

# **QED Radiative Corrections to DVCS: Outstanding Issues**

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**Deeply Virtual Compton Scattering: From Observables to  
GPDs**

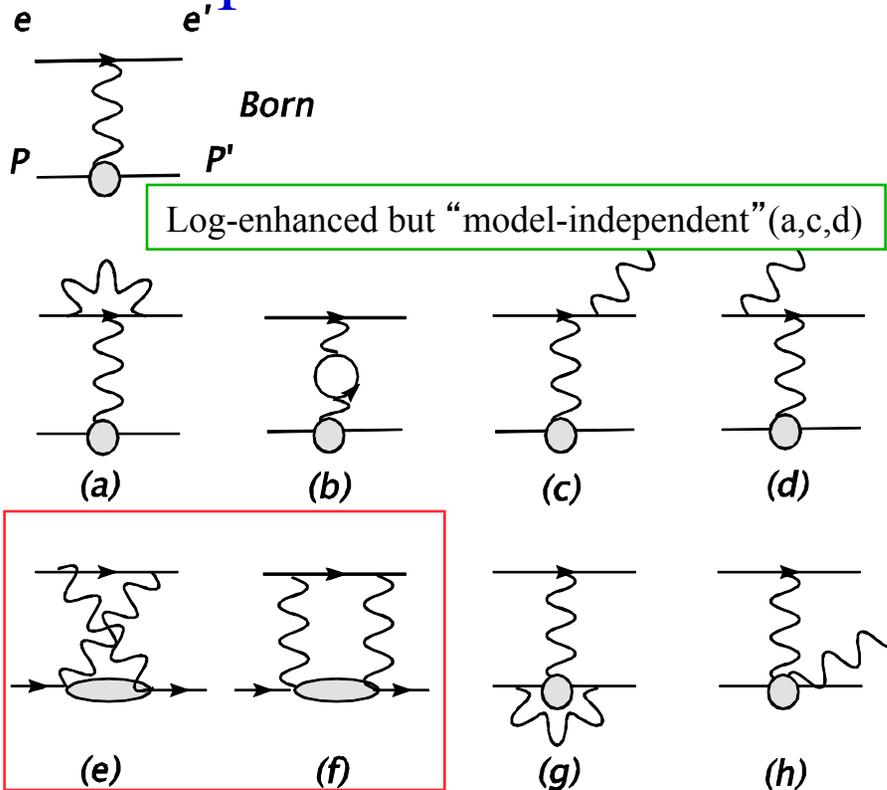
**Ruhr-Universität Bochum, February 10-12, 2014**

# Plan of talk

## **Radiative corrections for charged lepton scattering**

- Soft photon emission, spin independence
- Single-Spin Asymmetries of a Bethe-Heitler process
- Two-Photon exchange for DVMP
- Implications for DVCS
- Outlook

# Complete radiative correction in $O(\alpha_{\text{QED}})$



## Radiative Corrections to elastic ep:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and Virtual Compton (g,h)
- Corrections (e-h) depend on the nucleon structure

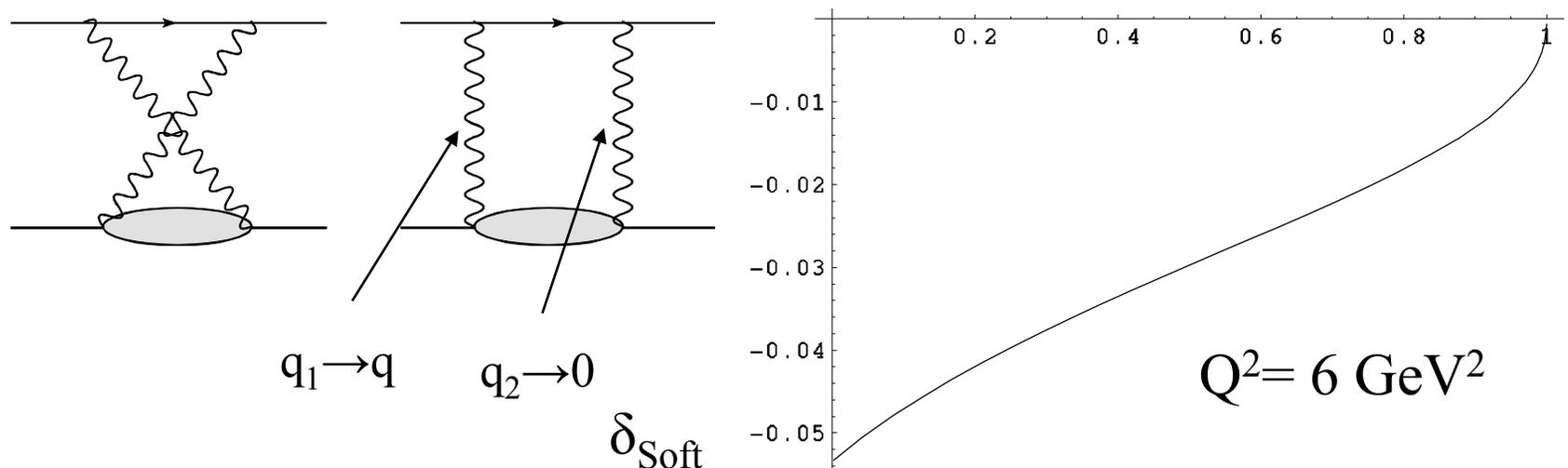
**Two-photon corrections: no large logs, but dependent on nucleon structure**

# Basic Approaches to QED Corrections

- L.W. Mo, Y.S. Tsai, Rev. Mod. Phys. 41, 205 (1969); Y.S. Tsai, Preprint SLAC-PUB-848 (1971).
  - Considered both elastic and inelastic inclusive cases. No polarization.
- D.Yu. Bardin, N.M. Shumeiko, Nucl. Phys. B127, 242 (1977).
  - Covariant approach to the IR problem. Later extended to inclusive, semi-exclusive and exclusive reactions with polarization.
    - RADGEN: Monte Carlo of  $p(e, e')X$  including radiative events
- E.A. Kuraev, V.S. Fadin, Yad.Fiz. 41, 7333 (1985); E.A. Kuraev, N.P.Merenkov, V.S. Fadin, Yad. Fiz. 47, 1593 (1988).
  - Developed a method of electron structure functions based on Drell-Yan representation; currently widely used at  $e^+e^-$  colliders.

