Orbital angular momentum in twist-three DVCS observables

Aurore Courtoy

Université de Liège, Belgium & INFN-LNF, Italy

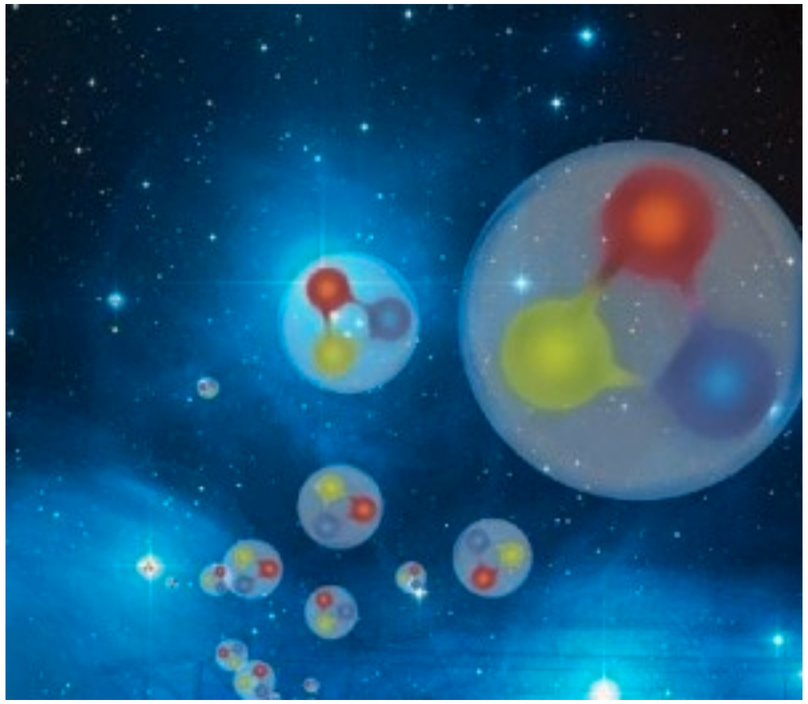
DVCS 2014

Ruhr University, Bochum

February 12th, 2014

Outline

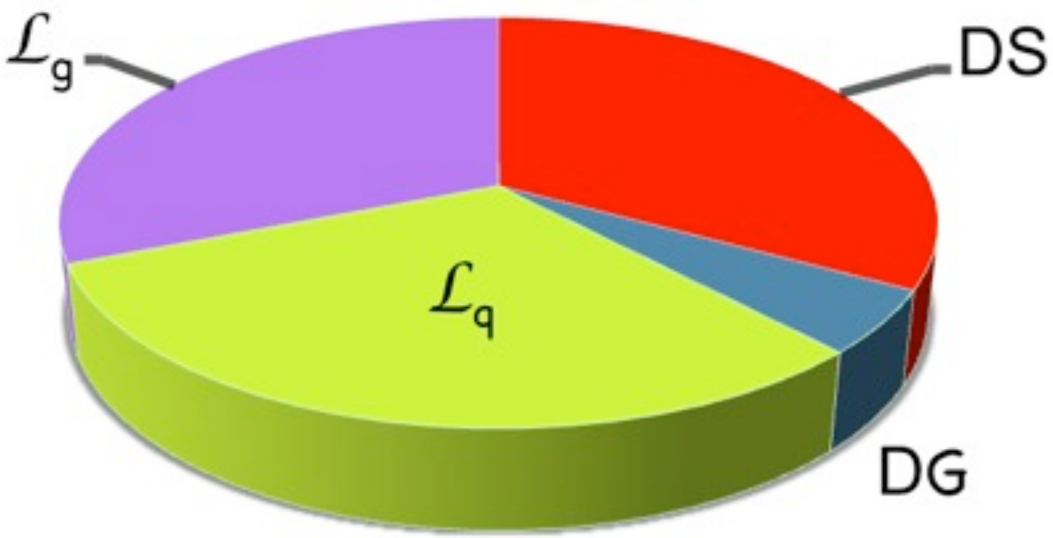
- **Hadron Structure**
  - (Generalized)$^?$ distribution functions
- **Where is the Orbital Angular Momentum?**
  - ...theoretically
    - helicity amplitudes
    - twist
  - ...experimentally

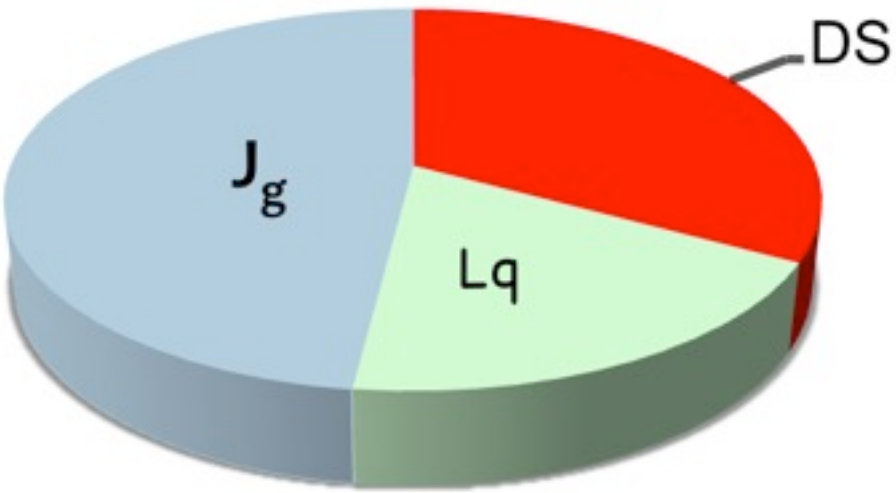


Based on arXiv:1310.5157 (accepted at PLB)

with
Gary Goldstein
Osvaldo González Hernández
Simonetta Liuti
Abha Rajan

Proton spin decomposition





Jaffe-Manohar

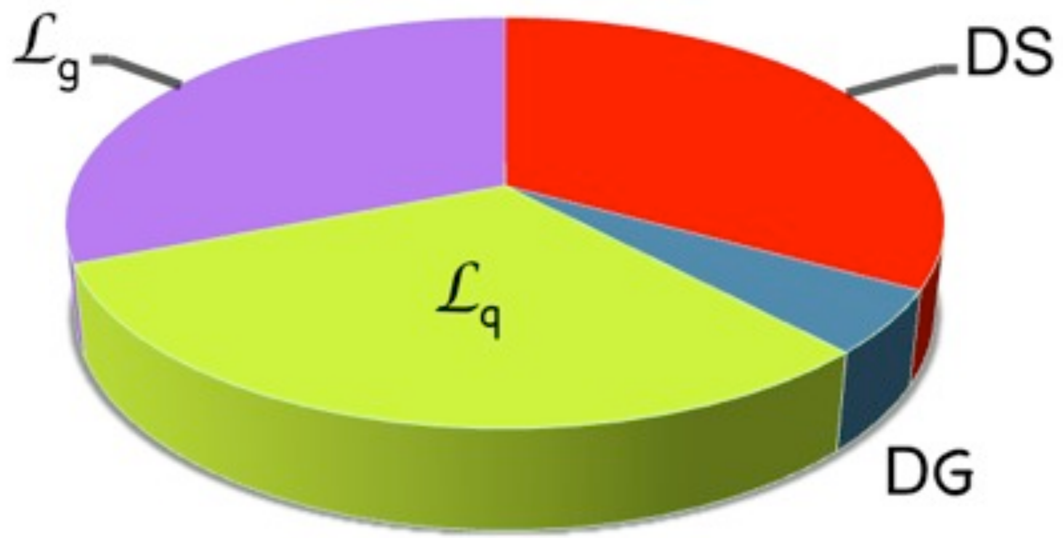
Find the differences!

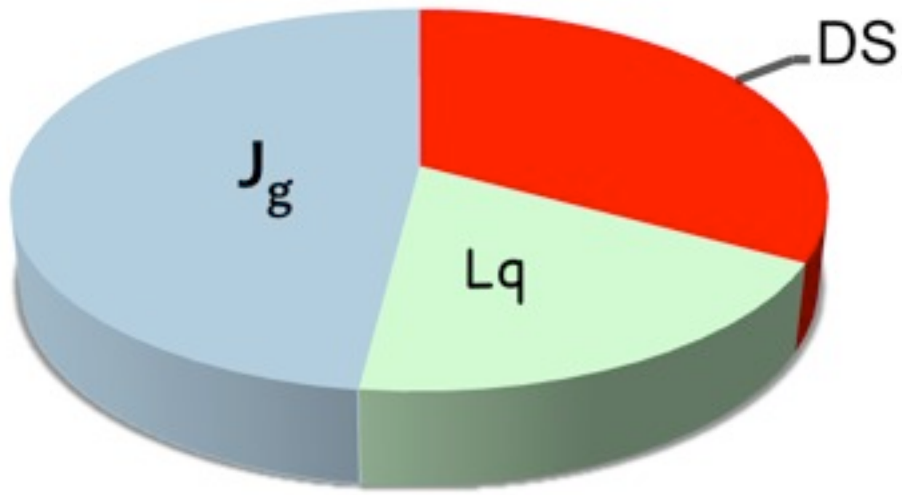
Ji

Saga or ``roman-fleuve" on parton OAM (to be continued)

Recent review: Leader & Lorcé

Proton spin decomposition





Jaffe-Manohar

Find the differences!

Ji

Saga or ``roman-fleuve" on parton OAM (to be continued)

Recent review: Leader & Lorcé

Beyond the OAM chronicle:
what can we experimentally access (=related to observable) **and what is its physical content?**

That's our pragmatic approach

(Generalized)$^?$ distribution functions

$$\sum_q \int_{-1}^{1} dx\, g_1^q(x) = \Delta\Sigma$$

≈0.3≠1

- Transverse spin?
  - higher-twist: g_T
  - role of $k_\perp$ highlighted long ago (e.g. Jackson, Ross & Roberts, PLB226)
  - formalized by Mulders & Tangerman, NPB461 → TMDs
- Nucleon spin decomposition
  - Ji PRL78: related fo Form Factors of non-forward matrix elements → GPDs

(Generalized)$^?$ distribution functions

$$\sum_q \int_{-1}^{1} dx\, g_1^q(x) = \Delta\Sigma$$

≈0.3≠1

- Transverse spin?
  - higher-twist: g_T
  - role of $k_\perp$ highlighted long ago (e.g. **Jackson, Ross & Roberts, PLB226**)
  - formalized by **Mulders & Tangerman, NPB461** → **TMDs**

any relation?

