Deeply Virtual Compton Scattering to the twist-four accuracy: Impact of finite-t and target mass corrections

V. M. Braun

University of Regensburg

based on

V. Braun, A. Manashov, D. Müller, B. Pirnay, arXiv:1401.7621

Bochum, 12.02.2014

How to calculate effects $\sim m^2/Q^2$ and t/Q^2 in DVCS?

Early work:

- **DVCS:**

— Extension of Nachtmann's approach to target mass corrections in DIS
— Spin-rotation (Wandzura-Wilczek)

Blümlein, Robaschik: NPB581 (2000) 449
Radyushkin, Weiss: PRD63 (2001) 114012
Belitsky, Müller: NPB589 (2000) 611
...

— Results not gauge invariant
— Results not translation invariant

- **B-decays:**

Ball, Braun: NPB543 (1999) 201

— Problem localized but not solved

Contributions of different twist are intertwined by symmetries:

- Conservation of the electromagnetic current and translation invariance

$$
\begin{aligned}
\partial^\mu T\left\{j_\mu^{em}(x)j_\nu^{em}(0)\right\} &= 0 \\
T\left\{j_\mu^{em}(2x)j_\nu^{em}(0)\right\} &= e^{-i\mathbf{P}\cdot x}\, T\left\{j_\mu^{em}(x)j_\nu^{em}(-x)\right\} e^{i\mathbf{P}\cdot x}
\end{aligned}
$$

  are valid in the sum of all twists but not for each twist separately

- Higher-twist contributions that restore gauge/translation invariance are due to descendants of leading-twist operators obtained by adding total derivatives

$$
T\left\{j_\mu^{em}(x)j_\nu^{em}(0)\right\} = \underbrace{\sum_N a_N \mathcal{O}_N}_{\text{leading-twist}} + \sum_N \left(b_N \partial^2 \mathcal{O}_N + c_N (\partial\mathcal{O})_N\right) + \text{ other operators}
$$

- These operators contribute to finite-t and target mass corrections

Task: Find the contributions to the OPE of all descendants of leading-twist operators

Guidung principle:

PRL 107 (2011) 202001

— “kinematic” approximation amounts to the assumption that genuine twist-four matrix elements are zero

— for consistency, they must remain zero at all scales

— they must not reappear at higher scales due to mixing with “kinematic” operators

- **“Kinematic” and “Dynamic” contributions must have autonomous scale-dependence**

Indeed, otherwise:

$$\Big(\langle\bar q F q\rangle + c\langle(\partial\mathcal{O})\rangle\Big)^{\mu^2} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0}\Big(\langle\bar q F q\rangle + c\langle(\partial\mathcal{O})\rangle\Big)^{\mu_0^2}$$

$$\Longrightarrow$$

$$\langle\bar q F q\rangle^{\mu^2} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0}\langle\bar q F q\rangle^{\mu_0^2} + c\left[\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0} - \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_2/\beta_0}\right]\langle(\partial\mathcal{O})\rangle^{\mu_0^2}$$

[The “kinematic” approximation corresponds to taking into account *all* operators with the same anomalous dimensions as the leading twist operators]

- Explicit diagonalization of the mixing matrix for twist-4 operators not feasible

- In a conformal theory

$$\Big(\partial_\mu + \beta(\alpha)\partial_\alpha + \mathbb{H}\Big)\, O_j \;=\; 0 \quad\Longrightarrow\quad \langle T\{O_{j_1}(x)O_{j_2}(0)\}\rangle \sim \delta_{j_1 j_2}$$

therefore

$$T\Big\{j(x)j(0)\Big\} \;=\; \sum_N C_N(x,\partial)\mathcal{O}_N + \ldots\,,$$

$$C(x,\partial)\mathcal{O}_N = a_N\mathcal{O}_N + b_N\partial^2\mathcal{O}_N + c_N(\partial\mathcal{O})_N + \ldots$$

$\Longrightarrow$

$$\langle T\Big\{j(x)j(0)\mathcal{O}_N(y)\Big\}\rangle \;=\; C_N(x,\partial)\langle T\Big\{\mathcal{O}_N(0)\mathcal{O}_N(y)\Big\}\rangle + 0$$

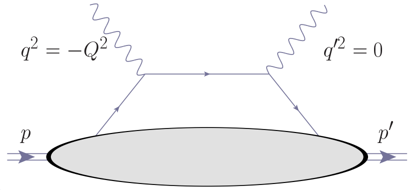
— might work in QCD, need NLO expressions in $d = 4 - \epsilon$ dimensions

- Orthogonality of eigenoperators suggests that $\mathbb{H}$ is a hermitian operator w.r.t. a certain scalar product

  Braun, Manashov, Rohrwild, Nucl. Phys. **B807** (2009) 89; Nucl. Phys. **B826** (2010) 235.