# Separating *soft* 2-photon exchange

- Tsai; Maximon & Tjon ( $k \rightarrow 0$ ); similar to Coulomb corrections at low  $Q^2$
- Grammer & Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to [cross section](#)
- **Already included in experimental data analysis**
- **NB:** Corresponding effect to polarization transfer and/or asymmetry is zero
- **Correction is independent of lepton mass: same for electrons or muons**



**A similar approach can be applied for any exclusive reaction**

**Semi-inclusive? Problem: soft photons do not resolve short scales**

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# Exchange of two hard photons

- 2-photon exchange contributions for non-soft intermediate photons
  - Can estimate based on a text-book example from *Berestetsky, Lifshitz, Pitaevsky: Quantum Electrodynamics - originally due to Gorshkov, Gribov, Lipatov, Frolov (1967)*
  - Double-log asymptotics of electron-quark backward scattering

$$\delta = -\frac{e_q e}{8\pi^3} \log^2 \frac{s}{m_q^2}$$

- Negative sign for backward ep-scattering; zero for forward scattering → Can (at least partially) mimic the electric form factor contribution to the Rosenbluth cross section
- Numerically ~3-4% (for SLAC kinematics and  $m_q \sim 300$  MeV)
- **Motivates a more detailed calculation of 2-photon exchange at quark level**

# Full Calculation of Bethe-Heitler Contribution

Additional work by AA et al., using MASCARAD (*Phys.Rev.D64:113009,2001*)  
 Full calculation including soft and hard bremsstrahlung

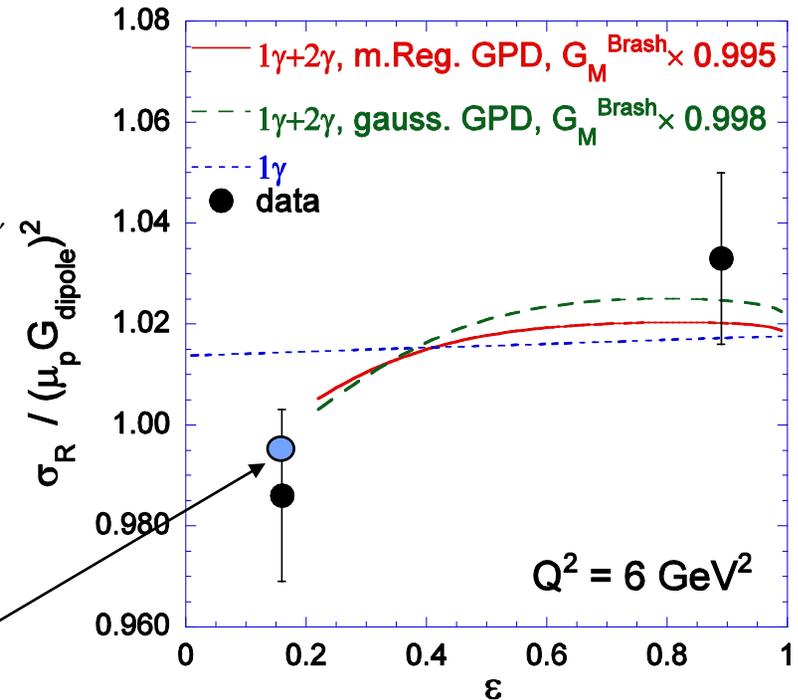
Radiative leptonic tensor in full form  
 AA et al, *PLB 514, 269 (2001)*

$$L^r_{\mu\nu} = -\frac{1}{2} \text{Tr}(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1 + \gamma_5 \hat{S}_e)(\hat{k}_1 + m)\bar{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left( \frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\mu - \frac{\gamma_\mu \hat{k} \gamma_\alpha}{2k \cdot k_1} - \frac{\gamma_\alpha \hat{k} \gamma_\mu}{2k \cdot k_2}$$

$$\Gamma_{\alpha\nu} = \left( \frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\nu - \frac{\gamma_\alpha \hat{k} \gamma_\nu}{2k \cdot k_1} - \frac{\gamma_\nu \hat{k} \gamma_\alpha}{2k \cdot k_2}$$

Cross section for ep elastic scattering



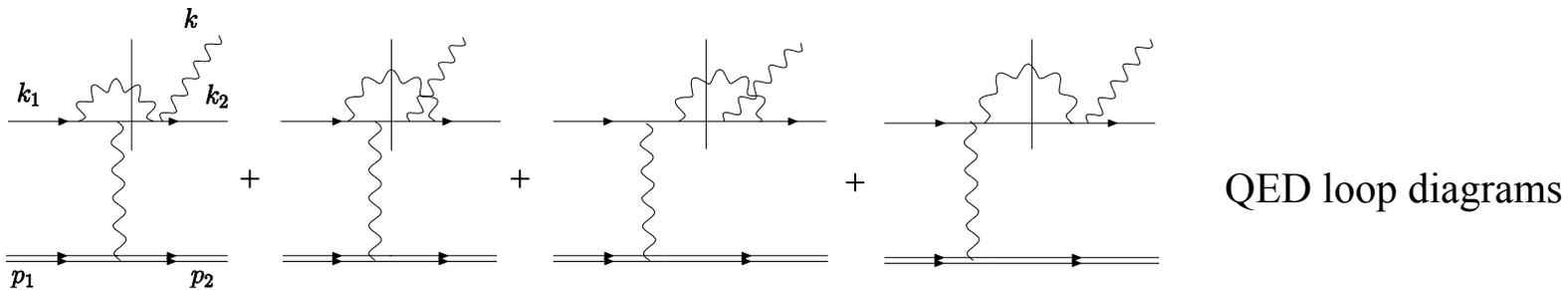
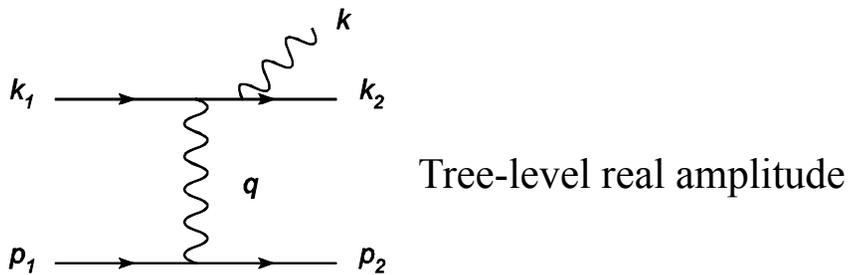
Additional effect of full soft+hard brem  $\rightarrow$  +1.2% correction to  $\epsilon$ -slope

# Rad. Correction to Single-Spin Asymmetries of VCS

- Evaluation of QED radiative corrections for single-spin asymmetries in Virtual Compton Scattering experiments with CLAS (see also earlier calculations by Vanderhaeghen et al. (2000) for beam SSA in VCS)
  - Since VCS is studied through interference with Bethe-Heitler process, its properties need to be understood precisely
  - If the QED correction to the asymmetries is a few per cent, it alters interpretation of VCS measurements in terms of GPDs
  - Earlier calculations for a related process of radiative Moller scattering  $e^-+e^- \rightarrow e^-+e^-+\gamma$  show large SSA (up to 20%, see Arbuzov et al., Phys. Atom. Nuclei, **59**, 841 (1996))

# Feynman Diagrams

- SSA in Bethe-Heitler process is due to interference between (real) tree-level amplitude and QED loops =  $O(\alpha)$  correction that contain absorptive parts



# Formalism

AA, Konchatnij, Merenkov, *Single-spin asymmetries in the Bethe-Heitler process  $e^- + p \rightarrow e^- + \text{gamma} + p$  from QED radiative corrections*, J.Exp.Theor.Phys.102:220-233, 2006; hep-ph/0507059

- Beam SSA

$$A^e = \frac{\alpha}{4\pi} \frac{\text{Re}(P_{\mu\nu}^{(1)} H_{\mu\nu})}{B_{\mu\nu} H_{\mu\nu}}$$

- $H_{\mu\nu}$  and  $B_{\mu\nu}$  are standard hadronic and leptonic tensors in the leading order
- $P_{\mu\nu}$  is calculated from loop diagrams using Cutkosky cuts and doing analytic 2-dimensional integration

