Radiative Corrections in DVCS

Igor Akushevich

Duke University, Durham, NC

and

Jefferson Laboratory, Newport News, VA

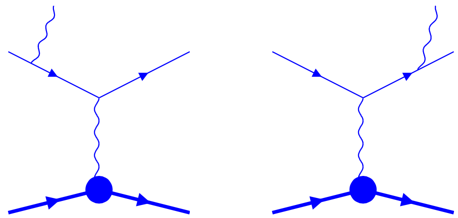
e-mail: igor.akushevich@duke.edu *or* ia6@duke.edu

Outline

- Contributions to RC in $e + p \to e' + p' + \gamma$
  - Feynman graphs of RC to BH and DVCS processes
  - Leading Log and Next-To-Leading Contributions
- Radiative Corrections to BH with Leading Accuracy
  - Details of Analytic Calculation;
  - Numeric Results.
- Radiative Corrections to BH with Next-to-Leading Accuracy
  - Details of Analytic Calculation (loop contributions, infrared divergence, total cross section, etc.);
  - Numeric Results: Comparison LO and NLO
- Radiative Corrections to DVCS
  - Contributions to RC and Covariant Hadronic Tensor
  - Calculation in with Leading Accuracy and Numerical Results
- Codes for Numerical Calculation of RC
  - Analytical codes for RC to DVCS in LO and NLO
  - Monte Carlo Generators: approach and available codes

Base processes: BH and DVCS

- BH and DVCS amplitudes



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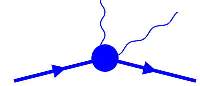
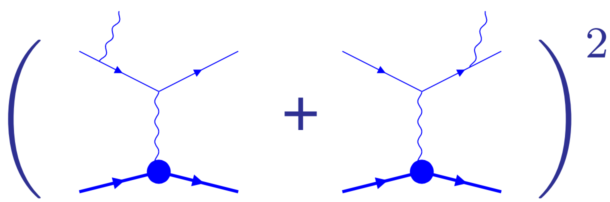
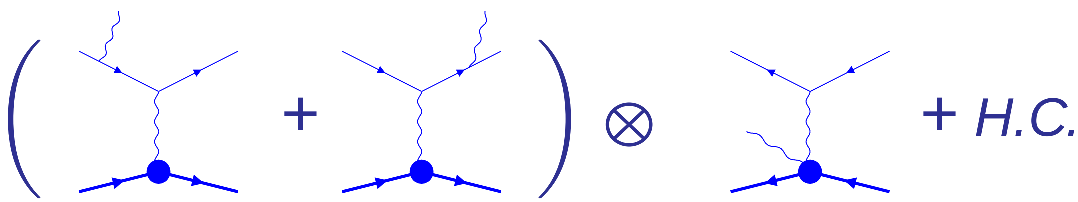


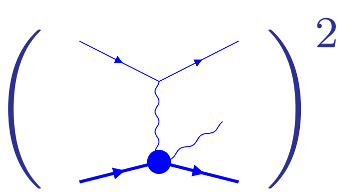
- Three respective contributions to the cross section of the process $e + p \rightarrow e' + p' + \gamma$
  - BH process



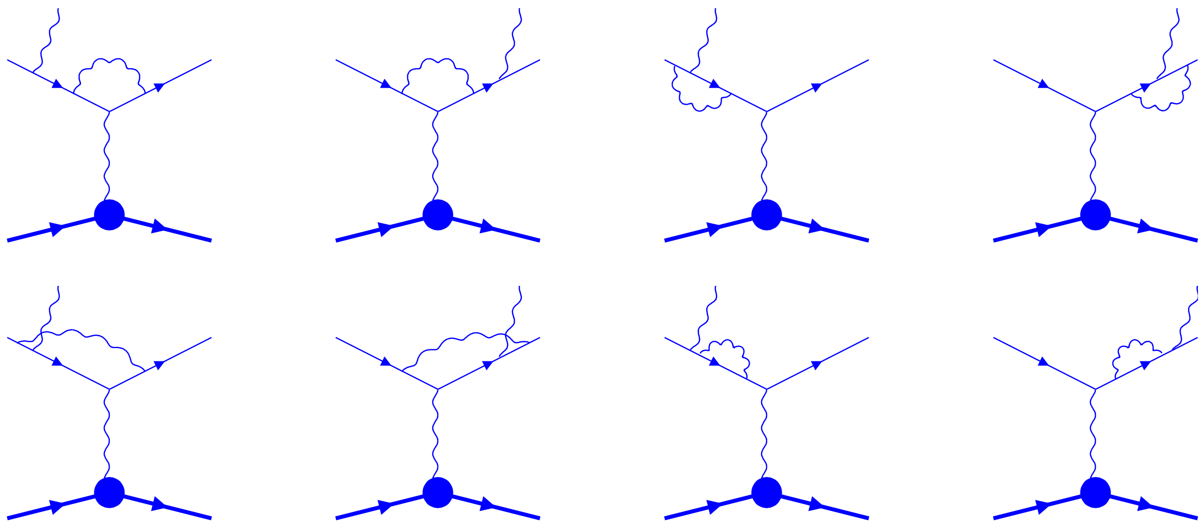
  - Interference of BH and DVCS amplitudes



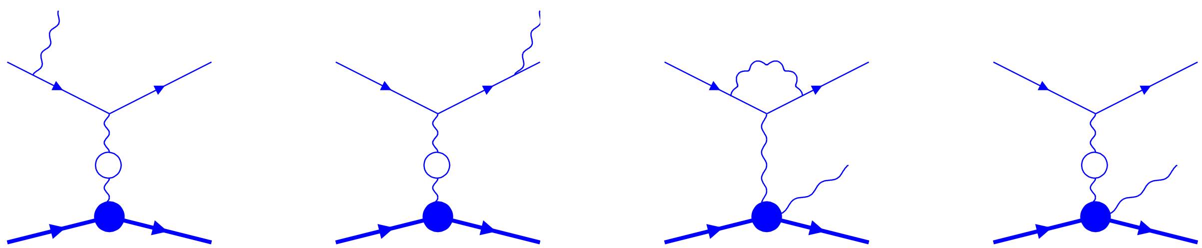
  - Pure DVCS process



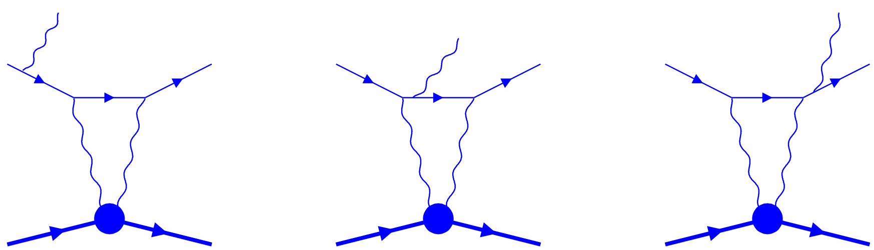
Structure of RC: One-loop contribution



One-loop Correction: Emission of real and additional virtual photons from leptonic line



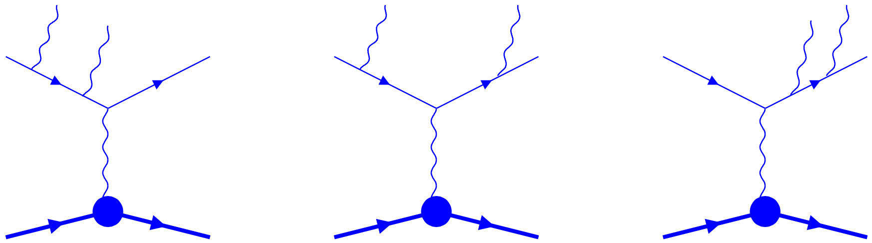
Correction due to vacuum polarization and One-loop correction with real photon emission from hadron line



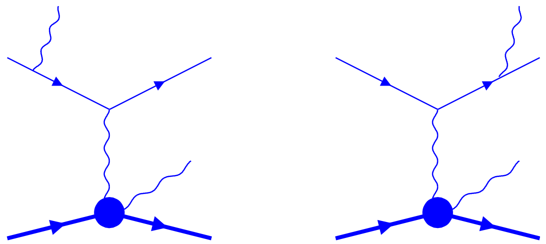
Box-diagram contribution.

Structure of RC: Emission of 2γ

Two real photon emission from lepton line

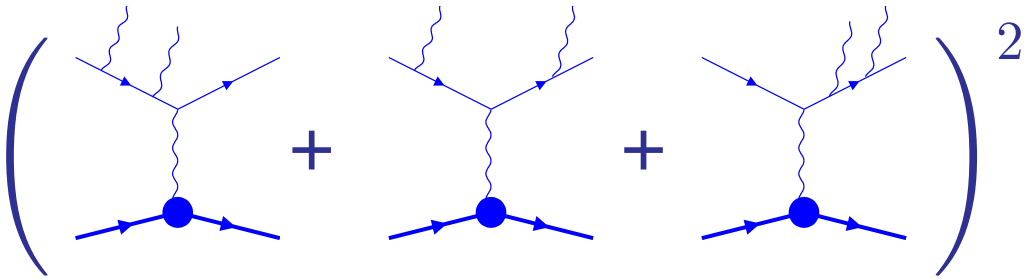


One real photon emission from lepton line and one real photon emission from hadronic line

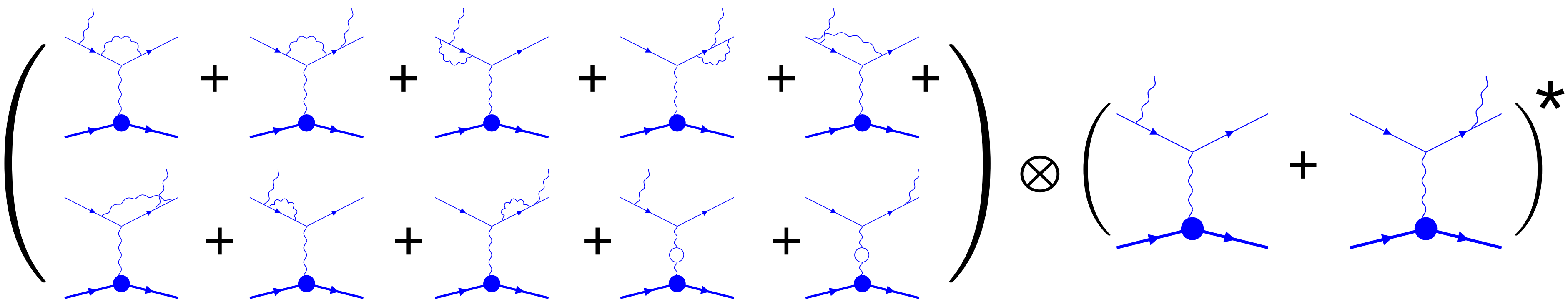


Structure of RC to BH

The contribution to the BH cross section due to additional photon emission:



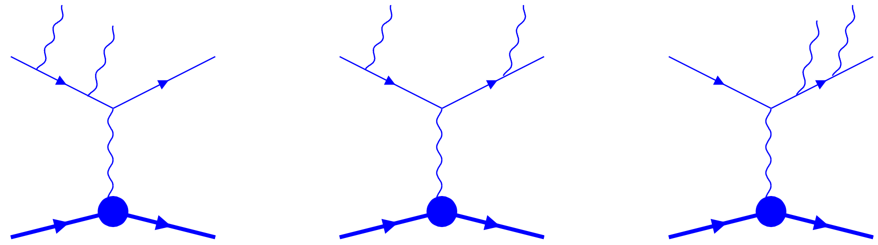
The contribution of loops:



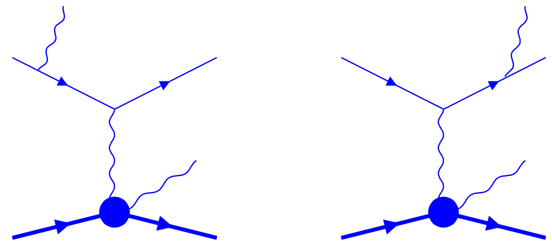
plus complex conjugate

RC to the interference of BH and DVCS: Emission of 2γ

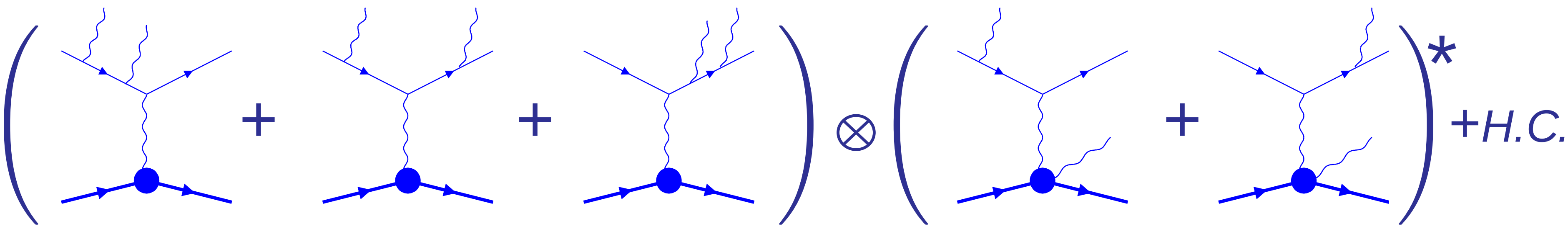
Two real photon emission from lepton line



