QED Radiative Corrections to DVCS: Outstanding Issues

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Deeply Virtual Compton Scattering: From Observables to GPDs
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Plan of talk

Radiative corrections for charged lepton scattering
- Soft photon emission, spin independence
- Single-Spin Asymmetries of a Bethe-Heitler process
- Two-Photon exchange for DVMP
- Implications for DVCS
- Outlook
Complete radiative correction in $O(\alpha_{\text{QED}})$

Radiative Corrections to elastic $ep$:
- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and Virtual Compton (g,h)
- Corrections (e-h) depend on the nucleon structure

Two-photon corrections: no large logs, but dependent on nucleon structure
Basic Approaches to QED Corrections

  - Considered both elastic and inelastic inclusive cases. No polarization.
  - Covariant approach to the IR problem. Later extended to inclusive, semi-exclusive and exclusive reactions with polarization.
  - RADGEN: Monte Carlo of p(e,e’)X including radiative events
  - Developed a method of electron structure functions based on Drell-Yan representation; currently widely used at e⁺e⁻ colliders.
Separating *soft* 2-photon exchange

- Tsai; Maximon & Tjon ($k \to 0$); similar to Coulomb corrections at low $Q^2$
- Grammer & Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- **Already included in experimental data analysis**
- **NB:** Corresponding effect to polarization transfer and/or asymmetry is zero
- **Correction is independent of lepton mass: same for electrons or muons**

\[ \delta_{\text{Soft}} \]

\[ q_1 \to q \quad q_2 \to 0 \]

A similar approach can be applied for any exclusive reaction

**Semi-inclusive? Problem:** soft photons do not resolve short scales
Exchange of two hard photons

- 2-photon exchange contributions for non-soft intermediate photons
  - Can estimate based on a text-book example from *Berestetsky, Lifshitz, Pitaevsky: Quantum Electrodynamics - originally due to Gorshkov, Gribov, Lipatov, Frolov (1967)*
  - Double-log asymptotics of electron-quark backward scattering
    \[
    \delta = -\frac{e^2}{8\pi^3} \log^2 \frac{s}{m_q^2}
    \]
  - Negative sign for backward ep-scattering; zero for forward scattering → Can (at least partially) mimic the electric form factor contribution to the Rosenbluth cross section
  - Numerically ~3-4% (for SLAC kinematics and m_q~300 MeV)
  - **Motivates a more detailed calculation of 2-photon exchange at quark level**
Full Calculation of Bethe-Heitler Contribution

Additional work by AA et al., using MASCARAD (Phys. Rev. D64: 113009, 2001)
Full calculation including soft and hard bremsstrahlung

Radiative leptonic tensor in full form
AA et al, PLB 514, 269 (2001)

\[ L'_{\mu\nu} = -\frac{1}{2} Tr(\hat{k}_2 + m) \Gamma_{\mu\alpha}(1 + \gamma_5 \xi_e)(\hat{k}_1 + m) \Gamma_{\alpha\nu} \]

\[ \Gamma_{\mu\alpha} = \left( \frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\mu - \frac{\gamma_\mu \hat{k} \gamma_\alpha}{2k \cdot k_1} - \frac{\gamma_\alpha \hat{k} \gamma_\mu}{2k \cdot k_2} \]

\[ \Gamma_{\alpha\nu} = \left( \frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\nu - \frac{\gamma_\alpha \hat{k} \gamma_\nu}{2k \cdot k_1} - \frac{\gamma_\nu \hat{k} \gamma_\alpha}{2k \cdot k_2} \]

Additional effect of full soft+hard brem → +1.2% correction to \( \varepsilon \)-slope

Cross section for ep elastic scattering

\[ Q^2 = 6 \text{ GeV}^2 \]

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Rad. Correction to Single-Spin Asymmetries of VCS