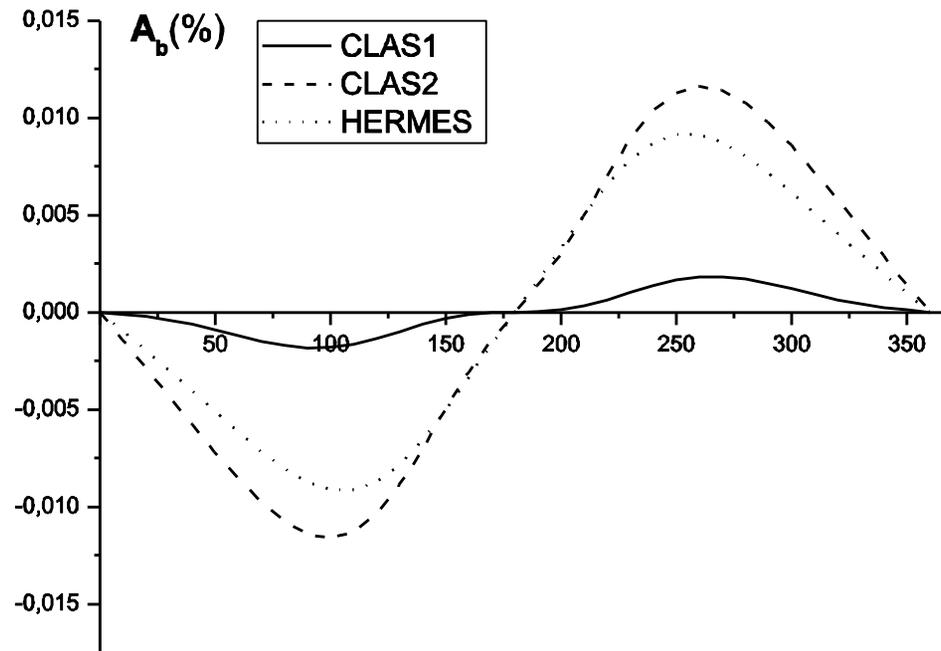
$$P_{\mu\nu}^{(1)} = i(k_1 k_2 q \nu) [B_1 \tilde{k}_{1\mu} + B_2 \tilde{k}_{2\mu}] - i(k_1 k_2 q \mu) [B_1^* \tilde{k}_{1\nu} + B_2^* \tilde{k}_{2\nu}]$$

# Expression for beam SSA

$$P_{\mu\nu}^{(1)} H_{\mu\nu} = \frac{2\pi(k_1 k_2 q p_1)}{st} (F_1^2 - \frac{q^2}{4M^2} F_2^2) [(2V - s + q^2) \bar{B}_1 + (2X - s - u) \bar{B}_2],$$
$$\bar{B}_1 = \frac{2(u^2 - 2s^2 - su)}{uc} + \frac{2bc}{c^2} + \frac{4b^2}{t^2} - \frac{4b}{t} (1 + \frac{b}{t}) \log(1 + \frac{t}{u}),$$
$$\bar{B}_2 = \frac{6s}{c} - \frac{2(2b - t)}{t} + 4(-1 + \frac{ub}{t^2} - \frac{s}{t}) \log(1 + \frac{t}{u})$$

- Results are expressed in terms of analytic functions of Mandelstam invariants
- Free of infrared and mass singularities
- No large logarithms appear
- In addition to  $\alpha$ , proportional to  $q^2$  that is small in DVCS kinematics
- Similar formulas obtained for target SSA; similar suppression takes place

# Numerical results



Asymmetry less than 0.015% due to  $O(\alpha)$ +additional kinematic suppression

# RC for Exclusive Electroproduction of Pions

- AA, Akushevich, Burkert, Joo, Phys.Rev.D66, 074004 (2002)
  - Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation; Can be used for any exclusive electroproduction of 2 hadrons, e.g.,  $d(e,e' p)n$  (EXCLURAD code)

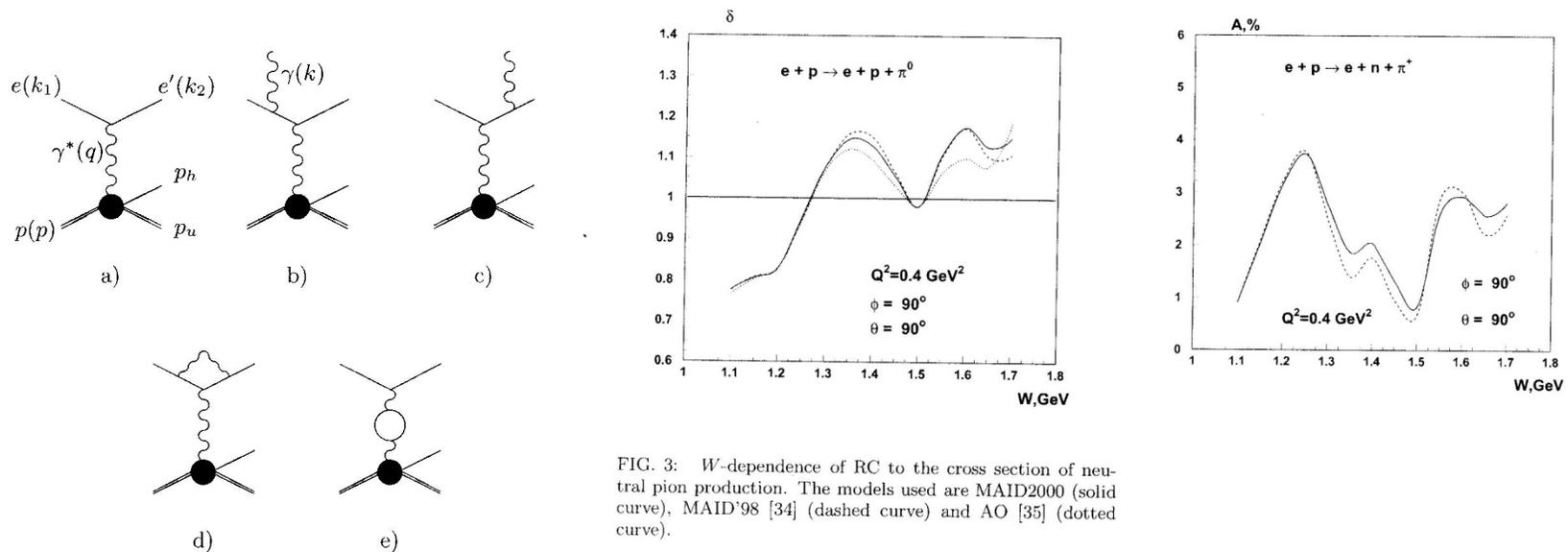


FIG. 3:  $W$ -dependence of RC to the cross section of neutral pion production. The models used are MAID2000 (solid curve), MAID'98 [34] (dashed curve) and AO [35] (dotted curve).

Used in data analysis at Jlab (and MIT, HERMES, MAMI,...)