One real photon emission from lepton line and one real photon emission from hadronic line

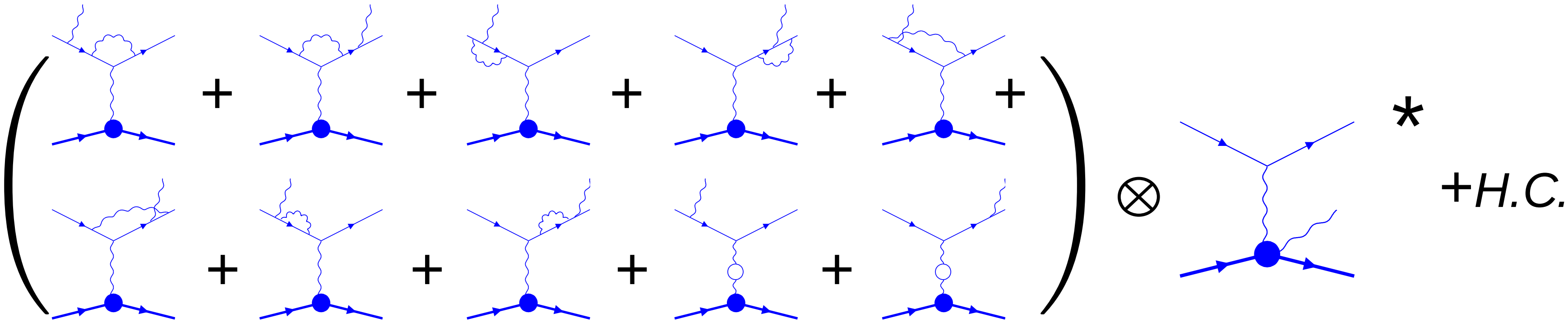


The contribution to the helicity dependent part of DVCS cross section

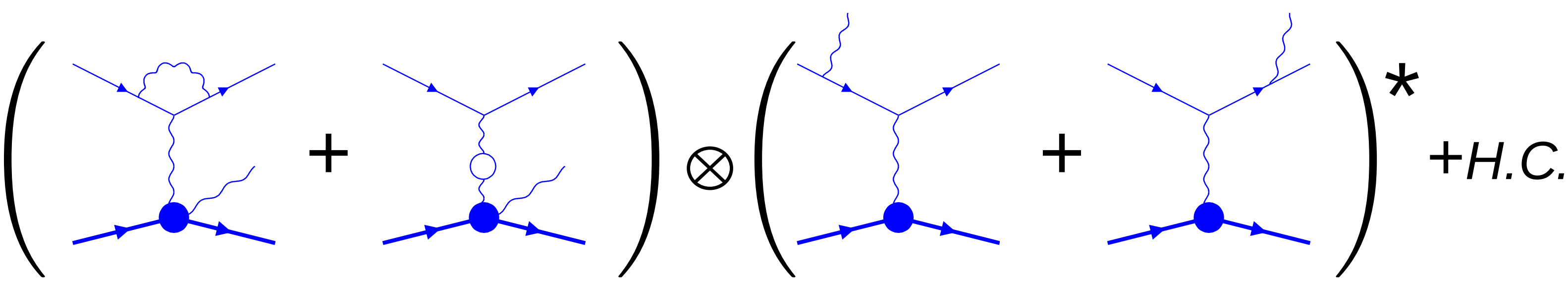


RC to the interference of BH and DVCS: Loop Effects

Two contributions need to be taken into account



The second contribution



Steps in RC calculation

- Matrix element squared.
- Integration over loops and taking care on ultraviolet divergence (i.e., making the electron charge and mass renormalization).
- Phase space parametrization and integration over a part of kinematical variables of an additional photon
  - BH cross section is defined by four kinematical variables: x, Q^2, t and ϕ.
  - The cross section with two photons emitted is defined by seven kinematical variables:
    - *the same four variables: x, Q^2, t and ϕ and*
    - *three additional variables: two-photon invariant mass V^2 and two angles of the photon pair.*
- Extract and cancel the infrared divergence without making new assumptions.
- Add a contribution of higher order corrections (calculated approximately).
- Code the results to have
  - A program for RC calculation in a kinematical point defined by x, Q^2, t and ϕ.
  - Monte Carlo Generator with inclusion of RC contributions.
- Analyze uncertainties in RC calculation.

Leading, Next-to-Leading, and Exact Contributions to RC

By “exactly” calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy.

The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where A and B do not depend on the electron mass.

$\log(\frac{Q^2}{m^2}) \sim 15$ for $Q^2 \sim 1\mathrm{GeV}^2$

- If only A is kept, this is the leading log approximation.
- If both contributions are kept (i.e., contained A and B), this is the calculation with the next-to-leading accuracy, practically equivalent to exact calculation.

Theoretical Background

One-loop correction and soft photon emission, Vanderhaeghen et al. Phys.Rev. C62(2000)025501

- Some ideas of one-loop correction calculation including ultraviolet and infrared renormalization using dimensional regularization.

Calculation in leading approximation, Bytev, Kuraev, Tomasi-Gustafsson, Phys.Rev. C77, 055206 (2008)

- Approach for the calculation in leading log approximation, shifted kinematics, expression for loops effect in leading approximation.

The calculations of the next order corrections to the radiative tails from elastic peaks, Akhundov, Bardin, and Shumeiko, Yad. Fiz. 44, 1517 1986 (Sov. J. Nucl. Phys. 44, 988 1986)

- Phase space parametrization of two photons, exact approach for extraction of infrared divergence.

Table of integrals. Arbuzov, Belitsky, Kuraev, Shaikhatdenov, JINR E2-98-53, hep-ph/0703048

- Asymptotical expressions for loop integrals in non–collinear kinematics.

The theory of DVCS from Belitsky, Mueller, Kirchner (Nucl. Phys. B629(2002)323)

- Expressions for cross check of BH and DVCS cross section, hadronic tensor for DVCS.

Our recent calculation in leading log approximation, Akushevich, Ilyichev, Phys.Rev. D85 (2012) 053008

- Results for leading log approximation.

Part II:

Radiative Corrections to BH with Leading Accuracy

Two-photon emission: Matrix elements

Six matrix elements of the process are denoted $\mathcal{M}_{1-6} = e^4 t^{-1} J^h_\mu J_{1-6,\mu}$, where

$$J_{1\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_2 \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1 \qquad J_{2\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_1 \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1$$

$$J_{3\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1 \qquad J_{4\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1$$

$$J_{5\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1 \qquad J_{6\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1$$

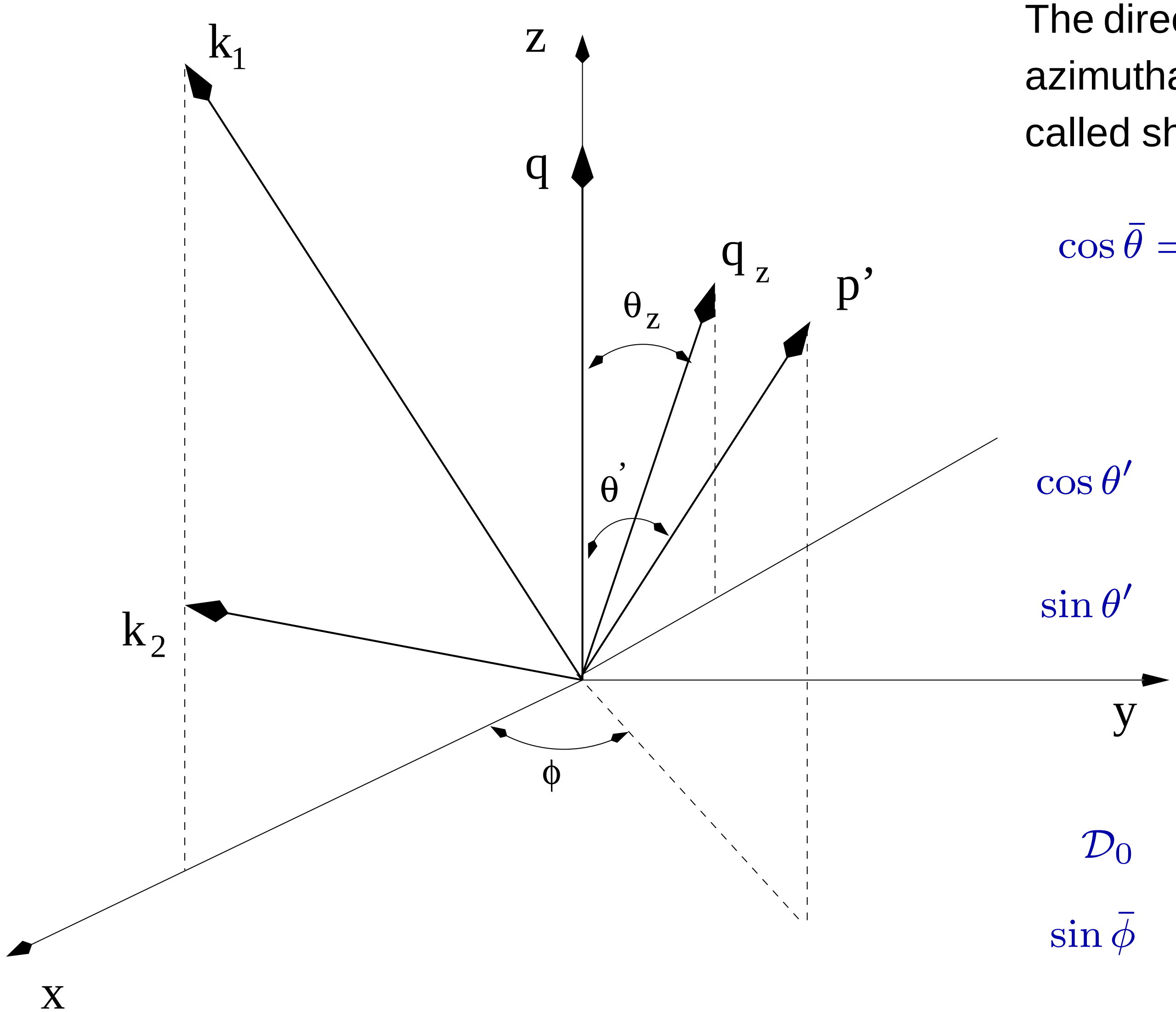
where $V^2 = \kappa^2 = (\kappa_1 + \kappa_2)^2$.

For s-peak (p-peak) the additional unobserved photon is emitted in the direction of the initial (final) lepton. Therefore,

$$\Big(\sum_{i=1}^{6} \mathcal{M}_i\Big)^2 = \mathcal{M}^2_{1s} + \mathcal{M}^2_{1p} + \mathcal{M}^2_{2s} + \mathcal{M}^2_{2p}$$

where indices correspond to the unobserved photon, e.g., $1s$ means that the photon with momentum κ_1 is unobserved and in the s-peak.