- Nucleon spin decomposition
  - **Ji PRL78**: related fo Form Factors of non-forward matrix elements→ **GPDs**

OAM definitions (1 episode)

See Cédric Lorcé's talks & publications
and
Ji, Xiong & Yuan, PRL109

Wigner functions, natural framework

$$\hat{\mathcal{W}}(\vec{r},k) = \int d^4\xi\, e^{ik\cdot\xi}\bar{\Psi}_{GL}(\vec{r}-\xi/2)\gamma^+\Psi_{GL}(\vec{r}+\xi/2)$$

GL=choice of gauge link

quantum average →

$$\langle\hat{O}\rangle = \int d\vec{r}dk\, O(\vec{r},k)\,\hat{\mathcal{W}}(\vec{r},k)$$

OAM definitions (1 episode)

See Cédric Lorcé's talks & publications
and
Ji, Xiong & Yuan, PRL109

Wigner functions, natural framework

$$\hat{\mathcal{W}}(\vec{r},k) = \int d^4\xi\, e^{ik\cdot\xi}\,\bar{\Psi}_{GL}(\vec{r}-\xi/2)\gamma^+\Psi_{GL}(\vec{r}+\xi/2)$$

GL=choice of gauge link

quantum average →

$$\langle\hat{O}\rangle = \int d\vec{r}dk\, O(\vec{r},k)\,\hat{\mathcal{W}}(\vec{r},k)$$

- gauge-invariant

- canonical

OAM definitions (1 episode)

See Cédric Lorcé's talks & publications
and
Ji, Xiong & Yuan, PRL109

Wigner functions, natural framework

$$\hat{\mathcal{W}}(\vec{r},k) = \int d^4\xi \, e^{ik\cdot\xi}\bar{\Psi}_{GL}(\vec{r}-\xi/2)\gamma^+\Psi_{GL}(\vec{r}+\xi/2)$$

GL=choice of gauge link

quantum average →

$$\langle\hat{O}\rangle = \int d\vec{r}dk \, O(\vec{r},k)\,\hat{\mathcal{W}}(\vec{r},k)$$

- **gauge-invariant**

$$L_{FS} = \frac{\langle PS|\int d^3\vec{r}\,\bar{\psi}(\vec{r})\gamma^+(\vec{r}_\perp\times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle} = \int(\vec{b}_\perp\times\vec{k}_\perp)W_{FS}(x,\vec{b}_\perp,\vec{k}_\perp)dxd^2\vec{b}_\perp d^2\vec{k}_\perp$$

Ji, Xiong & Yuan, PRL109

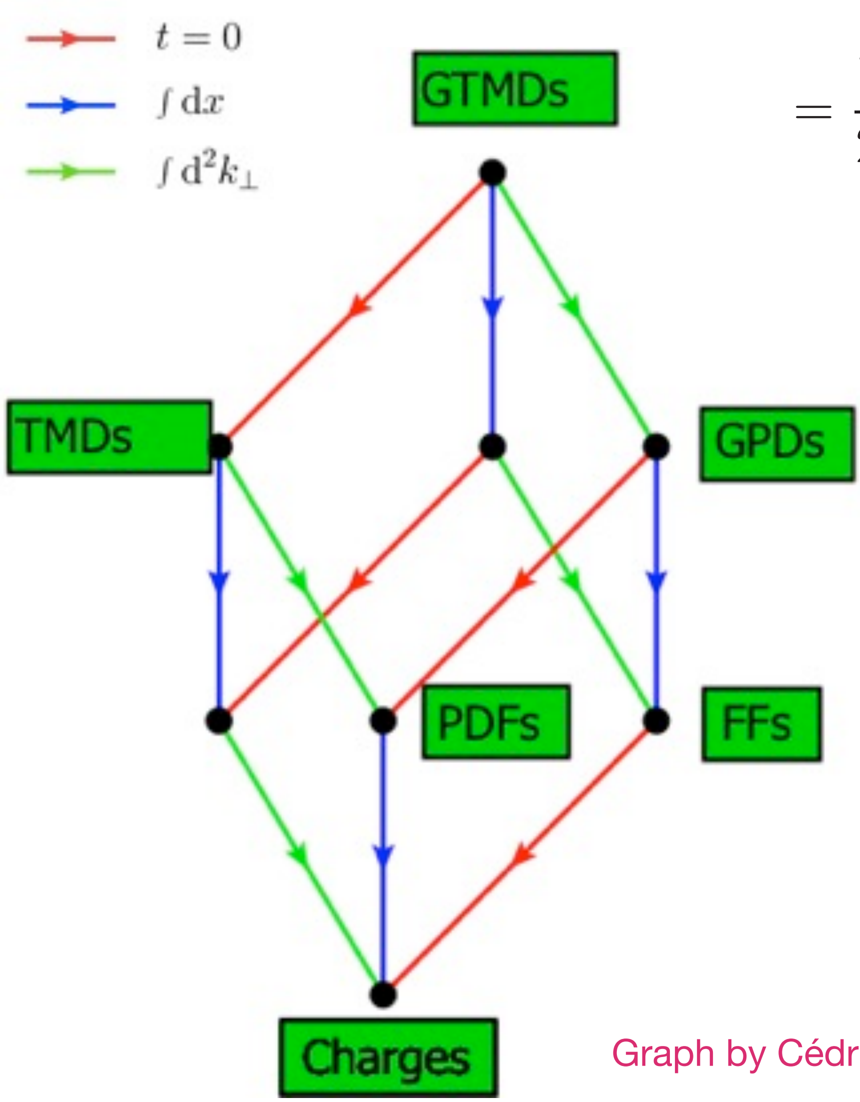
- **canonical**

$$l_q = \frac{\langle PS|\int d^3\vec{r}\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp\times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle} = \int(\vec{b}_\perp\times\vec{k}_\perp)W_{\rm LC}(x,\vec{b}_\perp,\vec{k}_\perp)dxd^2\vec{b}_\perp d^2\vec{k}_\perp$$

(Generalization)^2 of distributions

Wigner function quantized at light-cone time ⟶ Generalized TMDs

$$W^{[\Gamma]}_{\lambda\lambda'}(P,x,\vec{k}_T,\Delta,N;\eta) = \int dk^-\, W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta)$$
$$= \frac{1}{2}\int \frac{dz^-\, d^2\vec{z}_T}{(2\pi)^3}\, e^{ik\cdot z}\, \langle p',\lambda'|\,\bar{\psi}\Big(-\frac{1}{2}z\Big)\,\Gamma\,\mathcal{W}\Big(-\frac{1}{2}z,\frac{1}{2}z\,|\,n\Big)\,\psi\Big(\frac{1}{2}z\Big)\,|p,\lambda\rangle\Big|_{z^+=0}$$



Graph by Cédric Lorcé

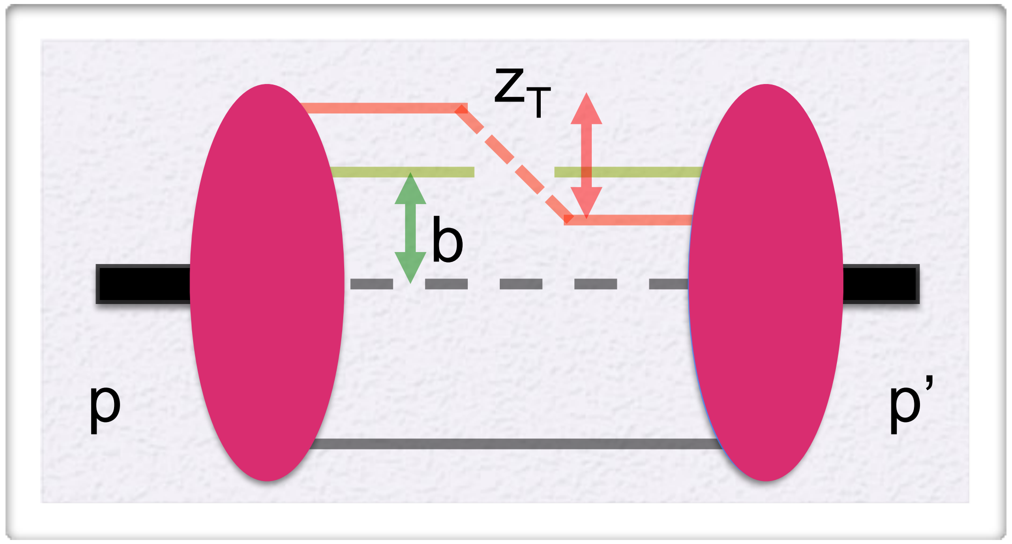
GTMDs account for both $k_\perp$ & Δ

Meissner, Metz, Schlegel & Goeke, JHEP 0808 (2008) 038
Meissner, Metz & Schlegel, JHEP 0908 (2009) 056

Partonic meaning

GTMDs account for both $k_\perp$ & Δ

2 transverse momenta



$$\bar{k}_T = \frac{k_T + k'_T}{2} \Rightarrow z_T = b_{T,in} - b_{T,out}$$

$$\Delta_T = k'_T - k_T \Rightarrow b_T = \frac{b_{T,in} + b_{T,out}}{2}$$

Average + Shift

Impact parameter space

OAM and GTMDs

- **Still need to define/find process related to GTMD** (à la Goloskokov & Kroll, EPJC 53 ??)
- **Purely theoretical object**
- **Don't know behavior with a possible factorization**
- **No constraint from pQCD so far**

- **Only limits to GPDs & TMDs** (well defined?)

OAM and GTMDs

- **Still need to define/find process related to GTMD** (à la Goloskokov & Kroll, EPJC 53 ??)
- **Purely theoretical object**
- **Don't know behavior with a possible factorization**
- **No constraint from pQCD so far**

- **Only limits to GPDs & TMDs** (well defined?)

Main motivations:

- **Orbital Angular Momentum**
- **Wigner functions b/c of quantum average**

OAM and GTMDs

- **Still need to define/find process related to GTMD** (à la Goloskokov & Kroll, EPJC 53 ??)
- **Purely theoretical object**
- **Don't know behavior with a possible factorization**
- **No constraint from pQCD so far**

- **Only limits to GPDs & TMDs** (well defined?)

Lorcé & Pasquini, PRD84
Lorcé, Pasquini, Xiong & Yuan, PRD85

Main motivations:

- **Orbital Angular Momentum**
- **Wigner functions b/c of quantum average**

$$\ell_z^q \equiv \langle \hat{L}_z^q \rangle^{[\gamma^+]}(\vec{e}_z) = \int dx d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \rho^{[\gamma^+]q}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{e}_z)$$

$$\ell_z^q = -\int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^q(x, 0, \vec{k}_\perp^2, 0, 0)$$

New structure only from GTMD

OAM and GTMDs

- **Still need to define/find process related to GTMD** (à la Goloskokov & Kroll, EPJC 53 ??)
- **Purely theoretical object**
- **Don't know behavior with a possible factorization**
- **No constraint from pQCD so far**

- **Only limits to GPDs & TMDs** (well defined?)

Lorcé & Pasquini, PRD84
Lorcé, Pasquini, Xiong & Yuan, PRD85

Main motivations:

- **Orbital Angular Momentum**
- **Wigner functions b/c of quantum average**

$$\ell_z^q \equiv \langle \hat{L}_z^q \rangle^{[\gamma^+]}(\vec{e}_z)$$
$$= \int dx d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \rho^{[\gamma^+]q}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{e}_z)$$

$$\ell_z^q = -\int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^q(x, 0, \vec{k}_\perp^2, 0, 0)$$

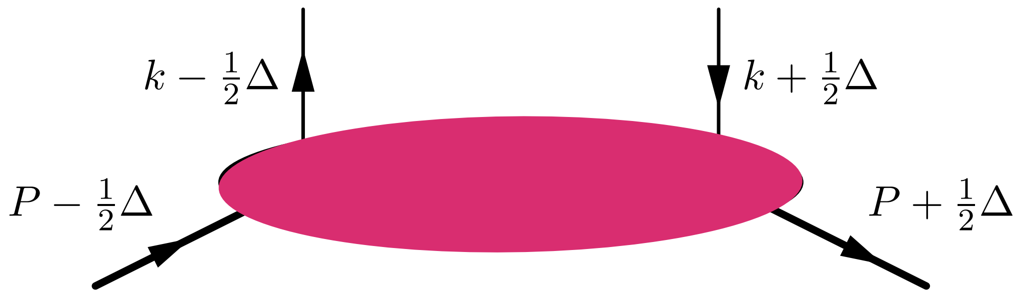
New structure only from GTMD

But something doesn't match our intuition...