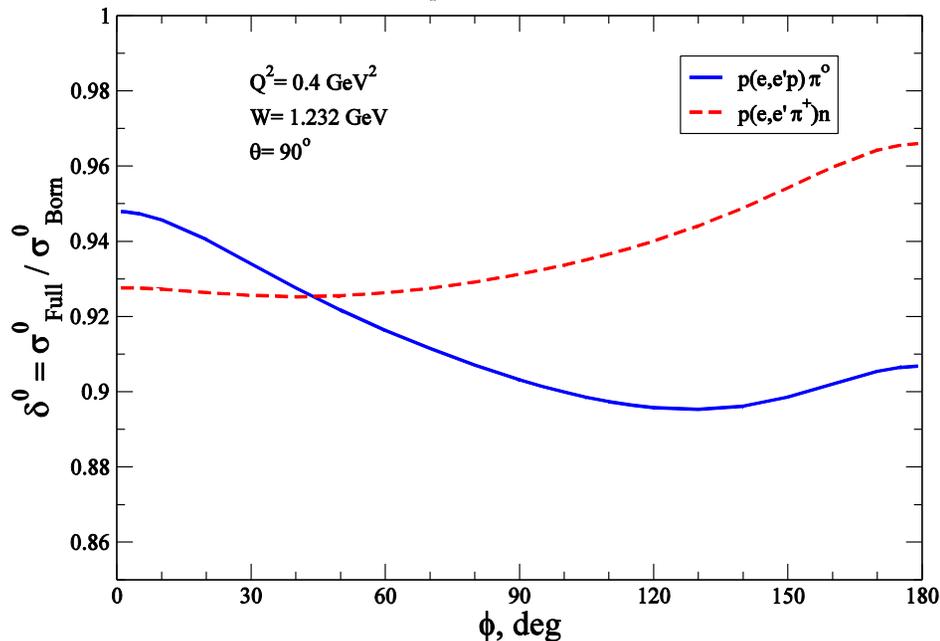
# Radiative Corrections for Exclusive Processes

- Photon emission is a part of any electron scattering process:  
accelerated charges radiate
- Exclusive electron scattering processes such as  $p(e, e' h_1)h_2$  are in fact inclusive  $p(e, e' h_1)h_2 n\gamma$ ,  
where we can produce an infinite number of low-energy photons
- But low-energy photons do not affect polarization observables, thanks to Low theorem

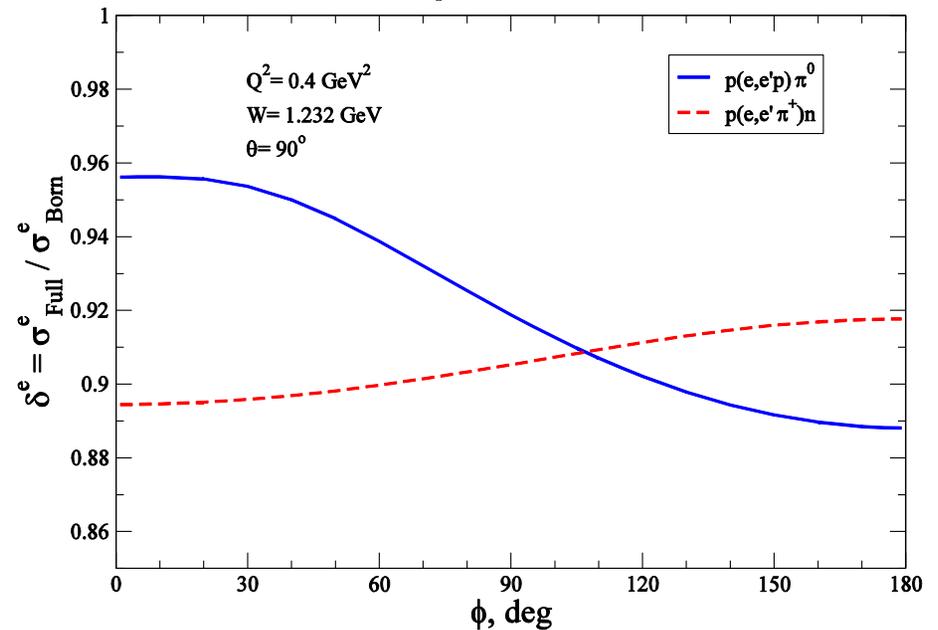
# Exclurad updated to include polarization (work with K. Joo)

- Corrections to single-spin beam and target asymmetries and double-spin beam-target asymmetry
  - Target polarized along the beam direction

Radiative Correction  
Unpolarized cross section



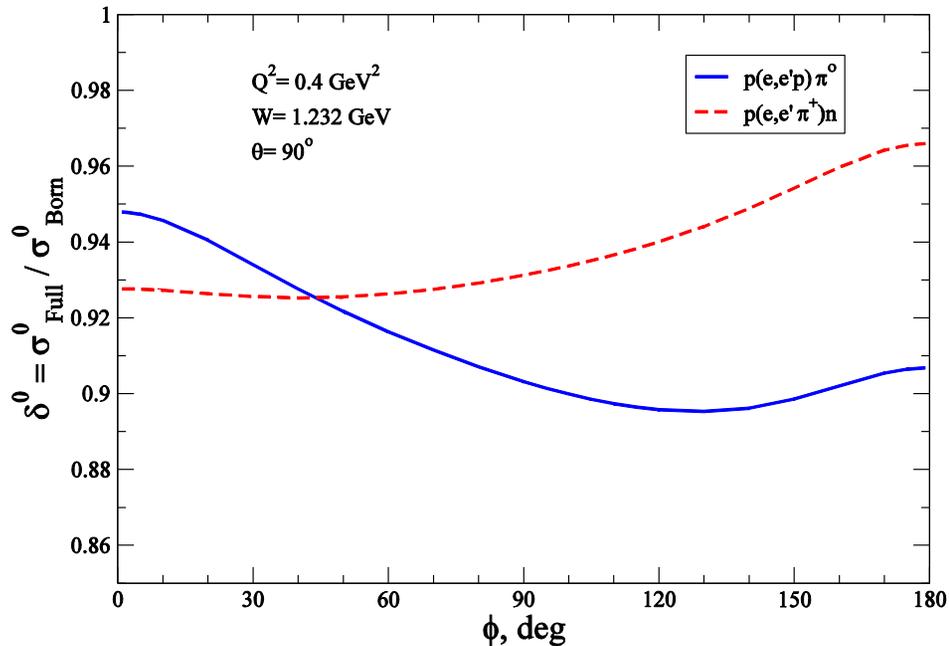
Radiative Correction  
Beam polarization correlation