Definitions of vectors and angles in the Lab. frame



The direction of q_z defines new polar ($\bar\theta$) and azimuthal ($\bar\phi$) angles of the final proton (so-called shifted kinematics):

$$\cos\bar\theta = \cos\theta'\cos\theta_z - \sin\theta'\sin\theta_z\cos\phi$$

$$\cos\theta' = \frac{A\cos\theta_z + \sqrt{\mathcal{D}_0}\sin\theta_z\cos\phi}{\cos^2\theta_z + \sin^2\theta_z\cos^2\phi}$$

$$\sin\theta' = \frac{\cos\theta_z\sqrt{\mathcal{D}_0} - A\sin\theta_z\cos\phi}{\cos^2\theta_z + \sin^2\theta_z\cos^2\phi}$$

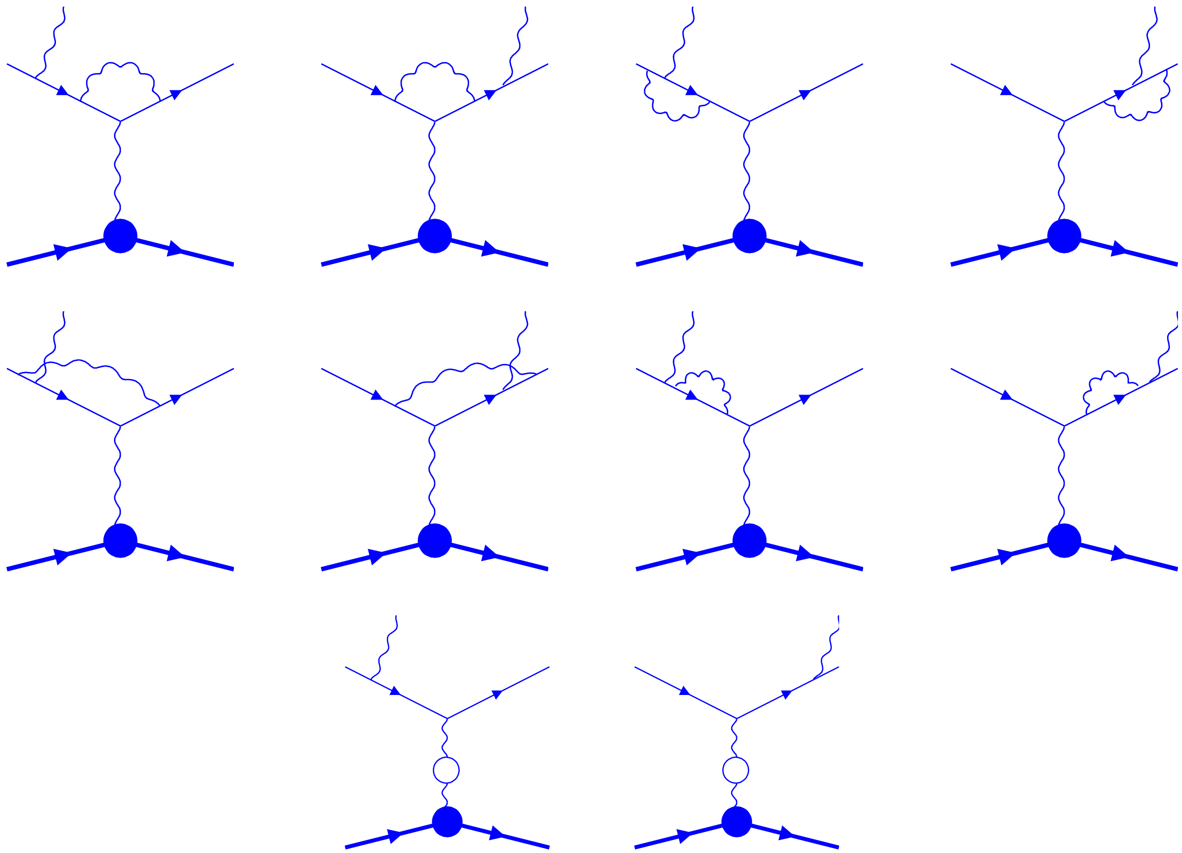
$$\mathcal{D}_0 = \cos^2\theta_z + \sin^2\theta_z\cos^2\phi - A^2$$

$$\sin\bar\phi = \frac{\sin\theta'\sin\phi}{\sin\bar\theta}$$

The “shifted” kinematics is completely calculatable (Bytev, Kuraev, Tomasi-Gustafsson, PR C77 (2008) 055206; Akushevich, Ilyichev, PR D85 (2012) 053008)

Loop diagrams

Feynman graphs of one-loop effects for the BH cross section



$$\sigma_V = \frac{\alpha}{\pi}\left(\log\frac{4M^2\omega^2_{min}}{SX} + \frac{3}{2}\right)L\sigma_{BH} = -\frac{\alpha L}{2\pi}\sigma_{BH}\left(\int\limits_0^{1-\Delta_1} dz_1 \frac{1+z_1^2}{1-z_1} + \int\limits_0^{1-\Delta_2} dz_2 \frac{1+z_2^2}{1-z_2}\right)$$

The lowest order RC

$$\sigma_{obs}(S,x,Q^2,t,\phi) = (1+2\Pi(t))\sigma_{BH}(S,x,Q^2,t,\phi) + \frac{\alpha}{2\pi}L\Bigg[$$

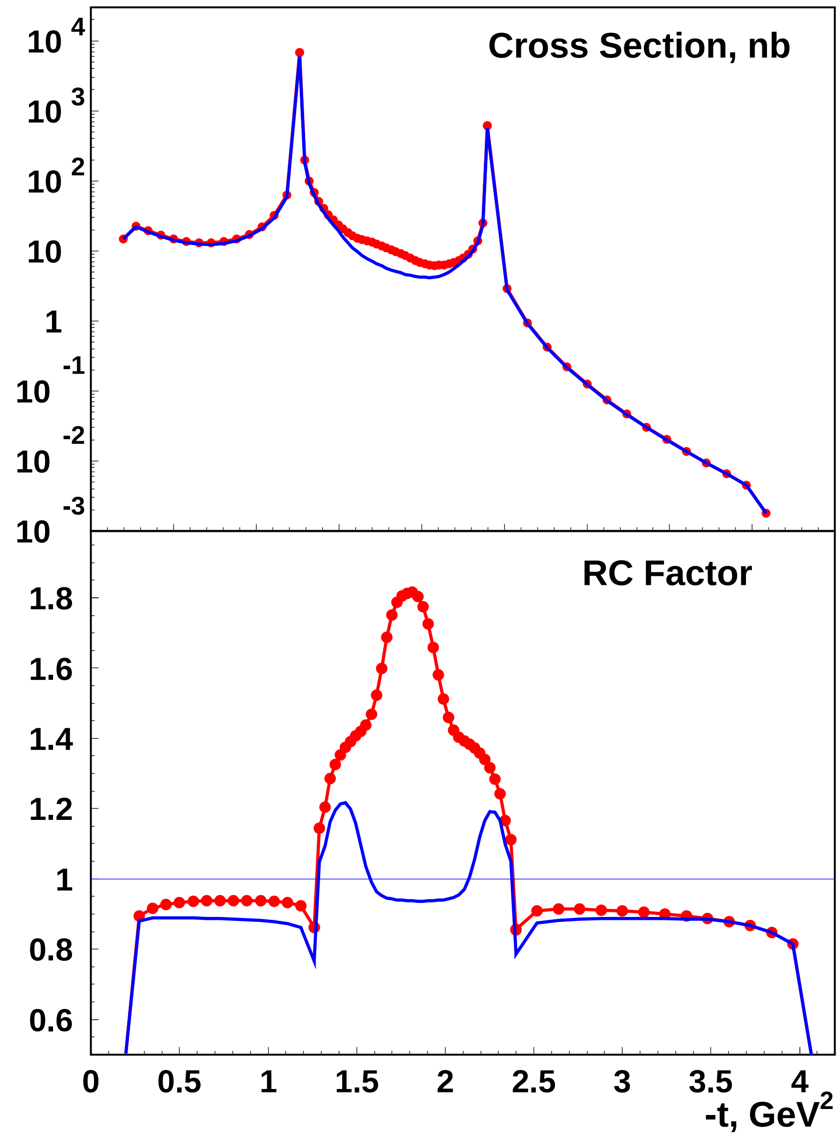
$$\int\limits_0^1 dz_1 \left(\frac{1+z_1^2}{1-z_1}\right)\left(\frac{\sin\theta_s'}{\mathcal{D}_{0s}^{1/2}}\theta(z-z_1^m)\left(\frac{x_s}{x}\right)^2\sigma_{BH}(z_1S,x_s,z_1Q^2,t,\bar{\phi}_s) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)$$

$$+\int\limits_0^1 dz_2 \left(\frac{1+z_2^2}{1-z_2}\right)\left(\frac{\sin\theta_p'}{\mathcal{D}_{0p}^{1/2}}\theta(z-z_2^m)\frac{1}{z_2}\left(\frac{x_p}{x}\right)^2\sigma_{BH}(S,x_p,z_2^{-1}Q^2,t,\bar{\phi}_p) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)\Bigg]$$

where $x_s = z_1Q^2/(z_1S-X)$ and $x_p = Q^2/(z_2S-X)$ are Bjorken x in shifted kinematics.

Integration limits $z_{1,2}^m$ are defined by experimental cuts (e.g., on missing mass) or kinematics.

Numerical results: Cross section and RC factor

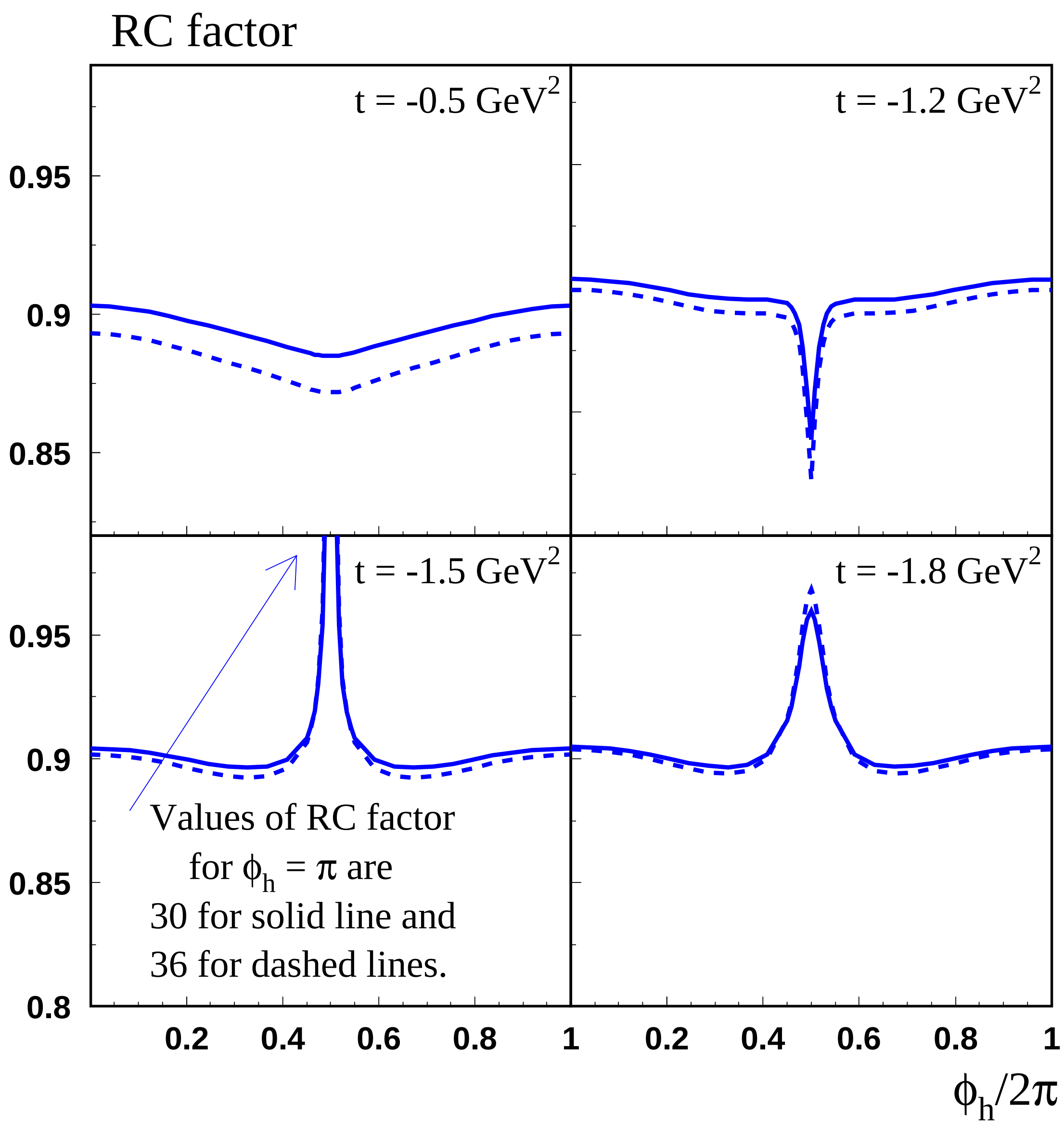


The observed cross sections of the BH process (upper plot) and respective RC factors (lower plot) for beam energy 5.77 GeV, x=0.4, and Q^2=1.8GeV^2.

The red (blue) line shows the results of calculation without (with) the cut on missing energy ($E_\gamma <$0.3 GeV).