Classification of GTMDs

MMS

- Lorentz scalar
- Hermiticity
- Charge-conjugation
- Parity conservation



$$
\begin{aligned}
W^{\gamma^+}_{\Lambda\Lambda'} &= \frac{1}{2P^+}\Big[\boxed{\overline{U}(p',\Lambda')\gamma^+U(p,\Lambda)F_{11} + \overline{U}(p',\Lambda')\frac{i\sigma^{i+}\Delta^i_T}{2M}U(p,\Lambda)(2F_{13}-F_{11})} \\
&+ \boxed{\overline{U}(p',\Lambda')\frac{i\sigma^{i+}\bar{k}^i_T}{2M}U(p,\Lambda)(2F_{12})} + \boxed{\overline{U}(p',\Lambda')\frac{i\sigma^{ij}\bar{k}^i_T\Delta^j_T}{M^2}U(p,\Lambda)\,F_{14}}\Big] \\
&= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}i\Lambda\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14}
\end{aligned}
$$

related to GPDs

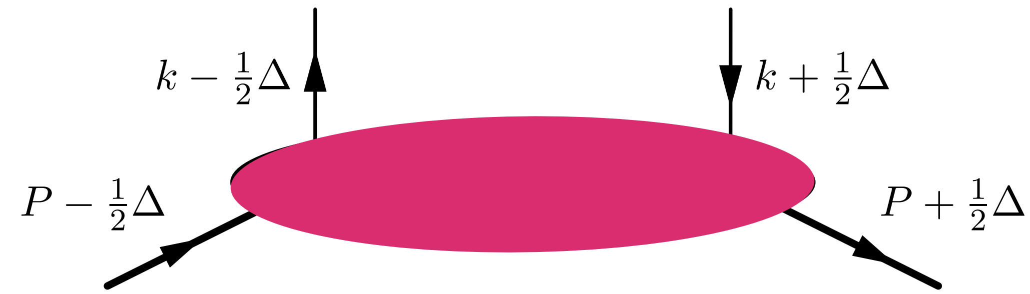
related to the Sivers fct

``new" correlation of $k_\perp$ & Δ

Classification of GTMDs

MMS

- Lorentz scalar
- Hermiticity
- Charge-conjugation
- Parity conservation



$$
\begin{aligned}
W^{\gamma^+}_{\Lambda\Lambda'} &= \frac{1}{2P^+}\Big[\boxed{\overline{U}(p',\Lambda')\gamma^+U(p,\Lambda)F_{11}+\overline{U}(p',\Lambda')\frac{i\sigma^{i+}\Delta_T^i}{2M}U(p,\Lambda)(2F_{13}-F_{11})} \\
&\quad + \boxed{\overline{U}(p',\Lambda')\frac{i\sigma^{i+}\bar{k}_T^i}{2M}U(p,\Lambda)(2F_{12})}+\boxed{\overline{U}(p',\Lambda')\frac{i\sigma^{ij}\bar{k}_T^i\Delta_T^j}{M^2}U(p,\Lambda)\,F_{14}}\Big] \\
&= \delta_{\Lambda,\Lambda'}F_{11}+\delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1-i\Delta_2}{2M}(2F_{13}-F_{11})+\delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1-i\bar{k}_2}{2M}(2F_{12})+\delta_{\Lambda,\Lambda'}i\Lambda\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}F_{14}
\end{aligned}
$$

related to GPDs

related to the Sivers fct

``new" correlation of $k_\perp$ & Δ

How do we interpret that ``new" correlation in terms of 2-body scattering?

Parity relations

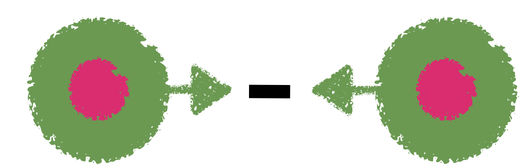
- **Helicity amplitudes of 2-body scattering** → $A_{\Lambda',\,\lambda';\,\Lambda,\,\lambda} : q'(k',\lambda') + N(p,\Lambda) \to q(k,\lambda) + N'(p',\Lambda')$

- **16 HA related through parity relations** → $A_{-\Lambda',\,-\lambda';\,-\Lambda,\,-\lambda} = (-1)^{\eta}\, A^{*}_{\Lambda',\,\lambda';\,\Lambda,\,\lambda}$

leaving 8 independent amplitudes.

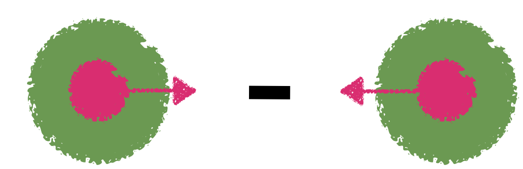
- **so, the combinations** →

$$-i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14} \;=\; \left(A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}\right)/4$$



$$i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}G_{11} \;=\; \left(A_{++,++} - A_{+-,+-} + A_{-+,-+} - A_{--,--}\right)/4$$

... are Not indpt in CoM frame



Parity relations

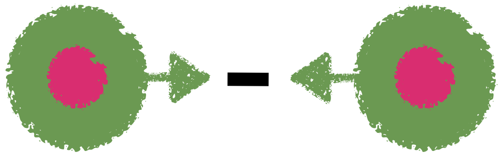
- Helicity amplitudes of 2-body scattering → $A_{\Lambda',\,\lambda';\,\Lambda,\,\lambda} : q'(k',\lambda') + N(p,\Lambda) \to q(k,\lambda) + N'(p',\Lambda')$

- 16 HA related through parity relations → $A_{-\Lambda',\,-\lambda';\,-\Lambda,\,-\lambda} = (-1)^{\eta}\, A^{*}_{\Lambda',\,\lambda';\,\Lambda,\,\lambda}$

leaving 8 independent amplitudes.

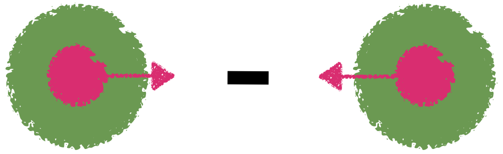
- so, the combinations →

$$-i\,\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2} F_{14} \;=\; \left(A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}\right)/4$$

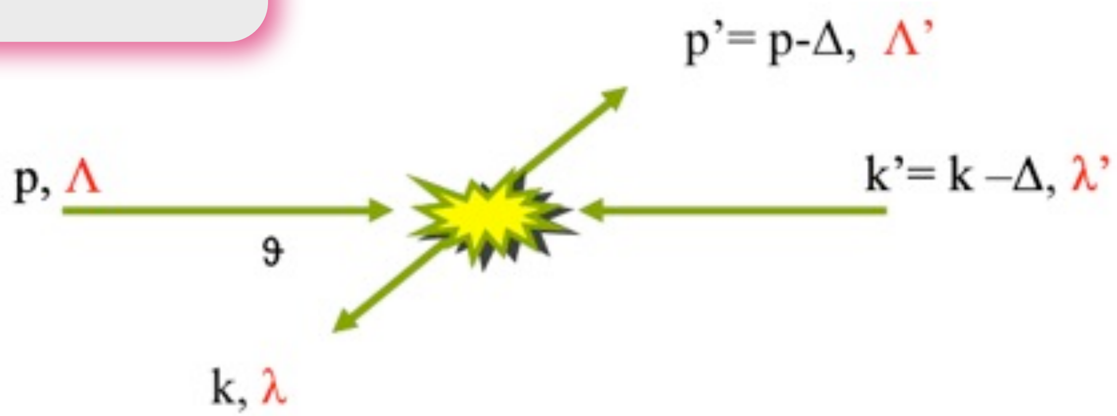


$$i\,\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2} G_{11} \;=\; \left(A_{++,++} - A_{+-,+-} + A_{-+,-+} - A_{--,--}\right)/4$$

... are Not indpt in CoM frame



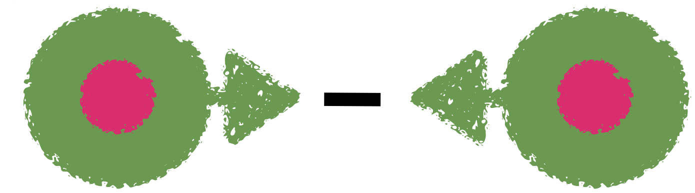
$\langle \mathbf{S}_L \cdot \bar{\mathbf{k}}_T \times \mathbf{\Delta}_T \rangle$ is parity violating



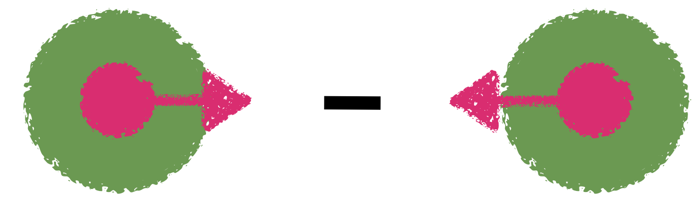
→ CoM 2-body scattering must occur on a plane
→ CoM frame with p in z direction:
it leaves 1 transverse direction (related to θ)

U/L polarized quarks in L/U polarized target

$$-i\,\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}F_{14}=\left(A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}\right)/4$$



$$i\,\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}G_{11}=\left(A_{++,++}-A_{+-,+-}+A_{-+,-+}-A_{--,--}\right)/4$$



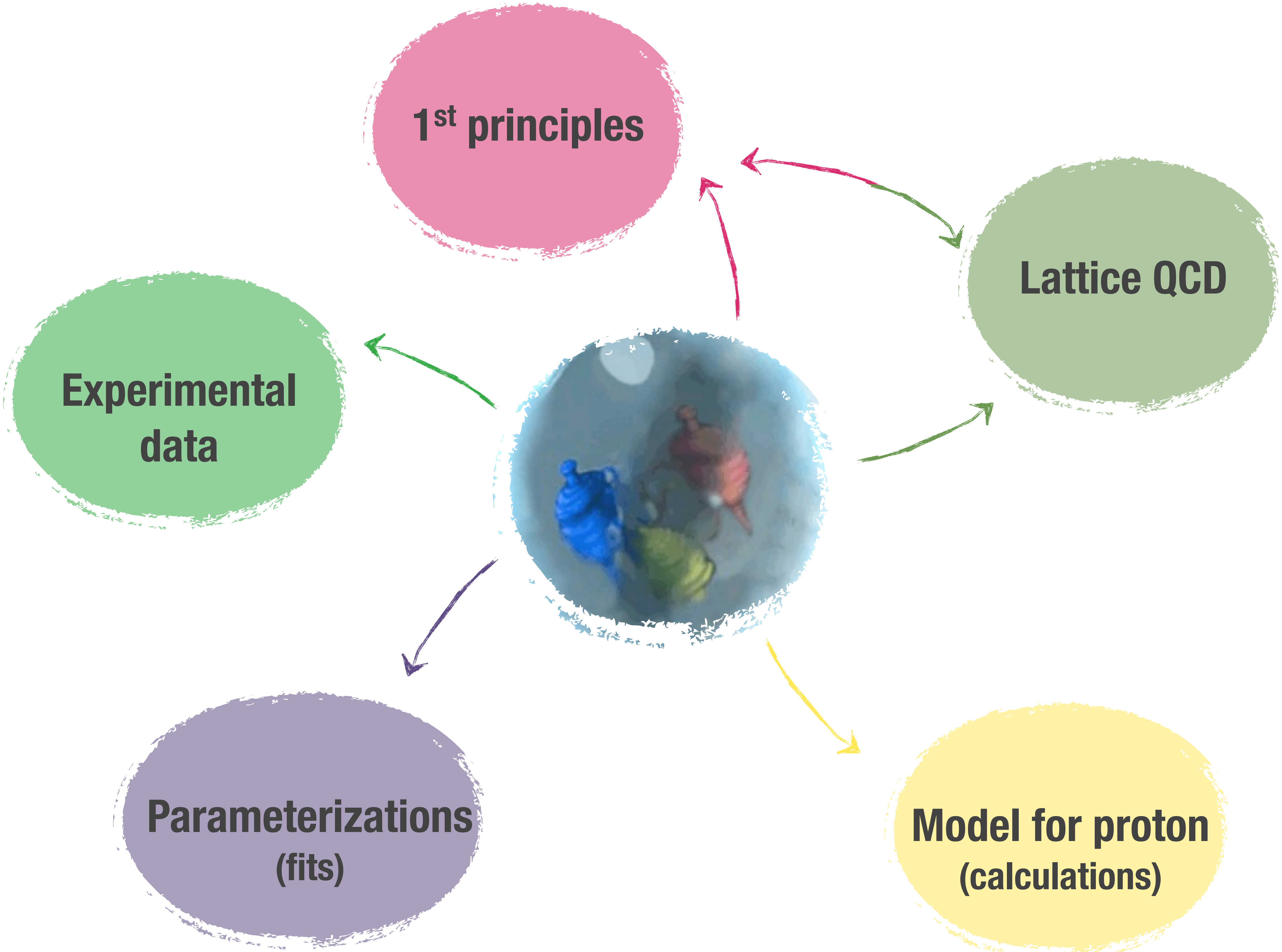
In terms of Generalized Parton Correlation Functions...