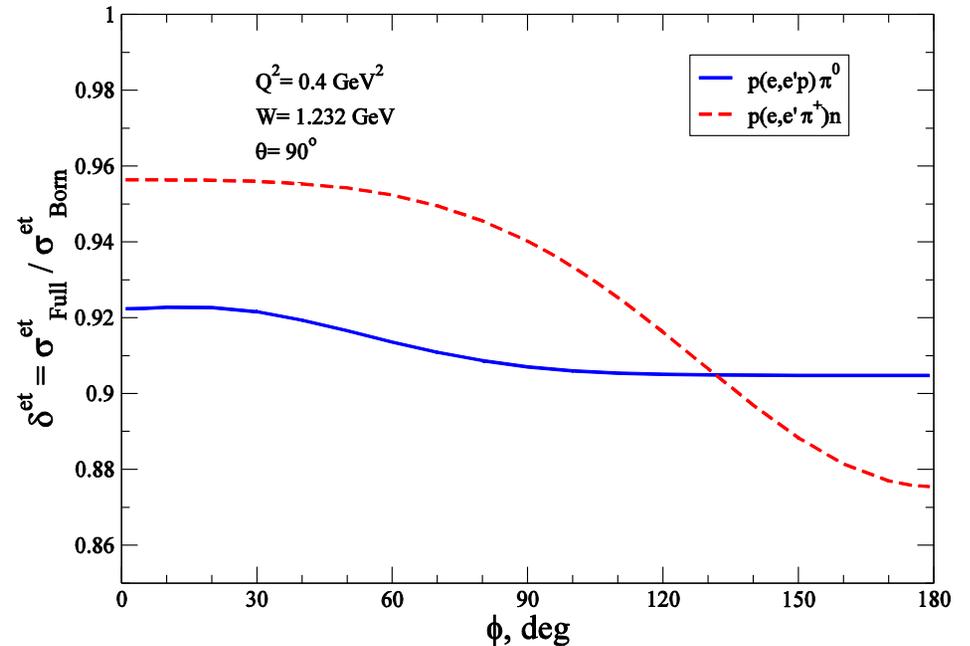
# RC for beam-target asymmetry

If kinematic cuts for the radiated photon are tight (below 2<sup>nd</sup> pion production threshold, correction to polarization asymmetry is under <1%)

**Radiative Correction**  
Unpolarized cross section



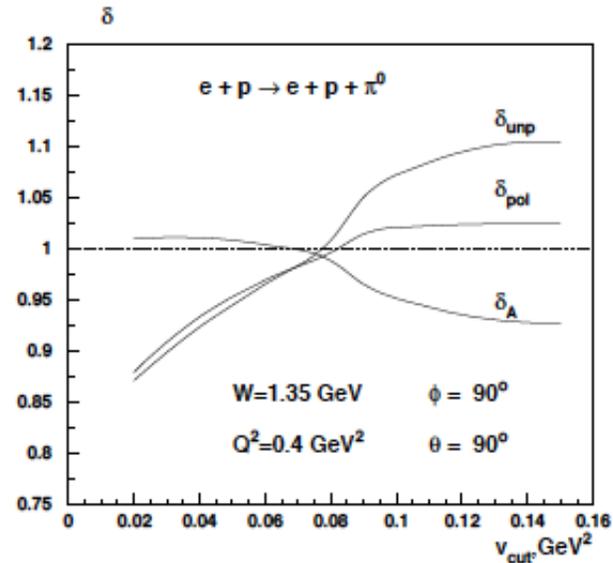
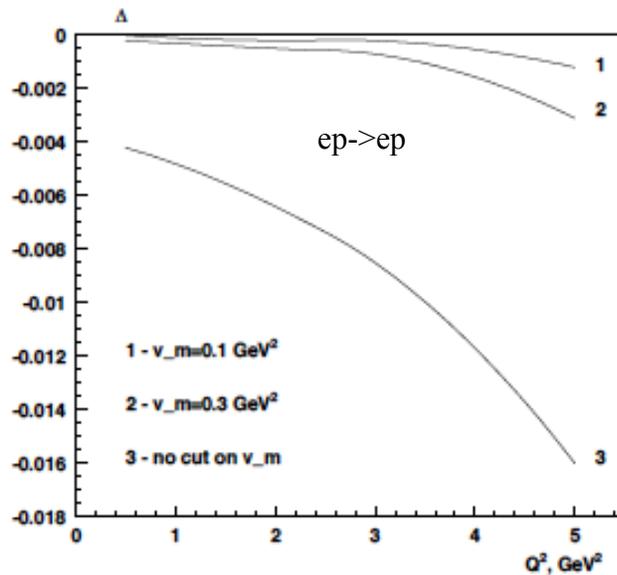
**Radiative Correction**  
Beam-target polarization correlation



# RC for Spin Asymmetries

- RC is zero for soft photons (can be enforced by kinematic cuts for brem photons, but not for TPE)
  - =>RC to spin asymmetries strongly depend on kinematic cuts
- Important to use no soft approximation for calculations of spin asymmetries

## RC dependence on the cuts

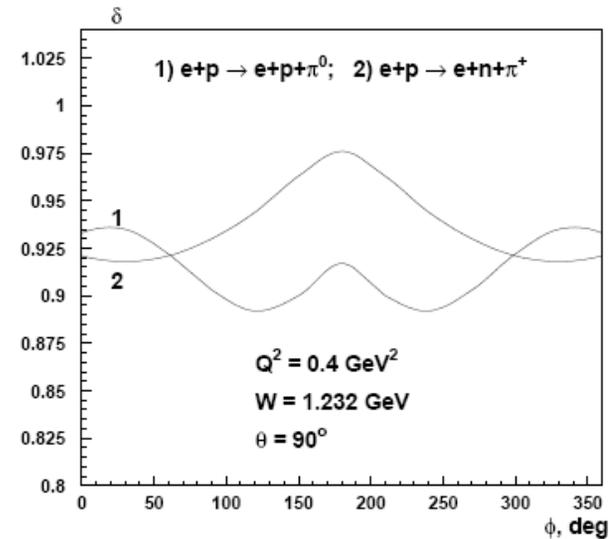
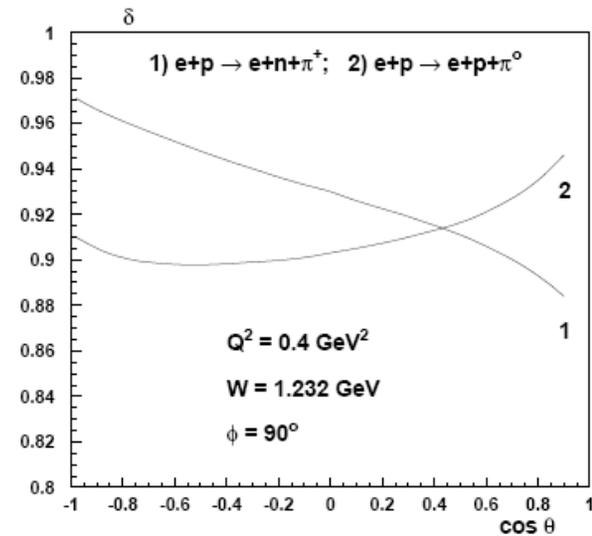


# Spin-independence of soft terms in RC

- Soft-photon corrections are spin-independent
  - Corrections due to (double)bremstrahlung were calculated by Vanderhaeghen et al Phys.Rev.C62:025501,2000 in a soft-photon approximation.- estimated corrections to cross section  $\sim 25\%$ ; to single-spin asymmetry  $\sim 5\%$  (AA: *seems too large to me*)

# Angular Dependence of Rad. Corrections

- Rad. Corrections introduce additional angular dependence on the experimentally observed cross section of electroproduction processes, both exclusive and semi-inclusive



# Rad. Corrections to $e^+e^-$ pair production

- Usual corrections+charge asymmetric corrections

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## Second Born Corrections to Wide-Angle High-Energy Electron Pair Production and Bremsstrahlung\*

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Stanford University, Stanford, California 94305

(Received 15 April 1968)