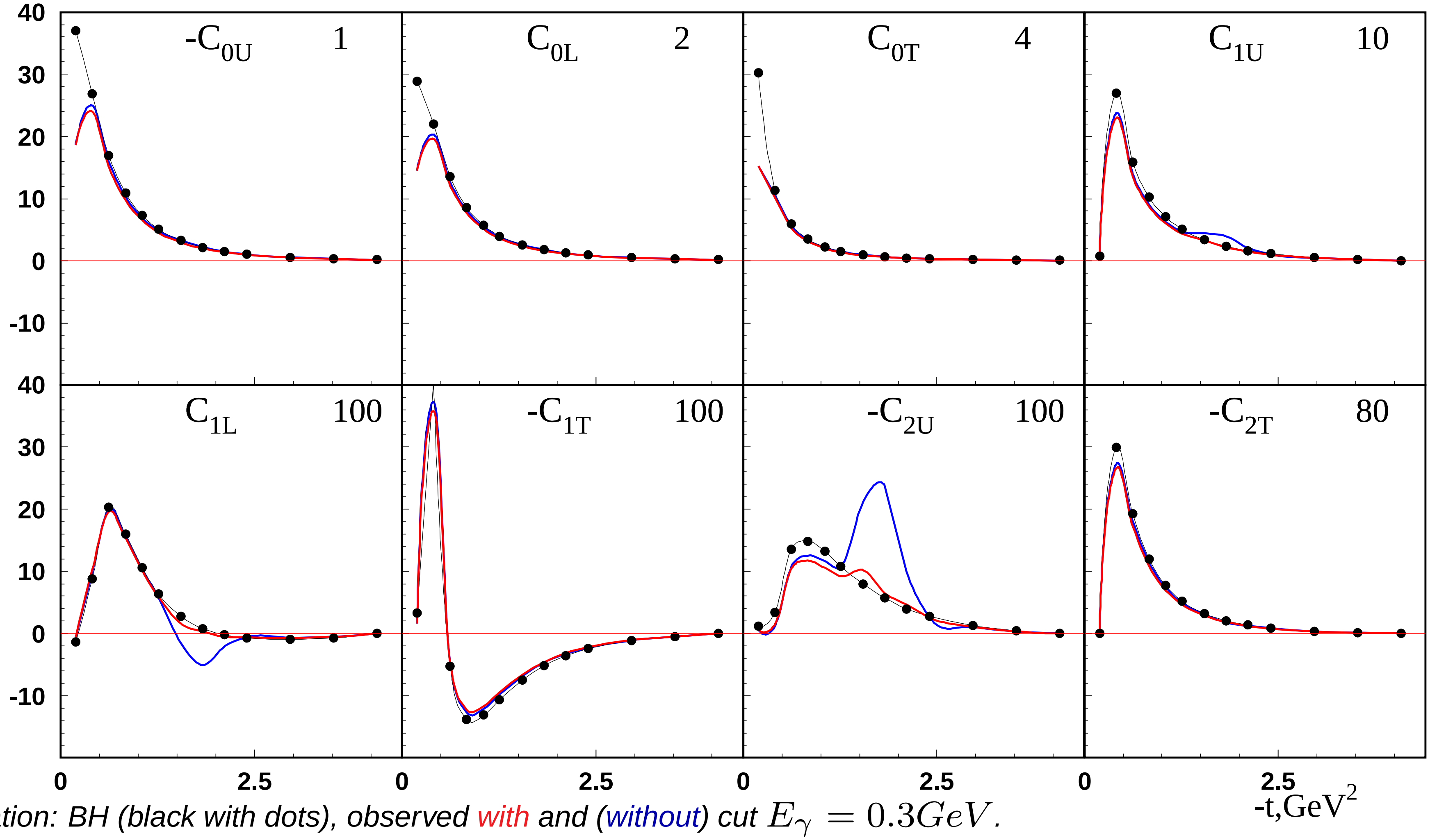
$$\mathrm{RCfactor} = \frac{\sigma_{observed}}{\sigma_{BH}}$$

Numerical results: ϕ dependence of RC factor



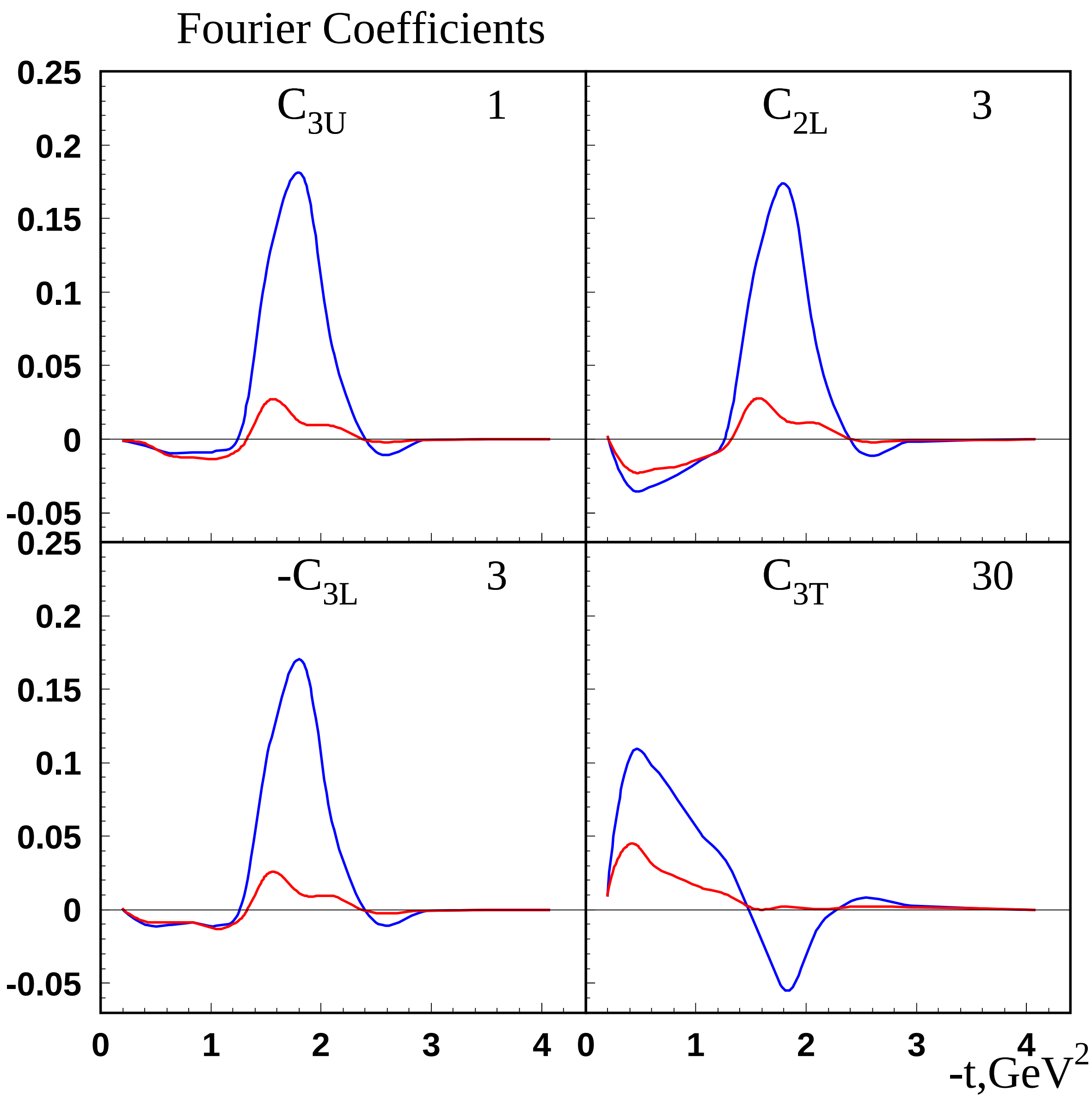
Numerical results: Fourier Coefficients

$$C_{nPol} = \frac{1}{2\pi f} \int_0^{2\pi} d\phi \cos(n\phi) \ \mathcal{P}_1 \mathcal{P}_2 \ \sigma_{BH,Pol}, \qquad Pol = U, L, P$$



Notation: BH (black with dots), observed with and (without) cut $E_\gamma = 0.3 GeV$.

Numerical results: ϕ dependence



Part III:

Radiative Corrections to BH with Next-to-Leading Accuracy

Why we need an exact calculation

By "exactly" calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy.

The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where A and B do not depend on the electron mass.

$\log(\frac{Q^2}{m^2}) \sim 15$ for $Q^2 \sim$ 1GeV^2

If only A is kept, this is the leading log approximation.

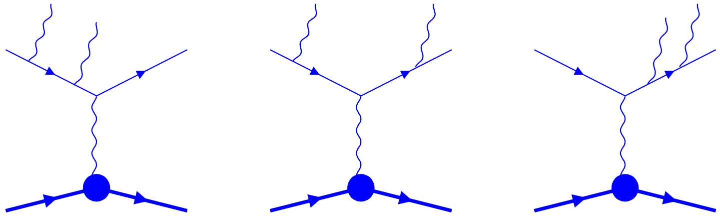
Unpleasant feature is that $B > A$ or even $B \gg A$, especially for the contribution with additional photon emission.

Approximate Calculation: Strengths and Limitations

- Strengths:
  - Expressions for RC are clear, transparent and easy to code.
  - The results are independent on the model of hadronic structure in the larger extent. Therefore they allow for calculations for all contributions to the total cross sections (e.g., for BH squared contribution, the interference term) when respective code for the Born cross section is available.
  - The formulas allow for generalization for the next-to-leading order and multiple photon emission.
- Limitations:
  - Only leading contribution is reconstructed exactly, i.e., if the correction is of the form $\sigma_{RC} = A\log(Q^2/m^2) + B + O(m^2/Q^2)$ then the approach reconstruct A exactly and B in some approximation. The quality of this approximation is difficult to control without exact formulas.
  - Implementation of experimental cuts usually focused on suppression of main contribution could lead to inexact results for RC evaluation.

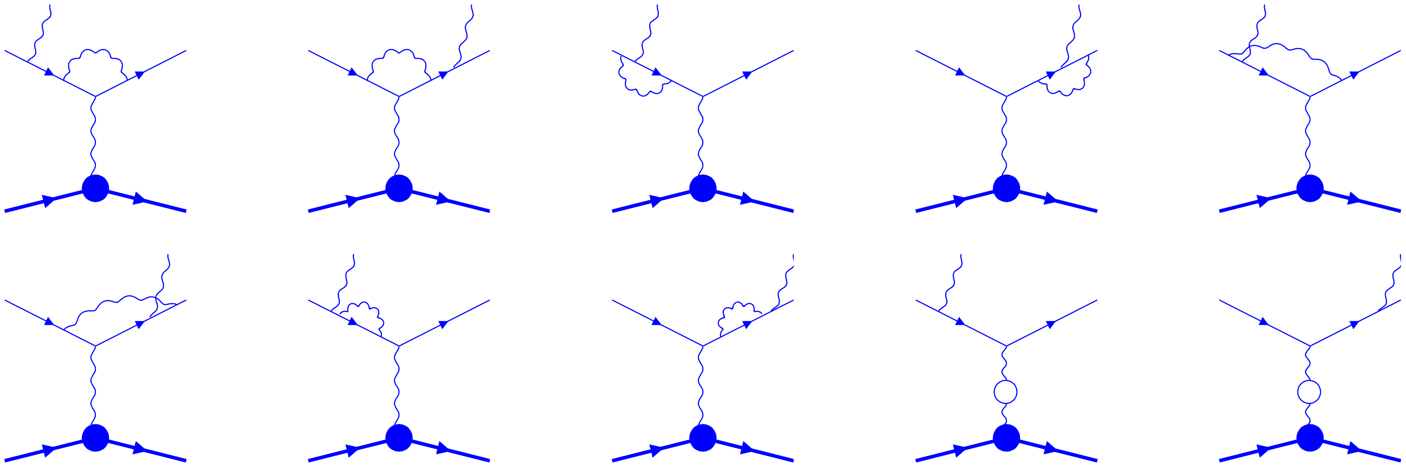
Steps in RC calculation to BH process

- Cross section of Two-Photon Emission process



  - Matrix element squared.
  - Phase space parametrization and integration over part of kinematical variables of an additional photon

- Loops and infrared divergence



Loop effects: Details of Calculation

- The ultraviolet divergence cancels with respective contributions of counterterms that are obtained in the form:

$$\delta_1 = \delta_2 = 2(2P_{IR}+P-2), \quad \delta_m = 2(2P_{IR}+4P-4), \quad P = P_{IR} = \frac{1}{d-4}+\frac{\gamma}{2}+\log\Big(\frac{m}{2\mu\sqrt{\pi}}\Big)$$

- The cross section of loop effects is found in the form:

$$\sigma_{loop} = \frac{\alpha}{\pi}\Big(\delta^v_{IR}+\delta^v_m\Big)\sigma^{BH}+\sigma_F$$

where $\delta^v_{IR} = -\frac{1}{2}\big(4P_{IR}(L_m-1)+1\big)$ and $\delta^v_m = -\frac{1}{2}\big(L_m^2-3L_m-\pi^2/3\big)$.