BMP reference frame

Braun, Manashov, Pirnay: PRD **86** (2012) 014003



longitudinal plane (q, q')

$$n = q', \qquad \tilde{n} = -q + \frac{Q^2}{Q^2+t}\, q'$$

with this choice $\Delta = q - q'$ is longitudinal and

$$|P_\perp|^2 = -m^2 - \frac{t}{4}\frac{1-\xi^2}{\xi^2} \sim t_{\rm min} - t$$

where

$$P = \frac{1}{2}(p+p'), \qquad \xi_{\rm BMP} = -\frac{(\Delta\cdot q')}{2(P\cdot q')} = \frac{x_B(1+t/Q^2)}{2-x_B(1-t/Q^2)}$$

photon polarization vectors

$$\varepsilon^0_\mu = -\Big(q_\mu - q'_\mu q^2/(qq')\Big)\,/\sqrt{-q^2}\,,$$
$$\varepsilon^\pm_\mu = (P^\perp_\mu \pm i\bar{P}^\perp_\mu)/(\sqrt{2}|P_\perp|)\,, \qquad \bar{P}^\perp_\mu = \epsilon^\perp_{\mu\nu}P^\nu$$

BMP helicity amplitudes

Braun, Manashov, Pirnay: PRD **86** (2012) 014003

$$\begin{aligned}\mathcal{A}_{\mu\nu}(q,q',p) &= i\int d^4x\, e^{-i(z_1 q - z_2 q')x}\langle p', s'|T\{J_\mu(z_1 x)J_\nu(z_2 x)\}|p, s\rangle \\ &= \varepsilon^+_\mu\varepsilon^-_\nu\,\mathcal{A}^{++} + \varepsilon^-_\mu\varepsilon^+_\nu\,\mathcal{A}^{--} + \varepsilon^0_\mu\varepsilon^-_\nu\,\mathcal{A}^{0+} \\ &+ \varepsilon^0_\mu\varepsilon^+_\nu\,\mathcal{A}^{0-} + \varepsilon^+_\mu\varepsilon^+_\nu\,\mathcal{A}^{+-} + \varepsilon^-_\mu\varepsilon^-_\nu\,\mathcal{A}^{-+} + q'_\nu\mathcal{A}^{(3)}_\mu\end{aligned}$$

for the calculation to the twist-4 accuracy one needs

- $\mathcal{A}^{++}, \mathcal{A}^{--}$: $1 + \frac{1}{Q^2}$
- $\mathcal{A}^{0+}, \mathcal{A}^{0-}$: $\frac{1}{Q}$ $\longleftarrow$ agree with existing results
- $\mathcal{A}^{-+}, \mathcal{A}^{+-}$: $\frac{1}{Q^2}$ $\longleftarrow$ straightforward

BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

$$\mathcal{A}_q^{a\pm} = \mathbb{H}^q_{a\pm}\, h + \mathbb{E}^q_{a\pm}\, e \mp \widetilde{\mathbb{H}}^q_{a\pm}\, \tilde{h} \mp \widetilde{\mathbb{E}}^q_{a\pm}\, \tilde{e}$$

with, e.g. Belitsky, Müller, Ji: NPB **878** (2014) 214

$$h = \frac{\bar{u}(p')\,(\not{q} + \not{q}')u(p)}{P\cdot(\not{q} + \not{q}')} \qquad \ldots$$