$$\begin{aligned}
W^{\gamma^+}_{\Lambda',\Lambda} &= \overline{U}(p',\Lambda')\Bigg[\overbrace{\frac{P^+}{M}(A_1^F+xA_2^F-2\xi A_3^F)}^{type1}+\overbrace{\frac{i\sigma^{+k}}{M}A_5^F+\frac{i\sigma^{+\Delta}}{M}A_6^F}^{type2}+\boxed{\overbrace{\frac{P^+i\sigma^{k\,\Delta}}{M^3}(A_8^F+xA_9^F)}^{type3}} \\
&\quad \underbrace{+\frac{P^+i\sigma^{k\,N}}{M^3}(A_{11}^F+xA_{12}^F)+\frac{P^+i\sigma^{\Delta\,N}}{M^3}(A_{14}^F-2\xi A_{15}^F)}_{type4}\Bigg]U(p,\Lambda) \\
&= A^{[\gamma^+]}_{\Lambda'+;\Lambda+}+A^{[\gamma^+]}_{\Lambda'-;\Lambda-}
\end{aligned}$$

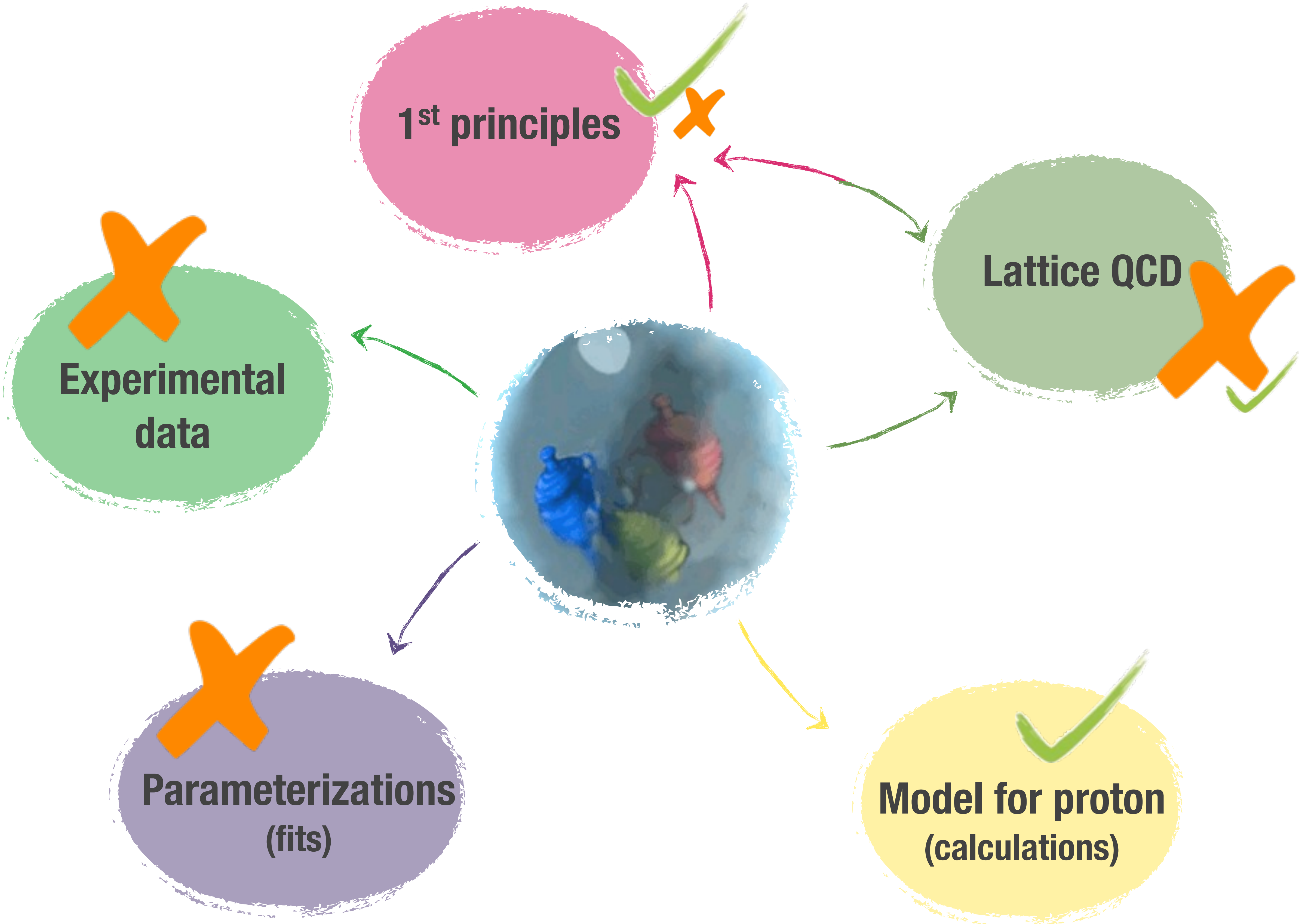
F_{14}

has a different behavior under Parity

How can we access OAM?



How can we access OAM?



Higher-twist contributions

- Helicity amplitude combinations exists
  - Final state interactions transform differently under parity
  - It comes at twist-3 with the structure $\langle \mathbf{S}_L \times \mathbf{\Delta}_T \rangle$
  - Helicity amplitudes here follow $A^{tw3}_{\Lambda'\pm,\Lambda\pm} \rightarrow A^{tw2}_{\Lambda'\pm,\Lambda\mp}$
  - so that we can build the ``LU" structure in terms of twist-3 GTMDs

$$-\frac{4}{P^+}\left[\frac{\bar{\mathbf{k}}_T \cdot \mathbf{\Delta}_T}{\Delta_T} F_{27} + \Delta_T F_{28} - \left(\frac{\bar{\mathbf{k}}_T \cdot \mathbf{\Delta}_T}{\Delta_T} G_{27} + \Delta_T G_{28}\right)\right] = A^{tw3}_{++,++} + A^{tw3}_{+-,+-} - A^{tw3}_{-+,-+} - A^{tw3}_{--,--}$$

- from unp. quarks in L pol proton to transverse direction corr. with FSI/3rd body

Let's go back to GPDs...

Great news is that those GTMDs do admit a GPD limit!

twist-3 GPDs

$$
\begin{aligned}
2\widetilde{H}_{2T} + E_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \boldsymbol{\Delta}_T}{\Delta_T^2} \right) F_{21} + F_{22} \right] \\
\widetilde{E}_{2T} &= -2 \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \boldsymbol{\Delta}_T}{\Delta_T^2} \right) F_{27} + F_{28} \right] = G_2 = \widetilde{H}^3_- \\
2\widetilde{H}'_{2T} + E'_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \boldsymbol{\Delta}_T}{\Delta_T^2} \right) G_{21} + G_{22} \right] \\
\widetilde{E}'_{2T} &= -2 \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \boldsymbol{\Delta}_T}{\Delta_T^2} \right) G_{27} + G_{28} \right]
\end{aligned}
$$

MMS
Kiptily & Polyakov EPJC37
Belitsky *et al* NPB629

Let's go back to GPDs...

Great news is that those GTMDs do admit a GPD limit!

twist-3 GPDs

$$
\begin{aligned}
2\widetilde{H}_{2T} + E_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) F_{21} + F_{22} \right] \\
\widetilde{E}_{2T} &= -2 \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) F_{27} + F_{28} \right] = G_2 = \widetilde{H}_-^3 \\
2\widetilde{H}'_{2T} + E'_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) G_{21} + G_{22} \right] \\
\widetilde{E}'_{2T} &= -2 \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) G_{27} + G_{28} \right]
\end{aligned}
$$

MMS
Kiptily & Polyakov EPJC37
Belitsky *et al* NPB629

- gauge-invariant
- canonical

Let's go back to GPDs...

Great news is that those GTMDs do admit a GPD limit!

twist-3 GPDs

$$
\begin{aligned}
2\widetilde{H}_{2T} + E_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) F_{21} + F_{22} \right] \\
\widetilde{E}_{2T} &= -2 \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) F_{27} + F_{28} \right] = G_2 = \widetilde{H}_-^3 \\
2\widetilde{H}'_{2T} + E'_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) G_{21} + G_{22} \right] \\
\widetilde{E}'_{2T} &= -2 \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) G_{27} + G_{28} \right]
\end{aligned}
$$

MMS
Kiptily & Polyakov EPJC37
Belitsky *et al* NPB629

- **gauge-invariant**

$$
L_{FS} = \frac{\langle PS| \int d^3\vec{r}\, \bar{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}
= \int (\vec{b}_\perp \times \vec{k}_\perp) W_{FS}(x, \vec{b}_\perp, \vec{k}_\perp)\, dx\, d^2\vec{b}_\perp d^2\vec{k}_\perp
$$

Ji, Xiong & Yuan, PRL109

- **canonical**

$$
l_q = \frac{\langle PS| \int d^3\vec{r}\, \overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}
= \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp)\, dx\, d^2\vec{b}_\perp d^2\vec{k}_\perp
$$

Let's go back to GPDs...