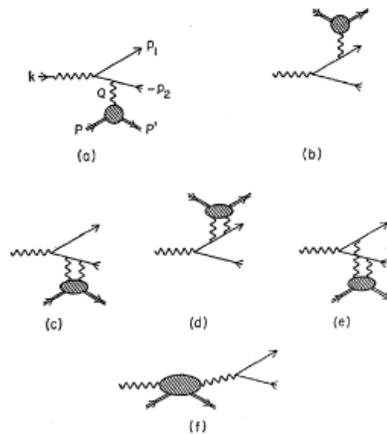
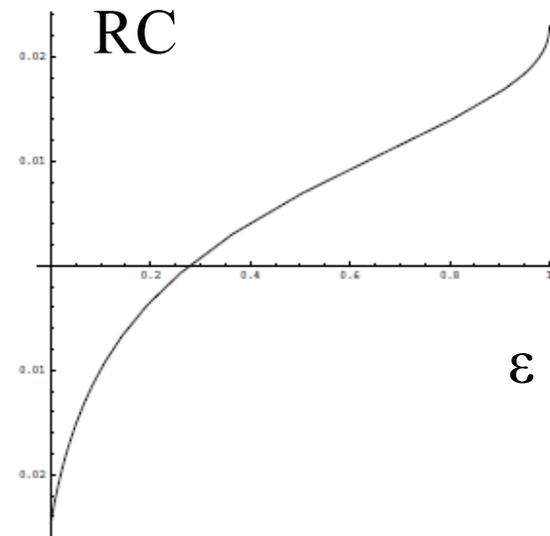
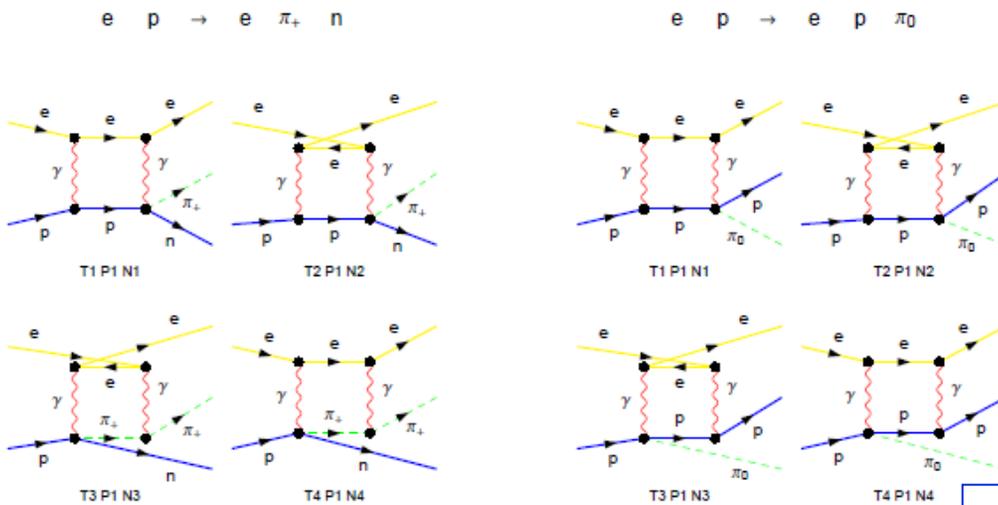


FIG. 1. Feynman diagrams for electron pair production. (a)–(e) give the Bethe-Heitler amplitude through second order in the electromagnetic interaction with the nucleus. Diagram (f) represents the virtual Compton contribution to pair production and includes contributions from the nuclear-pole terms, nucleon and nuclear excitations, and neutral vector-meson production.

*Need to be re-visited in view  
of time-like DVCS measurements  
at JLAB*

# Two-Photon Exchange in Exclusive Electroproduction of Pions (same for muons!)

- Standard contributions: EXCLURAD
- Additional contributions due to two-photon exchange, calculated by AA, Aleksejevs, Barkanova, arXiv:1207.1767 (Phys.Rev. D88 (2013) 053008) Calculated in soft-photon approximation



Calculated  $\epsilon$ -dependence of TPE correction.  
 $Q^2=6 \text{ GeV}^2$ ,  $W=3.2 \text{ GeV}$ ,  $E_e=5.5 \text{ GeV}$ .  
 Shows  $\pm 2\%$  variation with  $\epsilon$ .

# TPE for Pion Production: IR regularization

- AA, Aleksejevs, Barkanova, arXiv:1207.1767 (Phys.Rev. D88 (2013) 053008)
- Need to add real photon emission to cancel IR divergence
- Use a finite photon mass for intermediate steps; photon mass dependence cancels in the end after adding TPE and real-photon emission
- Expressed results in terms of Passarino-Veltman integrals
- Obtained analytic results for the limit

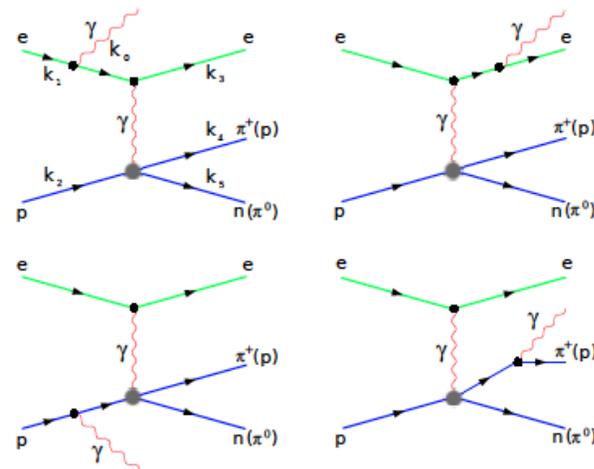
of zero electron mass

“Soft” TPE: a necessary step before includ

“hard” TPE, need to subtract soft terms

at the quark level and add at the hadron

level



# TPE: some details of the calculation

- Brem+TPE, neglecting the electron mass

- Soft photons factorize at the amplitude level,

$$M_1^{SPT} = -\frac{\alpha}{2\pi} S \cdot C_0(\{k_1, m_1\}, \{-k_2, m_2\}) \cdot M_0.$$

- Passarino-Veltman 3-point scalar integral

$$C_0(\{k_i, m_i\}, \{k_j, m_j\}) = \frac{1}{i\pi^2} \int d^4q \frac{1}{q^2} \cdot \frac{1}{(k_i - q)^2 - m_i^2} \cdot \frac{1}{(k_j - q)^2 - m_j^2}.$$

$$\delta_{\text{boz}}^{SPT} = -\frac{\alpha}{\pi} \text{Re} [S \cdot C_0(\{k_1, m_1\}, \{-k_2, m_2\}) + X \cdot C_0(\{k_3, m_3\}, \{k_2, m_2\}) + V_3 \cdot C_0(\{k_3, m_3\}, \{-k_4, m_4\}) + V_1 \cdot C_0(\{k_1, m_1\}, \{k_4, m_4\})].$$