- The results read Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\mathbb{H}_{++} = T_0 \circledast H + \frac{t}{Q^2}\Big[-\frac{1}{2}T_0 + T_1 + 2\xi \mathbf{D}_\xi\, T_2\Big] \circledast H + \frac{2t}{Q^2}\xi^2\partial_\xi \xi\, T_2 \circledast (H+E)$$

$$\mathbb{H}_{0\,+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\Big[\xi\partial_\xi T_1 \circledast H + \frac{t}{Q^2}\partial_\xi \xi\, T_1 \circledast (H+E)\Big] - \frac{t}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \Big[\xi\,(H+E) - \widetilde{H}\Big]$$

$$\mathbb{H}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\Big[\xi\partial_\xi^2 \xi\, T_1^{(+)} \circledast H + \frac{t}{Q^2}\partial_\xi^2\xi^2\, T_1^{(+)} \circledast (H+E)\Big] + \frac{2t}{Q^2}\xi\Big[\xi\partial_\xi \xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\, \xi\, T_1 \circledast \widetilde{H}\Big]$$

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$$h = \frac{\bar{u}(p')\,(\not{q} + \not{q}')u(p)}{P\cdot(\not{q}+\not{q}')} \qquad \ldots$$

- The results read Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\mathbb{E}_{++} = T_0 \circledast E + \frac{t}{Q^2}\left[-\frac{1}{2}T_0 + T_1 + 2\xi \mathbf{D}_\xi\, T_2\right] \circledast E - \frac{8m^2}{Q^2}\xi^2\partial_\xi \xi\, T_2 \circledast (H+E)$$

$$\mathbb{E}_{0+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\left[\xi\partial_\xi T_1 \circledast E\right] + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \left[\xi\,(H+E) - \widetilde{H}\right]$$

$$\mathbb{E}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\left[\xi\partial_\xi^2 \xi\, T_1^{(+)} \circledast E\right] - \frac{8m^2}{Q^2}\xi\left[\xi\partial_\xi\xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\, \xi\, T_1 \circledast \widetilde{H}\right]$$

etc.

where $F = H, E, \widetilde{H}, \widetilde{E}$ **are** C**-even GPDs**

$$T \circledast F = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{2\xi}\, T\left(\frac{\xi + x - i\epsilon}{2(\xi - i\epsilon)}\right) F(x,\xi,t)$$

the coefficient functions $T_k^{\pm}$ **are given by the following expressions:**

$$T_0(u) = \frac{1}{1-u}$$

$$T_1(u) \equiv T_1^{(-)}(u) = -\frac{\ln(1-u)}{u}$$

$$T_1^{(+)}(u) = \frac{(1-2u)\ln(1-u)}{u}$$

$$T_2(u) = \frac{\mathrm{Li}_2(1) - \mathrm{Li}_2(u)}{1-u} + \frac{\ln(1-u)}{2u}$$

and

$$\mathbf{D}_\xi = \partial_\xi + 2\frac{|\xi P_\perp|^2}{t}\partial_\xi^2 \xi \;=\; \partial_\xi - \frac{t - t_{\min}}{2t}(1-\xi^2)\partial_\xi^2 \xi$$

Main features:

- Two expansion parameters

$$\frac{t}{Q^2}; \qquad \frac{t-t_{\rm min}}{Q^2} \sim \frac{|\xi P_\perp|^2}{Q^2}$$

- All mass corrections for scalar targets absorbed in $t_{\rm min} = -4m^2\xi^2/(1-\xi^2)$; always overcompensated by finite-t corrections in the physical region
- Some extra m^2/Q^2 corrections for nucleon due to spinor algebra; disappear in certain CFF combinations
- Factorization checked to $1/Q^2$ accuracy
- Gauge and translation invariance checked to $1/Q^2$ accuracy
- Correct threshold behavior $t \to t_{\rm min}$, $\xi \to 1$

From CFFs to DVCS observables

- **The only existing calculation to the required accuracy: BMJ**

Belitsky, Müller, Ji: NPB **878** (2014) 214

!!! **Subtlety: BMJ use a different reference frame to define photon helicity amplitides; hence a different set of CFFs (calligraphic) related to BMP CFFs (blackboard bold) by a kinematic trafo**

$$\mathcal{F}_{\pm+} = \mathbb{F}_{\pm+} + \frac{\varkappa}{2}\Big[\mathbb{F}_{++} + \mathbb{F}_{-+}\Big] - \varkappa_0\,\mathbb{F}_{0+}\,,$$
$$\mathcal{F}_{0+} = -\,(1+\varkappa)\,\mathbb{F}_{0+} + \varkappa_0\Big[\mathbb{F}_{++} + \mathbb{F}_{-+}\Big]$$

$$\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

where

$$\varkappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}\,, \qquad \varkappa \sim (t_{\min} - t)/Q^2$$