- The approach of Arbuzov, Belitsky, Kuraev, Shaikhatdenov was used for two-, three-, and four-denominator. Vector and tensor integration is performed for integrals containing l_μ and $l_\mu l_\nu$ in numerator.

$$J_{012q} = -\frac{P_{IR}L_m}{w}+\frac{1}{wQ^2}\left(2L_mL_w-L_t^2-\Phi\Big(1-\frac{t}{Q^2}\Big)-\frac{\pi^2}{6}\right)$$

$$J^\mu_{0q} = \left(-P+1-\frac{1}{2}L_w\right)(k_{1\mu}-k_\mu)$$

where $L_m = \log(Q^2/m^2)$, $L_w = \log(w/m^2)$, $L_t = \log(-t/m^2)$.

- Important is that all integrals can be calculated analytically.

Two-photon emission: Phase Space

The phase space for the cross section $d\sigma_r = \frac{M_r^2}{2S(2\pi)^8} d\Gamma$ of the process $e + p \to e' + p' + \gamma_1 + \gamma_2$ is parametrized as

$$d\Gamma = \frac{dp_2}{2E_2}\frac{dk_1}{2\omega_1}\frac{dk_2}{2\omega_2}\frac{dp'}{2E'}\delta(p + p_1 - p_2 - p' - k_1 - k_2) = d\Gamma_0 dV^2 d\Gamma_{2\gamma}$$

where V is invariant mass of two photons and

$$d\Gamma_{2\gamma} = \frac{dk_1}{2\omega_1}\frac{dk_2}{2\omega_2}\delta(p + p_1 - p_2 - p' - k_1 - k_2) = \frac{1}{8} d\Omega_R = \frac{1}{8}\cos(\theta_R) d\phi_R$$

The four kinematical variables to describe the kinematics (and phase space) of one-photon emission process are usual:

$$Q^2, t, x_B, \phi_h$$

Three additional variables to describe the kinematics of the additional photon is

$$V^2, \phi_R, \theta_R.$$

Analytical Integration

- Integration of the matrix element squared over $d\Gamma_{2\gamma}$ (or over ϕ_R and θ_R) can be performed analytically.
- 69 specific integrals including vector and tensor integrals were calculated and combined in the table of integrals.
- All integrals were calculated analytically and the results of analytical integration were tested numerically.
- Note, the integration performed exactly: even the approximation of the small lepton mass is not required.
- The integrals were calculated using the system of center-mass of two photons. The result of the integration is represented in covariant form, and therefore can be presented in Lab. system.

Example of Analytical Integration

$$\mathrm{J}[A] = \frac{2}{\pi}\int d\Gamma_{2\gamma}\; A = \frac{1}{4\pi}\int d\Omega_R\; A = \frac{1}{4\pi}\int d\cos\theta_R\phi_R\; A.$$

the angles θ_R and ϕ_R define the orientation of momenta of photons in the system where $\vec{k}=0$, i.e., in the two-photon central mass system.

$$\mathrm{J}[1]=1; \qquad \mathrm{J}[\frac{1}{w_1}] = \mathrm{J}[\frac{1}{w_2}] = L_1 = \frac{1}{\sqrt{\lambda_1}}\log\frac{w+\sqrt{\lambda_1}}{w-\sqrt{\lambda_1}}; \qquad \mathrm{J}[\frac{1}{w_1^2}] = \mathrm{J}[\frac{1}{w_2^2}] = \frac{1}{m^2V^2}$$

$$\mathrm{J}[\frac{1}{u_1^2w_2}] = \mathrm{J}[\frac{1}{u_2^2w_1}] = \frac{1}{\lambda_I}(\frac{u_I}{m^2V^2}+w_IL_I); \quad \mathrm{J}[\frac{D}{u_1^2w_2}] = -\mathrm{J}[\frac{D}{u_2^2w_1}] = \frac{1}{2\lambda_I}\left(\frac{\Phi_{I2}}{m^2V^2}-\Phi_{I1}L_I\right)$$

The new variables are functions of kinematical variables, e.g.,

$$w_I = w(wu - V^2Q^2) - 2m^2V^2(w+u), \quad V_I = wu - V^2Y_m, \quad \lambda_I = V_I^2 - 4m^4V^4,$$

$$W_{Ip} = w^2u - V^2w_Y, \quad U_{Ip} = u^2w - V^2u_Y, \quad L_I = \frac{1}{\sqrt{\lambda_I}}\log\frac{V_I+\sqrt{\lambda_I}}{V_I-\sqrt{\lambda_I}},$$

$$\Phi_{I1} = S_{xt}W_{Ip} - 2V^2(SV_I + 2m^2V^2X), \quad \Phi_{I2} = S_{xt}U_{Ip} - 2V^2(XV_I + 2m^2V^2S), \quad \ldots$$

Infrared Divergence

The cross section containing IR is represented in the form:

$$\frac{d\sigma_{IR}}{d\Gamma_0} = \frac{\alpha}{\pi}\delta_R^{IR}\frac{d\sigma_0}{d\Gamma_0}, \quad \delta_R^{IR} = \frac{1}{4\pi}\int\limits_0^{V_m^2} dV^2 \int d\Gamma_{2\gamma} 4(F_1^{IR} + F_2^{IR})$$

where $F_{1,2}^{IR} = \left(\frac{k_2}{u_{1,2}} - \frac{k_1}{w_{1,2}}\right)^2 = \frac{Q^2+2m^2}{u_{1,2}w_{1,2}} - \frac{m^2}{u_{1,2}^2} - \frac{m^2}{w_{1,2}^2}$, $w_{1,2} = 2k_1\kappa_{1,2}$, and $u_{1,2} = 2k_2\kappa_{1,2}$.

The integration over the 3-momentum of one of photons and then to over V^2 is performed using the δ-function from phase space. The integration region over the momentum of remaining photon (denoted by κ_{cm}) in the two-photon center-mass system can be split into two parts by an infinitesimal parameter $\bar{\kappa}$ resulting in $\delta_R^{IR} = \delta_1 + \delta_2$ with

$$\delta_1 = \frac{1}{4\pi}\int\limits_0^{V_m^2} dV^2 \int d\Gamma_{2\gamma} 4(F_1^{IR}+F_2^{IR})\theta(\bar{\kappa}-\kappa_{cm}), \quad \delta_2 = \frac{1}{4\pi}\int\limits_0^{V_m^2} dV^2 \int d\Gamma_{2\gamma} 4(F_1^{IR}+F_2^{IR})\theta(\kappa_{cm}-\bar{\kappa})$$

- The second term does not contain infrared divergence and is calculated straightforwardly

$$\delta_2 = 2\log\left(\frac{V_m^2}{4\bar{\kappa}^2}\right)(L_m - 1)$$

Infrared Divergence (cont.)

- The calculation of the first term is performed in the dimensional regularization. The phase space of remaining photon (after integration using the δ-function) is rewritten in d-dimensional space as :

$$\delta_1 = \frac{(2\sqrt{\pi}\mu)^{4-d}}{\Gamma(d/2-1)} \int_0^1 d\alpha \int_0^{\bar{\kappa}} \frac{d\kappa_{cm}}{\kappa_{cm}^{5-d}} \int_{-1}^1 d\zeta (1-\zeta^2)^{\frac{n}{2}-2} \left(\frac{Q^2+2m^2}{(E_\alpha - p_\alpha\zeta)^2} - \frac{m^2}{(E_1-p_1\zeta)^2} - \frac{m^2}{(E_2-p_2\zeta)^2} \right)$$

Energies are taken in the system of center mass of two photons:

$$E_1 = \frac{w}{4k_{cm}}, \quad E_2 = \frac{u}{4k_{cm}}, \quad E_\alpha = \frac{w\alpha + u(1-\alpha)}{4k_{cm}}$$

and $p_\alpha^2 = E_\alpha^2 - m_\alpha^2$, $m_\alpha^2 = m^2 + \alpha(1-\alpha)Q^2$.

- The first step in the calculation is the integration over ζ. The result of this integration involves the hyperheometric function, however allows for expansion over k_{cm}. The forthcoming integration over k_{cm}, extraction of IRD terms, integration over α, and expansion over m keeping only leading and next-to-leading terms result in

$$\delta_R^{IR} = \left(2P_{IR} + \log\left(\frac{V_m^4}{u_0 w_0} \right) \right) (L_m - 1) + \frac{1}{2} L_m^2 - \frac{\pi^2}{6} - \frac{1}{2} \log^2 \frac{u_0}{w_0}$$

RC in the Next-to-Leading Approximation

Combining all contributions we have

$$\frac{d\sigma}{d\Gamma_0} = A \log\frac{Q^2}{m^2} + B + \sum_{i=1}^{4} T_i^v \mathcal{F}_i + \int_0 \frac{dV^2}{V^2} \sum_{i=1}^{4} \Big(T_i^F(V^2) - T_i^F(0) \Big) \mathcal{F}_i$$

- A and B are resulted from the sum of (and infrared divergent) terms of the loop and two-gamma constitutions. They do not depend on the lepton mass.
- $\mathcal{F}_i$ are squared combinations of formfactors, (e.g., $\mathcal{F}_1 = (F_1 + F_2)^2$, $\mathcal{F}_2 = F_1^2 + \tau F_2^2$, $\mathcal{F}_3 = (F_1 + F_2)(F_1 + \tau F_2)$, and $\mathcal{F}_4 = (F_1 + F_2)F_2$. Terms with i=1,2 correspond to unpolarized case , and i=3,4—polarized case.

RC in the Next-to-Leading Approximation

Combining all contributions we have

$$\frac{d\sigma}{d\Gamma_0} = A \log \frac{Q^2}{m^2} + B + \sum_{i=1}^{4} T_i^v \mathcal{F}_i + \int\limits_0 \frac{dV^2}{V^2} \sum_{i=1}^{4} \Big(T_i^F(V^2) - T_i^F(0) \Big) \mathcal{F}_i$$

- A and B are resulted from the sum of (and infrared divergent) terms of the loop and two-gamma constitutions. They do not depend on the lepton mass.
- $\mathcal{F}_i$ are squared combinations of formfactors.
- T_i^v came from nonfactorized (and lepton mass independent) part of the loop cross section:

$$T_i^v = T_{i0}^v + T_{i1}^v P_{yt} + T_{i2}^v P_w + T_{i3}^v P_u + T_{i4}^v F_{1y} + T_{i5}^v F_{2w} + T_{i6}^v F_{2u},$$

where where $\Phi(x)$ is the Spence function defined as $\Phi(x) = -\int_0^x t^{-1} \log|1-t| dt$, T_{ij}^v are rational functions of w_0, u_0, and t and $P_{yt} = \frac{\pi^2}{3} + 2\Phi\left(1 - \frac{t}{Q^2}\right) - \log^2 \frac{t}{Q^2}$, $P_w = \log \frac{t}{Q^2} \log \frac{w_0}{Q^2} - \Phi\left(1 - \frac{w_0}{t}\right)$, $P_u = \log \frac{t}{Q^2} \log \frac{u_0}{Q^2} - \Phi\left(1 + \frac{u_0}{t}\right)$, $F_{1y} = \frac{1}{t_y} \log \frac{t}{Q^2}$, $F_{1w} = \frac{1}{t_w} \log \frac{t}{w_0}$, $F_{1u} = \frac{1}{t_u} \log \frac{u_0}{t}$, $F_{2y} = \frac{F_{1y}-1}{t_y}$, $F_{2w} = \frac{F_{1w}-1}{t_w}$, and $F_{2u} = \frac{F_{1u}-1}{t_u}$.