- Evaluation of QED radiative corrections for single-spin asymmetries in Virtual Compton Scattering experiments with CLAS (see also earlier calculations by Vanderhaeghen et al. (2000) for beam SSA in VCS)
  - Since VCS is studied through interference with Bethe-Heitler process, its properties need to be understood precisely
  - If the QED correction to the asymmetries is a few per cent, it alters interpretation of VCS measurements in terms of GPDs
  - Earlier calculations for a related process of radiative Moller scattering $e^-+e^-\rightarrow e^-+e^-+\gamma$ show large SSA (up to 20%, see Arbuzov et al., Phys. Atom. Nuclei, 59, 841 (1996))
Feynman Diagrams

- SSA in Bethe-Heitler process is due to interference between (real) tree-level amplitude and QED loops = $O(\alpha)$ correction that contain absorptive parts

Tree-level real amplitude

QED loop diagrams
Formalism


- Beam SSA

\[ A^e = \frac{\alpha}{4\pi} \frac{\text{Re}(P^{(1)}_{\mu\nu}H_{\mu\nu})}{B_{\mu\nu}H_{\mu\nu}} \]

- \(H_{\mu\nu}\) and \(B_{\mu\nu}\) are standard hadronic and leptonic tensors in the leading order

- \(P_{\mu\nu}\) is calculated from loop diagrams using Cutkosky cuts and doing analytic 2-dimensional integration

\[ P^{(1)}_{\mu\nu} = i(k_1 k_2 q_{\nu})[B_1 \tilde{k}_{1\mu} + B_2 \tilde{k}_{2\mu}] - i(k_1 k_2 q_{\mu})[B_1^* \tilde{k}_{1\nu} + B_2^* \tilde{k}_{2\nu}] \]
Expression for beam SSA

\[ P_{\mu\nu}^{(1)} H_{\mu\nu} = \frac{2\pi(k_1 k_2 q p)}{s t} \left( F_1^2 - \frac{q^2}{4M^2} F_2^2 \right) [(2V - s + q^2) B_1 + (2X - s - u) B_2], \]

\[ B_1 = \frac{2(u^2 - 2s^2 - su)}{uc} + \frac{2bc}{c^2} + \frac{4b^2}{t^2} - \frac{4b}{t} \left( 1 + \frac{b}{t} \right) \log \left( 1 + \frac{t}{u} \right), \]

\[ B_2 = \frac{6s}{c} - \frac{2(2b - t)}{t} + 4(-1 + \frac{ub}{t^2} - \frac{s}{t}) \log \left( 1 + \frac{t}{u} \right). \]

• Results are expressed in terms of analytic functions of Mandelstam invariants
• Free of infrared and mass singularities
• No large logarithms appear
• In addition to \( \alpha \), proportional to \( q^2 \) that is small in DVCS kinematics
• Similar formulas obtained for target SSA; similar suppression takes place
Numerical results

Asymmetry less than 0.015% due to $O(\alpha)$+additional kinematic suppression
RC for Exclusive Electroproduction of Pions


Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation; Can be used for any exclusive electroproduction of 2 hadrons, e.g., \( d(e,e'p)n \) (EXCLURAD code)

FIG. 3: \( W \)-dependence of RC to the cross section of neutral pion production. The models used are MAID2000 (solid curve), MAID'98 [34] (dashed curve) and AO [35] (dotted curve).

Used in data analysis at Jlab (and MIT, HERMES, MAMI,...)
Radiative Corrections for Exclusive Processes

• Photon emission is a part of any electron scattering process: accelerated charges radiate.
• Exclusive electron scattering processes such as $p(e,e' h_1)h_2$ are in fact inclusive $p(e,e' h_1)h_2 n\gamma$, where we can produce an infinite number of low-energy photons.
• But low-energy photons do not affect polarization observables, thanks to Low theorem.
Excluirad updated to include polarization (work with K. Joo)

- Corrections to single-spin beam and target asymmetries and double-spin beam-target asymmetry
- Target polarized along the beam direction
RC for beam-target asymmetry

If kinematic cuts for the radiated photon are tight (below 2nd pion production threshold, correction to polarization asymmetry is under <1%)
RC for Spin Asymmetries

- RC is zero for soft photons (can be enforced by kinematic cuts for brems photons, but not for TPE)
  =>RC to spin asymmetries strongly depend on kinematic cuts
- Important to use no soft approximation for calculations of spin asymmetries