Great news is that those GTMDs do admit a GPD limit!

twist-3 GPDs

$$
\begin{aligned}
2\widetilde{H}_{2T} + E_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T\cdot\mathbf{\Delta}_T}{\Delta_T^2}\right) F_{21} + F_{22}\right] \\
\widetilde{E}_{2T} &= -2\int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T\cdot\mathbf{\Delta}_T}{\Delta_T^2}\right) F_{27} + F_{28}\right] = G_2 = \widetilde{H}^3_- \\
2\widetilde{H}'_{2T} + E'_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T\cdot\mathbf{\Delta}_T}{\Delta_T^2}\right) G_{21} + G_{22}\right] \\
\widetilde{E}'_{2T} &= -2\int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T\cdot\mathbf{\Delta}_T}{\Delta_T^2}\right) G_{27} + G_{28}\right]
\end{aligned}
$$

MMS
Kiptily & Polyakov EPJC37
Belitsky *et al* NPB629

- gauge-invariant

$$
L_{FS} = \frac{\langle PS|\int d^3\vec{r}\,\bar{\psi}(\vec{r})\gamma^+(\vec{r}_\perp\times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle} = \int(\vec{b}_\perp\times\vec{k}_\perp)W_{FS}(x,\vec{b}_\perp,\vec{k}_\perp)dxd^2\vec{b}_\perp d^2\vec{k}_\perp
$$

related to twist-2 & twist-3 GPDs

Ji, Xiong & Yuan, PRL109

- canonical

$$
l_q = \frac{\langle PS|\int d^3\vec{r}\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp\times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle} = \int(\vec{b}_\perp\times\vec{k}_\perp)W_{LC}(x,\vec{b}_\perp,\vec{k}_\perp)dxd^2\vec{b}_\perp d^2\vec{k}_\perp
$$

related to twist-2 GPDs
&
its gauge-invariant extension is twist-3

Relation to GPDs

- **Ji's Sum Rule PRL97**

$$J_{q(g)} = \frac{1}{2}\int_{-1}^{1} dx\, x(H_{q(g)}(x) + E_{q(g)}(x))$$

Sum Rule

$$\Rightarrow L_q = \frac{1}{2}\int_{-1}^{1} dx\, x\,(H_q(x) + E_q(x)) - \frac{1}{2}\int_{-1}^{1} dx\,\widetilde{H}(x)$$

- **Penttinen *et al* PLB491**

Sum Rule

$$\int dx\, x\, G_2^q(x) = \frac{1}{2}\left[-\int dx x(H^q(x) + E^q(x)) + \int dx \tilde{H}^q(x)\right]$$

$$= -L_q$$

- **Hatta *et al* JHEP10**

WW approx

$$L_q^{WW}(x) = x\int_x^1 \frac{dy}{y}\,(H_q(y) + E_q(y)) - x\int_x^1 \frac{dy}{y^2}\,\widetilde{H}(y)$$

Canonical vs. gauge-invariant

- In WW approximation, doesn't matter

$$L_q(x) = L_q^{WW}(x) + \boxed{\overline{L}_q(x)}$$
$$\mathcal{L}_q(x) = L_q^{WW}(x) + \boxed{\overline{\mathcal{L}}_q(x)}$$

genuine twist-3 contribution

- Anyway, we know very little about twist-3 GPDs, so WW is fine for now

except for some model calculations

Canonical vs. gauge-invariant

- In WW approximation, doesn't matter

$$L_q(x) = L_q^{WW}(x) + \boxed{\overline{L}_q(x)}$$
$$\mathcal{L}_q(x) = L_q^{WW}(x) + \boxed{\overline{\mathcal{L}}_q(x)}$$

genuine twist-3 contribution

- Anyway, we know very little about twist-3 GPDs, so WW is fine for now

except for some model calculations

- To evaluate $L_q^{WW}(x)$, we can
  - use a parameterization for twist-2 GPDs (Goldstein, Gonzalez-Hernandez & Liuti, PRD84)
  - apply WW formula

OAM density

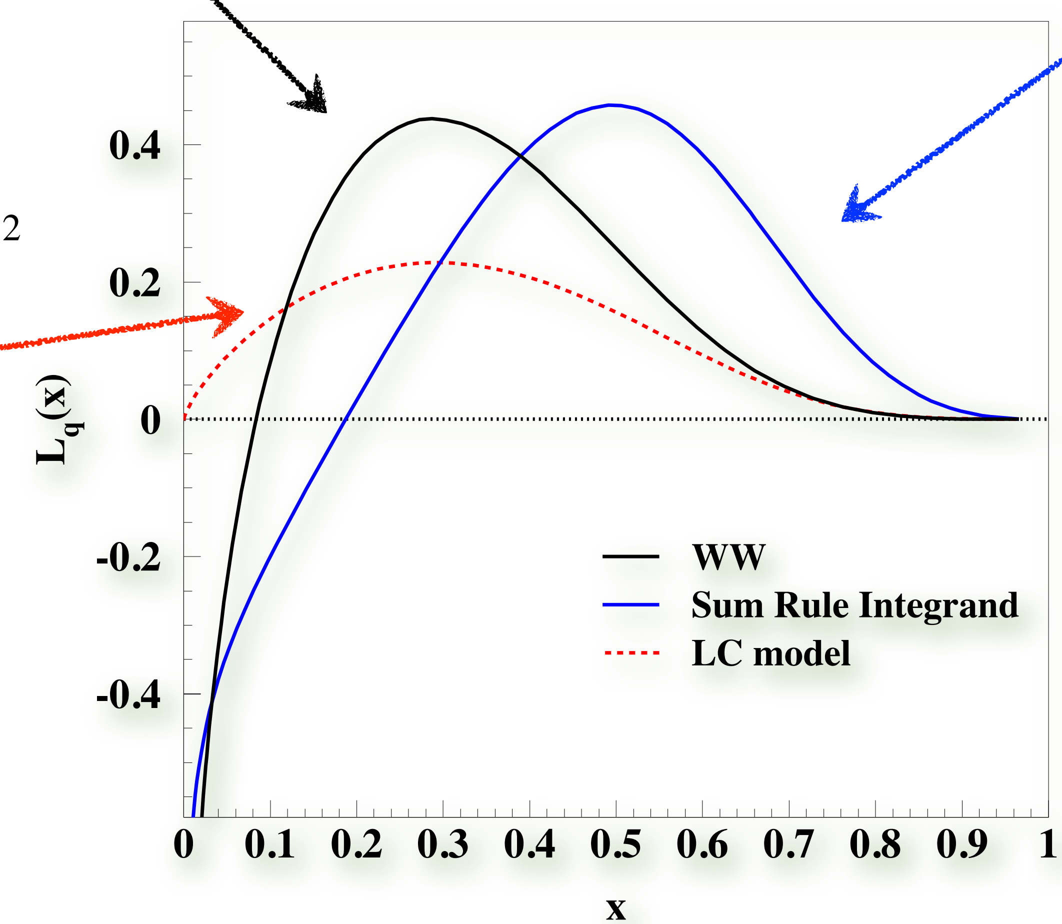
$$L_q^{WW}(x) = x\int_x^1 \frac{dy}{y}\left(H_q(y)+E_q(y)\right) - x\int_x^1 \frac{dy}{y^2}\,\widetilde{H}(y)$$

$$\text{“}L_q(x)\text{”} \rightarrow \frac{1}{2}\,x(H_q(x)+E_q(x)) - \frac{1}{2}\,\widetilde{H}(x)$$

$$\mathscr{L}_q^z = \frac{g^2}{16\pi^3}\int_0^1 dx \int d^2\vec{k}_\perp (1-x)\left|\Psi^{\uparrow}_{\frac{-1}{2}}\right|^2$$

=0.11

Burkardt & BC



Black and blue give the same integrated result L_q=0.13

OAM density

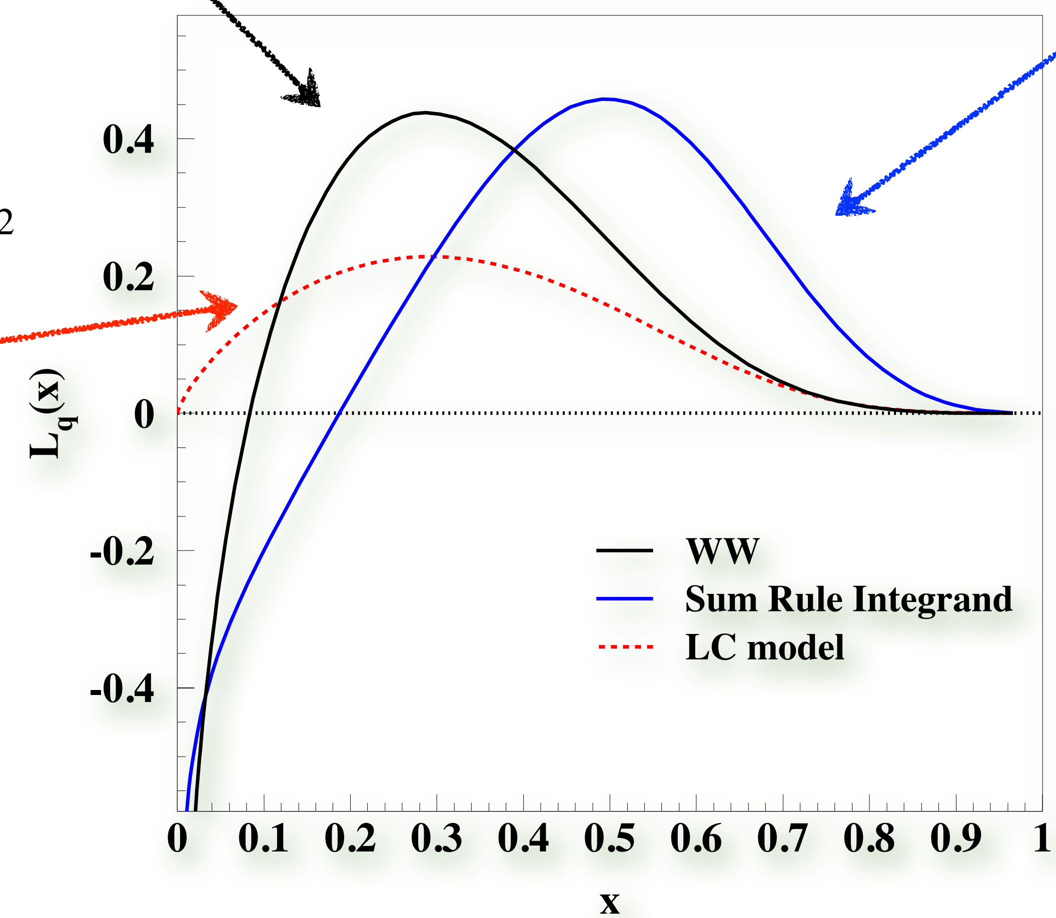
$$L_q^{WW}(x) = x\int_x^1 \frac{dy}{y}\,(H_q(y)+E_q(y)) - x\int_x^1 \frac{dy}{y^2}\,\widetilde{H}(y)$$

$$\text{“}L_q(x)\text{”} \to \frac{1}{2}\,x(H_q(x)+E_q(x)) - \frac{1}{2}\,\widetilde{H}(x)$$

$$\mathscr{L}_q^z = \frac{g^2}{16\pi^3}\int_0^1 dx \int d^2\vec{k}_\perp (1-x)\left|\Psi^{\uparrow}_{\frac{-1}{2}}\right|^2$$

=0.11

Burkardt & BC



Black and blue give the same integrated result L_q=0.13

Ok, so, now, is G_2 related to any observable?