RC in the Next-to-Leading Approximation

Combining all contributions we have

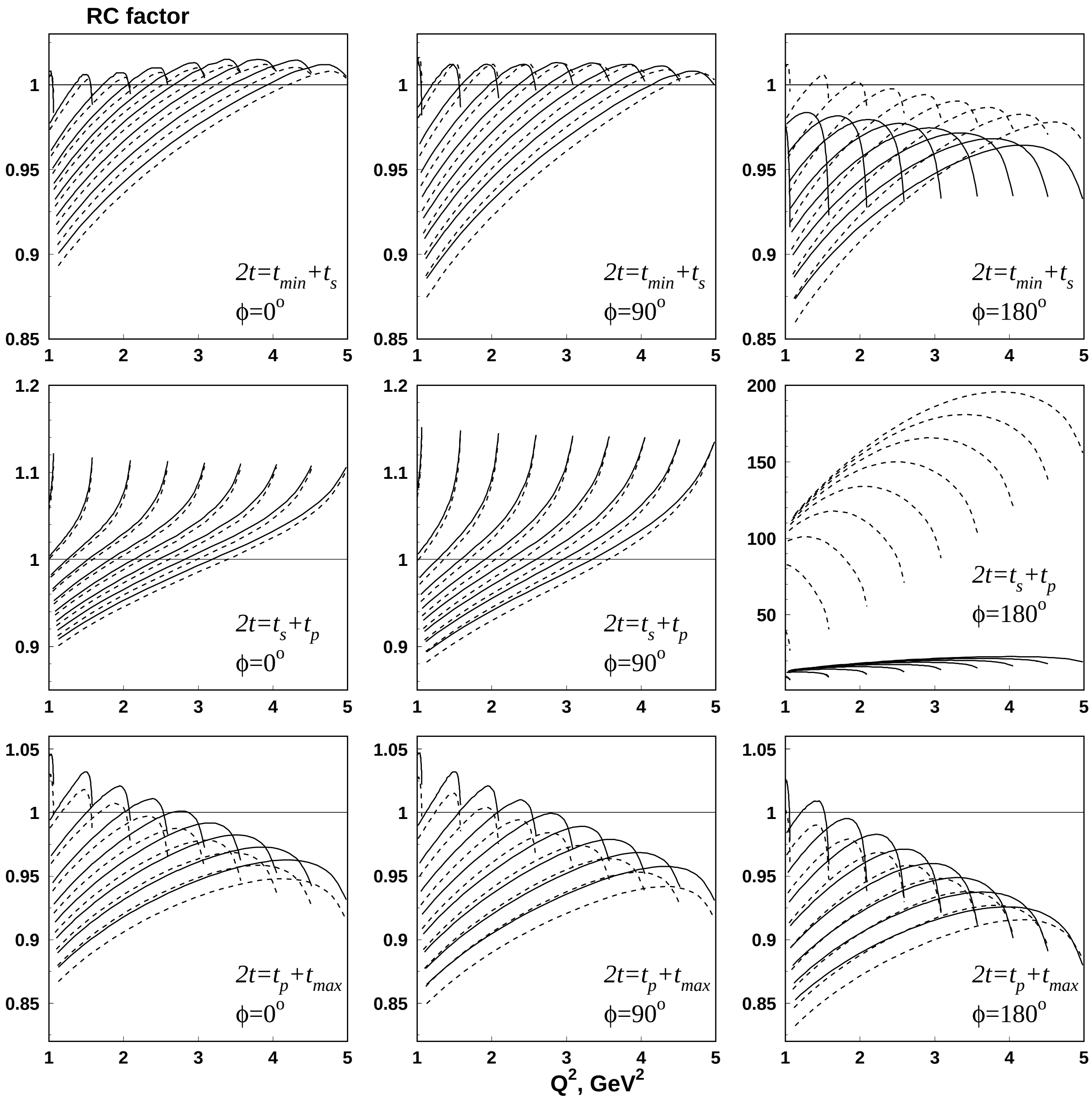
$$\frac{d\sigma}{d\Gamma_0} = A \log \frac{Q^2}{m^2} + B + \sum_{i=1}^{4} T_i^v \mathcal{F}_i + \int_0 \frac{dV^2}{V^2} \sum_{i=1}^{4} \Big(T_i^F(V^2) - T_i^F(0) \Big) \mathcal{F}_i$$

- A and B are resulted from the sum of (and infrared divergent) terms of the loop and two-gamma constitutions. They do not depend on the lepton mass.
- $\mathcal{F}_i$ are squared combinations of formfactors.
- T_i^v came from nonfactorized (and lepton mass independent) part of the loop cross section.
- $T_i^F(V^2)$ came from nonfactorized (can have mass-dependence) part of the contribution of two-gamma contribution:

$$T_i^F = \frac{T_{i1}^F}{w} \log \frac{w^2}{m^2 V^2} + \frac{T_{i2}^F}{u} \log \frac{u^2}{m^2 V^2} + T_{i3}^F \log \frac{V_{\rm I}}{m^2 V^2} + T_{i4}^F \log \frac{Q^2}{m^2} + T_{i5}^F$$

Quantities T_{ij}^F are rational (mass independent) functions of Q^2, u, w, and V^2.

Leading and Next-to-Leading Contributions

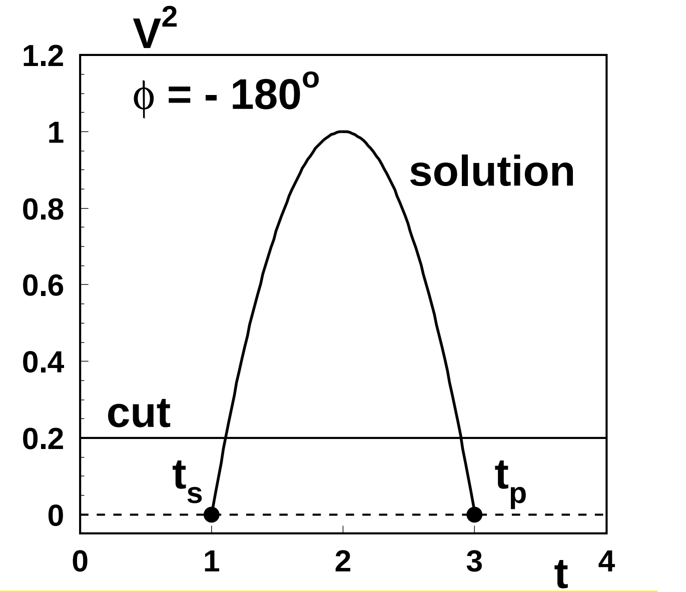


The Q^2 dependence of LO (dashed) and NLO (solid) RC factor for several x, t and ϕ calculated for beam energy equaling 5.75 GeV and without any cuts on the the invariant mass of two photons.

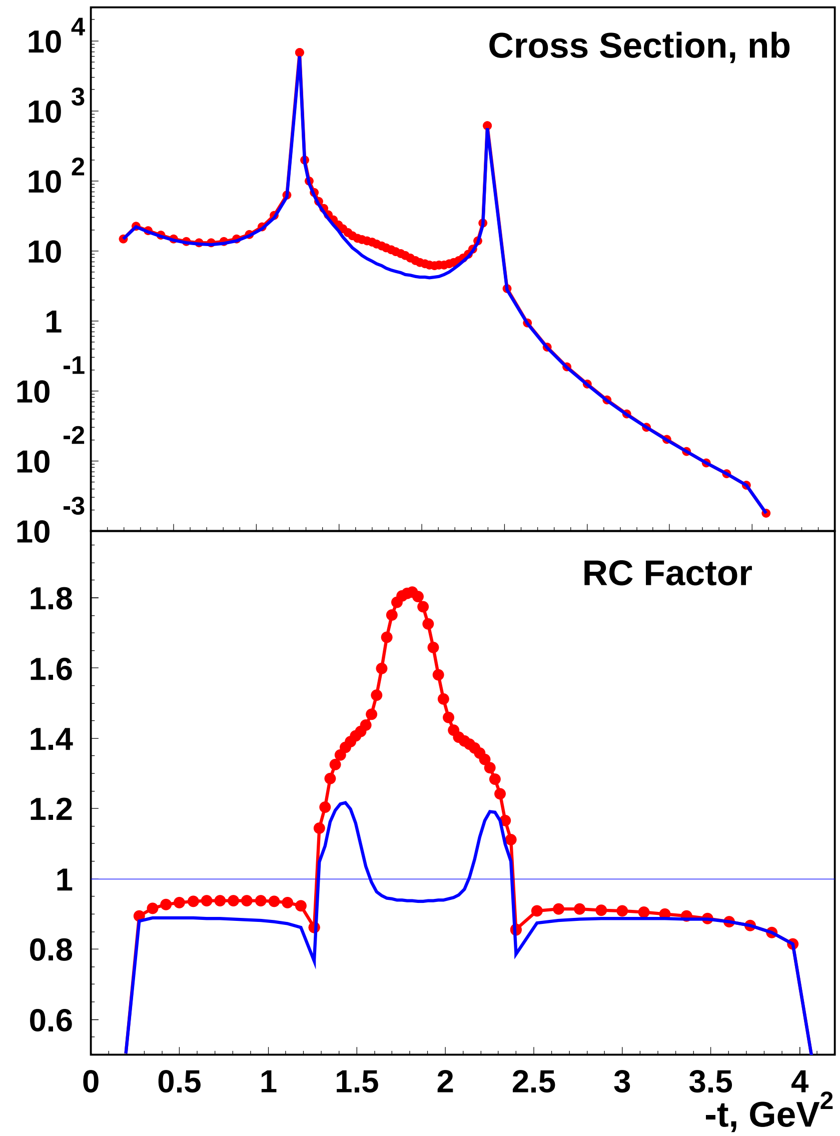
Nine curves at each plot correspond (from the left to the right) to nine values of x: 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, and 0.5. Q^2_{max} is defined by kinematics.

Large effect for ϕ =180^o and $t = (t_s + t_p)/2$

- The large effect comes from the two-photon emission process when both two irradiated photons are collinear: one is collinear to the initial electron and and another is collinear to the final electron.
- The corresponding BH process (i.e., one photon emission process) is the process with the emitted photon with 4-momentum corresponding to the sum of momenta of the two collinear photons. This photon is not collinear and therefore the respective cross section of the BH process is not large.
- This is visible for both sets of formulas:
  - *The first term in LO RC corresponds to s-peak of one of the photon and the second term describe the p-peak. If another photon is collinear to another electron (i.e., final electron for the first term and initial electron for the second term), respective scalar products of the photon with electron momenta occurred in denominator of the BH cross section has to be small.*
  - *In the NLO formulas the double collinear kinematics resulted in* $V_{\rm I} = 0$ *for* $m \to 0$ *(because* $u = V_{\rm I}/w$, $w = V_{\rm I}/u$*). The solution is possible only for* $t_s \le t \le t_p$ *and* $\phi = 180^o$



Numerical results: Cross section and RC factor



The observed cross sections of the BH process (upper plot) and respective RC factors (lower plot) for beam energy 5.77 GeV, x=0.4, and Q^2=1.8GeV^2.

The red (blue) line shows the results of calculation without (with) the cut on missing energy ($E_\gamma <$0.3 GeV).

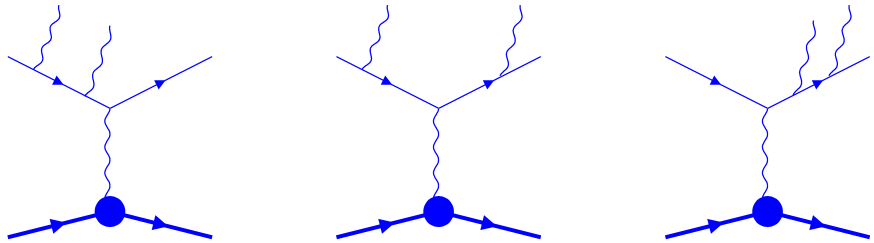
$$\text{RCfactor} = \frac{\sigma_{observed}}{\sigma_{BH}}$$

Part IV:

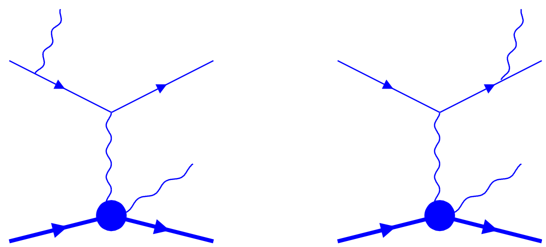
Radiative Corrections to DVCS

RC to DVCS: Emission of two hard photons

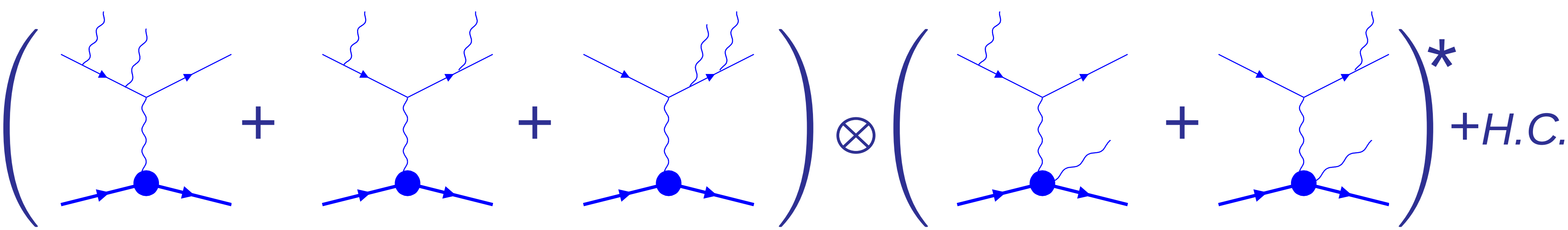
Two real photon emission from lepton line



One real photon emission from lepton line and one real photon emission from hadronic line