Adopted strategy is, thus,

$$\text{BMP CFFs} \xrightarrow{\text{exact}} \text{BMJ CFFs} \xrightarrow{\text{exact}} \text{observables}$$
$$\nwarrow\ \mathcal{O}(1/Q^2)$$

Defining the Leading Twist approximation

Kumerički-Müller convention (KM)

$$\mathrm{LT}_{\mathrm{KM}}: \begin{cases} \mathcal{F}_{++} = T_0 \circledast F, & \mathcal{F}_{0+} = 0, \\ \mathcal{F}_{-+} = 0, & \xi = \xi_{\mathrm{KM}} \end{cases}$$

Braun-Manashov-Pirnay convention (BMP)

$$\mathrm{LT}_{\mathrm{BMP}}: \begin{cases} \mathbb{F}_{++} = T_0 \circledast F, & \mathbb{F}_{0+} = 0, \\ \mathbb{F}_{-+} = 0, & \xi = \xi_{\mathrm{BMP}} \end{cases}$$

$$\Downarrow$$

$$\mathrm{LT}_{\mathrm{BMP}}: \begin{cases} \mathcal{F}_{++} = \left(1 + \frac{\varkappa}{2}\right) \mathbb{F}_{++} & \mathcal{F}_{0+} = \varkappa_0 \, \mathbb{F}_{++}, \\ \mathcal{F}_{-+} = \frac{\varkappa}{2} \mathbb{F}_{++}, & \xi = \xi_{\mathrm{BMP}}, \end{cases}$$

Changing frame of reference results in

- **Different skewedness parameter** $\xi_{KM} = \dfrac{x_B}{2 - x_B}$ **vs.** $\xi_{BMP} = \dfrac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$
- **Numerically significant excitation of helicity-flip CFFs** $\mathcal{F}_{0+}, \mathcal{F}_{-+}$

Numerical analysis: Unpolarized target (1) GPD model: GK12

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

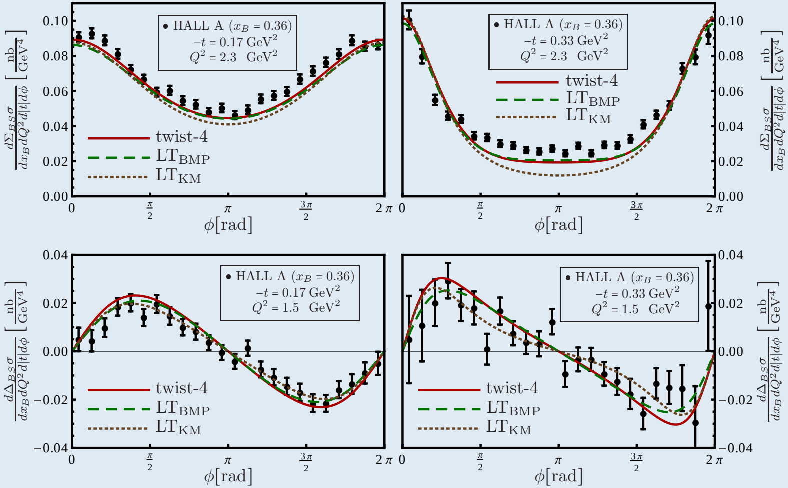

Figure: Unpolarized cross section [upper panels] and electron helicity dependent cross section difference [lower panels] from HALL A

Unpolarized target (2)

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

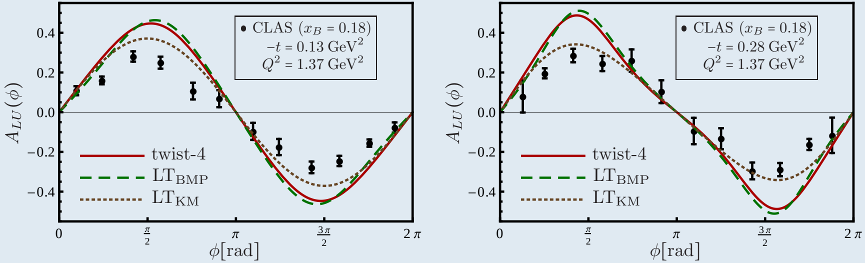

Figure: Single electron beam spin asymmetry by CLAS collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