DVCS formalism

formalism from BKM [NPB629]

$$\widetilde{\mathcal{H}}^{eff} = -2\xi\left(\frac{1}{1+\xi}\widetilde{\mathcal{H}} + \widetilde{\mathcal{H}}_3^{+} - \widetilde{\mathcal{H}}_3^{-}\right)$$

with $\widetilde{\mathcal{H}}_3^{-} = C^{-} \otimes \left(\widetilde{E}_{2T} = G_2 = \widetilde{H}_3^{-}\right)$

$$\left\{\begin{matrix} c^{\mathcal{I}}_{2,\mathrm{LP}} \\ s^{\mathcal{I}}_{2,\mathrm{LP}} \end{matrix}\right\} = \frac{16\Lambda K^2}{2-x_{\mathrm{B}}}\left\{\begin{matrix} -\lambda y \\ 2-y \end{matrix}\right\}\left\{\begin{matrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{matrix}\right\}\mathcal{C}^{\mathcal{I}}_{\mathrm{LP}}(\mathcal{F}^{\mathrm{eff}}),$$

$$\mathcal{C}^{\mathcal{I}}_{\mathrm{LP}} = \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}(F_1+F_2)\left(\mathcal{H}+\frac{x_{\mathrm{B}}}{2}\mathcal{E}\right) + F_1\widetilde{\mathcal{H}} - \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}\left(\frac{x_{\mathrm{B}}}{2}F_1 + \frac{\Delta^2}{4M^2}F_2\right)\widetilde{\mathcal{E}},$$

HERE IS THE OBSERVABLE

$$\mathcal{A}_{\mathrm{UL}}(\phi) \simeq \sum_{n=1}^{3} A_{\mathrm{UL}}^{\sin(n\phi)}\sin(n\phi) + A_{\mathrm{UL}}^{\cos(0\phi)},$$

$$\mathcal{A}_{\mathrm{LL}}(\phi) \simeq \sum_{n=0}^{2} A_{\mathrm{LL}}^{\cos(n\phi)}\cos(n\phi)\,.$$

Asymmetry Amplitude	Contributory Fourier-Coefficients	Power of $\frac{1}{Q}$ Suppression	Dominant CFF Dependence	Twist Level
$A_{\mathrm{UL}}^{\sin(2\phi)}$	$s^{\mathrm{I}}_{2,\mathrm{LP}}$	2	$\mathrm{Im}\,\mathcal{C}^{\mathrm{I}}_{\mathrm{LP}}$	3
	$s^{\mathrm{DVCS}}_{2,\mathrm{LP}}$	2	$\mathrm{Im}\,\mathcal{C}^{\mathrm{DVCS}}_{\mathrm{T,LP}}$	2
$A_{\mathrm{LL}}^{\cos\phi}$	$c^{\mathrm{I}}_{1,\mathrm{LP}}$	1	$\mathrm{Re}\,\mathcal{C}^{\mathrm{I}}_{\mathrm{LP}}$	2
	$c^{\mathrm{DVCS}}_{1,\mathrm{LP}}$	3	$\mathrm{Re}\,\mathcal{C}^{\mathrm{DVCS}}_{\mathrm{LP}}$	3

OAM from sin(2φ) modulation

formalism from BKM [NPB629]

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} \longrightarrow \boxed{A_{UL} = \frac{a\sin\phi + b\sin 2\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi}}$$

CLAS12
denominator's functional form ➜ unstability of fit (private info CLAS)
need for cross section measurement

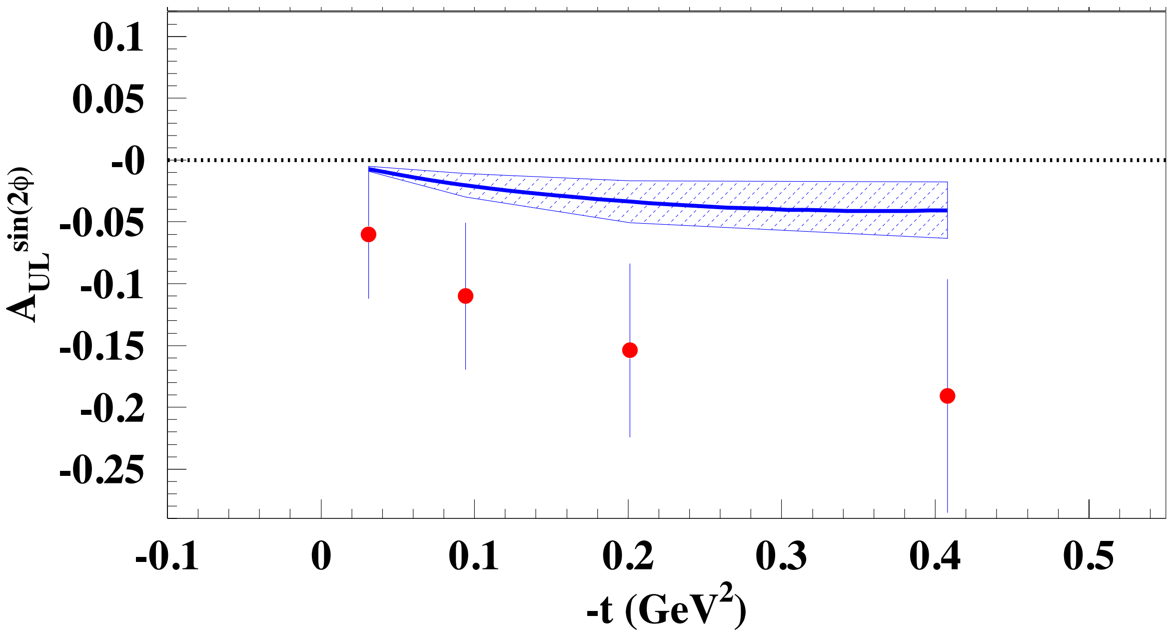
$$a \approx s^{\mathcal{I}}_{1,LP} \propto F_1(t)\Im m\widetilde{\mathcal{H}}$$

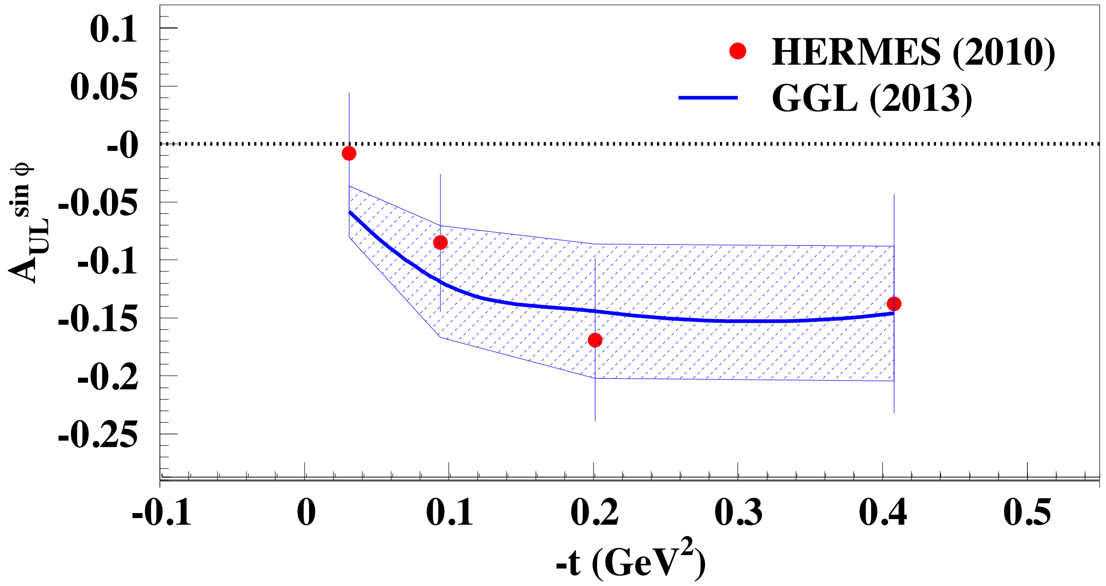
$$b \approx s^{\mathcal{I}}_{2,LP} \propto F_1(t)\Im m\widetilde{\mathcal{H}}^{eff}$$

- in WW approximation, we can use twist-2 GPDs
  - for *a* and *b*
- some plots were shown during the workshop
- we illustrated using GGL GPD set [Goldstein, Gonzalez Hernandez & Liuti, PRD84]

OAM from sin(2φ) modulation @ HERMES (JHEP06)

- the sin(φ) is prediction and autoconsistency check.
- the sin(2φ) is ``prediction"





sin(2φ) is huge at HERMES
Really intricated observable but still...

- only interference term, ~ξ=0, WW approximation...to be improved

Conclusions

OAM

- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ is parity-odd at twist-2
- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ is not parity-odd at twist-3
  - from GTMDs to GPDs

- What does it mean in terms of Wigner functions? (à la Ji, does the gauge link matter?, ...)
- Can we go beyond WW approximation? (start with evaluation in models?)

Conclusions

OAM

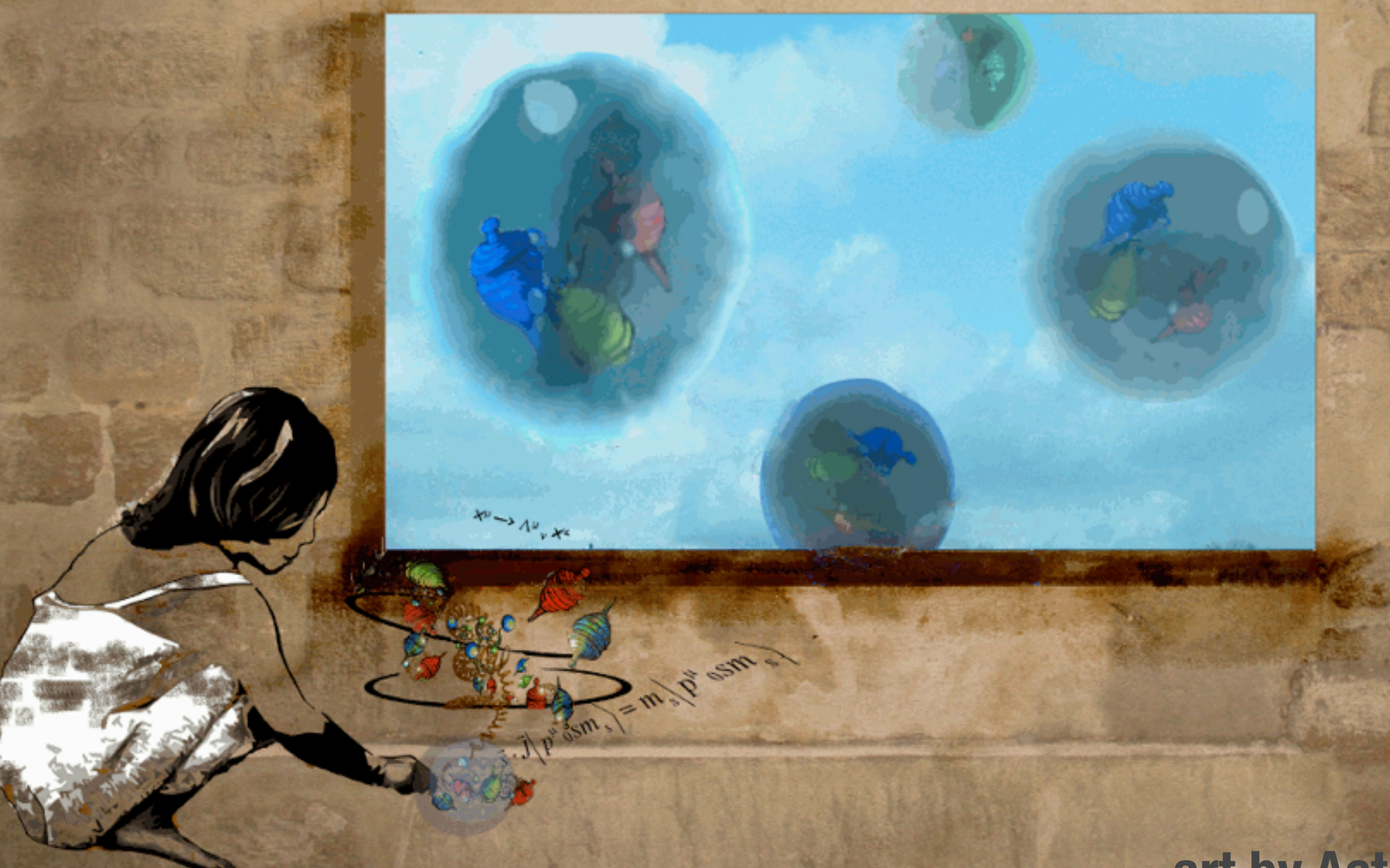
- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ is parity-odd at twist-2
- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ is not parity-odd at twist-3
  - from GTMDs to GPDs

- What does it mean in terms of Wigner functions? (à la Ji, does the gauge link matter?, ...)
- Can we go beyond WW approximation? (start with evaluation in models?)