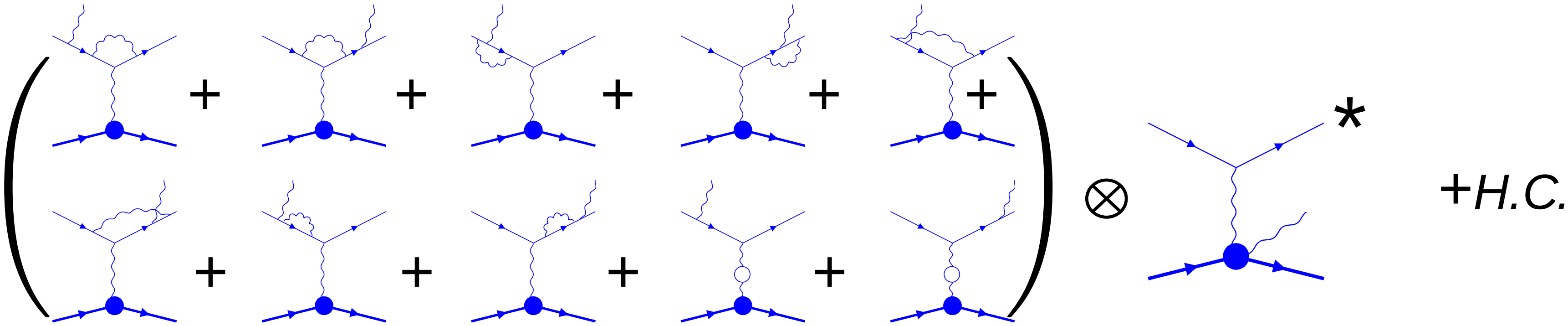


The contribution to the helicity dependent part of DVCS cross section

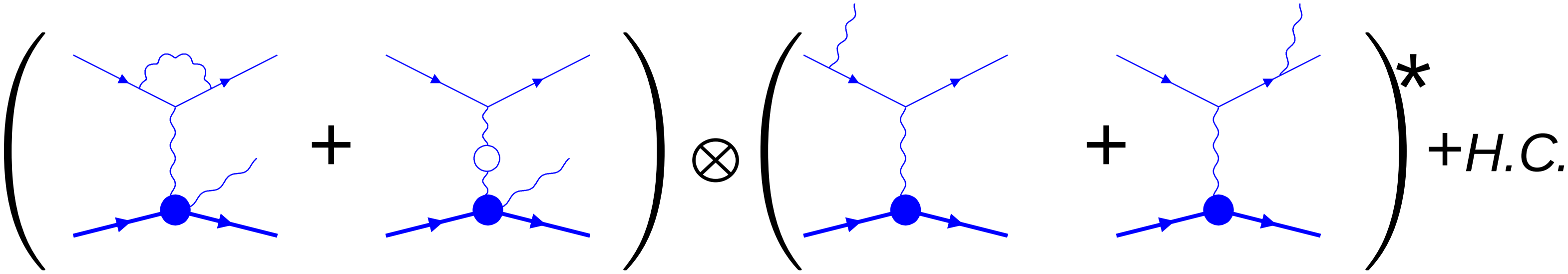


RC to DVCS: Loop Effects

Two contributions need to be taken into account



The second contribution



Calculation of RC to DVCS

Steps in the calculation of RC to DVCS:

- Make sure that the results of analytical calculation of the 1γ-emission cross section in the BMK-approximation involving the covariant hadronic tensor as an intermediate quantity coincide with results of the original paper
- Calculate matrix element squared controlling all assumptions and approximations.
- Parametrize the phase space of all final particles (use the shifted kinematics).
- Add the contribution of loop diagram.
- Implement and estimate the effects of higher order corrections.

The result in leading approximation is (quite expected):

- The expression of the observed cross section through integral (over additional photon energies) on the base DVCS cross cross is in exactly same form as for BH process.

Therefore, we can simply calculate the cross section for complete one-photon-emission process including BH and DVCS contributions and use this formula to calculate the next order RC.

The lowest order RC

$$\sigma_{obs}(S,x,Q^2,t,\phi) = (1+2\Pi(t))\sigma_{BH}(S,x,Q^2,t,\phi) + \frac{\alpha}{2\pi}L\Bigg[$$

$$\int\limits_0^1 dz_1 \left(\frac{1+z_1^2}{1-z_1}\right)\left(\frac{\sin\theta'_s}{\mathcal{D}_{0s}^{1/2}}\theta(z-z_1^m)\left(\frac{x_s}{x}\right)^2\sigma_{BH}(z_1S,x_s,z_1Q^2,t,\bar{\phi}_s) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)$$

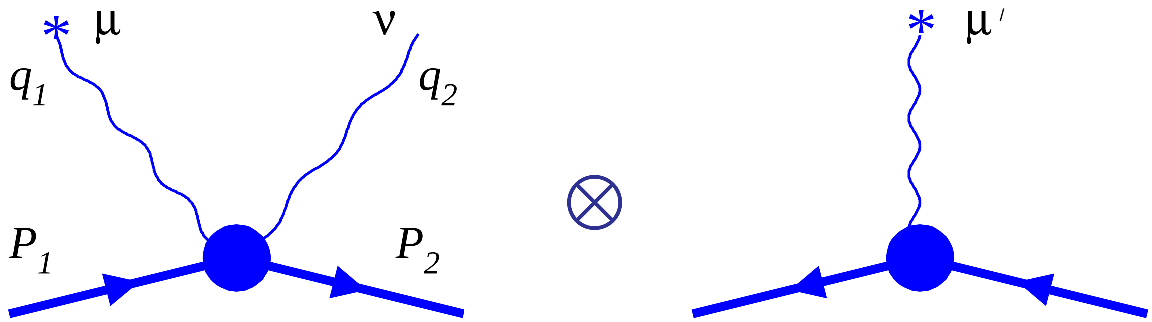
$$+\int\limits_0^1 dz_2 \left(\frac{1+z_2^2}{1-z_2}\right)\left(\frac{\sin\theta'_p}{\mathcal{D}_{0p}^{1/2}}\theta(z-z_2^m)\frac{1}{z_2}\left(\frac{x_p}{x}\right)^2\sigma_{BH}(S,x_p,z_2^{-1}Q^2,t,\bar{\phi}_p) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)\Bigg]$$

where $x_s = z_1Q^2/(z_1S-X)$ and $x_p = Q^2/(z_2S-X)$ are Bjorken x in shifted kinematics. Integration limits$z_{1,2}^m$ are defined by experimental cuts (e.g., on missing mass) or kinematics.

This formula is also valid for interference of BH and DVCS amplitudes and for pure DVCS contribution.

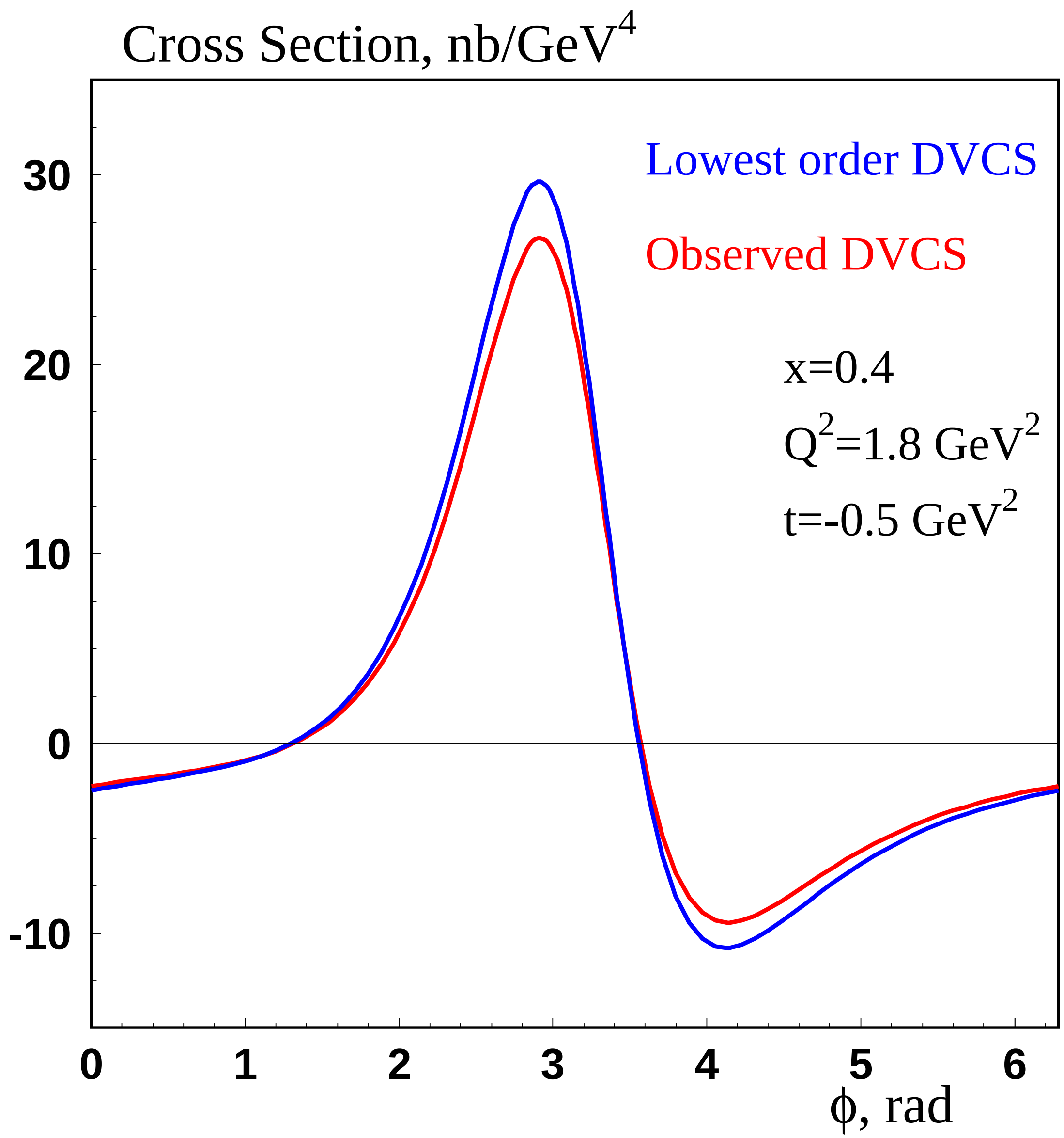
Exact Calculation of RC to DVCS

The calculation of RC for interference term with next-to-leading accuracy requires appealing the model for hadronic state.

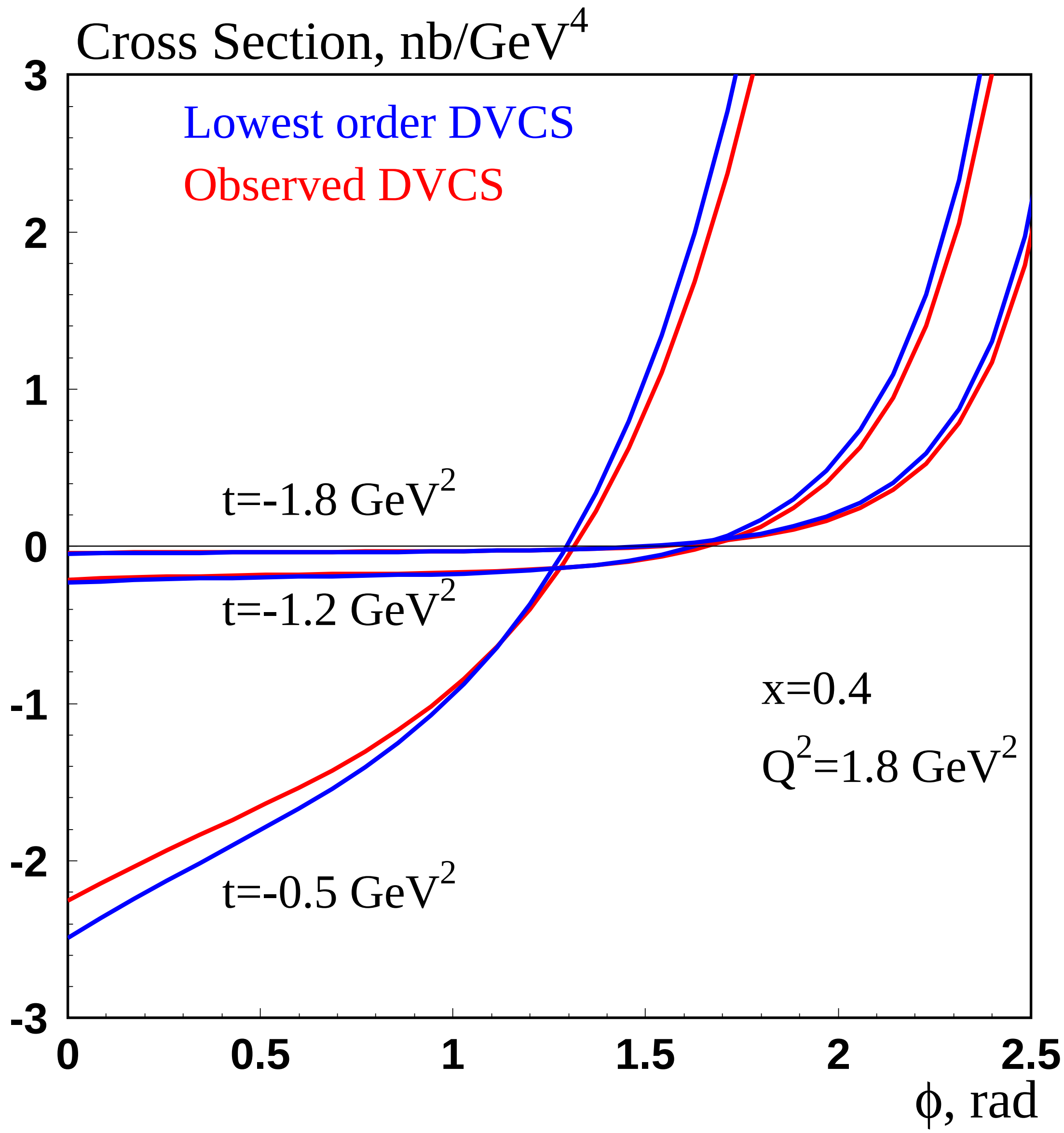


The theory of DVCS from Belitsky, Mueller, Kirchner (*Nucl. Phys. B629(2002)323*) are used to construct covariant hadronic tensor $T_{\mu\nu,\mu'}$ in terms of nucleon formfactors $F_{(p,d)}$, GDPs ($\mathcal{E}$, $\mathcal{H}$, $\tilde{\mathcal{E}}$, $\tilde{\mathcal{E}}^3_+$, etc.), four-vectors ($P = P_1 + P_2$, $\Delta = P_2 - P_1$, $q = (q_1 + q_2)/2$), and kinematic variables (ξ, η, Δ^2, Q^2).