Simplified in a small-mass limit, final result reads

$$\delta_{\text{tot}}^{SPT} = \delta_{\text{tot}}^{IR} + \delta_{\text{tot}}^F \quad \delta_{\text{boz}}^F = -\frac{\alpha}{\pi} \left[ \frac{1}{2} \ln \frac{S}{X} \cdot \ln \frac{S \cdot X}{m_2^2} + \frac{1}{2} \ln \frac{V_3}{V_1} \cdot \ln \frac{V_1 \cdot V_3}{m_4^2} - \pi^2 - Li_2 \left( \frac{S + m_2^2}{S} \right) + Li_2 \left( \frac{X - m_2^2}{X} \right) + \right.$$

$$\left. \delta_{\text{boz}}^{IR} = -\frac{\alpha}{\pi} \ln \frac{m_2^2}{\lambda^2} \left[ \ln \frac{S}{X} - \ln \frac{V_1}{V_3} \right] \quad Li_2 \left( \frac{V_1 - m_4^2}{V_1} \right) - Li_2 \left( \frac{V_3 + m_4^2}{V_3} \right) \right]$$

$$\delta_{\gamma}^{IR} = -\frac{\alpha}{\pi} \ln \frac{4\Delta\varepsilon^2}{\lambda^2} \left[ -\ln \frac{S}{X} + \ln \frac{V_1}{V_3} \right]. \quad \delta_{\gamma}^F = -\frac{\alpha}{\pi} \left[ Li_2 \left( 1 - \frac{\beta_2 \cdot (u_1 - V_1)}{S \cdot m_5^2} \right) + Li_2 \left( 1 - \frac{m_2^2 \cdot (u_1 - V_1)}{S \cdot \beta_2} \right) - Li_2 \left( 1 - \frac{\beta_4 \cdot (u_1 - V_1)}{V_1 \cdot m_5^2} \right) - \right.$$

$$\left. Li_2 \left( 1 - \frac{m_4^2 \cdot (u_1 - V_1)}{V_1 \cdot \beta_4} \right) - Li_2 \left( 1 - \frac{\beta_2 \cdot (u_3 - V_3)}{X \cdot m_5^2} \right) - Li_2 \left( 1 - \frac{m_2^2 \cdot (u_3 - V_3)}{X \cdot \beta_2} \right) + \right.$$

$$\left. Li_2 \left( 1 - \frac{\beta_4 \cdot (u_3 - V_3)}{V_3 \cdot m_5^2} \right) + Li_2 \left( 1 - \frac{m_4^2 \cdot (u_3 - V_3)}{V_3 \cdot \beta_4} \right) \right].$$

## TPE at higher $Q^2$

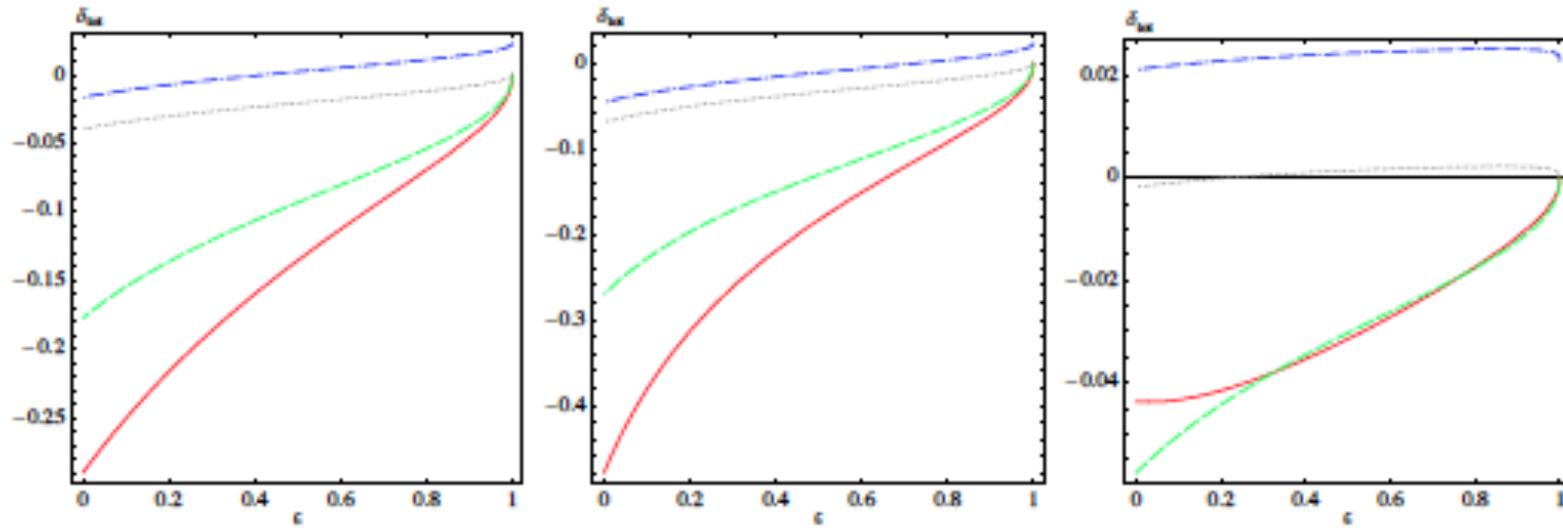


Figure 5:  $\pi^0$  electroproduction two-photon box correction (for detected proton) dependencies on virtual photon degree of polarization parameter  $\epsilon$  for momentum transfers  $Q^2 = 3.0 \text{ GeV}^2$  (left plot),  $Q^2 = 7.0 \text{ GeV}^2$  (middle plot) and  $Q^2 = 0.4 \text{ GeV}^2$  (right plot). All plots are given for  $\phi_4 = 90^\circ$  and  $\theta_4 = 90^\circ$  and  $W = 1.232 \text{ GeV}$ . Dot-dashed curve - SPT, dotted curve - SPT with  $\alpha\pi$  subtracted, dashed curve - SPMT, solid curve - FM approach.

TPE effects increase at higher  $Q^2$ ; SPMT (Maximon-Tjon soft-photon prescription) results in abnormally large corrections