RC dependence on the cuts
Spin-independence of soft terms in RC

- Soft-photon corrections are spin-independent
  - Corrections due to (double)bremsstrahlung were calculated by Vanderhaeghen et al Phys.Rev.C62:025501,2000 in a soft-photon approximation.- estimated corrections to cross section ~25%; to single-spin asymmetry ~5% (AA: *seems too large to me*)
Angular Dependence of Rad. Corrections

- Rad. Corrections introduce additional angular dependence on the experimentally observed cross section of electroproduction processes, both exclusive and semi-inclusive.
Rad. Corrections to $e^+e^-$ pair production

- Usual corrections + charge asymmetric corrections

Need to be re-visited in view of time-like DVCS measurements at JLAB
Two-Photon Exchange in Exclusive Electroproduction of Pions (same for muons!)

- Standard contributions: EXCLURAD

Calculated $\varepsilon$-dependence of TPE correction. $Q^2=6$ GeV$^2$, $W=3.2$ GeV, $E_e=5.5$ GeV . Shows ±2% variation with $\varepsilon$. 
TPE for Pion Production: IR regularization

- Need to add real photon emission to cancel IR divergence
- Use a finite photon mass for intermediate steps; photon mass dependence cancels in the end after adding TPE and real-photon emission
- Expressed results in terms of Passarino-Veltman integrals
- Obtained analytic results for the limit of zero electron mass
- “Soft” TPE: a necessary step before including “hard” TPE, need to subtract soft terms at the quark level and add at the hadron level
TPE: some details of the calculation

- Brem+TPE, neglecting the electron mass
- Soft photons factorize at the amplitude level,
  \[
  M_{1}^{\text{SPT}} = -\frac{\alpha}{2\pi} S \cdot C_{0}([k_{1}, m_{1}], [-k_{2}, m_{2}]) \cdot M_{0}.
  \]
- Passarino-Veltman 3-point scalar integral
  \[
  C_{0}([k_{1}, m_{1}], [k_{2}, m_{2}]) = \frac{1}{i\pi^{2}} \int d^{4}q \frac{1}{q^{2}} \left( \frac{k_{i} - q}{k_{i}^{2} - m_{i}^{2}} \right) \left( \frac{k_{j} - q}{k_{j}^{2} - m_{j}^{2}} \right).
  \]

Simplified in a small-mass limit, final result reads

\[
\delta_{\text{box}}^{\text{SPT}} = -\frac{\alpha}{\pi} \text{Re} \left[ S \cdot C_{0}([k_{1}, m_{1}], [-k_{2}, m_{2}]) + X \cdot C_{0}([k_{3}, m_{3}], [k_{2}, m_{2}]) + V_{3} \cdot C_{0}([k_{3}, m_{3}], [-k_{4}, m_{4}]) + V_{1} \cdot C_{0}([k_{1}, m_{1}], [k_{4}, m_{4}]) \right].
\]
TPE at higher $Q^2$

TPE effects increase at higher $Q^2$; SPMT (Maximon-Tjon soft-photon prescription) results in abnormally large corrections
Angular dependence of “soft” corrections
arXiv:1207.1767 (~same for DVMP and DVCS)
Soft TPE for VCS

- Photon coupling to external charged lines
- Results are independent of hadronic models
- IR-finite due to cancellation with real-photon emission
TPE to Bethe-Heitler

For BH+VCS
obtain common “soft factors” that are straightforward to include in RC codes

Important:
TPE corrections to VCS-BH
intereference terms are C-even
Summary for TPE calculations

- Demonstrated an ongoing program of TPE calculations for exclusive reactions
- Two-photon exchange calculated in soft approximation for pion electroproduction
  => Work in progress: TPE effect for VCS+BH.
- Soft-photon contributions expressed in terms of Passarino-Veltman integrals
- Can be added to existing codes and/or generators and studied for specific experimental conditions
- Equally applicable to muon scattering (important for DVMP and DVCS at COMPASS)