Numerical results: observed cross sections



Numerical results: observed cross sections



Part V:

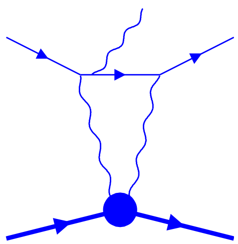
Analytical Codes and Monte Carlo Generators

Codes for Numerical Calculation of RC in a kinematical point

- **DVCSLL** is the code to calculate RC to BH process in leading approximation:
  - *The BH cross section of the lowest order in a shifted kinematical point is factorized in integrand. No any assumptions about hadronic structure (except of choosing a specific form for nucleon formfactors) are required.*
  - *Cases of longitudinal and transverse target polarization are included.*
  - *Higher order correction are included in terms of electron structure functions.*
  - *Cut on missing energy is implemented.*
  - *Both numeric and Monte Carlo integration methods are implemented*
  - *Integration over ϕ is implemented.*
- **DVCSLL_second** is the code to calculate RC to BH and DVCS in leading approximation:
  - *The same analytic formula for RC is valid.*
  - *BMK approximation is used to describe hadronic structure.*
- **BHexact** is the code to calculate RC to BH with the next-to-leading accuracy:
  - *Calculate leading log and next-to-leading contribution separately.*
  - *Numeric integration is used.*
  - *Cases of longitudinal and transverse target polarization are included.*

Uncertainties in RC calculation

- Accuracy of theoretical calculation
  - *Leading log (DVCSLL) and next-to-leading (BHexact) accuracy of the calculation of the additional photon emission (deserve careful consideration)*
  - *Higher order corrections through exponentiation procedure (not so high effect is expected)*
  - *Accuracy of numeric integration (under control)*
  - *Approximations made when experimental cuts are implemented (could be tested (and finally resolved) using Monte Carlo generators)*
- Model dependence
  - *Model for nucleon formfactors (essential effect is not expected, but needed to be checked for each specific data analysis)*
  - *BMK approximation (could be important for RC to DVCS)*
- Physical contributions not taken into account yet
  - *Pentagon (or 5-point) diagrams (importance is not known), e.g.,*



Monte Carlo Generator to BH: Approach

- To represent the observed cross section as a sum of positively definite contributions:
  - Original formula: $\sigma_{obs} = (1+\delta)\sigma_{BH} + \int_{z_s}^{1} \frac{K_s(z)-K_s(1)}{1-z} + \int_{z_p}^{1} \frac{K_p(z)-K_p(1)}{1-z}$,
  - Define Δ as a minimal energy of the photon we want to generate (i.e., calorimeter resolution),
  - Split each integral as

$$\int_{z_s}^{1} \frac{K_s(z)-K_s(1)}{1-z} = \int_{z_s}^{1-\frac{\Delta}{E}} \frac{K_s(z)}{1-z} - \int_{z_s}^{1-\frac{\Delta}{E}} \frac{K_s(1)}{1-z} + \int_{1-\frac{\Delta}{E}}^{1} \frac{K_s(z)-K_s(1)}{1-z},$$

  - Calculate analytically second integral and neglect third integral resulting in

$$\int_{z_s}^{1} \frac{K_s(z)-K_s(1)}{1-z} = \sigma_s(\Delta) + \delta_s(\Delta)\sigma_{BH}$$

and

$$\int_{z_p}^{1} \frac{K_p(z)-K_p(1)}{1-z} = \sigma_p(\Delta) + \delta_p(\Delta)\sigma_{BH}$$

  - combining all together $\sigma_{obs} = (1+\delta(\Delta))\sigma_{BH} + \sigma_s(\Delta) + \sigma_p(\Delta)$, where each contribution is positively definite. *The price for this representation is the dependence on* Δ

Monte Carlo Generator to BH: Numeric Implementation

Generation of an event by BHRADGEN

- Calculate observed cross section in the kinematical point (external x, Q^2, t, and ϕ_h and uniformly simulated ϕ_e) according $\sigma_{obs} = (1 + \delta(\Delta))\sigma_{BH} + \sigma_s(\Delta) + \sigma_p(\Delta)$
- calculate probabilities of all three channels: nonradiated, radiated in s-peak, and radiated in p-peak:

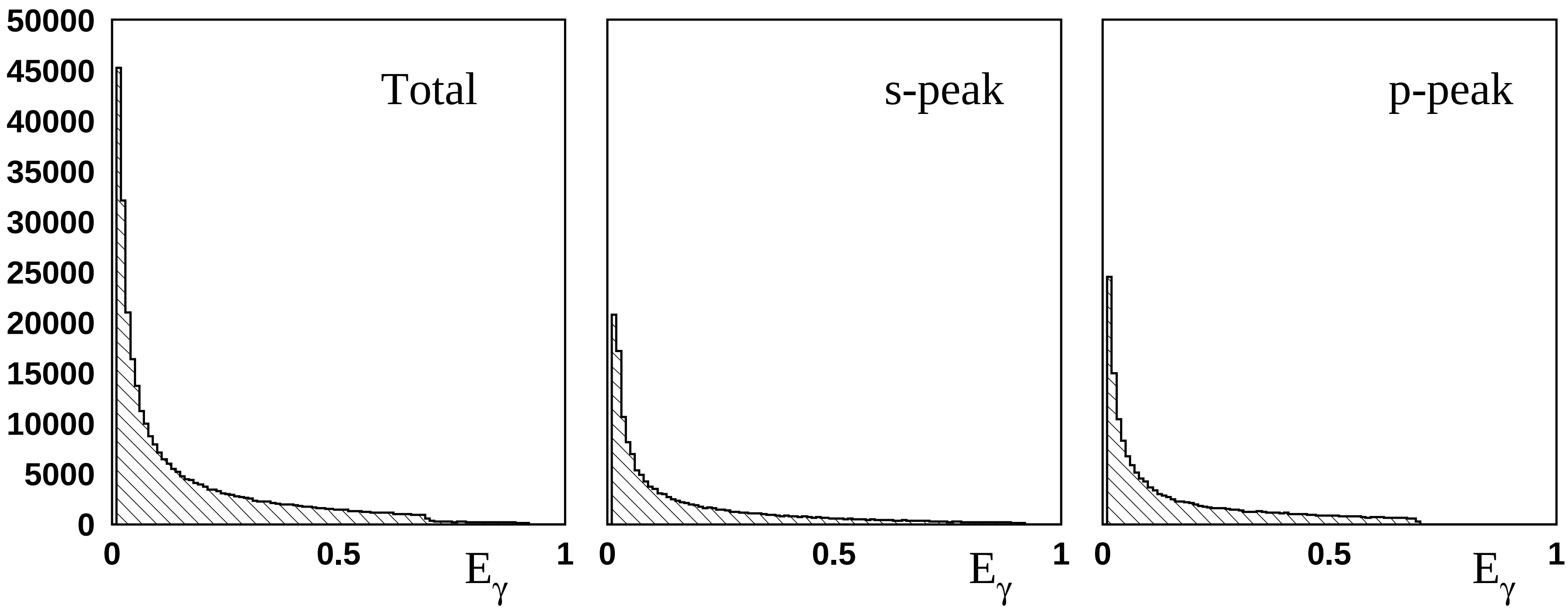
$$p_{nonrad} = \frac{(1 + \delta(\Delta))\sigma_{BH}}{\sigma_{obs}}, \qquad p_{s-peak} = \frac{\sigma_s(\Delta)}{\sigma_{obs}}, \qquad p_{p-peak} = \frac{\sigma_p(\Delta)}{\sigma_{obs}}.$$

- Generate the scattering channel according these probabilities
  - if base BH event (i.e., with one photon in the final state) is simulated then no additional variables are needed to be simulated
  - if two-photon event is simulated (i.e., s-peak or p-peak events), then kinematical variables of an additional photon are needed to be simulated
    - Photon energy is simulated through the variable $z_{1,2}$ according their distributions in integrand of expressions for s- and p-peak contributions.
    - Photon angles are simulated in s- or p-peaks.

Monte Carlo Generator BHRadgen: Numeric Example

- Kinematical point: x=0.4, Q^2=1.8 GeV^2, t=-0.2 GeV^2, ϕ_h=160^o.
- Probabilities:

$$p_{nonrad} = 0.686, \qquad p_{s-peak} = 0.152, \qquad p_{p-peak} = 0.162.$$



BHRadgen: Design and Limitations of Current Version

- The design of the Monte Carlo Generator BHRadgen is:
  - Input is four kinematical variables x, Q^2, t, and ϕ.
  - Output is
    - *Generated channel of scattering for an event, i.e., "radiated" (two photons in final state) or "non-radiated" (one photon in final state).*
    - *Three additional kinematical variables (to describe an additional photon) generated for "radiated" event.*
    - *Cross section of RC for any event.*
  - Cross sections and distributions over additional kinematical variables are calculated for the given kinematical point (x, Q^2, t, and ϕ). Then any number of events are simulated using this information. If simulation of many events is required for a certain kinematical point, then the program is efficient. However, the computation is not so fast if the point needs to be simulated for each event.
  - Approaches to accelerate generation of an event:
    - *Look-up Table.*
    - *Relaxation of requirements to the accuracy of Monte Carlo integration.*
    - *Using a numeric approach for integration and calculation of distribution over additional photonic variables.*

BHRadgen: Limitations of Current Version (cont.)

- Collinear kinematics is used for simulation of photonic angles.
  - Instead, the distribution can be used from integrand over photonic angles
- The calculation is based on the leading log approximation.
  - Next-to-leading corrections can be implemented using results for the RC calculation with the next-to-leading accuracy.
  - In this case new analytical results for the distribution over additional photonic variables need to be obtained and implemented. Current code was obtained using the results integrated over two angles of an additional photon.
- Only BH is implemented. Contributions of DVCS can be added (in the BMK approximation)

Discussion about priorities in further development of BHRadgen would be helpful.

Conclusion and Further plans

- Leading log approximation provides compact expressions and relatively good estimate of RC for BH and DVCS. Hadronic structure is incorporated in these expressions through the “base” cross section (i.e., the cross section with one photon emission)
- Main conclusion is that the analytic calculation of the radiative corrections in the BH process for unpolarized, longitudinally polarized, and transversely polarized targets within next-to-leading accuracy (practically “exactly”) has been completed.
- The most important feature of the calculation is that complete integration for loop contributions and integration over angles of an additional photon for two-gamma contribution was performed analytically.
- Large effects are predicted when both photons are collinear (one is collinear to the initial lepton and another is collinear to the final lepton). Since the photon in respective BH process is not collinear (its momentum is the sum of two collinear photons), the BH cross section is not so large.
- All conclusions are valid for the specific way of reconstruction of kinematic variables: leptonic and hadronic momenta are used to reconstruct the kinematics of the BH process. Kinematical variables of the photon were assumed to be unmeasured (or used only in the calculation of kinematical cuts).
- Universal way to avoid multiple calculation to cover all possibilities for data analysis designs is the development of the Monte Carlo generator of the BH process with the additional process with two photons.