# Angular dependence of “soft” corrections arXiv:1207.1767 (~same for DVMP and DVCS)

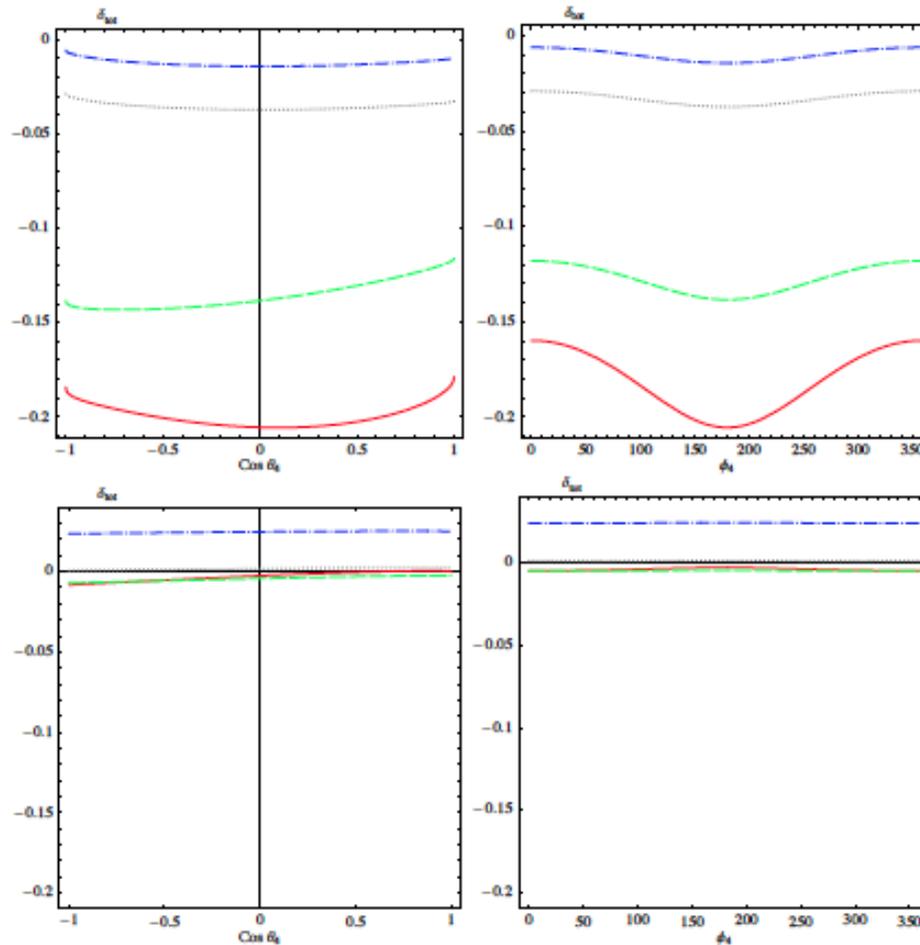
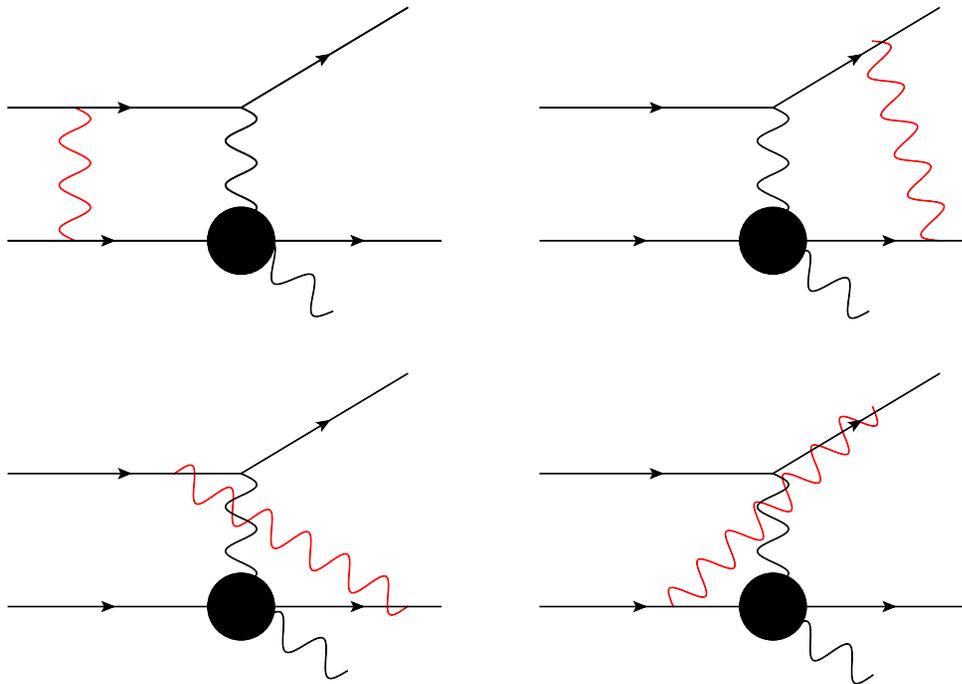


Figure 3:  $\pi^0$  electroproduction two-photon box correction angular dependencies for the high  $Q^2 = 6.36 \text{ GeV}^2$  (top row) and low  $Q^2 = 0.4 \text{ GeV}^2$  (bottom row) momentum transfers,  $W = 1.232 \text{ GeV}$  and  $E_{\text{lab}} = 5.75 \text{ GeV}$ . Left column: dependence on  $\cos \theta_4$  with  $\phi_4 = 180^\circ$ . Right column: dependence on  $\phi_4$  with  $\theta_4 = 90^\circ$ . Dot-dashed curve - SPT, dotted curve - SPT with  $\alpha$  subtracted, dashed curve - SPMT, solid curve - FM approach.

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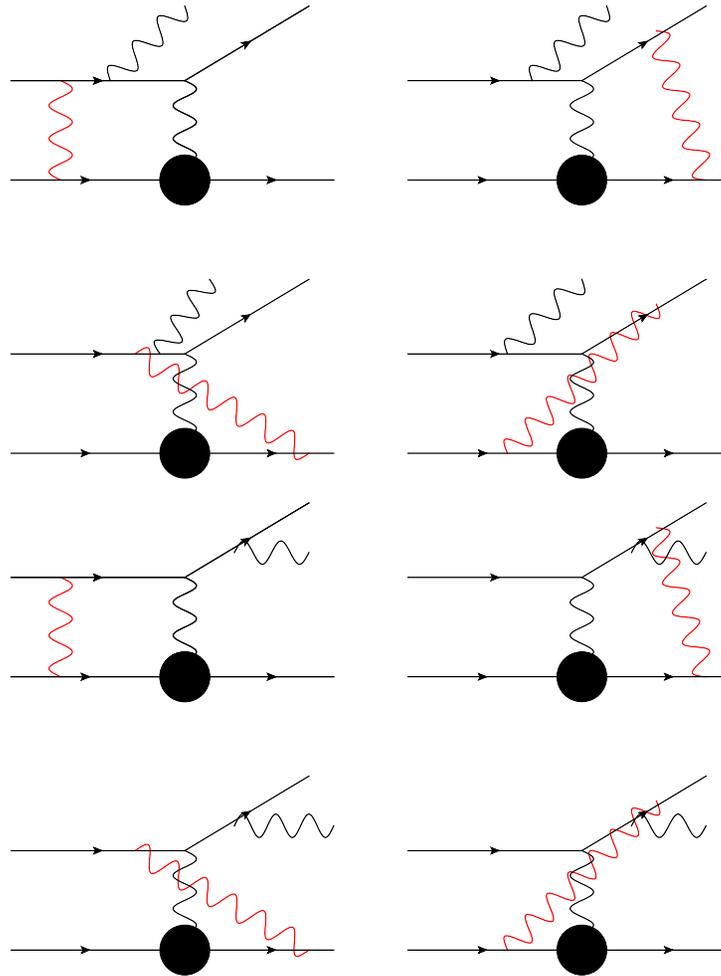
# Soft TPE for VCS

- Photon coupling to external charged lines
- Results are independent of hadronic models
- IR-finite due to cancellation with real-photon emission



VCS

# TPE to Bethe-Heitler



For BH+VCS  
obtain common “soft factors” that are  
straightforward to include in RC codes

Important:  
TPE corrections to VCS-BH  
interference terms are C-even

# Summary for TPE calculations

- Demonstrated an ongoing program of TPE calculations for exclusive reactions
- Two-photon exchange calculated in soft approximation for pion electroproduction
  - =>Work in progress: TPE effect for VCS+BH.
- Soft-photon contributions expressed in terms of Passarino-Veltman integrals
- Can be added to existing codes and/or generators and studied for specific experimental conditions
- Equally applicable to muon scattering (important for DVMP and DVCS at COMPASS)