Differential distributions

$$I(E,\Delta E,\delta)\,d(\Delta E) = \delta\left(\frac{\Delta E}{E}\right)^{\delta}\frac{d\Delta E}{E}$$

  - Monte-Carlo produces yield in experimental M_X^2 acceptance for unit Born-approximation cross section (Real Radiative-effects only)

Analytic Approximation to Virtual Radiative Corrections

- These formulae do not include self-energy and proton radiation terms included in Vanderhaeghen *et al* code. 2-4% discrepancy from full calculations
- Correction to **VCS** is independent of structure of VCS amplitude:

$$\frac{1}{2}\delta^{\mathrm{VCS}}_{\mathrm{vertex}} = \frac{\alpha}{4\pi}\left\{-\left[\ln\frac{Q^2}{m_e^2}\right]^2 + \frac{\pi^2}{3} - 4 + 3\ln\frac{Q^2}{m_e^2}\right\} = \frac{1}{2}\delta^{\mathrm{Elastic}}_{\mathrm{vertex}}$$

- Correction to BH is different!

$$\frac{1}{2}\delta^{\mathrm{BH}}_{\mathrm{vertex}} = \frac{\alpha}{4\pi}\left\{-\left[\ln\frac{Q^2}{m_e^2}\right]^2 + \frac{\pi^2}{3} - 6 - 2\ln\frac{Q^2}{m_e^2}\right\}$$

Vacuum Polarization

- VCS:

$$\Pi(Q^2) = \frac{\alpha}{3\pi}\left\{\ln\frac{Q^2}{m^2} - \frac{5}{3}\right\}$$

- BH

$$\Pi(-t) = \frac{\alpha}{3\pi}\left\{\ln\frac{-t}{m^2} - \frac{5}{3}\right\}$$

Correction to Cross section does not factorize (P. Guichon version of code)

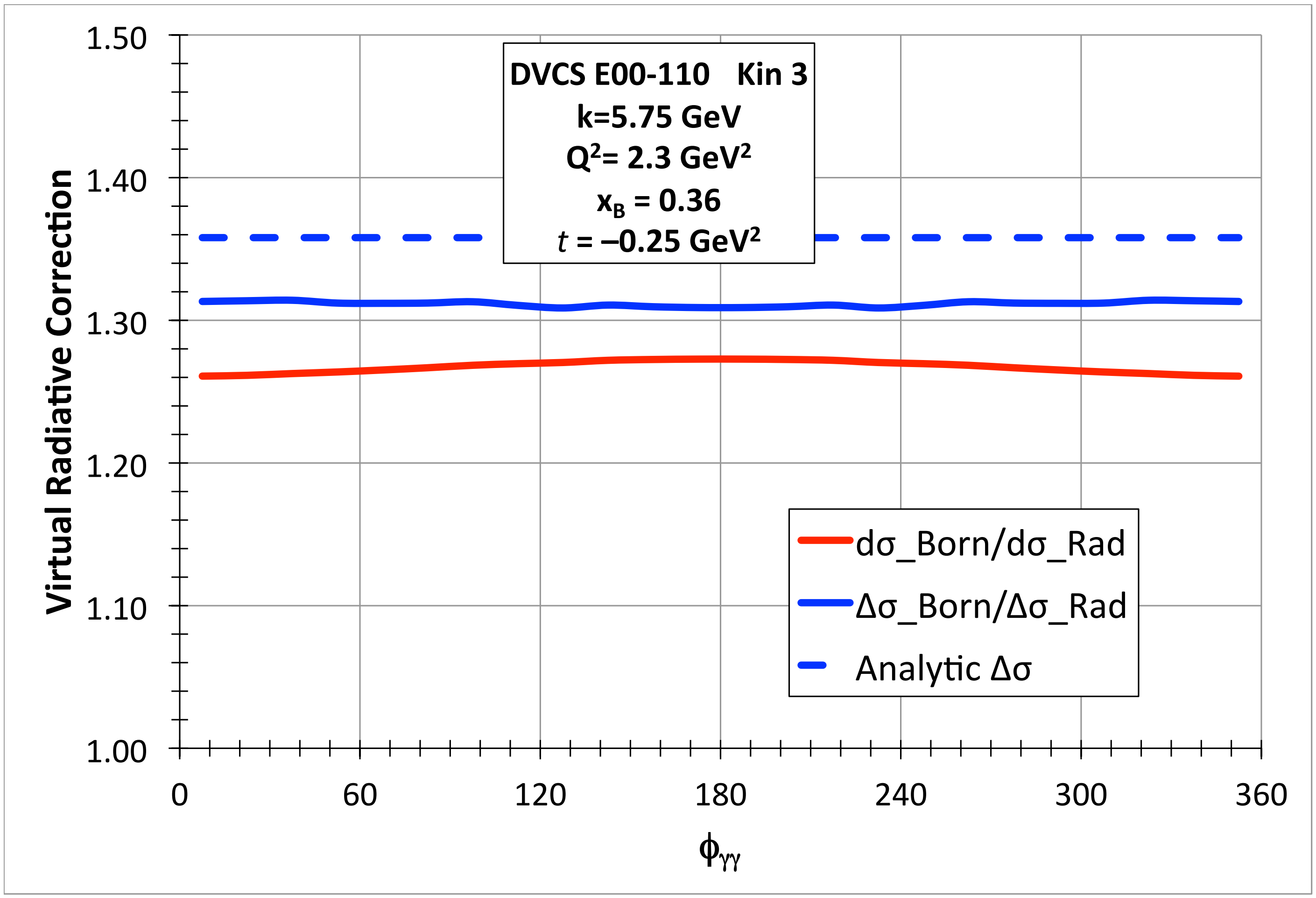
$$d\sigma = d\sigma^{\rm VCS} + d\sigma^{\rm BH} + \left[VCS^\dagger BH\right]$$