- Anyhow, we've spotted an observable!
  - sin(2φ) modulation of TSA for DVCS
- help experimentalist with input model results (cross sections, range of parameters?)
- Global fits with twist-3 observables? (improved GGL)

¿waiting for the CLAS data?

Thank you



art by Astrid Morreale

http://amorreal.web.cern.ch/amorreal/artpQCD

Twist-3 helicity amplitudes

$$W^{\gamma^i}_{\Lambda'\Lambda} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\frac{\bar{k}_T^i}{M}F_{21} + \frac{\Delta_T^i}{M}(F_{22}-2F_{26}) + \frac{i\sigma^{ji}\bar{k}_T^j}{M}F_{27} + \gamma^i(2F_{28})\right.$$
$$\left. + \frac{Mi\sigma^{i+}}{P^+}F_{23} + \frac{\bar{k}_T^i}{M}\frac{i\sigma^{k+}\bar{k}_T^k}{P^+}F_{24} + \frac{\Delta_T^i}{M}\frac{i\sigma^{k+}\bar{k}_T^k}{P^+}F_{25} + \frac{\Delta_T^i}{P^+}\gamma^+(2F_{26})\right]U(p,\Lambda)$$

$$A^{tw3}_{\Lambda'\lambda',\Lambda\lambda} = \int \frac{dz^-\, d^2\mathbf{z}_T}{(2\pi)^3} e^{ixP^+z^- - i\bar{\mathbf{k}}_T\cdot\mathbf{z}_T} \, \langle p',\Lambda' \mid \mathcal{O}_{\lambda'-\lambda}(z) \mid p,\Lambda\rangle|_{z^+=0}$$

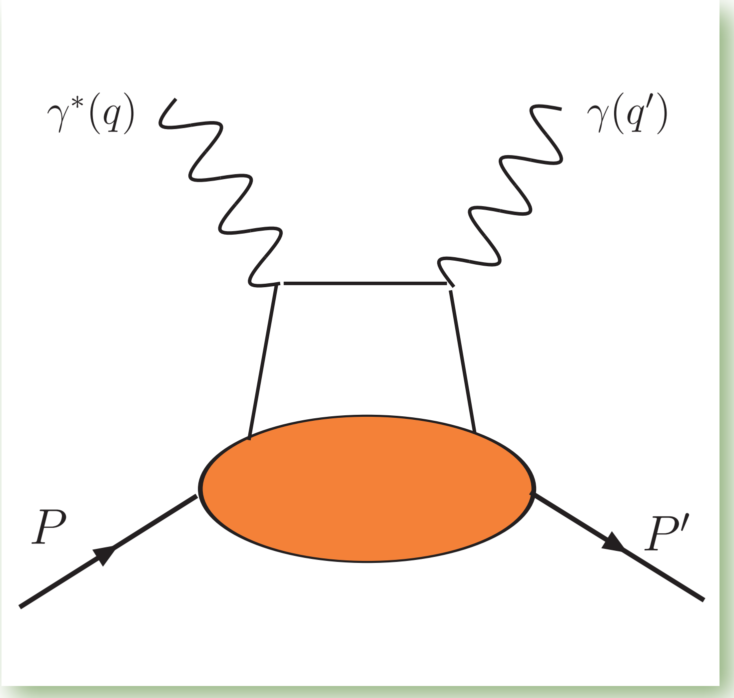
$$\mathcal{O}^q_{\pm\mp}(z) = \phi^\dagger_\pm\left(-\frac{z}{2}\right)\chi_\pm\left(\frac{z}{2}\right) \pm \chi^\dagger_\pm\left(-\frac{z}{2}\right)\phi_\pm\left(\frac{z}{2}\right).$$

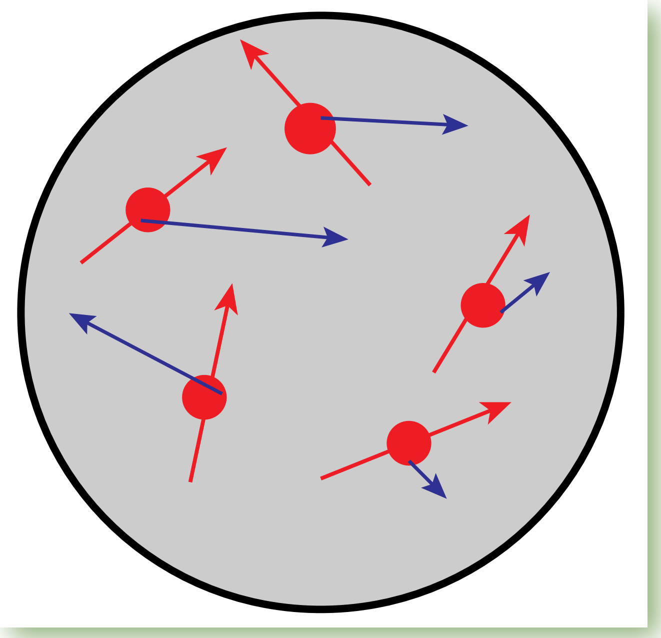
$$\boxed{A^{tw3}_{\Lambda'\pm,\Lambda\pm} \to A^{tw2}_{\Lambda'\pm,\Lambda\mp}}$$

Generalized Functions

Momentum transfer b/w initial and final state

Intrinsic quark transverse motion





- Generalized Parton Distributions

$f(x) \rightarrow f(x, (P'-P)^2, n.(P'-P))$

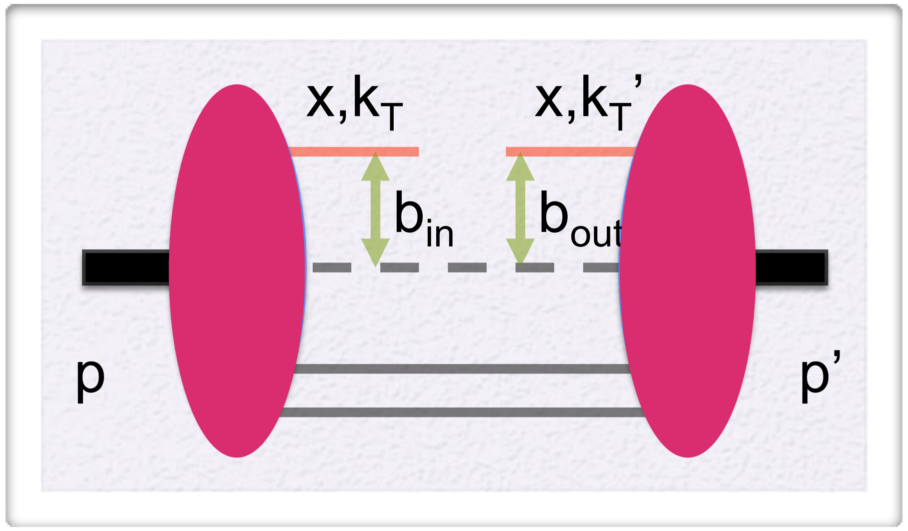
- Transverse Momentum Distributions

$f(x) \rightarrow f(x, k_\perp)$

Partonic meaning

- Generalized Parton Distributions

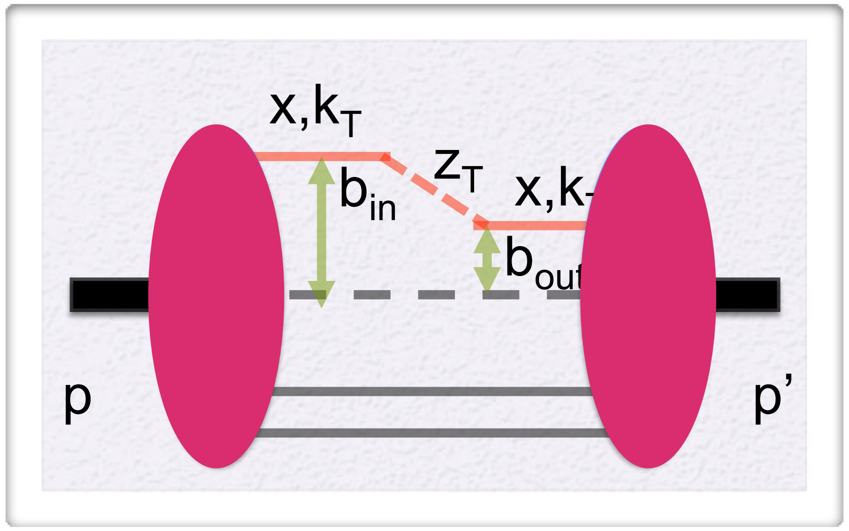
f(x)→f(x, (P'-P)^2, n.(P'-P))



$$\Delta_T = P'_T - P_T = k'_T - k_T$$
$$\int \frac{d^2\Delta_T}{(2\pi)^2}\, e^{i\Delta_T\cdot b_T} \Rightarrow b_T = \frac{b_{T,in} + b_{T,out}}{2}$$

- Transverse Momentum Distributions

f(x)→f(x, k$_\perp$)



$$\bar{k}_T = \frac{k_T + k'_T}{2}$$
$$\int d^2k_T\, e^{-ik_T\cdot z_T} \Rightarrow z_T = b_{T,in} - b_{T,out}$$