Unpolarized target (3)

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

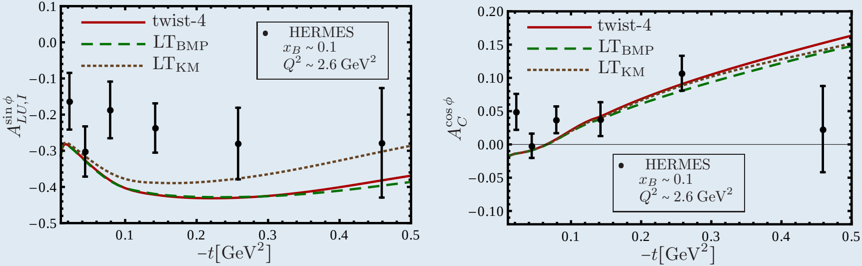

Figure: The single electron beam spin asymmetry [left panel] in the charge-odd sector and the unpolarized beam charge asymmetry [right panel] measured by the HERMES collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

Longitudinally polarized targets

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

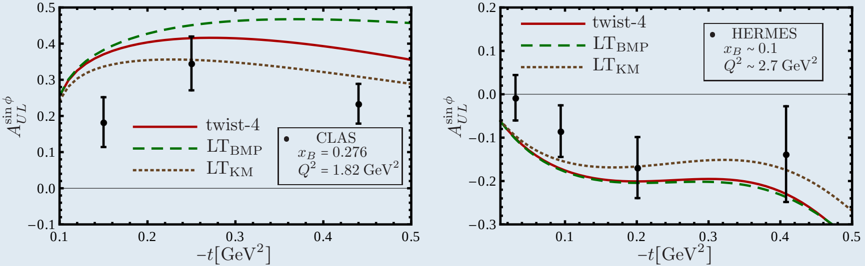

Figure: Longitudinal proton spin asymmetry from CLAS [left panel], measured with an electron beam, and HERMES [right panel], measured with a positron beam

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

Collider kinematics

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

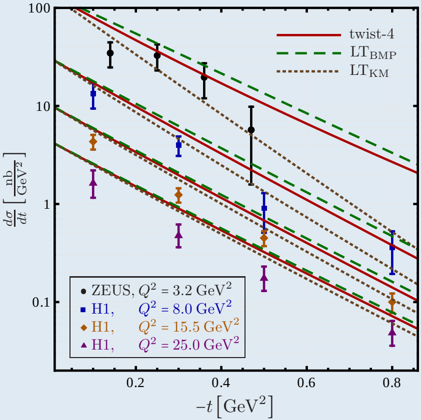

Figure: The DVCS cross section from H1 (squares, diamonds, triangles) and ZEUS (circles)

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

Summary and conclusions

- **Target mass and finite-t corrections to DVCS are known to twist-4 accuracy They are relatively simple and can be implemented with moderate effort**
- **Premium:**

  Gauge and translation invariance of the Compton tensor is restored to $1/Q^2$ accuracy
  Convention-dependence of the common leading-twist calculations is removed
  Theoretically motivated limits $-t/Q^2 \lesssim 1/4$
- **For several key observables, the lion share of the twist-4 effects is captured by going over to the BMP frame**
- **Observables for transversely polarized targets may have larger $m^2/Q^2, t/Q^2$ corrections; a separate study required**
- **Standartization badly needed for all steps, starting from the Compton tensor**

$$\begin{aligned}\mathcal{A}_{\mu\nu}(q,q',p) &= \varepsilon^+_\mu \varepsilon^-_\nu \mathcal{A}^{++} + \varepsilon^-_\mu \varepsilon^+_\nu \mathcal{A}^{--} + \varepsilon^0_\mu \varepsilon^-_\nu \mathcal{A}^{0+} \\ &+ \varepsilon^0_\mu \varepsilon^+_\nu \mathcal{A}^{0-} + \varepsilon^+_\mu \varepsilon^+_\nu \mathcal{A}^{+-} + \varepsilon^-_\mu \varepsilon^-_\nu \mathcal{A}^{-+} + q'_\nu \mathcal{B}_\mu\end{aligned}$$