$$\Rightarrow d\sigma^{\rm VCS}\left[1+\frac{\delta^{\rm VCS}_{\rm vertex}/2}{1-\Pi(Q^2)}\right]^2 + d\sigma^{\rm BH}\left[1+\frac{\delta^{\rm BH}_{\rm vertex}/2}{1-\Pi(-t)}\right]^2 + \left[VCS^\dagger BH\right]\left[1+\frac{\delta^{\rm VCS}_{\rm vertex}/2}{1-\Pi(Q^2)}\right]\left[1+\frac{\delta^{\rm BH}_{\rm vertex}/2}{1-\Pi(-t)}\right]$$

- Each term, VCS, BH, Interference,
  - different radiative corrections.
- This form differs slightly from the usual correction factor of $(1+\delta_{\rm vertex}+2\Pi)$

Sample Virtual Radiative Corrections

BH + DVCS (factorized GPD model)



Asymmetries have a ~5% radiative correction

Questions

from Experimentalists to Theorists

- Confirm, VCS Radiative-corrections are model independent
- Confirm VCS and BH Radiative corrections are different
- How to exponentiate virtual corrections?
  - M. Vanderhaegen *et al*., Phys Rev C **62,** 025501 (2000):

$$\frac{e^{\delta(\text{vertex+self energy})}}{(1-\Pi)^2}$$

  - M. Vanderhaegen *et al*. code seems to be different.

Answers

from Experimentalists to Theorists

- Published Radiatively Corrected Cross Sections
  - Real Radiative Corrections calculated in purely factorized approximation: Cancels exactly in all asymmetries
  - Hall-A: Virtual Radiative corrections applied based on average correction to $|VCS+BH|^2$
    - Dependent on VCS model
    - For better than ~ 5% precision: Theorists/GPD-fitters must undo our 'experimental' correction, (virtual part only)
      - Apply QED corrections at amplitude level to DVCS +BH model in fit.
  - Relative Asymmetries (Hall-B, HERMES) generally published without radiative corrections.
    - There is a ~5% radiative correction
    - Model independent *iff* $A = [DVCS \bullet BH]/|BH|^2$

UV Divergences of Virtual Corrections

- *e.g.* vertex graph on BH initial state radiation *(V1i):*

$$M_{v1i} \sim \int d^4 l \frac{\gamma^{\alpha}\left(l\cdot\gamma\right)\left(\varepsilon\cdot\gamma\right)\gamma_{\alpha}}{l^2\left(l^2-2l\cdot k\right)\left(l^2-2l\cdot k-2k\cdot q\right)} \xrightarrow[l^2\to\infty]{} \int \frac{dl^2}{l^2}$$

- Q.E.D. is renormalizable! Infinities are tamable.
  - To each order in perturbation theory, redefine Fields, Masses, and Couplings
  - $A^{\mu}_{\text{Bare}} = Z_3^{1/2} A^{\mu}_{\text{Physical}}$ $\psi_{\text{Bare}} = Z_2^{1/2}\psi_{\text{Physical}}$
    $m_{\text{Bare}} = Z_m\, m$ $e_{\text{Bare}} = Z_g\, e_{\text{Physical}}$
  - This cancels all UV divergences, but creates IR-divergences

IR Divergence of Real Radiative Correction

- *e.g.* VCS Bremsstrahlung graphs

$$M_{b4}+M_{b5}\sim\bar{u}(k')\left[\gamma^{\mu}\frac{1}{(k-l)\cdot\gamma-m}\left(\gamma\cdot\varepsilon_f^{*}(l)\right)+\left(\gamma\cdot\varepsilon_f^{*}(l)\right)\frac{1}{(k'+l)\cdot\gamma-m}\gamma^{\mu}\right]u(k)\left[\frac{-1}{(k-l-k')^2}H_{\mu\nu}\varepsilon_f^{*}(q')\right]$$

$$\sim\bar{u}(k')\gamma^{\mu}u(k)\left[\frac{2k\cdot\varepsilon_f^{*}(l)}{-2k\cdot l-m^2}+\frac{2k'\cdot\varepsilon_f^{*}(l)}{2k'\cdot l-m^2}\right]\left[\frac{-1}{(k-l-k')^2}H_{\mu\nu}\varepsilon_f^{*}(q')\right]+\ldots$$

- Must always integrate cross section over photon energies dl less than the experimental resolution:

$$\int\frac{d^3l}{l}\left|M_{b4}+M_{b5}\right|^2\sim d\sigma^{\text{No Radiation}}\left[\int_0^{\Delta E}\frac{dl}{l}\right](\ldots)$$

  - IR-divergence, but cancels order-by-order with IR-divergences of renormalized Virtual corrections.