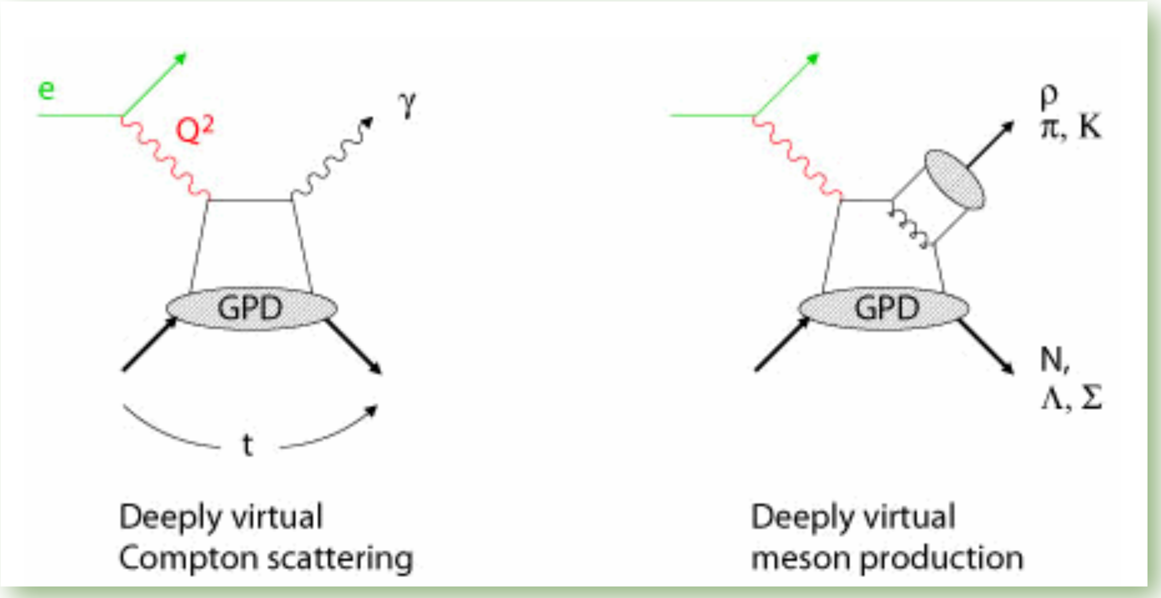
Average

Shift

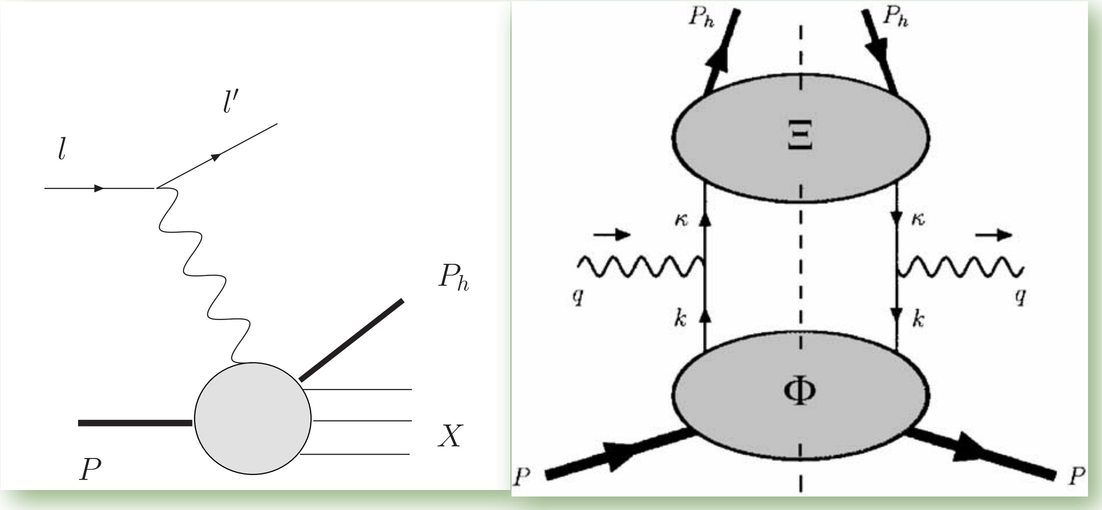
Impact parameter space

Generalized Functions

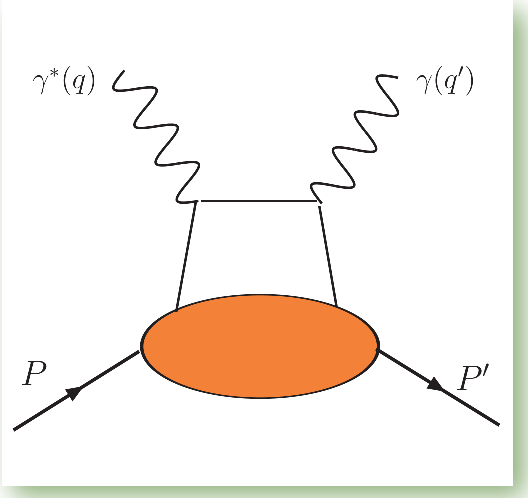
Exclusive processes



Semi-inclusive processes



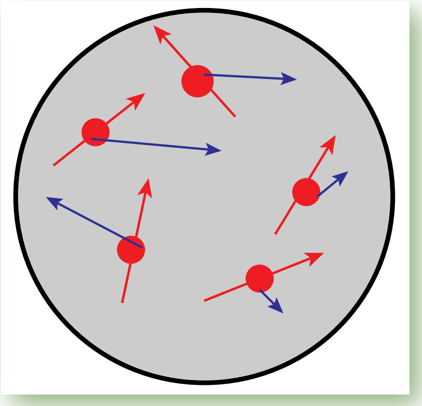
Momentum transfer b/w initial and final state



- Generalized Parton Distributions

$f(x) \to f(x, \Delta^2, n.\Delta)$

Intrinsic quark transverse motion

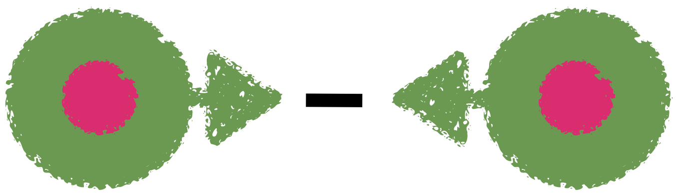


- Transverse Momentum Distributions

$f(x) \to f(x, k_\perp)$

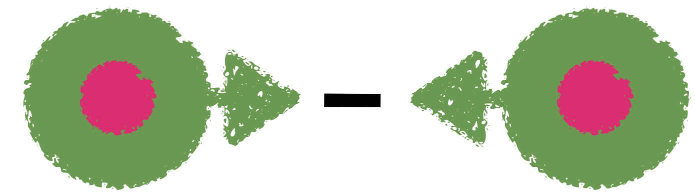
Models

$$-i\,\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14} = \left(A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}\right)/4$$



Models

$$-i\,\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}F_{14}=\left(A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}\right)/4$$



What happens in models? Is it zero?

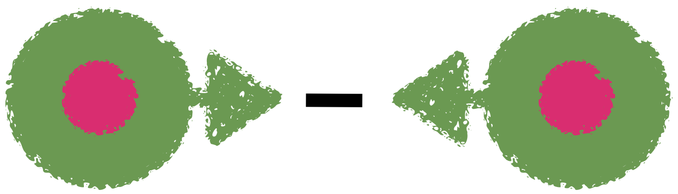
Not in quark models
E.g. in the bag...

$$i\frac{\bar{k}_x\Delta_y-\bar{k}_y\Delta_x}{M^2}F_{14}^u\ \propto\ 16i\times\left[\frac{\left(\vec{k}'\times\vec{k}_3\right)_z}{k_3k'}t_1(k_3)t_1(k')\right]$$

AC, Liuti, Rajan..., in preparation

Models

$$-i\,\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}F_{14} = \left(A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}\right)/4$$



What happens in models? Is it zero?

Not in quark models
E.g. in the bag...

$$i\frac{\bar{k}_x\Delta_y-\bar{k}_y\Delta_x}{M^2}F^u_{14} \;\propto\; 16i\times\left[\frac{\left(\vec{k}'\times\vec{k}_3\right)_z}{k_3k'}t_1(k_3)t_1(k')\right]$$

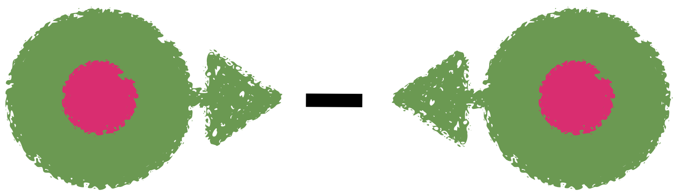
AC, Liuti, Rajan..., in preparation

Why is it so?

We think
- **the ``low-energy quark"** (constituent, preconfined...) **implies complex dynamics**
- **expansion in twist does not match model content**

Models

$$-i\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}F_{14} = \left(A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}\right)/4$$



What happens in models? Is it zero?

Not in quark models
E.g. in the bag...

$$i\frac{\bar{k}_x\Delta_y-\bar{k}_y\Delta_x}{M^2}F^u_{14} \propto 16i\times\left[\frac{\left(\vec{k}'\times\vec{k}_3\right)_z}{k_3k'}t_1(k_3)t_1(k')\right]$$

AC, Liuti, Rajan..., in preparation

Why is it so?

We think
- **the ``low-energy quark" (constituent, preconfined...) implies complex dynamics**
- **expansion in twist does not match model content**

Calculate "canonical" twist-3 functions. Compare ...

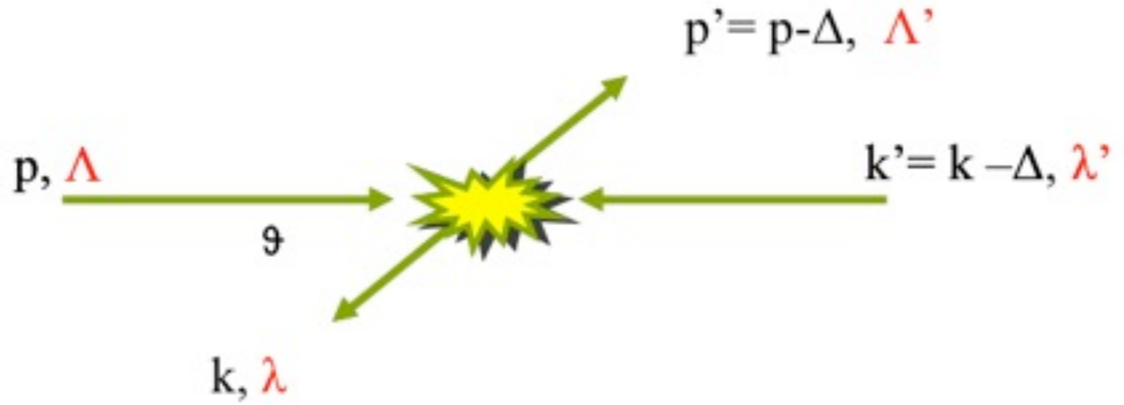
Where is the OAM?

Leading-order ⇔ 2-body scattering

Landshoff, Polkinghorne and Short, NPB28

CoM 2-body scattering must occur on a plane

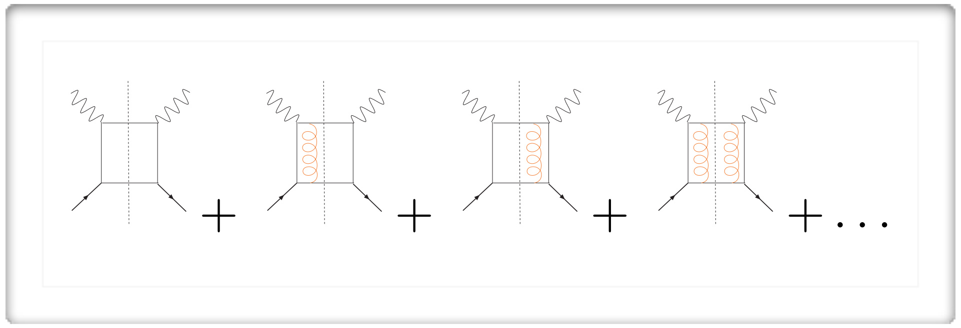
CoM frame with p in z direction: leaves 1 transverse direction (related to θ)



Example:

Sivers function

- T-odd, so not allowed as if 2-body scattering
- Needs a third body: different symmetries, more flexibility
- That 3rd body comes from the gauge link→ final state interaction



Here?

What is the concept of *twist* when adding (hard) scales?